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On Intuitionistic Fuzzy β Generalized α Normal Spaces

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Abstract. In this paper a new concept of generalized intuitionistic fuzzy topological space called intuitionistic fuzzy β generalized α normal space is introduced. Several characterizations of intuitionistic fuzzy β generalized α normal space, intuitionistic fuzzy strongly β generalized α normal and intuitionistic fuzzy strongly β generalized α regular spaces are studied. Moreover, the related intuitionistic fuzzy functions with intuitionistic fuzzy β generalized α normal spaces are investigated.

1. Introduction

The notion of intuitionistic fuzzy set was first defined by Atanassov [7, 8] as a generalization of Zadeh [21] fuzzy set. This notion of intuitionistic fuzzy set has been developed by the same author and appeared in the literature [7,8]. Using the notion of intuitionistic fuzzy sets, Coker [13] introduced the notion of intuitionistic fuzzy topological spaces as a generalization of Chang [11] fuzzy topological spaces. Recently many concepts of fuzzy topological space have been extended in intuitionistic fuzzy topological spaces. Separation axioms in intuitionistic fuzzy topological space have been studied by some authors [1–4, 6, 9, 10]. Jayanthi [17] introduced the generalized β closed set in intuitionistic fuzzy topological spaces and intuitionistic fuzzy generalized closed sets are introduced by Saranya and Jayanthi [18]. Then Gomathi and Jayanthi [15, 16] have studied intuitionistic fuzzy β generalized α closed sets and intuitionistic fuzzy β generalized α continuous functions respectively. Thanh and Quang [19] have studied πgp-normality in topological spaces by using πgp-closed and πgp-open sets. In this paper, we introduce a new class of spaces called an intuitionistic fuzzy β generalized α normal

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spaces, and investigate some of their properties. Some interesting characterizations such as an IF β generalized α-normality is hereditary property with respect to an open and IF β generalized α-closed subspace, and equivalently of intuitionistic fuzzyβ generalized α-normal space and other meanings are introduced.

In addition, we introduce the concept of intuitionistic fuzzy β generalized α-regular spaces, some of their properties are investigated such as showing that an IF strongly-regular β^*- $T_{1/2}$ space is an IF β generalized α-regular, and every IF β generalized α-normal - R_0 space is an IF β generalized α-regular space.

In the last section of this paper, we study some properties of related intuitionistic fuzzy functions with intuitionistic fuzzy β generalized α normal spaces.

2. Preliminaries

Definition 2.1 [7] Let T be a non empty fixed set. An intuitionistic fuzzy set H (IFS for short)in T is an object having the form $H = \{\langle t, \mu_H(t), \gamma_H(t)\rangle : t \in T\}$ where the function $\mu_H : T \to I$ and $\gamma_H : T \to I$ denote the degree of membership (namely $\mu_H(t)$) and the degree of non-membership (namely $\gamma_H(t)$) of each element $t \in T$ to the set H respectively, and $0 \le \mu_H(t) + \gamma_H(t) \le 1$, for each $t \in T$.

Definition 2.2 [7] Let H and J be IF sets of the form $H = \{\langle t, \mu_H(t), \gamma_H(t)\rangle : t \in T\}$ and $J = \{\langle x, \mu_J(t), \gamma_J(t)\rangle : t \in T\}$. Then

(a) $H \subseteq J$ if and only if $\mu_H(t) \le \mu_J(t)$ and $\gamma_H(t) \ge \gamma_J(t)$ for all $t \in T$.

(b) $H = J$ if and only if $H \subseteq J$ and $J \subseteq H$.

(c) $H^c = \{\langle t, \gamma_H(t), \mu_H(t)\rangle : t \in T\}$.

(d) $H \bigcap J = \{\langle t, \mu_H(t) \wedge \mu_J(t), \gamma_H(t) \vee \gamma_J(t)\rangle : t \in T\}$.

(e) $H \bigcup J = \{\langle t, \mu_H(t) \vee \mu_J(t), \gamma_H(t) \wedge \gamma_J(t)\rangle : t \in T\}$.

(f) $0^{\sim}_T = \{\langle t, , 0, 1\rangle : t \in T\}$ and $1^{\sim}_T = \{\langle t, 1, 0\rangle : t \in T\}$.

Definition 2.3 [5] Let $\{H_i : i \in I\}$ be an arbitrary family of IFS in T. Then

(a) $\bigcap H_i = \{\langle t, \wedge\mu_{H_i}(t), \vee\gamma_{H_i}(t) : t \in T\}$.

(b) $\bigcup H_i = \{t, \vee\mu_{H_i}(t), \wedge\gamma_{H_i}(t) : t \in T\}$.

Definition 2.4 [12] Let $\alpha, \beta \in [0, 1], \alpha + \beta \le 1$ An intuitionistic fuzzy point (IFP for short) of nonempty set T is an IFS of T denoted by $P = t_{(\alpha,\beta)}$ and defined by

$$P = t_{(\alpha,\beta)}(y) = \begin{cases} (\alpha, \beta) if t = y \\ (0, 1) if t \neq y \end{cases} \tag{2.1}$$

In this case, t is called the support of $t(\alpha,\beta)$ and α,β are called the value and no value of $t(\alpha,\beta)$ respectively. Clearly an intuitionistic fuzzy point can be represented by an ordered pair of fuzzy point as follows: $t_{(\alpha,\beta)} = (t_\alpha, 1 - t_{(1-\beta)})$. An IFP,$t_{(\alpha,\beta)}$ is said to belong to an IFS $H = \{\langle t, \mu_H(t), \gamma_H(t)\rangle : t \in T\}$ denoted by $P = t_{(\alpha,\beta)} \in H(or P \subseteq H), if \alpha \leq \mu_H(t)$ and $\beta \geq \gamma_H(t)$.

We identify a fuzzy point t_r in T by the IF point $t_{(r,(1-r))}$ in T.

For more details about operations on IF-sets, IF points and IF functions, we can see [7, 8, 12–14].

Definition 2.5 [13] An intuitionistic fuzzy topology (briefly IFT) on a nonempty set T is a family ψ of intuitionistic fuzzy sets in T satisfy the following axioms:

(T1) $0_T^\sim, 1_T^\sim \in \psi$.

(T2) If $H_1, H_2 \in \psi$, then $H_1 \bigcap H_2 \in \psi$.

(T3) If $H_\lambda \in \psi$ for each $\lambda in \Lambda$, then $\bigcup_{\lambda\in\Lambda} H_\lambda \in \psi$.

In this case the pair (T, ψ) is called an intuitionistic fuzzy topological space (briefly IFTS) denoted by **T**, and each intuitionistic fuzzy set in ψ is known as an intuitionistic fuzzy open set (briefly IFOS) of T. The complement H^c of an $IFOS$ H in $IFTS$ (T, ψ) is an intuitionistic fuzzy closed set (briefly IFCS) in T.

Definition 2.6. An IFS H of an IFTS (T, ψ) is an

(i) IF α-open set (resp.α - closed) if $H \subseteq (int(cl(int(H)))$ (resp.$cl(int(cl(H))) \subseteq H$. [14]

(ii) IF $\beta - open($ resp.β - closed) if $H \subseteq (cl(int(cl(H)))$ (resp.$int(cl(int(H))) \subseteq H$. [14]

(iii) IF semi-open if $H \subseteq cl(int(H))$. [14]

(iv) IF pre-open if $H \subseteq (int(cl(H)))$. [14]

(v) IF generalized closed (briefly IFgC) if $cl(H) \subseteq J$ whenever $H \subseteq J$ and J is an IFO. [18]

Definition 2.7 [5] Let H be any IFS in IFTS (T, ψ). Then the IF β closure and IF interior of H are defined as follows,

$IF\beta cl(H) = \bigcap\{F : H \subseteq F, F \ is \ IF\beta CS \ in \ T\}$.

$IF\beta int(H) = \bigcup\{J : J \subseteq H, J \ is \ IF\beta OS \ in \ T\}$.

Definition 2.8 [15] An IFS H of an IFTS (T, ψ) is said to be an IF β generalized α -closed set (IFβGαCS for short) if $\beta cl(H) \subseteq J$ whenever $H \subseteq J$ and J is an $IF\alpha OS$ in (T, ψ). The complement H^c of an IFβGαCS H in an IFTS (T, ψ) is called an IF β generalized α - open set(IFβGαOS for short) in T. The family of all IFβGα CS of an IFTS (T, ψ) is denoted by IFβGαCS(T).

Definition 2.9 Let H be any IFS in IFTS (T, ψ). Then the IF β generalized α closure and IF β generalized α interior of H are defined as follows,

$IF\beta g\alpha cl(H) = \bigcap\{F : H \subseteq F, F \ is \ IF\beta g\alpha CS \ in \ T\}$.

$IF\beta g\alpha int(H) = \bigcup\{J : J \subseteq H, J \ is \ IF\beta g\alpha OS \ in \ T\}$.

3. Intuitionistic Fuzzy β Generalized α Normal Spaces

In this section, we have introduced IF β generalized α -normal space and studied some of its characterizations.

Definition 3.1 An IF topological space T is said to be IF β generalized α-normal(in short IF β g α-normal) if for every pair of IF disjoint β generalized α- closed sets H_1 and H_2 of T, there exist IF disjoint β-open sets R_1, R_2 of T such that $H_1 \subseteq R_1$ and $H_2 \subseteq R_2$.

Example 3.2 Let $T = \{a, b\}$ and R_1, R_2 are IF sets on T defined as follows:
$R_1 = \langle t, (\frac{a}{1.0}, \frac{b}{0.0}), (\frac{a}{0.0}, \frac{b}{1.0})\rangle$.

$R_2 = \langle t, (\frac{a}{0.0}, \frac{b}{1.0}), (\frac{a}{1.0}, \frac{b}{0.0})\rangle$.

Then the family $\psi = \{0_T^\sim, 1_T^\sim, R_1, R_2\}$ is an IFT on T.
The IF sets R_1,R_2 in T are IF disjoint β open sets and the IF sets $H_1 = \langle t, (\frac{a}{0.7}, \frac{b}{0.0}), (\frac{a}{0.3}, \frac{b}{1.0})\rangle$,$H_2 = \langle t, (\frac{a}{0.0}, \frac{b}{0.6}), (\frac{a}{1.0}, \frac{b}{0.4})\rangle$ are IF β g α -CSs such tha $H_1 \bigcap H_2 = 0_T^\sim$ and $H_1 \subseteq R_1$ and $H_2 \subseteq R_2$.

Then **T** is an IF β g α-normal space.

Theorem 3.3 For an IFTS (T, ψ), the following are equivalent:
(1) **T** is β g α-normal.
(2) For any pair of IF disjoint β g α OSs R_1 and R_2 of T whose union is $1_{\tilde{T}}$, there exist an IF disjoint β-CSs H_1 and H_2 of T such that $H_1 \subseteq R_1$ and $H_2 \subseteq R_2$ and $H_1 \bigcup H_2 = 1_{\tilde{T}}$.
(3) For each IFβ g α-CS H and an IFβ g α-OS K containing H, there exists an $IF\beta$-OS R_2 such that $H \subseteq R_2 \subseteq IF\beta - cl(R_2) \subseteq K$.
(4) For any pair of IF disjoint β g α-CSs H and K of T there exists an $IF\beta$-OS R_2 of T such that $H \subseteq R_2$ and $IF\beta - cl(R_2) \bigcap K = 0_{\tilde{T}}$.
(5) For any pair of IF disjoint β gα-CSs H and K of T there exists an $IF\beta$-OSs R_1 and R_2 of T such that $H \subseteq R_1$, $K \subseteq R_2$ and $IF\beta - cl(R_1) \bigcap IF\beta - cl(R_2) = 0_{\tilde{T}}$.

Proof.(1) $\Rightarrow$ (2) Let R_1 and R_2 be two IF β g α- OSs in an IF β g α-normal space T such that $R_1 \bigcup R_2 = 1_{\tilde{T}}$. Then R_1^c, R_2^c are IF disjoint β g α- CSs. Since T is an IF β g α-normal space there exist IF disjoint β-OSs R and Q such that $R_1^c \subseteq R$ and $R_2^c \subseteq Q$. Let $H_1 = R^c$,$H_2 = Q^c$. Then H_1 and H_2 are IF β-CSs $H_1 \subseteq R_1$, $H_2 \subseteq R_2$ and $H_1 \bigcup H_2 = 1_{\tilde{T}}$.

(2) $\Rightarrow$ (3) Let H be an IFβ g α- CS and K be an IFβ g α- OS containing H. Then H^c and K are IFβ g α -OSs such that $H^c \bigcup K = 1_{\tilde{T}}$. Then by (2) there exist an IF β-CSs H_1 and H_2 such that $H_1 \subseteq H^c$ and $H_2 \subseteq K$ and $H_1 \bigcup H_2 = 1_{\tilde{T}}$. Thus, we obtain $H \subseteq H_1^c$, $K^c \subseteq H_2^c$ and $H_1^c \bigcap H_2^c = 0_{\tilde{T}}$. Let $R_2 = H_1^c$ and $R_1 = H_2^c$. Then R_1 and R_2 are IF disjoint β -OSs such that $H \subseteq H_1^c \subseteq R_2 \subseteq K$. As R_2^c an IF β-CS, we have $H \subseteq R_2 \subseteq IF\beta - cl(R_2) \subseteq K$.

(3) $\Rightarrow$ (4) Let H and K be IF disjoint β g α -CSs of T. Then $H \subseteq K^c$ where K^c is IFβ g α -open. By the part (3), there exists an IF β-OS R_2 of T such that $H \subseteq R_2 \subseteq \beta - cl(R_2) \subseteq K^c$. Thus, $IF\ \beta - c1(R_2) \bigcap K = 0_{\tilde{T}}$.

(4) $\Rightarrow$ (5) Let H and K be any IF disjoint β g α -CSs of T. Then by the part (4), there exists an IF β-OS R_1 containing H such that $IF\beta - c1(R_1) \bigcap K = 0_{\tilde{T}}$. Since $IF\beta - c1(R_1)$ is an IFβ g α - closed, then it is IFβ g α -closed. Thus $IF\beta - c1(R_1)$ and K are IF disjoint β g α-CSs of T. Again by the part (4), there exists an IF β-OS R_2 in T such that $K \subseteq R_2$ and $IF\beta - c1(R_1) \bigcap IF\beta - c1(R_2) = 0_{\tilde{T}}$.

(5) $\Rightarrow$ (1) Let H and K be any IF disjoint β g α-CSs of T. Then by the part (5), there exist IF β-OSs R_1 and R_2 such that $H \subseteq R_1$, $K \subseteq R_2$, and $IF\beta - c1(R_1) \bigcap IF\beta - c1(R_2) = 0_{\tilde{T}}$. Therefore,

we obtain that $R_1 \bigcap R_2 = 0_T^{\sim}$. Hence T is IF$\beta$ g α-normal. □

Definition 3.4 [16] A space (T, ψ) is called β^* - $T_{1/2}$ if every IF β g α-CS in T is β closed.

Definition 3.5 [15] If every IF β g α-CS is an IFCS in (T, ψ), then the space can be called as an IF β g α $T_{1/2}$ space.

For the IF regularity we give the following definition.

Definition 3.6 An IFTS T is said to be IF β g α-regular if for every β g α-CS F of T and an IF point $P = t_{(\alpha,\beta)}$ not in F there exist IF disjoint β-OSs R_1, R_2 of T such that $P \in R_1$ and $F \subseteq R_2$.

Example 3.7 Let T = {a,b,c} and M, N are IF sets on T defined as follows:
$R_1 = \langle t, (\frac{a}{1.0}, \frac{b}{0.0}, \frac{c}{1.0}), (\frac{a}{0.0}, \frac{b}{1.0}, \frac{c}{0.0})\rangle$.

$R_2 = \langle t, (\frac{a}{0.0}, \frac{b}{1.0}, \frac{c}{0.0}), (\frac{a}{1.0}, \frac{b}{0.0}, \frac{c}{1.0})\rangle$.

$R_1^c = \langle t, (\frac{a}{0.0}, \frac{b}{1.0}, \frac{c}{0.0}), (\frac{a}{1.0}, \frac{b}{0.0}, \frac{c}{1.0})\rangle$.

$R_2^c = \langle t, (\frac{a}{1.0}, \frac{b}{0.0}, \frac{c}{1.0}), (\frac{a}{0.0}, \frac{b}{1.0}, \frac{c}{0.0})\rangle$.

Let P = $t(\alpha, \beta)$ = $a(0.5, 0.3)$ with $P \nsubseteq R_1^c$. Then there exist IF disjoint β-OSs R_1, R_2 of T such that $P \subseteq R_1$, $R_1^c \subseteq R_2$, and $R_1 \bigcap R_2 = 0_T^{\sim}$.
Then the family $\psi = \{0_T^{\sim}, 1_T^{\sim}, R_1, R_2\}$ is an IFT on T. Which is an β g α-regular space.

Definition 3.8 [20] An IFTS T is called IF strongly-regular if for each IF β g α-CS H and an IF point $P = t_{(\alpha,\beta)}$ not in H, there exist an IF β g α-OSs U, V of T such that $P \in U$ and $H \subseteq V$.

Definition 3.9 [20] An IF topological space T is called IF strongly-normal if for each IF β g α-CSs H_1 and H_2, there exist an IF β g α-OSs U, V such that $H_1 \subseteq U$ and $H_2 \subseteq V$.

Since every IF β -OS is an IF β g α-OS then we have.
IF β g α normal (resp.regular) space $\Rightarrow$ IF strongly-normal (resp. regular) space.

Lemma 3.10 An IF strongly-regularβ^*-$T_{1/2}$ space is an IF β g α-regular.

Proof Let (T, ψ) be an IF strongly-regular space as well as β^* —$T_{1/2}$ space. Since, (T, ψ) is a β^*-$T_{1/2}$ space, then every IF β g α-CS in T is β closed i.e. the class of IF β g α-CSs and β-closed sets coincide. Now, (T, ψ) is strongly regular space which provides that for each IF β g α-CS H of T and an IF point $P = t_{(\alpha,\beta)}$ not in H there exist IF disjoint β-OSs U, V such that $p \in U$ and $H \subseteq V$. Combining these facts, it is concluded that for each IF β g α-CS H and each IF point $P = t_{(\alpha,\beta)}$ there exist an IF disjoint β-OSs U and V such that $H \subseteq U$ and $p \in V$, which turns (T, ψ) to be an IF β g α-regular.□

Definition 3.11 An IF β g α space is said to be IF R_0 if for IF β - OS R and each IF point $P = t_{(\alpha,\beta)} \in R$, then IF $\beta cl\{p\} \subseteq R$.

Theorem 3.12 Every IF β g α-normal-R_0 space is an IF β g α-regular space.

Proof Let H be an IF β gα-CS in T and an IF point $P = t_{(\alpha,\beta)}$ in T such that P is not in H. Then, $p \in H^c$, where H^c is an IF β g α OS in T. Since T is an IF β g α-normal R_0 space, we have $IF\beta cl\{p\} \subseteq H^c$, then $H \bigcap IF\beta cl\{p\} = 0_{\widetilde{T}}$. Thus H and $IF\beta cl\{p\}$ are IF disjoint β g α-CSs in T. By β g α-normality of T, there exist IF disjoint β-OSs R_1, R_2 of T such that $H \subseteq R_1$ and IF $\beta cl\{p\} \subseteq R_2$. Therefore, there exist an IFβ-OSs R_1, R_2 of T such that $H \subseteq R_1$ and $p \in R_2$. Hence, T is an IF β g α-regular space.□

Lemma (3.13) [17] Suppose $H \subseteq Y \subseteq T$ and (T, ψ) is an IF β g α space. If Y is open and an IF β g α-closed in (T, ψ) and H is an IF β g α-closed in (Y, T_Y), then H is also β g α-closed in (T, ψ).

IF β generalized α-normality is hereditary property with respect to an open and IF β generalized α-closed subspace.

Theorem 3.14 If (T, ψ) is an IF β generalized α-normal space and Y is an IF open and β g α-CS of (T, ψ), then (Y, T_Y) is an IF β generalized α-normal subspace.

Proof Let H_1 and H_2 be any two IF disjoint β g α-CSs of (Y, T_Y). Since Y is an IF open and β g α-CS of (T, ψ), hence, in view of Lemma (3.13), H_1 and H_2 are IF β g α -closed in (T, ψ), and since (T, ψ) is an IF β g α-normal, then there exist an IF disjoint β-OSs R_1 and R_2 of (T, ψ) such that $H_1 \subseteq R_1$ and $H_2 \subseteq R_2$. As Y is also an IF open so Y is an IF α- open and then we get $R_1 \bigcap Y$ and $R_2 \bigcap Y$ as an IF disjoint β-OSs of the IF subspace (Y, T_Y) such that $H_1 \subseteq R_1 \bigcap Y$ and $H_2 \subseteq R_2 \bigcap Y$. Hence, (Y, T_Y) is an IF β g α-normal space.□

4. The Related Intuitionistic Fuzzy Functions with Intuitionistic Fuzzy β Generalized α Normal Spaces

We start by the following definition.

Definition 4.1 A function $f:(T,\psi)\rightarrow(Y,\delta)$ is called:

(1) IF β g α-closed if $f(H)$ is IF β -gα-closed in Y for each IF β g α-CS H of T.
(2) IF M-β-open if $f(H)$ is an IFβ -open in Y for each IF β-OS H of T.
(3) IF β g α-irresolute if $f^{-1}(H)$ is IF β g α-closed in T for each IFβ g α - CS H in Y.

Definition 4.2 [16] An IF function $f:(T,\psi)\rightarrow(Y,\delta)$ is said to be an IF β g α-continuous function if $f^{-1}(F)$ is an IF β g α-CS in T for every IF-CS H in Y.

Theorem 4.3 Let $f:(T,\psi)\rightarrow(Y,\delta)$ be an IF continuous β gα -closed injection and if (Y,δ) is an IF β g α-normal, then (T,ψ) is an IF β g α -normal.

Proof Let H_1 and H_2 are IF disjoint β g α-CSs in (T,ψ), since f is injective, $f(H_1)$ and $f(H_2)$ are IF β g α -CSs in (Y,δ), there exist an IF disjoint β -OSs R_1 and R_2 such that $f(H_i)\subseteq R_i$ for $i=1,2$. Since f is an IF β g α-continuous,$f^{-1}(R_1)$ and$f^{-1}(R_2)$ are IF β g α -CSs in (T,ψ) and $H_i\subseteq f^{-1}(R_i)$ for $i=1,2$. Put $Q_i=IF\beta-int(f^{-1}(R_i)$ for $i=1,2$. Then Q_1 and Q_2 are IFβ OSs with $H_1\subseteq Q_1$ and $H_2\subseteq Q_2$, and $Q_1\bigcap Q_2=0_T^{\sim}$. Then (T,ψ) is an IF β g α -normal.□

The following important Lemma can be proved easily.

Lemma 4.4 (a) The image of IF β g α-OS under an IF-open continuous function is β g α-open.

(b) The inverse image of IF β g α-O(resp.β gα-C) set under an open continuous function is IF β g α-O (resp. β g α-C) set.

Proposition 4.5 The image of IF β g α-OS under IF-open and IF-closed continuous function is IF β g α-open.

Proof Clearly.□

Theorem 4.6 If $f : (T, \psi) \rightarrow (Y, \delta)$ be an IF-open and IF-closed continuous bijection function and H be a IF β g α -CS in (Y, δ), then $f^{-1}(H)$ is IF β g α-CS in (T, ψ).

Proof Let H be an IF β g α-CS in (Y, δ) and R be any IF β g α -OS of (T, ψ) such that $f^{-1}(H) \subseteq R$. Then by the Proposition (4.5), we have $f(R)$ is IF β g α-OS of (Y, δ) such that $H \subseteq f(R)$. Since H is an IF β g α -CS of (Y, δ) and $f(R)$ is IFβ g α-OS in (Y, δ), thus IF $\beta - cl(H) \subseteq R$. By Lemma (4.4) we obtain that $f^{-1}(H) \subseteq f^{-1}(IF\beta - cl(H)) \subseteq R$, where $f^{-1}(IF\beta - cl(H))$ is β- closed in (T, ψ). This implies that $IF\beta - cl(f^{-1}(H)) \subset R$. Therefore $f^{-1}(H)$ is IF β g α-CS in (T, ψ).□

We show that an IFβ generalized α-normality is a topological property with respect to an IF open-and-closed bijection continuous function.

Theorem 4.7 An IF β g α-normality is a topological property.

Proof Let (T, ψ) be an IF β g α-normal space and be an open-and-closed bijection continuous function. We need to show that (Y, δ) is IF β generalized α-normal. Let H_1 and H_2 be any IF disjoint β generalized α-CSs in (Y, δ). Then by the Theorem (4.6) $f^{-1}(H_1)$ and $f^{-1}(H_1)$ are IF disjoint β generalized α-CSs of (T, ψ). By IF β g α-normality of (T, ψ), there exist β-OSs R_1 and R_2 of (T, ψ) such that $f^{-1}(H_1) \subseteq R_1$, $f^{-1}(H_2) \subseteq R_2$ and $R_1 \bigcap R_2 = 0_{\widetilde{T}}$. Then, we have $H_1 \subseteq f(R_1)$, $H_2 \subseteq f(R_2)$ and $f(R_1) \bigcap f(R_2) = 0_{\widetilde{T}}$. Thus, $f(R_1)$ and $f(R_2)$ are IF disjoint β-OSs of (Y, δ) such that $H_1 \subseteq f(R_1)$ and $H_2 \subseteq f(R_2)$. Hence, (Y, δ) is IF β g α-normal. □.

Theorem 4.8 If $f : (T, \psi) \rightarrow (Y, \delta)$ be an IFβ g α -irresolute, M-β-open bijection function from an IF β g α -normal (T, ψ) to an IF space (Y, δ), then (Y, δ) is an IF β g α -normal space.

Proof Let H_1 and H_2 be any two IF disjoint β g α-CSs in (Y, δ). Since f is an IF β g α -irresolute, we have $f^{-1}(H_1)$ and $f^{-1}(H_2)$ are IF disjoint β g α-CSs in (T, ψ). By IF β gα-normality of (T, ψ), there exist β -OSs R_1 and R_2 in (T, ψ) such that $f^{-1}(H_1) \subseteq R_1$, $f^{-1}(H_2) \subseteq R_2$ and $R_1 \bigcap R_2 = 0_{\widetilde{T}}$. Since f is an IF M-β-open and bijection function, we have $f(R_1)$ and $f(R_2)$ are IF β -OSs in (Y, δ) such that $H_1 \subseteq f(R_1)$, $H_2 \subseteq f(R_2)$ and $f(R_1) \bigcap f(R_2) = 0_{\widetilde{T}}$. Therefore, (Y, δ) is an IF β g α -normal.□

Theorem 4.9 If $f : (T, \psi) \rightarrow (Y, \delta)$ is an IF β g α - closed continuous surjection and (T, ψ) is an IFβ normal, then (Y, δ) is an IF β g α-normal.

Proof Since every IF β normal is an IF β g α -normal, the proof is clear.$\square$

5. Conclusion

In this paper, we introduced the concept of IF β g α normal spaces with study some of its properties. We also investigated the related intuitionistic fuzzy functions with intuitionistic fuzzy β g α normal spaces. In the future, based on some recent intuitionistic fuzzy β g α spaces studies, we will expand the research content of this paper further. Also,the entire content will be a successful tool for the researchers for finding the path to obtain the results in the context of intuitionistic fuzzy strongly-regular and strongly-normal spaces.

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