APPROXIMATE FUNCTION FOR UNSTEADY AERODYNAMIC KERNEL FUNCTION OF AEROELASTIC LIFTING SURFACES

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ABSTRACT: Prediction of unsteady aerodynamic loads is still the most challenging tasks in flutter aeroelastic analysis. Generally, the numerical estimation of steady and unsteady aerodynamics of thin lifting surface is conducted based on an integral equation relating aerodynamic pressure and normal wash velocity. The present work attempts to increase the accuracy of the prediction by using an approximate approach to evaluate kernel function occurring in the integral equation in the form of cylindrical function. Following previous approximation approaches by other researchers to solve the cylindrical function for planar lifting surfaces, this paper extends such approaches to non planar lifting surfaces. To increase the accuracy of the method, the integration region of the kernel function is divided into two parts – near and far regions, where a nonlinear regression curve fitting technique is adapted to estimate the denominator part of the cylindrical function of each region.

ABSTRAK: Penelahan daya aerodinamik tidak stabil merupakan satu tugas yang mencabar dalam menganalisis getaran aeroanjalan. Umumnya, anggaran berangka untuk daya aerodinamik stabil dan tidak stabil pada permukaan mengangkat yang nipis, adalah berdasarkan kepada persamaan kamiran di antara tekanan aerodinamik dan halaju aliran udara pada garis normal yang terhasil di bawah sayap pesawat. Kajian ini adalah bertujuan untuk menghasilkan penelahan daya aerodinamik yang lebih tepat dengan menggunakan pendekatan kira hampir untuk menilai fungsi Kernel yang terdapat dalam persamaan kamiran dalam bentuk fungsi silinder. Dengan menggunakan pendekatan kira hampir yang digunakan oleh penyelidik sebelumnya untuk menyelesaikan fungsi silinder pada permukaan mengangkat satah, kajian ini mengembangkan pendekatan tersebut kepada permukaan mengangkat tak sesatah. Untuk meningkatkan lagi ketepatan penelahan, kawasan pengamiran fungsi Kernel dibahagikan kepada dua bahagian, kawasan hampir dan kawasan jauh, di mana penyuaian lengkung regresi tak linear digunakan untuk kiraan hampir penyebut pada fungsi silinder pada setiap kawasan.

KEYWORDS: *aeroelasticity; unsteady aerodynamics; kernel function; cylindrical function; lifting surface*

1. INTRODUCTION

Investigations are in progress for the improvement of accuracy and efficiency of the load prediction methods for countless types of aerodynamic configurations ranging from simple two dimensional airfoils to complicated full scale aircrafts [1]. Since its derivation in 1940, Küssner's governing formulation is the origin of most aerodynamic load formulations for thin lifting surfaces [2,3]. The fundamental part in the Küssner integral equation is the so-called kernel function which relates aerodynamic pressure and normal wash velocity. Several forms of the kernel function have been proposed in the past, including the formulation of Watkins et al. [4], Laschka [5], Yates [6], Landahl [7], and

Berman et al. [8]. The effect of acoustics in the integral equation is formulated by Yu et al. [9]. The widely accepted formulation of the kernel function is due to Landahl where the formula is used in aeroelastic analysis tool of MSC Nastran software. All of these formulations contain a hyper-geometric function of incomplete cylindrical function type whose solutions are not readily available in commercial software.

Most of the literatures use approximation methods to solve the incomplete cylindrical function [3, 8]. The first approximate method was adopted by Watkins et al. [10] which is used in unsteady aerodynamic doublet lattice method of Ref. 8. Laschka [11] presented both analytical and numerical solutions for the kernel function with three digit accuracy. Similar numerical approximation of series of fraction with constant numerators was offered by Dat and Malfois [12]. Least square technique is suggested by Desmarais [13,14] in order to minimize the error in the approximation. Ueda [15] presented an expansion series to solve analytically the problem. However, the Ueda series requires a large number of steps for a large number of arguments of oscillating functions as shown in [2]. To increase the accuracy of the kernel function evaluation, an analytical separation of singular and regular functions occurring in the incomplete cylindrical function is proposed in [16] by modifying the expansion series. Another analytical solution is presented by Bismarck-Nasr [17] based on differential equation approach. [18] described possible extension of the kernel function formulation to transonic flow and separated flow. Epstein and Bliss [2] suggested an approximation to the incomplete cylindrical function by dividing the regions of integration into two parts which are near and far fields. Their work in Ref. 2 is presented for the planar lifting surfaces. Following the Epstein-Bliss approach, in the present work such approach is extended to non planar lifting surfaces and the accuracy is increased by adding more terms to the approximation function.

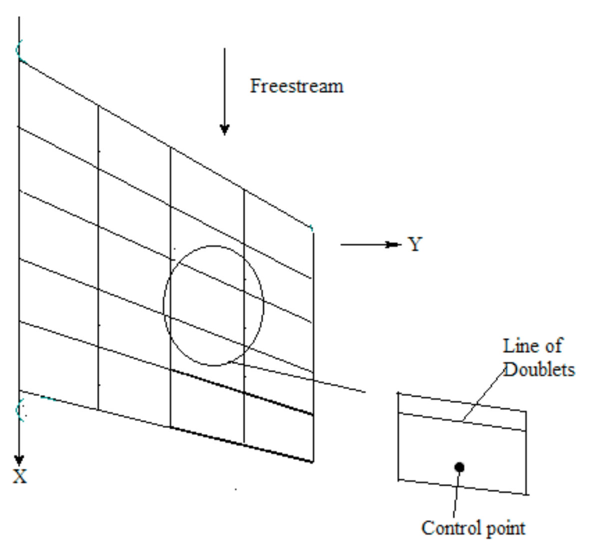

Fig. 1: Surface and Panel (box) geometry.

2. GOVERNING EQUATION

The Küssner integral equation relating the pressure and the normal wash distribution in the unsteady potential derivation is written as [3]:

$$\frac{w}{U} = \iint \left[\frac{\Delta p \;\; K\,(x,y,z,\xi,\eta,\zeta,k,M)}{8\pi\, q r^2}\right] d\xi\, d\eta \tag{1}$$

where w is the normal wash velocity at the control point (x,y,z) on the lifting surface as shown in Fig. 1, U is the free stream velocity of the unperturbated flow assumed in the x axis direction; and Δp is the nonstationary aerodynamic pressure difference at point $(\xi\,,\eta\,,\zeta)$ along the line of doublets of each trapezoidal aerodynamic model box shown in Fig. 1. The free stream dynamic pressure is denoted by q and is equal to $\rho U^2/2$ where ρ is the free stream air density. The kernel function of the integral K is a function of relative position, the free stream Mach number M, and the reduced frequency k. The relative position between the control point and the doublet pressure location can be expressed as:

$$x_0 = x - \xi$$

$$y_0 = y - \eta$$

$$z_0 = z - \zeta$$

and the reduced frequency k is defined as

$$k = \frac{\omega L}{U}$$

where L is the reference length.

2.1 Planar Surfaces

The kernel function K given in Epstein[2] for planar lifting surface is written as

$$K[\,x_0\,, y_0, k, M] = e^{-ikx_0}\frac{Me^{-ikX}}{R\sqrt{X^2+r^2}} + B(k,r,X) \tag{2}$$

where B is the incomplete cylindrical function formulated according to Landahl [7] and Ueda [15] as

$$B\,(k,r,x) = \int_{-\infty}^{X}\frac{e^{ikv}}{(r^2+v^2)^{\frac{3}{2}}}dv = \frac{1}{r^2}\int_{-\frac{X}{r}}^{\infty}\frac{e^{ikru}}{(1+u^2)^{\frac{3}{2}}}du \tag{3}$$

and the modified distance r , R and X are defined as

$$r = \sqrt{(y-\eta)^2} = |y_0|$$

$$R = \sqrt{x_0^2 + \beta^2 r^2}$$

$$X = \frac{x_0 - MR}{\beta^2} r^2$$

$$\beta^2 = 1 - M^2$$

Separating the real and imaginary terms [2] from Eqn (3) yields

$$B\,(k,r,X) = B_r + i\,B_i = \frac{1}{r^2}\int_{-\frac{X}{r}}^{\infty}\frac{\cos(kru)}{(1+u^2)^{\frac{3}{2}}} + \frac{i}{r^2}\int_{-\frac{X}{r}}^{\infty}\frac{\sin(kru)}{(1+u^2)^{\frac{3}{2}}} \tag{4}$$

and integrating the equation by parts gives

$$B(k,r,X) = \frac{1}{r^2}[\mathrm{V}]_{-|X/r|}^{\infty} - k^2 r^2 \int_{-\frac{X}{r}}^{\infty} e^{ikru} F(u) du \tag{5}$$

where the term V represents the non-integral term

$$V = e^{ikru}\left[\frac{u}{\sqrt{1+u^2}} - 1\right] - kre^{-ikru}\left[-u + \sqrt{1+u^2}\right] \tag{6}$$

and the function $F(u)$ is defined by

$$F(u) = -u + \sqrt{1+u^2} \tag{7}$$

In the present work, a simple curve fitting technique is utilized to approximate $F(u)$ as follows:

$$F(u) \cong f(u) = \begin{cases} f_a(u) & \text{for } u \leq 1.5 \\ f_b(u) & \text{for } u \geq 1.5 \end{cases} \tag{8}$$

where f_a (u) and f_b (u) are defined as

$$f_a(u) = \sum_{i=0}^{7} a_i u^i \tag{9a}$$

$$f_b(u) = \sum_{i=1}^{6} b_i u^{-i} \tag{9b}$$

and the coefficients a_i and b_i are defined in Table 1.

Table 1: Coefficients a_i and b_i of Eqn (8) and (9).

i	a_i	b_i
0	0.99999933	--
1	-0.99968242	0.49998705
2	0.49488907	0.00017800315
3	0.0303999182	-0.12594842
4	-0.21413252	0.00099707005
5	0.13894299	0.071073596
6	-0.041028643	-0.032656364
7	0.0048280266	--

The accuracy of the present approximation is demonstrated in Fig. 2 and Fig. 3. In Fig. 2, the plot of $f(u)$ resembles the target function $F(u)$. The plot of the error representing the difference between $F(u)$ and $f(u)$ is presented in Fig. 3. The maximum error for the present approximation is 3.5 x 10^{-6} at u = 1.4. Figure 3 also presents the plot of the error if the approximation of Epstein-Bliss [2] is used. The maximum error of their error is 0.015 which occurs at u = 0.5. Therefore, the present approximation provides much better accuracy compare to the approximation of Ref. 2. Note that the plot for the Epstein-Bliss approximation in Fig. 3 is multiplied by 10^{-3} whereas for the present approach is multiplied by 10^{-6} in order to show the sensitivity changes in the error.

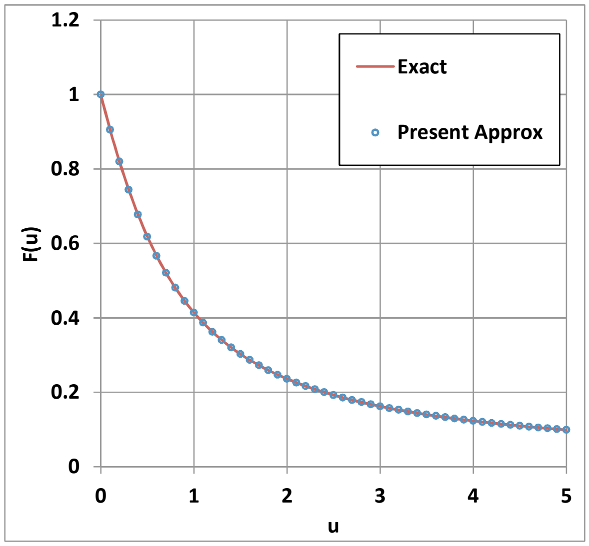

Fig. 2: Comparison between the present approach $f(u)$ and the target function $F(u)$.

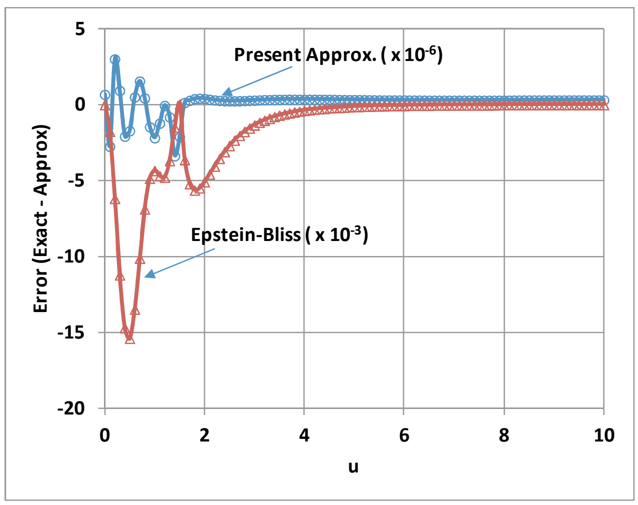

Fig. 3: Comparison between the error of the present approach $f(u)$ and the Epstein-Biss approach with respect to the target function.

Figure 4 shows comparison of $B_r(k,r,X)$ between the exact analytical solution of series expansion [14, 15] and the present approximation. Figure 5 shows comparison of $B_i(k,r,X)$ between the exact and the present approximation. To show the accuracy of the present approximation, the error or the difference between the analytical solution and the present approximation is plotted in Figs. 6 and 7 for B_r and B_i respectively. Both figures also present the error for B_r and B_i calculated using Epstein-Bliss approximation which demonstrates the improvement of the present approximation. Figure 6 shows that the maximum error of the Epstein-Bliss approximation for B_r is 0.0127 whereas the present approximation gives the maximum error of 0.00047. In Fig. 7, the maximum difference for B_i using the Epstein-Bliss approximation is 0.0094. But the new approximation plot shows the accuracy of 0.0000667 which increases the applicability of the new approximation to a good extent in solving the incomplete cylindrical function in the kernel function.

2.2 Nonplanar Surfaces

Rodemich [19] derived an expression in 1965 for the kernel function of nonplanar lifting surfaces as follow:

$$K = e^{-\left(\frac{i\omega x_0}{U}\right)}(K_1T_1 + K_2T_2)/r_1^2 \tag{10}$$

where

$$T_1 = cos(\gamma_s - \gamma_r)$$

$$T_2 = \frac{1}{{r_1}^2}(z_0\, cos\, \gamma_s - y_0\, sin\, \gamma_r)(z_0\, cos\, \gamma_r - y_0\, sin\, \gamma_s)$$

$$K_1 = I_1 + \left[\frac{Mr_1}{R}\right]\frac{e^{-ik_1u_1}}{(1+u_1^2)^{\frac{1}{2}}}$$

$$K_2 = -3I_2 - \frac{e^{-ik_1u_1}}{(1+u_1^2)^{\frac{1}{2}}}\left[\frac{ik_1M^2r_1^2}{R^2} - \frac{Mr_1}{R}\left\{\frac{(1+u_1^2)\beta^2r_1^2}{R^2} + 2 + \frac{Mr_1u_1}{R}\right\}\right]$$

where the incomplete cylindrical functions are given as

$$I_1 = \int_{u_1}^{\infty}\frac{e^{-ik_1u}}{(1+u_1^2)^{\frac{3}{2}}}\,du = r_1^2B_1 \tag{11}$$

$$I_2 = \int_{u_1}^{\infty}\frac{e^{-ik_1u}}{(1+u_1^2)^{\frac{5}{2}}}\,du = r_1^4\,B_2 \tag{12}$$

where

$$r_1 = (\,y_0^2 + z_0^2\,)^{1/2}$$

$$u_1 = \frac{MR - x_0}{\beta^2r_1}$$

$$k_1 = \frac{\omega r_1}{U}$$

The incomplete cylindrical function for nonplanar surfaces I_2 can be further separated into real and the imaginary parts as follows

$$I_2 = I_{2r} + I_{2i} = \int_{u_1}^{\infty}\frac{cos\,(k_1u)}{(1+u_1^2)^{\frac{5}{2}}}\,du + i\int_{u_1}^{\infty}\frac{sin\,(k_1u)}{(1+u_1^2)^{\frac{5}{2}}}\,du \tag{13}$$

The above given approximations in Eqn (8) and Eqn (9) could be utilized for nonplanar conditions. The solution will be extended towards the derivation of the function $F(u)$.

Figure 8 shows comparison of B_r with the exact analytical solution of the series expansion method [15]. The accuracy is quite considerable where it is shown in the Fig. 9 which shows the maximum error of 1.15 x 10^{-6}. In Fig. 10 the comparison of the Bi is compared with that of the expansion series method and the accuracy can be read in Fig. 11 which shows the maximum differences of 9 x 10^{-6} for the range until X = 10.

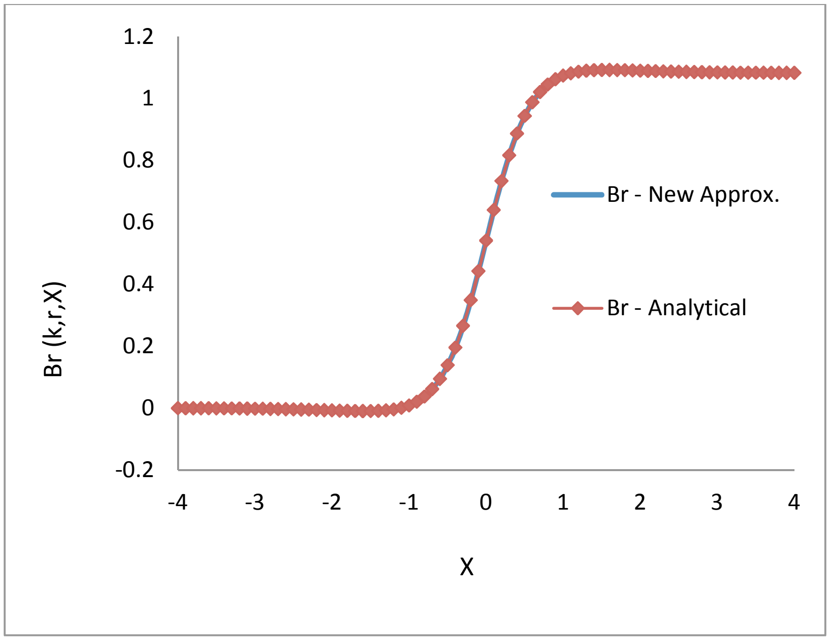

Fig. 8: Comparison of B_r (Nonplanar) between the present method and the series expansion method for k = 1.0 and r = 1.0.

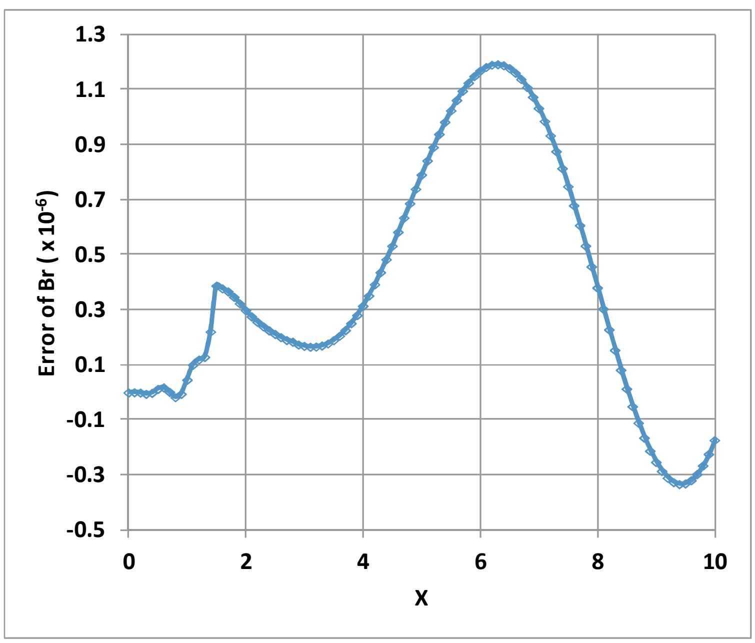

Fig. 9: Difference of B_r (Nonplanar) with the analytical solution for k = 1 and r = 1.

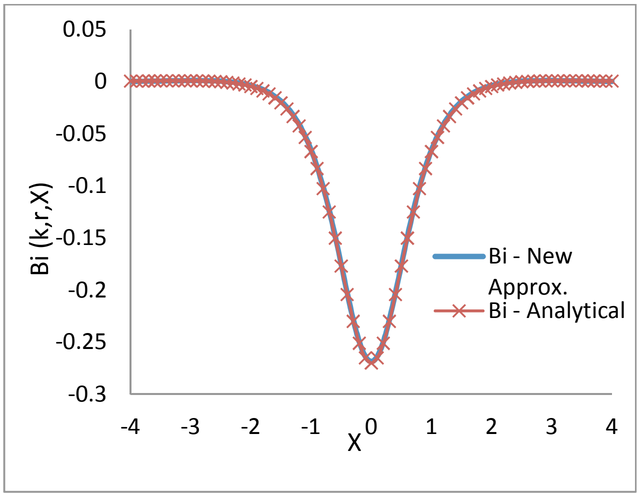

Fig. 10: Comparison of B_i (Nonplanar) between the present method and the series expansion method for k = 1.0 and r = 1.0.

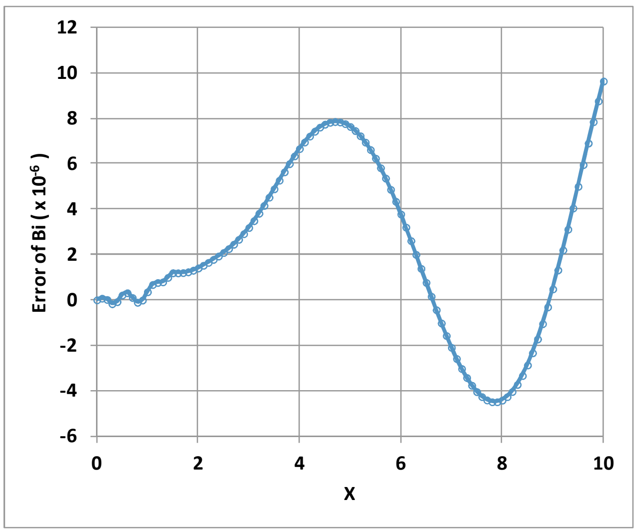

Fig. 11: Difference of B_i (Nonplanar) with the analytical solution for k = 1 and r = 1.

3. CONCLUSION

In the present method the solution to the incomplete cylindrical function is derived to a certain extent and the non-oscillatory part of the integrand is approximated using a simple curve fitting technique. The accuracy is appreciable with the comparison of analytical series expansion method. The applicability of the presented approximation to planar and non-planar configurations is augmented due to its plainness and the precision.

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