Hybrid Arguments

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Key Words: relevance, simple arguments, convergent arguments, linked arguments, hybrid arguments, linked sets, argument diagrams, supplementation, vulnerability, hypervulnerability.
Abstract: Sometimes logical support for a conclusion is provided exclusively by premises which are independently relevant to that conclusion. At other times, support is provided exclusively by independently irrelevant premises. On still other occasions, relevant and irrelevant premises may collectively offer a distinctive pattern of support. This paper provides a rigorous account of some of these differences in terms of a tripartite classification of convergent, linked and hybrid arguments. These various arguments are defined, diagrammed, and some of their logical properties are explored.

Premises which are irrelevant, in isolation, to an argument's conclusion may nonetheless indirectly provide logical support for that conclusion in significantly different ways. This paper provides a set theoretic account of some of these differences. The account is applicable to arguments with any finite number of premises, it marks in a nearly classical manner the distinction between convergent and linked arguments, and it highlights the existence of a hitherto unrecognized class of hybrid arguments.

For simplicity, relevance is treated here as a primitive dyadic relation obtaining in each instance between a set of propositions and a single proposition. When, for ease of exposition, I speak of a single premise P as being relevant to a conclusion C, this should be understood as shorthand for the claim that the unit set containing P is relevant to C. Finally, "C" is used throughout as a variable ranging over conclusions.

Df. 1 An argument A is *simple* iff A has exactly one conclusion. Otherwise, A is *complex*.

Simple arguments are the principal concern of this paper.

Df. 2 An argument A is *convergent* iff A is simple and each premise in A is relevant to C.

In light of definition 2, argument (A) below is convergent.

(A) (1) Data quacks.
(2) Data has webbed feet.
(3) Data is a duck.

Each premise, in isolation, is relevant to, or provides a reason in support of the conclusion.[1] (1) and (2) will be referred to as convergent premises, or as premises which converge on (3). Where downward pointing arrows represent relations of relevance, (A) can be diagrammed as follows.

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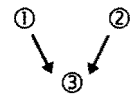


This diagram shows clearly that in (A) two separate reasons are offered in support of (3). Convergent arguments may of course have more than two premises and, although it stretches the ordinary meaning of the term somewhat, definition 2 allows, as a limiting case, convergent arguments with a single premise.

Not all arguments within which each premise is independently relevant to some conclusion are convergent. Chain (or extended) arguments such as

(B) (1) Data is a duck.
(2) Data's parents are ducks.
(3) Some of Data's ancestors are not mammals.

are not convergent, since they are complex. For the same reason, no divergent argument is convergent, in spite of the fact that it is possible for each premise in a divergent argument to be independently relevant to some conclusion.

The following *simple* argument, however, is also not convergent.

(C) (1) Data is by the pond.
(2) All the creatures by the pond are ducks.
(3) Data is a duck.

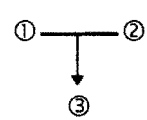
Neither premise, in isolation, is relevant to (3). Yet together (1) and (2) provide a reason in support of (3). According to certain conventions, it would be appropriate to say that (1) is linked to (2). However, since many premises can work together to provide the type of logical support exemplified in (C), it is in the long run simpler and more elegant to define a more general notion, and forgo talk of linkage as a relation between individual premises.

Df. 3 A set of premises Δ forms a *linked set* iff
(1) Δ contains at least two members,
(2) Δ is relevant to C, and
(3) no proper subset of Δ is relevant to C.

Without clause (1), every convergent argument with n premises would be composed of n linked sets.

Df. 4 An argument A is *linked* iff A is simple and each premise in A is a member of some linked set.

Thus (C) is a linked argument, where {1,2} forms a linked set. The following diagram of (C)



shows clearly that, although two premises are involved, only one reason is offered in support of (3).

Of course, not every linked set contains exactly two members, and not every linked argument contains a single linked set. (D), for example,

(D) (1) If Data's feathers are shiny, he is healthy.
(2) If Data is healthy, he is happy.
(3) Data's feathers are shiny.
(4) Most yellow ducks are happy.
(5) Data is a yellow duck.
(6) Data is happy.

contains the two linked sets {1,2,3} and {4,5}.

It is also possible for one or more premises to belong to more than one linked set. In the following argument

(E) (1) All the ducks for sale on the pond are expensive.
(2) Data is one of the ducks on the pond.
(3) Data is for sale.
(4) Most things that are for sale are expensive.
(5) Data is expensive.

{1,2,3} as well as {3,4} form linked sets.

Convergence and linkage, so characterized, represent two extreme cases. In the first case, each premise in a convergent argument is independently relevant to the conclusion. In the second case, no reason is offered for the conclusion of a linked argument until all of the premises within a linked set are considered together. Considered separately, or in any collection short of a linked set, the premises of a linked argument are irrelevant to the argument's conclusion.

For a variety of reasons, not all simple arguments are either convergent or linked. For example, any argument which contains a linked set as well as an independently relevant premise is neither linked nor convergent. However, a more interesting intermediate case is illustrated by argument (F) below.

(F) (1) All the ducks that I've seen on the pond are yellow.
(2) I've seen all the ducks on the pond.
(3) All the ducks on the pond are yellow.

Since (2) in isolation is not relevant to (3), (F) is not a convergent argument. Neither is it linked, since (1) is relevant to (3), and so (1) is not a member of any linked set. To understand how the premises of (F) offer a unique pattern of support for (3), it is useful to introduce the following relation of supplementation which, initially, is defined as a dyadic relation obtaining between individual premises.

Df. 5 A premise P *supplements* a premise Q iff

(1) P is not relevant to C,

(2) Q is relevant to C, and

(3) {P,Q} offers an additional reason R in support of C, which Q alone does not provide.

That is, in (F), while (2) on its own is not relevant to the conclusion, (1) and (2) together provide a better argument—an additional reason—for (3), than does (1) alone. Therefore, (2) supplements (1). Obviously, the supplementation relation is asymmetric.

Df. 5 needs to be generalized, however, since, as the following argument demonstrates, an irrelevant premise may also supplement a *set* of premises.

(G)

(1) My duck is yellow.

(2) Almost without exception, yellow ducks are migratory.

(3) My duck is no exception to any rule.

(4) My duck migrates.

In (G), (3), although irrelevant on its own, supplements the set {1,2}. The three premises together provide a better argument—an additional reason—for (4), than does the linked set {1,2}.

Furthermore, a *set* of irrelevant premises may also supplement a set of premises. Consider:

(H)

(1) My duck is yellow.

(2) Most yellow ducks, especially those born in Ontario, are migratory.

(3) My duck was born in Enterprise.

(4) Enterprise is in Ontario.

(5) My duck is migratory.

Here the set {3,4} is irrelevant to (5). Yet {3,4} supplements the linked set {1,2}. {1,2,3,4} provides an additional reason in support of (5) which {1,2} alone does not provide.

The complications introduced in the last two paragraphs can be accommodated as follows, where Σ and Δ are referred to respectively as the supplementing and the supplemented set.

Df. 6 A set of premises Σ *supplements* a set of premises Δ iff

(1) Σ is not relevant to C,

(2) Δ is relevant to C,

(3) $\Sigma \cup \Delta$ offers an additional reason R in support of C, which Δ alone does not provide, and

(4) Σ and Δ are the smallest sets yielding R which satisfy clauses (1), (2) and (3).

The need for clause (4) is partially explained by the following argument.

(I) (1) All the ducks that Data has seen on the pond are yellow.

(2) All the ducks that Dax has seen on the pond are yellow.

(3) Data has seen 95% of the ducks on the pond.

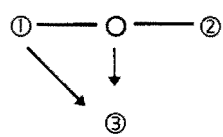
(4) All the ducks on the pond are yellow.

By definition 6, (3) supplements {1} in (I). Without clause (4) of definition 6, (3) would also supplement {1,2}. Intuitively, this seems incorrect. (3) provides an additional reason for believing (4), when considered together solely with (1). This additional reason is generated without considering the role played by (2) in the argument.

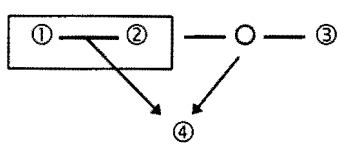
Finally, a hybrid argument can be defined as follows.

Df. 7 An argument A is a *hybrid* iff A is simple and contains at least one supplemented (or supplementing) set.

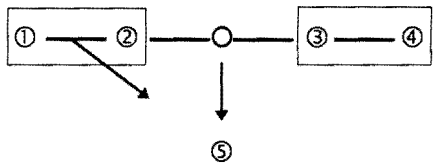
Since supplementation is a unique relation, hybrid arguments need to be diagrammed in a distinctive manner. The diagram of our first hybrid argument (F) follows, where the additional reason generated through supplementation is represented by an arrow originating in a small, unnumbered circle which connects the premises related by supplementation.



Two separate reasons are offered in support of (3). One reason is provided directly by (1). The other reason results from (1) being supplemented by the independently irrelevant premise (2). The diagram of (G) has a roughly similar structure, where a rectangle is used to illustrate the supplementation of the set of premises {1,2}.



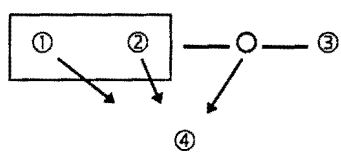
As noted earlier, {1,2} forms a linked set in (G). The supplemented set is also linked in (H), which may be diagrammed as follows.



However, not all supplemented sets are linked. Consider the following argument.

(J) (1) Data quacks.
(2) Data has webbed feet.
(3) 95% of those creatures who both quack and have webbed feet are ducks.
(4) Data is a duck.

(J) may be diagrammed as follows.



Here (3) supplements the set {1,2}, while (1) and (2) individually converge on (4).

Hybrid and linked arguments both genuinely employ independently irrelevant premises in constructing a relevant case for a conclusion. This fact prompts the following definition.

Df. 8 A premise P is *indirectly relevant* to a conclusion C if P is a member of either a linked or a supplementing set.

The use of "if" in definition 8 leaves open the possibility that irrelevant premises may indirectly provide logical support in still other ways.

One of the advantages of using argument diagrams to graphically represent relevance relations is that these diagrams perspicuously expose various structural weaknesses or vulnerabilities within individual arguments, and sometimes within classes of arguments. Two examples will suffice here.

Df. 9 An argument A is *vulnerable* iff there is at least one premise P in A such that the elimination of P from A would eliminate all relevant support for C.

Df. 10 An argument A is *hypervulnerable* iff the elimination of any single premise from A would eliminate all relevant support for C.

By definition, convergent, linked and hybrid arguments each provide some relevant support for their respective conclusions. In a vulnerable argument, this

support can be abolished altogether with one fatal blow. Hypervulnerability is, of course, merely one special case of vulnerability.

No convergent argument with more than one premise is vulnerable. On the other hand, every linked argument composed of a single linked set is vulnerable (and in fact hypervulnerable). And every hybrid argument composed of a single supplemented, linked set is also vulnerable (though not hypervulnerable). Moreover, while some linked arguments composed of more than one linked set, such as (D), are not vulnerable; some, such as (E), where the linked sets share a premise, are vulnerable. Accordingly, any hybrid argument which supplements one or more of those linked sets is also vulnerable. However, no hybrid argument which contains no supplemented sets other than ones composed of two or more convergent premises is vulnerable. And no hybrid argument is hypervulnerable.

Therefore, the following rough generalizations obtain. Linked arguments are more likely to be vulnerable than convergent arguments. And hybrid arguments are more likely to be vulnerable if they contain a supplemented set which is linked. The principal counterexample to the latter generalization concerns vulnerable hybrid arguments, such as (F), where the only supplemented set contains a single independently relevant premise.

Considerations about vulnerability are important since they may affect the structural classification of arguments. Obviously, no hybrid or linked argument is convergent. It may be thought that hybrid and linked arguments also form exclusive classes, since the irrelevant premises in a supplementing set cannot be members of any linked set. Perhaps surprisingly, these two claims are false. Consider the following analogical argument.

(K)	(1)	Data and Dax have the same diet.
	(2)	Data and Dax receive the same amount of exercise.
	(3)	Data is a healthy duck.
	(4)	Dax is a healthy duck.

(K) may be represented either as (K1) a linked argument composed of the linked sets {1,3} and {2,3}; or as (K2) a hybrid argument composed of, say, the linked set {1,3} and the supplementing premise (2). While these two interpretations make different substantive claims about relevance relations, they are not incompatible, and each set of claims may be defensible. One interpretation may be preferred on structural grounds, however. Viewed as a linked rather than a hybrid argument, (K) is less vulnerable. In (K1), the elimination of (3) alone would eliminate all relevant support for (4); whereas in (K2), the elimination of either (1) or (3) would have the same result. Other things being equal, charity would seem to dictate viewing (K) as a linked argument. As is often the case, evaluative considerations here impinge upon predominantly descriptive taxonomical concerns.

Much more work remains to be done in this area. Argument (L) is a simple illustration of one unresolved problem.

(L) (1) All the 1,000 ducks that Data has seen on the pond are yellow.

(2) Data has seen 1% of the ducks on the pond.

(3) All the ducks on the pond are yellow.

The premises of this argument provide some logical support for their conclusion. It is obvious that (L) is neither linked nor convergent. Is (L) a hybrid argument? It is easy to imagine contexts in which it is not. Charity would often support a reading of (L) whereby (1) independently provides substantial logical support for (3). However, (2) then not only fails to supplement (1); it may detract from, or weaken the support which (1) in isolation would offer (3). (2), however, is not altogether irrelevant to the argument. (2) clarifies precisely how much logical support (1) in fact offers (3). (2) provides extremely important information. Yet the contribution which (2) makes to (L) cannot be captured within the present tripartite classification. The above proposal therefore is incomplete.[2]

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[1] The crucial point, for the purposes of this paper, is that each premise in (A) is relevant to (3) in isolation from, or independently of the other premise. Whether, say, (1) is relevant to (3) in the stronger sense of being relevant independently of any other proposition whatsoever is a difficult question upon which I prefer to remain neutral. This comment applies *mutatis mutandis* to my claims about relevance in each of the remaining arguments (B) through (L).

[2] I thank Derek Allen, Ralph Johnson, Larry Powers and, especially, Bob Pinto for perceptive and helpful comments on various drafts of this paper.

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