Testing the Validity of Disjuctive Arguments Using Physical Models

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Introduction.

Methods for testing the validity of arguments often require formal techniques that cannot be applied without knowledge of symbolic logic. Simple physical models, such as Venn diagrams, that visually represent logical relationships can help students to test validity while they are still in the process of learning the formal tools. However, Venn Diagrams are restricted to categorical arguments.This motivated me to develop a similar method for arguments employing propositional logic.

In the previous issue of this journal, I introduced a method for using the physical model of a thermometer to test the validity of conditional arguments.The proposal advanced here uses diagrams of a scratch-and-win ticket for arguments based on disjunctions ("or" statements). Its guiding principle is the Counterexample Method, according to which an argument is invalid when it is possible for all of its premises to be true and its conclusion false. Analogously, if it is possible to diagram all and only the information expressed in the premises while excluding the information expressed in the conclusion, the diagram visually shows that the argument is invalid.

Scratch-and-win tickets are small rectangular cards containing a number of circles coated with paint. Under each may be written notice of a prize, which may be revealed when the paint is scratched off. For the purpose of our model we must assume that there is a prize somewhere on each ticket. According to the rules, a specific number of circles may contain a prize, but the number of painted circles that the player may scratch is specified. This model provides a relatively straightforward method for testing the validity of disjunctive arguments.

Before formulating the general method, it will be helpful to apply it to a few examples, including diagrams of each argument. The first thing we must determine is whether the "or" of a disjunction is exclusive or inclusive. A disjunction is exclusive whenever the truth of one disjunct excludes the truth of the other disjunct(s) (such as getting heads when flipping a coin makes it impossible that it is also tails). In such cases a scratch-and-win diagram that tests the validity of a disjunctive argument constructed from an exclusive "or" has only one prize. A

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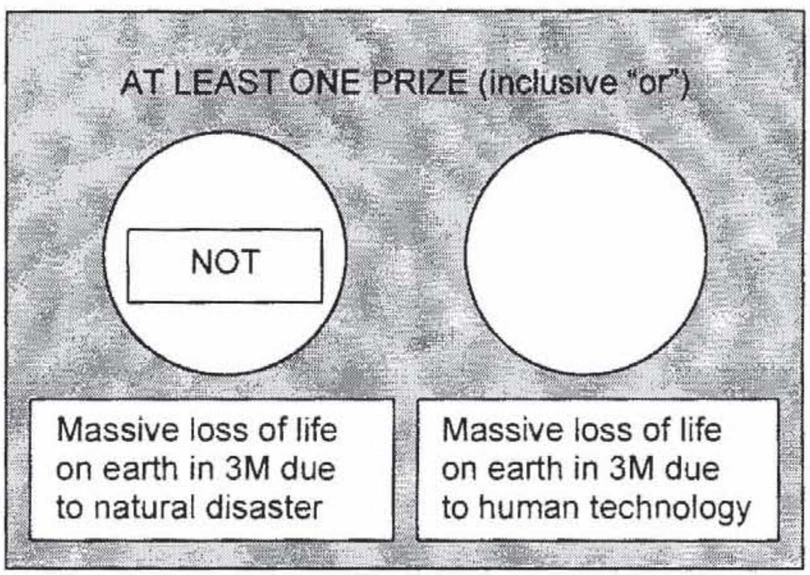
disjunction is inclusive when more than one disjunct may be true at the same time (such as tomorrow being windy or rainy). In such cases a scratch-and-win diagram may have more than one prize. When an argument denies disjuncts, rather than affirming them, the distinction between the inclusive and exclusive "or" is irrelevant. In contrast, as I will illustrate later, the distinction is relevant when an argument affirms the disjunct.

Examples

Example 1: Denying a disjunct of an inclusive "or" statement:

Either natural disaster or misuse of human technology will cause massive loss of life on Earth during the next millennium.
No natural catastrophe will occur during the next millennium.
Therefore, misuse of human technology will cause massive loss of life on Earth during next millennium.

Figure 1



VALID

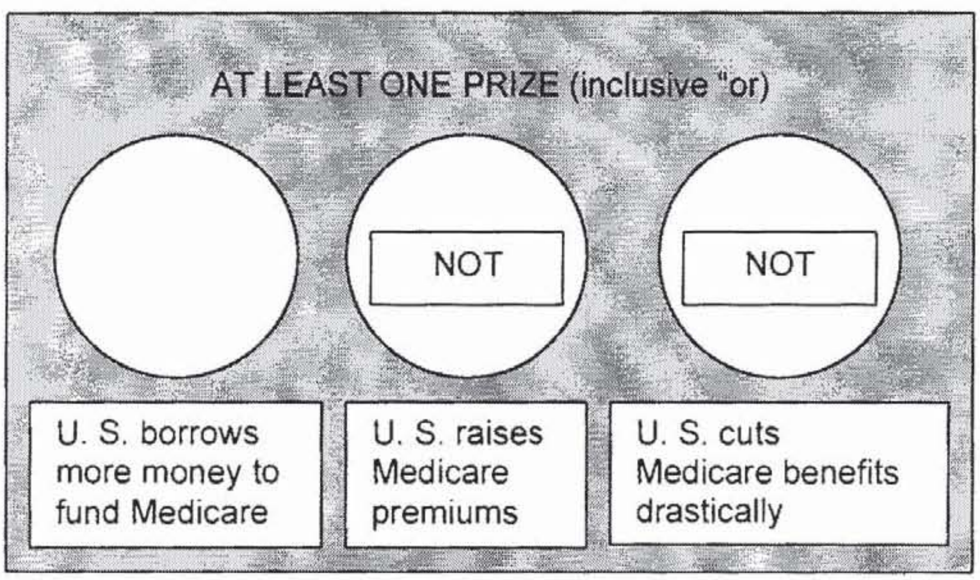
The rectangle represents a scratch-and-win ticket; its two circles represent the two possibilities given in the disjunctive premise, labeled accordingly. It is understood that the circles in the diagram are originally painted over. Because neither possibility that is stated in the disjunctive premise excludes the other, they are inclusive. Consequently, the ticket is labeled to show that it contains at least one prize.

A premise that either affirms or denies an alternative expressed in a disjunct is represented by scratching the paint off the corresponding circle. The second premise in Example 1 denies that the first disjunct is true, which means that there is no prize there. This is shown by the paint scratched off to reveal the word "NOT" in that circle. Because the ticket must contain a prize somewhere, it is obvious that it must lie under the other circle. (Insofar as no premise affirms or denies that possibility, the circle remains painted in the diagram.) According to the diagram, all and only the information in the premises contains (expresses) the information in the conclusion, so the argument is valid.

Example 2: Denying two disjuncts of an inclusive "or" statement:

The government will fund Medicare by borrowing more money or significantly increasing premiums or drastically reducing benefits.
The government will not significantly increase premiums (because seniors will never accept higher premiums).
The government will not drastically reduce benefits (because seniors will never accept reduced benefits).
Therefore, the government will fund Medicare by borrowing more money.

Figure 2



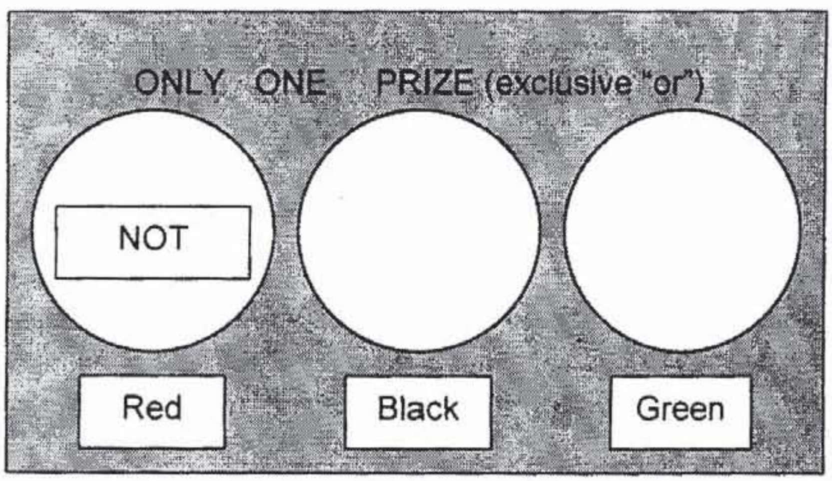
VALID

The above example illustrates how the principle used in the previous case may apply to more than two possibilities (disjuncts), and that their order on the ticket is logically irrelevant. In addition, it shows how practical arguments in ordinary language often rely on common assumptions concerning attitudes and values. In this case, assuming that the government will conform to the wishes of seniors, the two premises that imply the denial of two of the alternatives reveal no prize under the two corresponding circles in the diagram. So, the prize must be located under the remaining circle, which corresponds to what the conclusion asserts. This argument too is valid because the diagram that represents all and only the information in the premises cannot be constructed without also representing the information in the conclusion.

Example 3: Denying a disjunct of an exclusive "or" statement:

In roulette, the result of each play is red, black or (rarely) green.
My spin was not red.
Therefore, my spin was black.

Figure 3



INVALID

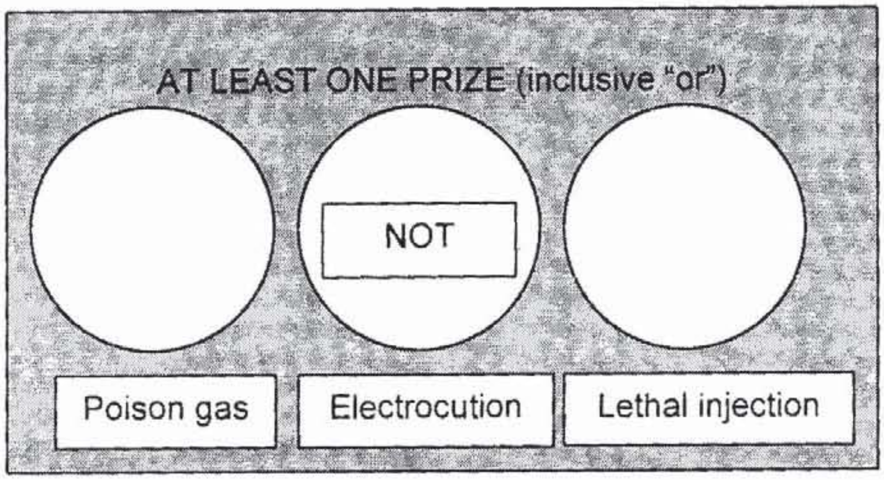
This example differs from Example 2 in that the premises have only eliminated one of the three possible locations for the prize, and that these possibilities are exclusive because only one of them can be true. The prize might be under either of the two remaining circles. Since it is still possible (however unlikely) that it might be under the "green" circle, the conclusion could be false. The diagram, constructed from all and only the information in the premises does not contain the information in the conclusion, so the argument is invalid.

Example 4: Denying a disjunct of an inclusive "or" statement, with a disjunctive conclusion:

Criminals sentenced to death should be executed by poison gas, electrocution, or lethal injection.

Criminals should not be electrocuted. Therefore, criminals should be executed either by poison gas or lethal injection.

Figure 4



VALID

This differs from the other examples in that there is more than one possibility that would make the conclusion true. Here the "or" is inclusive because there may be more than one method of execution that should be used, even though it would be bizarre to use more than one of them for a particular criminal. The possibility that the "electrocution" circle contains a prize has been denied, and the diagram illustrates that there must be a prize under at least one of the two remaining, unscratched circles. Since this is what the conclusion claims, the premises contain the information stated in the conclusion. Consequently, the argument is valid.

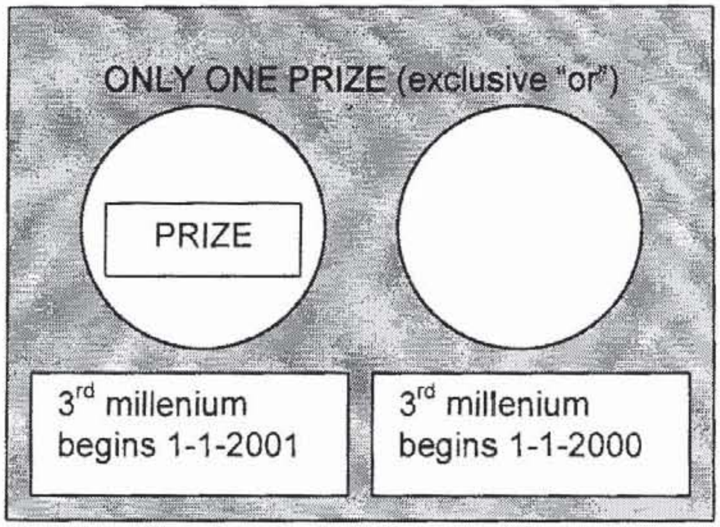
Example 5: Affirming a disjunct of an exclusive "or" statement:

Either the third millennium begins on January 1, 2001 or the third millennium begins on January 1, 2000.

The third millennium begins on January 1 2001 (because exactly 2000 years will have elapsed from 0, the beginning of the Common Era, until January 1, 2001).

Therefore, the new millennium did not begin on January 1, 2000.

Figure 5



VALID

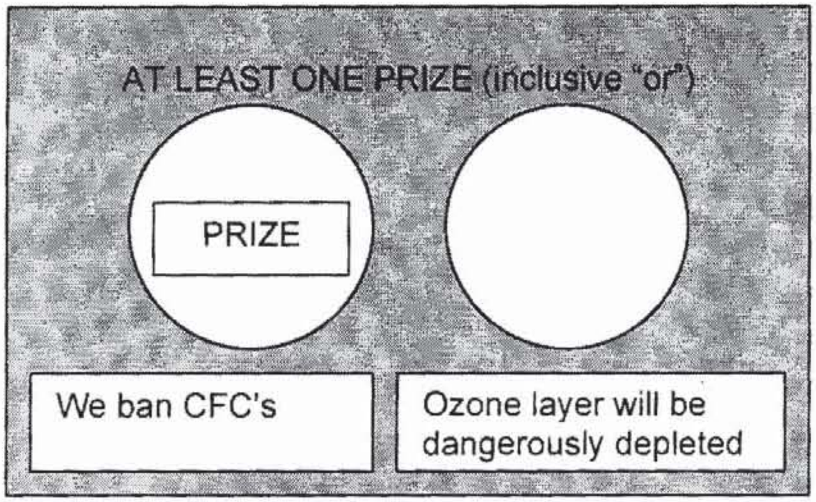
The difference from the previous example is that one of the disjuncts is affirmed, rather than denied. So, there is a prize located in the circle representing the affirmed disjunct. Hence, the word "PRIZE" appears in that circle. Next, we must determine whether the argument uses an exclusive or inclusive "or." Here, if either disjunct is true, the other must be false, so the disjuncts are exclusive. Consequently, the ticket is labeled to show that there is one, and only one, prize on the ticket. Since it has been determined that the first circle contains the prize, the other circle cannot. Because the conclusion denies that the unscratched circle contains a prize, all and only the information in the premises also contains the information in the conclusion. Thus, the argument is valid.

Example 6: Affirming a disjunct of an inclusive "or" statement:

Either we ban CFC's or the ozone layer will become dangerously depleted.
We have banned CFC's.
Therefore, the ozone layer will not become dangerously depleted.

Example 6 differs from the previous one in that both possibilities could be true, for we could ban CFC's and the ozone layer could still become dangerously depleted (from the effect of some other chemical, for instance). Hence, we are dealing with an inclusive "or" statement, so we label the ticket to show that it contains at least one prize. The second premise asserts that we have banned CFC's, which means that there is a prize under the corresponding circle, so we show the word "PRIZE"revealed there. The conclusion claims that there is no prize under the

Figure 6



INVALID

other circle, but because the argument uses an inclusive "or," it is possible that another prize could be found under the other circle. Hence, all and only the information in the premises does not contain the information in the conclusion: it is possible for the premises to be true and the conclusion false. Thus the argument is invalid.

Example 7. Affirming a disjunct of an exclusive "or" statement:

Presently, the largest country in the world is Russia or China or the U.S. or Australia or Brazil or Canada.
Canada has been shown to have the greatest land mass.
Therefore, Russia is not the largest country in the world.

Example 7 (see Figure 7 on page TS 80) is a complex example where several possibilities are mutually exclusive, and where one of them is asserted in order to conclude that one of the others is false. When a scratch-and-win ticket must represent these many possibilities, it is more convenient to use two rows of circles, just as do many actual tickets. Again, asserting that Canada has the largest land mass is equivalent to finding the prize under the corresponding circle. The word "PRIZE" thus appears there. Since these possibilities are exclusive, the ticket contains only one prize. The conclusion denies that Russia is the largest country, which means that there is no prize under that circle. Since all and only the information in the premises inescapably contains the information asserted in the conclusion, the argument is valid.

Figure 7

VALID

Method of Diagramming

Let us now consider the general rules for using the physical model of a scratch-and-win ticket for conducting validity tests.

1. Draw a large rectangle (ticket) to include all, and only, the information in the premises of the argument. It is crucial that we never use information from the conclusion to construct the diagram. The test is to determine whether the information contained in the premises includes that contained in the conclusion. No doubt, if we have already included information from the conclusion, we will find it in the diagram, but that will make the test worthless. By analogy, when testing a water sample for lead, if we added lead to the sample before doing the test, the results would be useless.
2. Draw and label as many circles as there are possibilities (disjuncts) in the disjunctive (. . .or. . .) premise. Each circle is understood to have either the word "PRIZE" (for a circle containing a prize) or the word "NOT" (for a circle that does not contain a prize) under the surface.

3. Determine whether, in reality, the possibilities given are inclusive or exclusive. Whenever the possibilities are exclusive, the truth of one possibility excludes the truth of the other possibility, only one of them can be true. This means that only one of the circles contains a prize. Whenever they are inclusive, that is, where more than one possibility may be true at the same time, then any number of circles may contain a prize. If exclusive, label the ticket "ONLY ONE PRIZE" at the top; if inclusive, label the ticket "AT LEAST ONE PRIZE" at the top.
4. Premises that either affirm or deny a given possibility are interpreted as analogous to scratching the surface of the corresponding circle to reveal the appropriate word. For each premise that affirms a disjunct, write the word "PRIZE" in the corresponding circle; for each premise that denies a disjunct, write the word "NOT" in that circle.

Method of Determining Validity

Given that the possibilities are either exclusive or inclusive, determine whether any of the circles that have not been scratched cannot/may/must contain a prize.

1. For "Only One Prize" tickets (exclusive "or" statements), whenever the word "PRIZE" has already appeared (because a disjunct has been affirmed), the remaining circles cannot contain a prize. (The single prize already has been won.)
2. For "At Least One Prize" tickets (inclusive "or" statements), whenever the word "PRIZE" has already appeared (a disjunct has been affirmed), any of the remaining circles may also contain a prize.
3. For either sort of ticket, whenever the word "PRIZE" has not appeared (disjuncts have only been denied—not affirmed): (1) If a single unscratched circle remains, it must contain the prize, but (2) if more than one unscratched circle remains, any of them may contain a prize (for exclusive "or," exactly one circle will).

We test for validity by determining whether the diagram (constructed by using all and only the information from the premises) corresponds to what the conclusion claims (contains the information found in the conclusion). If so, then the argument is valid; otherwise, it is invalid.

4. If the conclusion affirms that there is a prize within a single circle, the argument is valid whenever the diagram indicates that there must be a prize there (there must be a prize somewhere; it has not yet appeared; there is only one unscratched circle where it could be—Examples 1,2). Otherwise it is invalid (Example 3).
5. If the conclusion affirms that there is a prize under one of two or more unscratched circles, the argument is valid whenever a prize must be under at least one of them (there must be a prize somewhere; it has not yet appeared; the conclusion allows for it to be under any of the remaining circles—Example 4). Otherwise it is invalid.
6. If the conclusion denies that there is a prize under a single circle, the argument is valid whenever the circle cannot contain a prize (all of the prizes contained on

the ticket have already been won; the conclusion denies that there is a prize under a remaining unscratched circle—Examples 5,7). Otherwise it is invalid (Example 6).

Conclusions

The method for testing validity by using physical models has several advantages:

1. It provides a reliable procedure for determining the validity of disjunctive arguments.
2. The diagrams are easy to draw.
3. The method's simplicity makes it easy to use. In particular, it applies to an disjunction, regardless of whether its disjuncts are positive or negative.
4. Students need not comprehend the subject matter of the argument. Lack of background knowledge sometimes limits the test for validity that uses counterexamples. Where subject matter is technical or obscure, students may have little idea what it would mean for the premises or conclusion to be true or false.
5. Insofar as these physical models are analogous to logical forms, working with them will especially aid the visual learner to grasp the general logical forms underlying arguments having diverse content.