

NUMERICAL STUDY OF LAMINAR FORCED CONVECTION OF WATER/Al₂O₃ NANOFLUID IN AN ANNULUS WITH CONSTANT WALL TEMPERATURE

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ABSTRACT: In this paper, laminar forced convection of a nanofluid consisting of water and Al₂O₃ in a horizontal annulus has been studied numerically. A two-phase mixture model has been used to investigate thermal behaviors of the nanofluid over constant temperature thermal boundary condition and with different volume concentration of nanoparticles. Comparisons with previously published both experimental and analytical works on flow behavior in horizontal annulus show good agreements between the results as volume fraction is zero. In general, convective heat transfer coefficient increases with nanoparticle concentration.

ABSTRAK: Kertaskerja ini mengkaji secara numerik olakan paksa bendalir lamina yang mengandungi air dan Al₂O₃ didalam anulus mendatar. Model campuran dua fasa digunakan bagi mengkaji tingkah laku haba bendalir nano pada keadaan suhu malar dengan kepekatan nanopartikel berbeza. Perbandingan dengan karya eksperimen dan analitikal yang telah diterbitkan menunjukkan bahawa kelakuan aliran didalam anulus mendatar adalah baik apabila pecahan isipadu adalah sifar. Pada amnya, pekali pemindahan haba olakan meningkat dengan kepekatan nanopartikel.

KEYWORDS: *nanofluid; volume concentration; heat transfer enhancement; laminar flow convection; annulus.*

1. INTRODUCTION

Energy has been rated as the single most important issue currently facing humanity. Fundamentally, energy conversion and transportation occur at atomic or molecular levels, nanoscience and nanotechnology are expected to play a significant role in revitalizing the traditional energy industries and stimulating the emerging renewable energy industries.

Among all the forms of energy we are using today, over 70% are produced in or through the form of heat. In many industrial systems, heat must be transferred either to input energy into a system or to remove the energy produced in a system. Considering the rapid increase in energy demand worldwide, intensifying heat transfer process and reducing energy loss due to ineffective use have become increasingly important tasks. For achieving this important task, fluids in conversion and transportation of energy have important roles. Using of them have paid attention widely but for importing thermal characteristics of fluids, should be used new technology and produce nanofluid and replace it with common fluids [1].

Nanofluids are dilute suspensions of functionalized nanoparticles smaller than 100 nm, which belong a new type of functional composite materials developed about a decade ago with the specific aim of increasing the thermal conductivity of heat transfer fluids, which have now evolved into a promising nanotechnological area [1].

In less than several years since the seminal work by Choi concerning the concept of nanofluids, interest in this area has grown rapidly. However, investigators are having difficulty in understanding the anomalous behavior of nanofluids in regard to thermal conductivity and convection heat transfer coefficient [2].

These difficulties show the lack of agreement between results obtained in different laboratories, the often poor characterization of the suspensions, and the lack of theoretical understanding of the mechanisms responsible for the observed changes in properties [3].

Now we consider some of the researches in this field. Among common materials, in many energy conversion systems, only specific materials are used in the production of nanofluids having high thermal conductivity and other properties like cost, easy production etc. From these common materials, engine oil, ethylene glycol and water have the lowest thermal conductivity. On the other hand materials such as copper, aluminum and others have high thermal conductivity. This was the main idea of producing nanofluids by using base fluids like water, oil and nanoparticles of copper, aluminum or their oxides like CuO , Al_2O_3 . In an article from Pawel Keblinski, Jeffery A. Eastman, David G. Calill some of researches have been collected [3].

Masuda *et al.* [4] reported a 30% increase in the thermal conductivity of water with the addition of 4.3 vol. % Al_2O_3 nanoparticles. Xuan and Li [5] measured the turbulent friction factor of water-based nanofluids containing Cu nanoparticles in a volume fraction range of 1.0 – 2.0. Interestingly, they found that the friction factor for the nanofluids is approximately the same as that of water. Wang *et al.* [6] measured the viscosity of Al_2O_3 – water nanofluids and showed that nanofluids have lower viscosities when the particles are more dispersed. They also found an increase of about 30 % in viscosity at 3 vol. % Al_2O_3 , compared with that of water alone.

Also numerical works have paid attention beside experimental researches in recent years. For example, Maiga *et al.* [7] studied heat transfer enhancement by using nanofluids in forced convection flows. In this paper, the problem of laminar forced convection flow of nanofluids has been thoroughly investigated for two particular geometrical configurations, namely a uniformly heated tube and a system of parallel, coaxial and heated disks.

Numerical results, as obtained for water - Al_2O_3 and ethylene glycol - Al_2O_3 mixtures, have clearly shown that the inclusion of nanoparticles into the base fluids has been produced a considerable augmentation of the heat transfer coefficient that clearly increase with an increase of the particle concentration.

Xuan *et al.* [8] used an experimental system for investigation of convective heat transfer and flow features of nanofluids in a tube. Both the convective heat transfer coefficient and friction factor of the sample nanofluids for the turbulent flow are measured, respectively. The sample nanofluids are prepared by mixing the nano structured Cu particles below 100 nm diameter and deionized water. Nanofluids possess 0.3, 0.5, 0.8, 1.0, 1.2, 1.5, 2 % volume fractions of Cu nanoparticles. The Reynolds number, Re , varies between the range 10.000 ~ 25.000. In comparison with water, Nusselt number of nanofluids with 2 vol. % Cu nanoparticles is 60% greater. Lee *et al.* investigated the same issue [9].

Fully developed laminar mixed convection of a nanofluid consists of water and Al_2O_3 in horizontal and inclined tubes has been studied numerically by Akbari *et al.* [10]. It is shown that the nanoparticles concentration does not have significant effects on the hydrodynamics parameters. Heat transfer coefficient increases by 15% at 4 vol. % Al_2O_3 .

Lee et al. [11] researched effective viscosities and thermal conductivities of aqueous nanofluids containing low volume concentrations of Al_2O_3 nanoparticles.

The measured viscosities of the Al_2O_3 – water nanofluids show a nonlinear relation with the concentration even in the low volume concentration (0.01 % - 0.3 %) range, while the Einstein viscosity model clearly predicts a linear relation, and exceed the Einstein model predictions.

In contrast of viscosity, the measured thermal conductivities of the dilute Al_2O_3 – water nanofluids increase nearly linearly with the concentration, agree well with the predicted values by the Jong and Choi model [11], and are consistent in their overall trend with previous data at higher concentrations.

Kyo Sik Hwang et al. have measured the pressure drop and convective heat transfer coefficient of water based Al_2O_3 nanofluids flowing through a uniformly heated circular tube in the fully developed laminar flow regime [12]. The experimental results show that nanofluid friction factor shows good agreement with analytic prediction from the equation of Darcy for single – phase flow. However the convective heat transfer coefficient of the nanofluids increases by up to 8% at a concentration of 0.3 vol. % compared with that of pure water.

Behzadmehr et al. [13] did a numerical study of laminar mixed convection of a nanofluid in a horizontal tube by using two – phase mixture model. The nanofluid consists of water and Al_2O_3 . Comparisons with previously published experimental and numerical works on mixed convection in horizontal tubes show good agreements between the results. It is shown that at the fully developed region the nanoparticle concentration does not have significant effects on the hydrodynamics parameters. However its effects on the thermal parameters are important. Concentration of the nanoparticles is higher at the bottom and also at the near wall region.

Ho et al. [14] did numerical simulation of natural convection of nanofluid in a square enclosure. The Rayleigh number was varied between, $\text{Ra} = 10^3 - 10^6$ and the volumetric fraction of alumina nanoparticles between, $\Phi = 0 - 4\%$. Joseph et al. [15] studied heat transfer enhancement with the use of nanofluids in radial flow cooling systems considering temperature- dependent properties. Water / Al_2O_3 nanofluid with a volume fraction of nanoparticles as low as 4% can produce a 25% increase in the average wall heat transfer coefficient when compared to the base fluid alone.

Heris et al. [16] used nanofluids containing CuO and Al_2O_3 oxide nanoparticles in water as base fluid in different concentrations and investigated the laminar flow convective heat transfer through circular tube with constant wall temperature boundary condition. At low volume concentration the heat transfer coefficient is close to single phase result but with increasing volume concentration higher heat transfer enhancement for Al_2O_3 /water can be observed. Behzadmehr et al. [17] studied effect of nanoparticles mean diameter on mixed convection heat transfer of a nanofluid in a horizontal tube. Nanoparticles distribution at the tube cross section shows that the non–uniformity of the particles distribution augments when using larger nanoparticles and / or considering relatively high value of the Grashof numbers.

Annulus appears in many industrial heat exchangers. Therefore, many investigations have been done on the heat transfer mechanisms of an annulus. Among them Srivastara et al. [18] experimentally investigated the effect of an unheated length and the annulus ratio on the variations in heat transfer coefficient in the early entrance region of an annulus.

They showed that the effect of the shape of unheated section only becomes significant within $x/D = 2$.

However, thereafter the results correspond to the fully developed condition. Lu and Wang [19] experimentally studied the convective heat transfer of water flow in a narrow annulus. They showed that the thermal characteristics of fluid flow in an annulus are different from those in circular tubes. Transition from laminar to turbulent occurs at the lower Reynolds number compared to the one for circular tubes.

Gupta and Grag [20] numerically studied flow in the hydrodynamic entrance region of an annular tube by using an implicit finite difference. They found that for a very small annulus ratio the results depart significantly from those for a circular pipe. El – Shaarawi et al. [21] solved numerically the transient laminar forced convection in the entrance region of an annulus. For different initial thermal condition, they found that generally the responses associated with heating the outer boundary are more pronounced associated with heating the inner boundary are. Izadi et al. [22] investigated numerically laminar forced convection of a nanofluid of Al_2O_3 and water in an annulus. It is shown that the dimensionless axial velocity profile does not significantly change with the nanoparticle volume fraction. But, the temperature profiles are affected by the nanoparticle concentration. Also they showed that by increasing the heat fluxes ratio (outer to inner), the effect of one wall heat flux on the Nu of another wall increases via its effects on the bulk temperature.

As seen in these and similar works, heat transfer mechanisms in annulus could be very complex and this geometry is very common in many industrial installation. Therefore, the present work aims to investigate some behaviors of nanofluid flow into such a geometrical configuration. Also our study follows the research of Izadi et al. [22] and by using of their computational codes some studies have been done.

In this work, the effects of constant temperature thermal boundary condition on walls of annulus on thermodynamics properties of a laminar forced convection have been studied and hydrodynamics parameters do not change in comparing with studies of Izadi et al. [22].

2. MATHEMATICAL MODEL

Laminar forced convection of a nanofluid consisting of water and Al_2O_3 in horizontal concentric annulus with constant temperature at walls has been studied by developing a computational code. The effective viscosity and effective thermal conductivity of the fluid are varied with the temperature while the other properties are assumed to be constant. Dissipation and pressure work are neglected. With this assumption the dimensional conservation equation for steady state condition are as follows:

Continuity equation:

$$\nabla \cdot (\rho_{eff} V_m) = 0 \quad (1)$$

Momentum equation:

$$\nabla \cdot (\rho_{eff} V_m V_m) = -\nabla p + \nabla \cdot (\mu_{eff} \nabla V_m) \quad (2)$$

Energy equation:

$$\nabla \cdot (\rho_{eff} C V_m T) = \nabla \cdot (\mu_{eff} \nabla T) \quad (3)$$

The physical properties of the above equation are:

Effective density:

$$\rho_{eff} = (1 - \phi)\rho_f + \phi\rho_p \quad (4)$$

Chon et al. [23] correlation which considers the Brownian motion and nanoparticles mean diameter has been used for calculation the effective thermal conductivity

$$\frac{K_{eff}}{K_f} = 1 + 64.7 \times \phi^{0.7460} \left(\frac{d_f}{d_p} \right)^{0.3690} \left(\frac{K_s}{K_f} \right)^{0.7476} \times Pr^{0.9955} \times Re^{1.2321} \quad (5)$$

Where Pr and Re in Eq. (5) are defined as,

$$Pr = \frac{\mu}{\rho_f \alpha_f} \quad (6)$$

$$Re = \frac{\rho_f B_c T}{3\pi\mu^2 l_{BF}} \quad (7)$$

l_{BF} is the mean free path of water, B_c is Boltzman constant (1.3807×10^{-23} j/k) and μ is calculated by the following equation:

$$\mu = A \times 10^{\frac{B}{T-c}}, C = 140, B = 247, A = 2.414e-5 \quad (8)$$

Masoumi [24] correlation which considers the Brownian motion of nanoparticles, nanoparticles mean diameter and effect of temperature variation has been used for calculation of the effective nanofluid viscosity:

$$\mu_{eff} = \mu_{bf} + \mu_{app} \quad (9)$$

Where μ_{bf} and μ_{app} are respectively base fluid viscosity and apparent viscosity. Apparent viscosity is defined by the following equation:

$$\mu_{app} = \frac{\rho_p v_B d_p^2}{72\delta C} \quad (10)$$

The constant C depends on base fluid viscosity, diameter of nanoparticles and some constants. δ depends on diameter and volume fraction of nanoparticles. v_B is Brownian velocity that depends on temperature, diameter and density of nanoparticles.

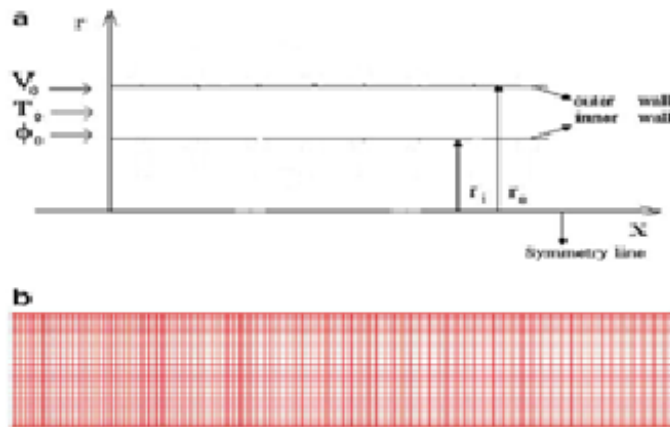


Fig. 1: (a) Schematic of the considered problem, (b) Grid distributions.

2.1 Boundary Condition

This set of nonlinear elliptical governing equations has been solved subject to following boundary conditions:

At the inlet annulus ($X=0$):

$$V_{mx} = V_i, \quad V_{mr} = 0 \quad \text{and} \quad T = T_0 \quad (11)$$

At the walls ($r = r_o, \quad r = r_i$)

$$V_{mx} = V_{mr} = 0, \quad i = \text{constant and } o = \text{constant} \quad (12)$$

At the annulus outlet ($X=L$): the diffusion flux in the direction normal to the exit plane is assumed to be zero for all variables except temperature that discrete fully developed energy equation is composed at outlet. An overall mass flux balance is also applied to correct velocities in outlet which are used for pressure correction.

2.2 Numerical Method

This set of coupled non-linear partial differential equations was discretized with the finite volume method. For the convective term a first order upwind method is used while the SIMPLE algorithm was introduced for the velocity-pressure coupling. Cells of Grid are non-uniform in two directions of axisymmetric geometry. It is finer near the wall and annulus entrance because the variables gradient is higher than other positions.

2.3 Grid Test and Results Validation

Several grid size have been tested to ensure that variation of grid numbers have no effect on the results. The selected grids consist of 15 and 200 cells, respectively in the radial and axial directions. Figure 2 compares the experimental results of frictional pressure drop at various Reynolds numbers [25] with the predicted results. While Fig. 3 compares the fully developed Nu profile with the corresponding analytical results for the state of constant heat flux thermal boundary condition [26]. Good agreements between the results are seen.

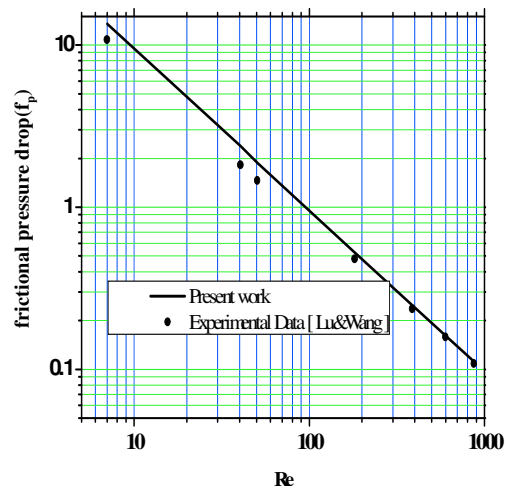


Fig. 2: Comparison of frictional pressure drops in horizontal annulus with experimental data [25].

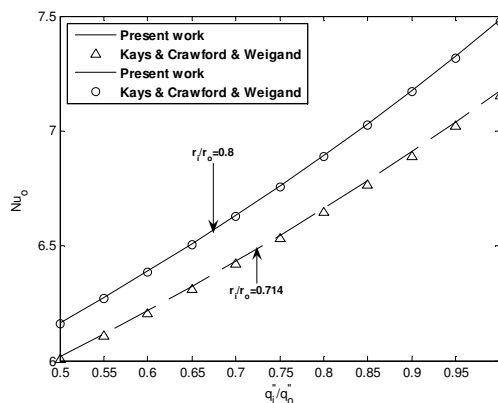


Fig. 3: Comparison of Nu_o in fully developed region of horizontal annulus with analytical calculation [27].

3. RESULTS AND DISCUSSION

Numerical simulation of forced convection of a nanofluid flow has been performed over Reynolds number, 900 and nanoparticle mean diameter, 25 nm. Wall temperatures were imposed at $T_i = 302\text{ K}$, $T_o = 298\text{ K}$ on inner and outer walls and the temperature of the inlet nanofluid is constant at 300 K. Hydrodynamics parameters such as velocities and friction coefficient at walls were like Izadi et al. [22] studies. Figure 4 shows heat flux behavior at inner wall. It is observed that heat flux decreases at the entrance region then increase in the length of the annulus. It is thought increasing in nanoparticle concentration increases heat flux.

Figure 5 shows the inner wall convective heat transfer coefficient at $Re=900$. It is seen that at such conditions the convective heat transfer coefficient reduces at the entrance region and goes to its minimum value then increases along the annulus.

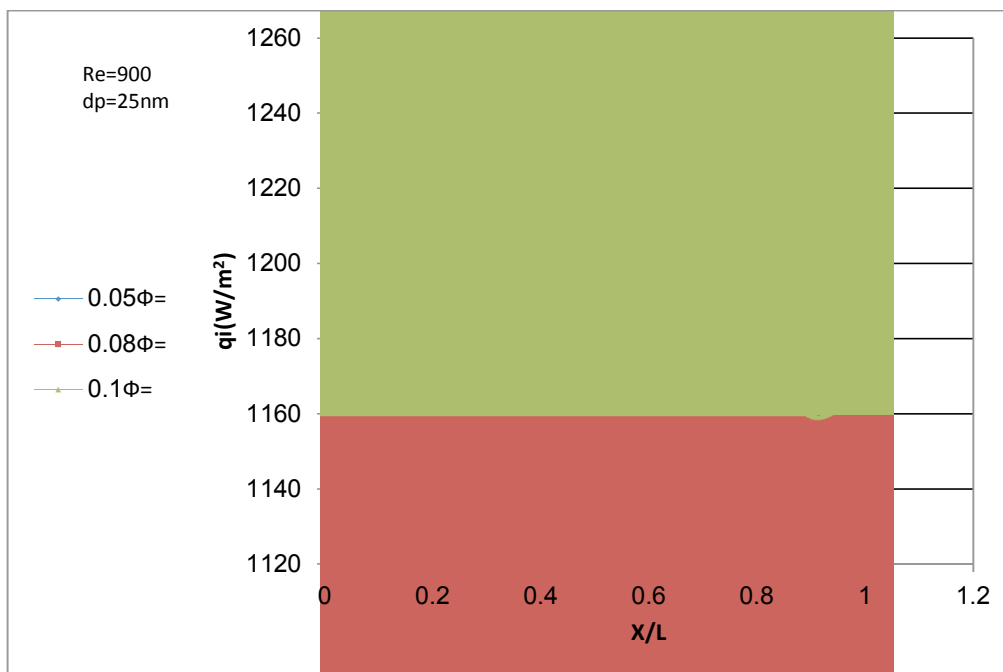


Fig. 4: Axial evolution of heat flux at the inner wall (W/m^2).

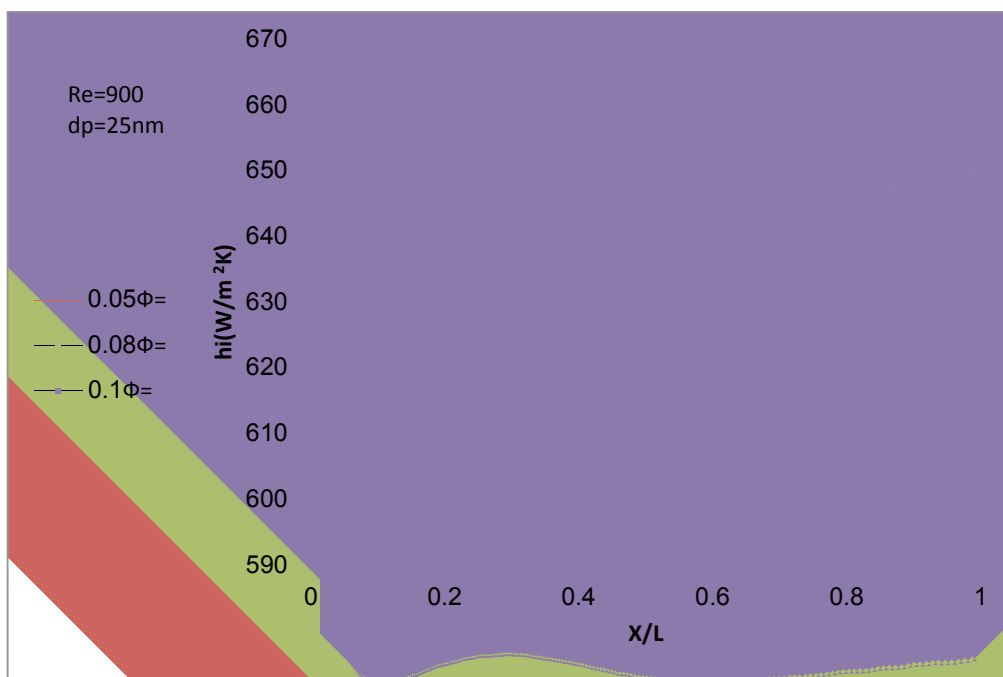


Fig. 5: Axial evolution of convective heat transfer coefficient at the inner wall (W/m^2K).

Figures 6 and 7 show the outer wall heat flux and convective heat transfer coefficient along the annulus for $Re=900$ respectively. The result is similar to the one obtained for the inner wall. Figures 8, 9, 10, 11 show the temperature distribution in the annulus for thermal boundary condition of the problem for nanoparticles volume concentration of 0.0, 0.05, 0.08, and 0.1 (In these figures $Z=X$).

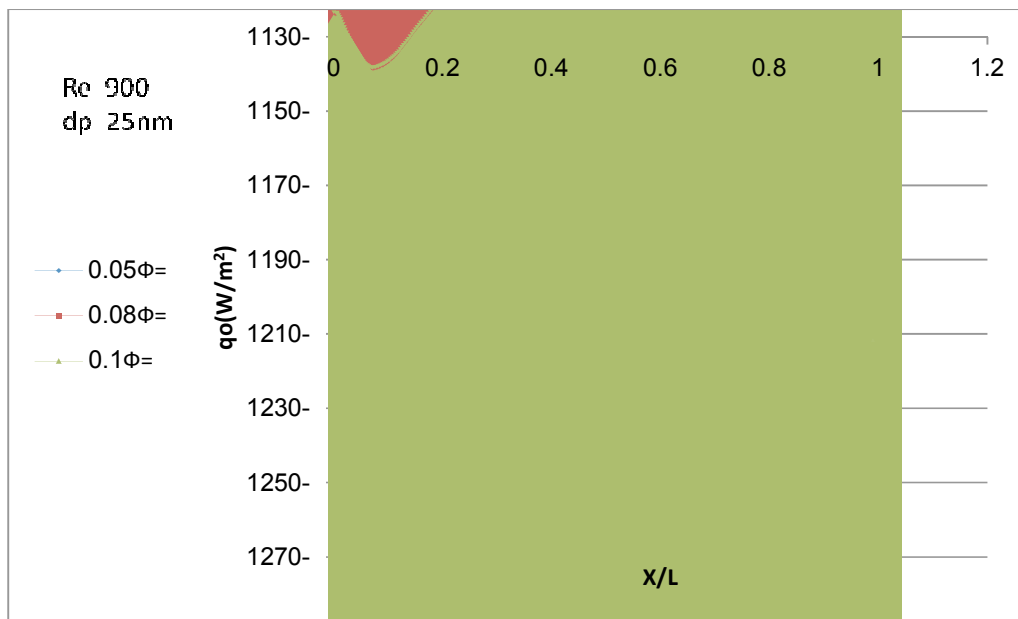


Fig. 6: Axial evolution of heat flux at the outer wall (W/m^2).

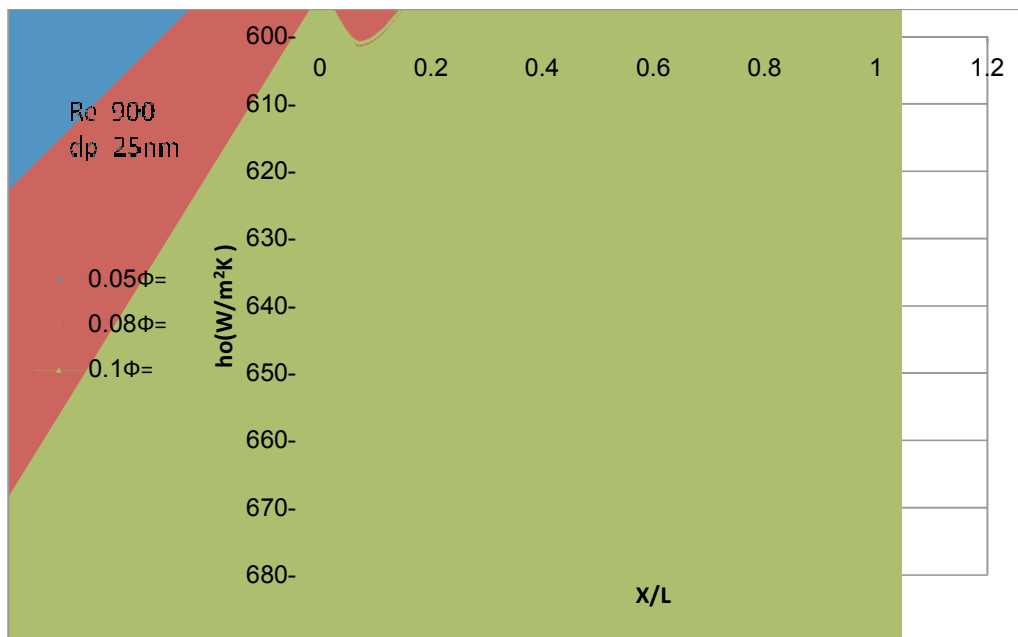
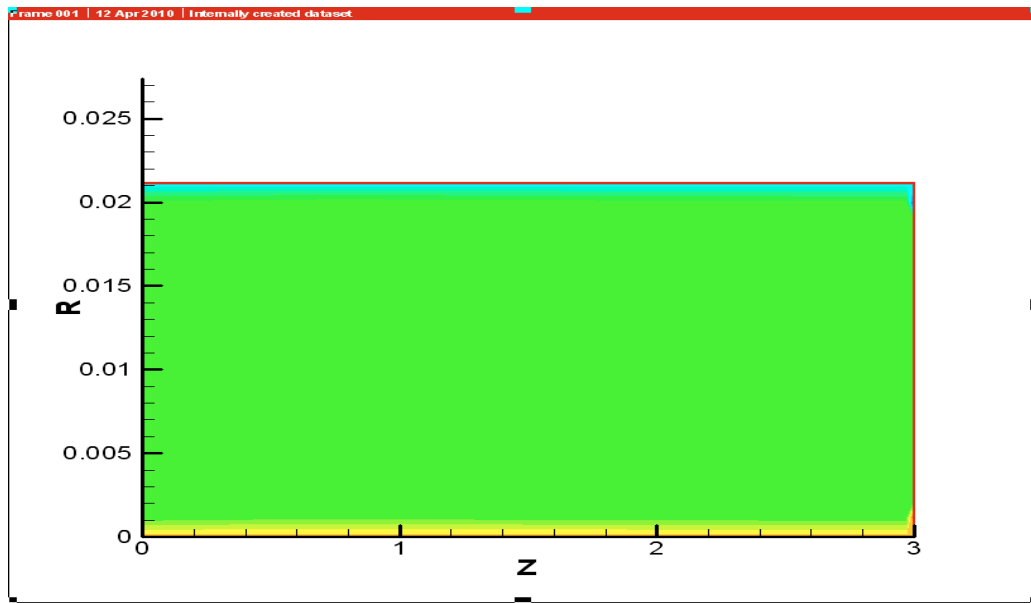
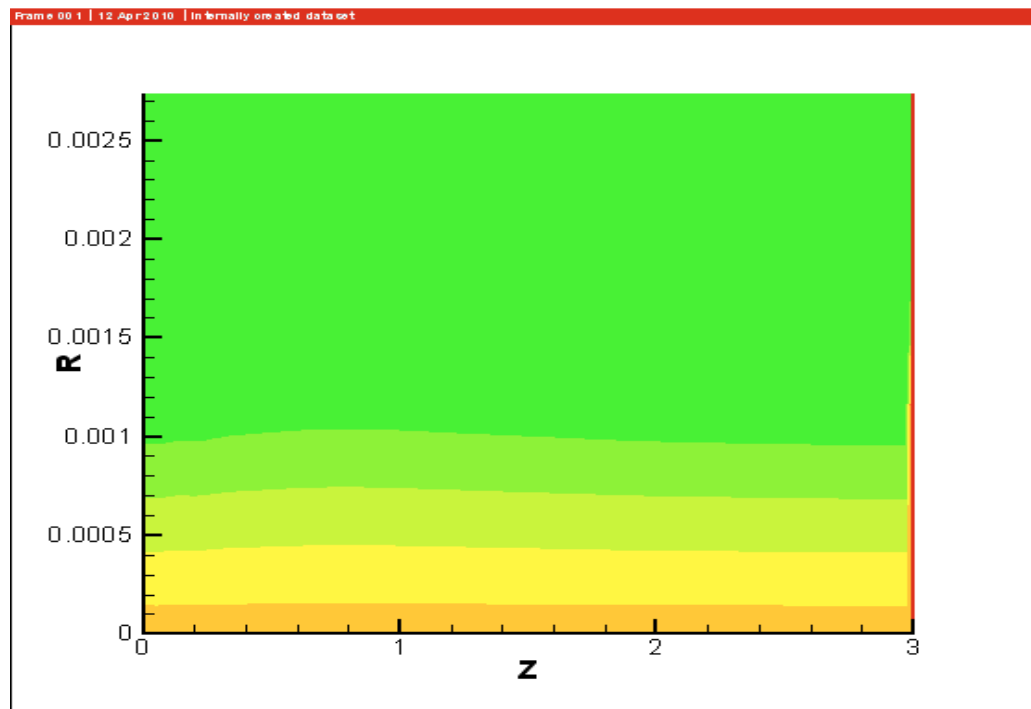


Fig. 7: Axial evolution of convective heat transfer coefficient at the outer wall (W/m^2K).

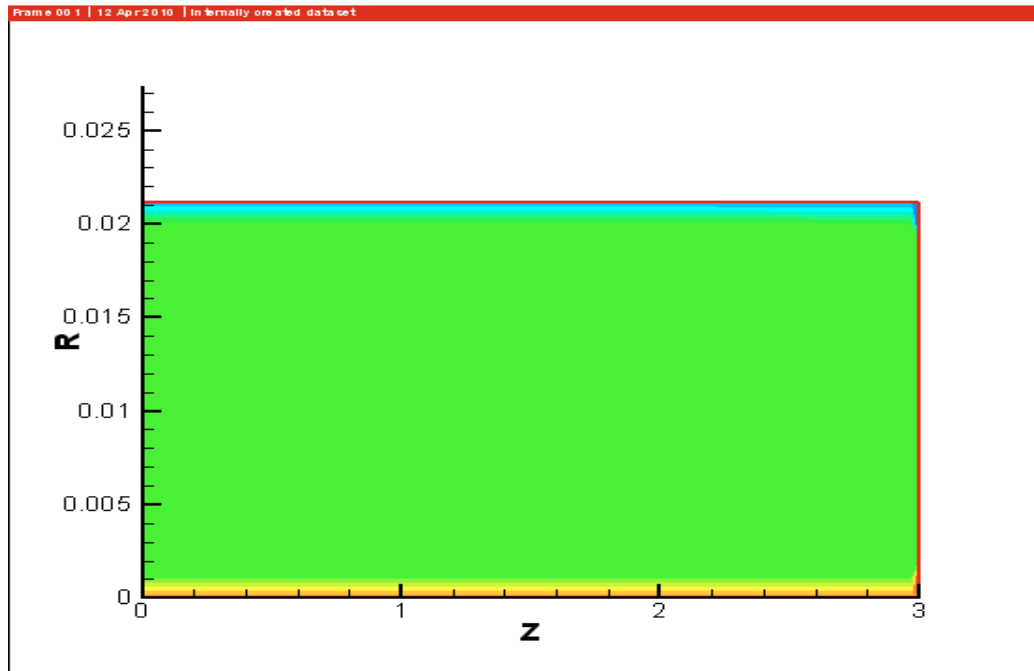


(a)

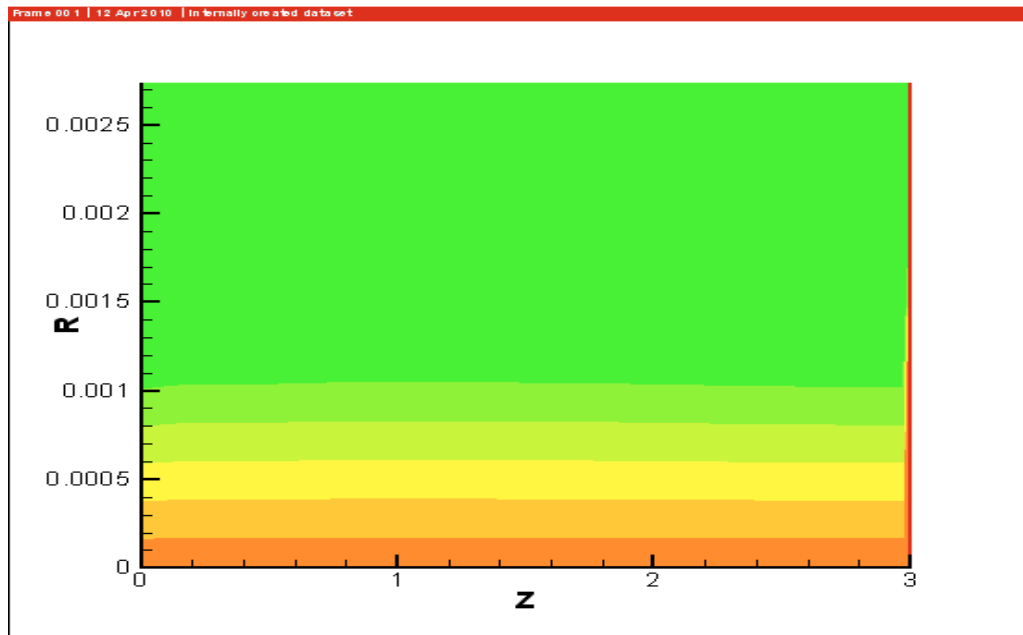


(b)

Fig. 8: (a) Temperature distribution in the annulus ($\Phi=0.0$), (b) the zoomed form of the inner wall zone in (a).

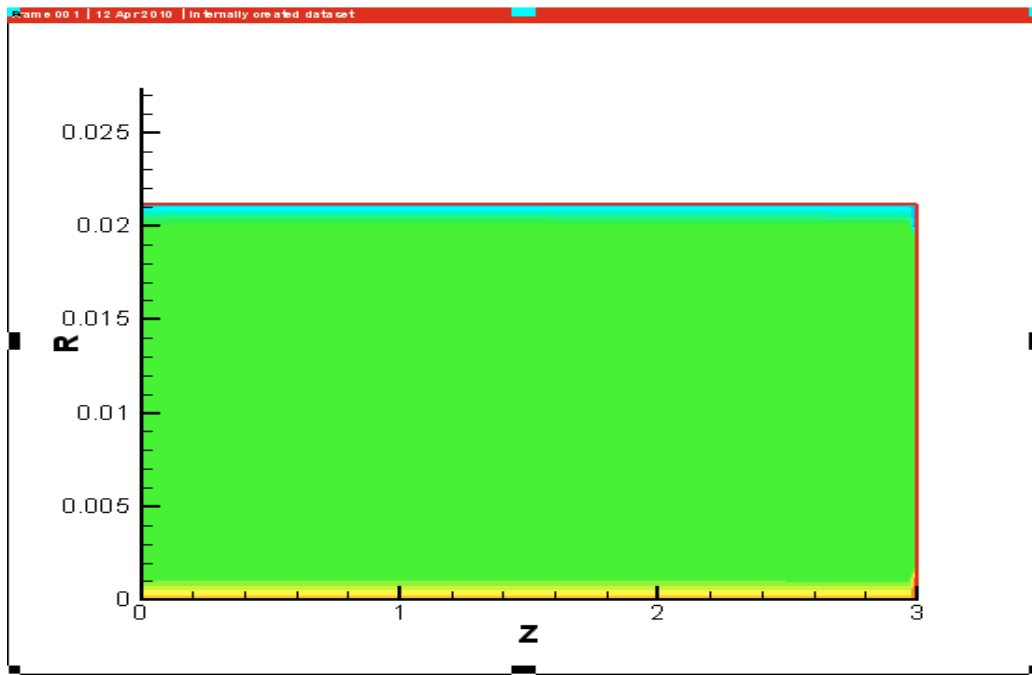


(a)

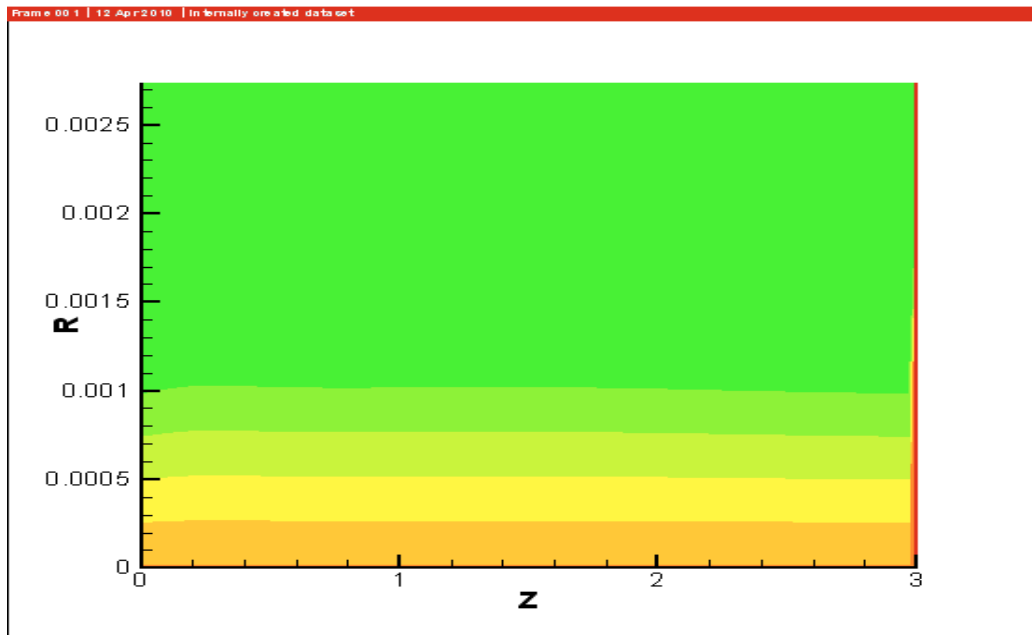


(b)

Fig. 9: (a) Temperature distribution in the annulus ($\Phi=0.05$), (b) the magnified form of the inner wall zone in (a).

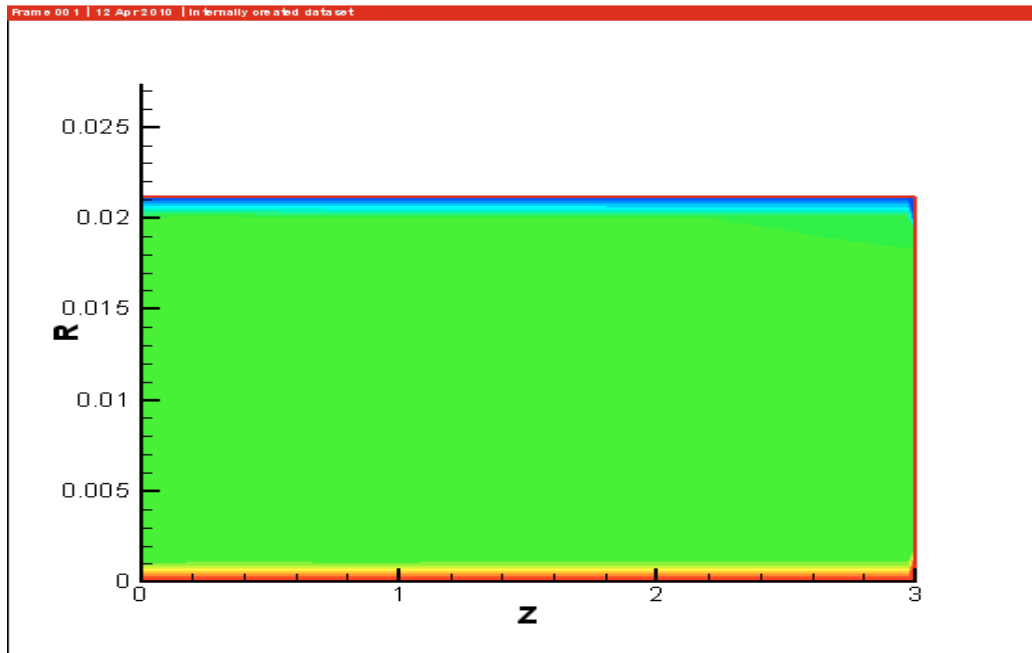


(a)

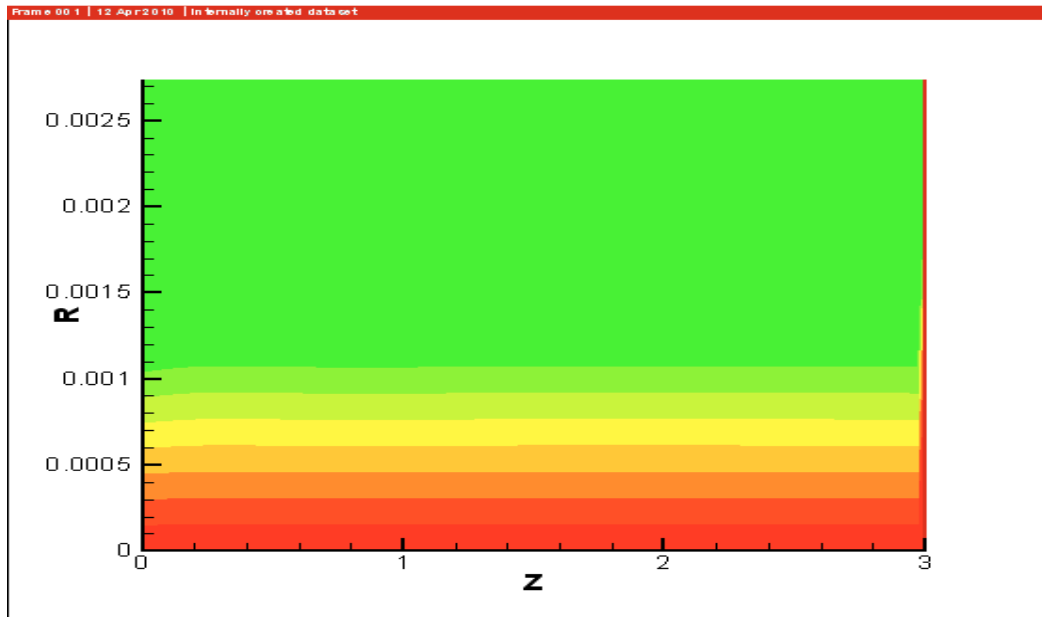


(b)

Fig. 10: (a) Temperature distribution in the annulus ($\Phi=0.08$), (b) the magnified form of the inner wall zone in (a).



(a)



(b)

Fig. 11: (a) Temperature distribution in the annulus ($\Phi=0.1$), (b) the magnified form of the inner wall zone in (a).

4. CONCLUSION

Numerical study of laminar forced convection of a nanofluid consists of water and Al_2O_3 in horizontal concentric annulus with constant temperature at walls have been studied by developing a computational code.

Two-phase model has been used for investigation of nanoparticles' (spherical shapes) volume concentration effects with above mentioned thermal boundary conditions, especially convective heat transfer coefficient. It is shown that increasing the nanoparticle volume concentration increases the convective heat transfer coefficient and also we observe that in the length of the annulus the heat flux and the convective heat transfer coefficient decreases in the entrance and then increases. Because of the aforementioned reasons, it can be said that in the length of the annulus there are sediments of nanoparticles that increase the heat transfer. And with increasing nanoparticles' volume concentration the effects of this phenomenon will be significant. These results have well accordance with experimental data. For example Nusselt number of Al_2O_3 -water nanofluid with the characteristics that have been written is brought here (Fig. 12).

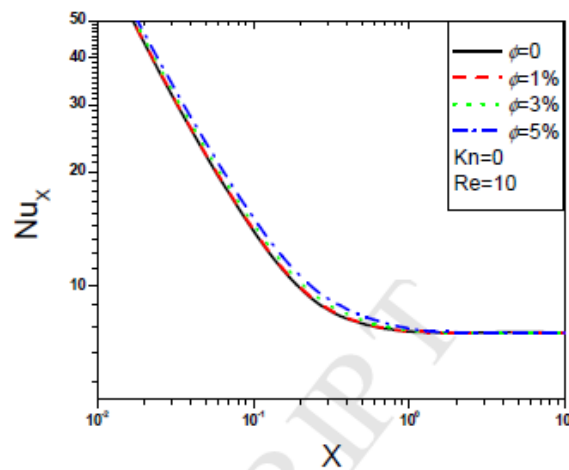


Fig. 12: The local Nusselt number (Nusselt number, $Nu = \frac{hD_h}{k}$ and Knudsen number, $Kn = \lambda/D_h$) [28].

Then there is a good accordance between Nusselt number (Fig. 12) and heat transfer coefficients (Figs. 5 and 7) of the wall in the entrance section of annulus. On the other hand there is a linear relation between h_i and h_o with q_i and q_o respectively. As a result numerical data are verifying with experimental data.

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NOMENCLATURE

B_C	Boltzman constant (J/K)	Kn	Knudsen number
C	Specific heat (J/kg K)	x	Axial direction
C_f	Area average friction coefficient (= $\zeta_i A_i + \zeta_o A_o / A_i + A_o / (\rho V_o^2 / 2)$)	<u>Greek letter</u>	
d_p	Nanoparticle diameter(nm)	ϕ	Volume fraction
d_f	Molecular diameter of base fluid(nm)	μ	Dynamic viscosity(Ns/m ²)
D_h	Hydraulic diameter(m)	ρ	Density(kg/m ³)
f_p	Friction coefficient	λ	Mean free path(m)
h_i	Inner wall convective heat transfer coefficient (W/m ² K)	<u>Subscripts</u>	
h_o	Outer wall convective heat transfer coefficient (W/m ² K)	app	appearance
k	Thermal conductivity (W/m K)	eff	effective
L_{Bf}	Mean free path of base fluid (m)	f	base fluid
N_i	Inner wall Nusselt number	i	inlet
N_o	Outer wall Nusselt number	m	mixture
P	Pressure (Pa)	mr	mixture in radial direction
Pr	Prandtl number	mx	mixture in axial direction
q_w	Uniform heat flux (W/m ²)	o	outlet
r	Radius(m)	p	particle
Re	Reynolds number	s	solid
T	Temperature (k)	w	wall
V	Velocity (m/s)	wi	inner wall
Nu	Nusselt number	wo	outer wall