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Identifying Process Deterioration in Weighted Exponentially Distributed Time Between Events

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ABSTRACT. In observational studies, the probability of selection of sampling units is not always equal. The recorded observations are biased in this scenario. The unweighted distributions in such situations are not useful until the inclusion probability of each item is same. The theory of weighted distributions offers a unifying approach for these types of conditions because it considers the adjustment bias. Failure to comply with such adjustment may lead to inappropriate results. In this article, an efficient mentoring scheme (Weighted-TBE chart) for time between events (TBE) using weighted exponential distribution has been proposed based on weighted variance (WV) method. A comparison has been established between CC based on weighted and unweighted probability distributions. The performance measure ARL has been calculated using Monte Carlo simulations. The Weighted-TBE chart has provided least values of ARL in the presence of unwanted process variations and proved to be more effective than the existing scheme. Further the proposed control chart has been applied to time between failures data to show its practical applicability.

1. Introduction

In any industrial sectors, the deviation of a characteristic of interest from its target has a key importance for its acceptability in the market. This variation can be categorized into natural and assignable variations. Natural variations, often called common cause variations, are an unavoidable

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feature of every process that cannot be avoided. On the other hand, assignable variations are due to external sources like negligence of operators, failure of components of system and power breakdown etc. such variations may cause the working process to be shifted from its target [1].

In the manufacturing sector, CC are extensively used as an assessment tool that serve a crucial role in detecting unwanted variations. The main purpose of control charting techniques is to assist quality engineers in tracking unexpected changes in any manufacturing process.

TBE CC are broadly used to supervise the occurrence of defects or non-conforming articles in an industrial process. Apart from manufacturing processes, any process with inter-arrival time or TBE random variable can be monitored by the TBE CC. For example, time between monitoring of regular maintained system [2], consecutive radiation pulses [3], time between medical errors [4] and time between a non-parametric CUSUM scheme [5]. The concept of Time between events control chart was first time presented by [6] and [7]. The TBE CC that is based on the inter-arrival of non-conforming articles follow the exponential random variables. Due to this, the TBE CC are commonly known as exponential control charts [8].

The exponential distribution has been considered the most suitable model to access the lifespan of an element or a product when the failure of an element is possible at any time regardless the age of an item. In designing of any control chart for detecting the shift in a process, quality control researchers may be deceived by the assumption of normality. Because the data gathered in the subgroups may be extremely skewed standard CC do not yield useful findings, hence alternative approaches should be used [9]. This issue is also discussed by [10] and [11].

To model the TBE, the exponential distribution is generally used and assumed to be the better model for skewed data sets. It is right to say that scientists are often unable to pick sampling units with the same probability in observational studies. For these issues, A unifying answer can be found in the theory of weighted distributions. Adjustment bias is considered in weighted distributions. If researchers don't make this Adjustment, the results are misleading.

The idea of weighted distribution was first time presented by [12]. Rao, on the other hand, explored weighted distributions in a unified manner. It was indicated by [13], We cannot argue that the recorded numbers are a random sample from the true distribution because of a variety of factors. For example, when certain occurrences are unobservable, the original distribution is damaged or uneven probability sampling is used. During the past few decades, different TBE CC using exponential distribution have been proposed. The theme of this research is to utilize weighted probability distribution of quality characteristics instead of using usual probability distribution that remained still unaddressed.

If the quality characteristic of any product follows an asymmetric distribution, Shewhart type CC might be misleading. Even so, if we use Shewhart CC the chances of type-I error increases due to increase in skewness because of the variability in the population. The most common ways to dealing with such skewed scenarios are heuristic CC, transformation and increasing the sample size. Three methods are used to create heuristic CC. [14] proposed the semi variance approximation. [15] formed the weighted variance methodology based on semi variance approximation. This technique depends on standard deviation of sample ranges and means, they derived the skewed control limits for such distributions. A simple and useful method of constructing CC by using weighted variance method was established by [16]. Skewness leads to biased ARL, especially for Shewhart control chart [17] and [18]. To overcome the problem, weighted variance method has been utilized to develop the control limits for Weighted-TBE chart for weighted exponential distribution.

2. Material and Methods

The design structure of the Weighted-TBE chart is presented in this section. The weighted variance method has been adopted to address the skewness of weighted exponential distribution

2.1 Control charts based on unweighted distribution

Assume that the variable of interest such as the TBE designated by X, follows an exponential distribution having scale parameter θ. Then the pdf of the distribution is by.

$$f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, \qquad x > 0,$$

where θ represents the mean and standard deviation.

According to [19] the transformed random variable $X^* = X^{\frac{1}{\beta}}$ follows the Weibull distribution with two parameters, shape (β) and scale $(\theta^{\frac{1}{\beta}})$ respectively. , the Weibull distribution follows the approximately normal distribution when we have $\beta = 3.6$ [20]. the CC for the exponential distribution were suggested by [9] and [21] by applying the same transformation. Santiago and Smith [9], suggested the following control limits of Shewhart-type control chart.

$$LCL = \theta^{\frac{1}{3.6}}\left[\Gamma\left(1+\frac{1}{3.6}\right)\right] - k\sqrt{\Gamma\left(1+\frac{2}{3.6}\right) - \Gamma^2\left(1+\frac{1}{3.6}\right)}, \qquad 2.1$$

$$UCL = \theta^{\frac{1}{3.6}}\left[\Gamma\left(1+\frac{1}{3.6}\right)\right] + k\sqrt{\Gamma\left(1+\frac{2}{3.6}\right) - \Gamma^2\left(1+\frac{1}{3.6}\right)}. \qquad 2.2$$

2.2 Proposed Weighted-TBE chart based on WV Method.

The weighted exponential distribution is a generalized form of exponential distribution. The probability density function of weighted exponential distribution with parameter $\theta > 0$ is

$$f(x) = \frac{1}{\theta^2} x e^{-\frac{x}{\theta}}, \qquad t \geq 0. \qquad 2.3$$

The corresponding CDF is

$$F(X) = 1 - \frac{e^{-\frac{x}{\theta}}[x+\theta]}{\theta}. \qquad 2.4$$

The mean and the variance of weighted exponential distribution are

$$E(X) = 2\theta,$$

$$Var(X) = 2\theta^2.$$

The weighted variance method adjusts the control limits for skewed distribution according to the skewness of the underlying population with no assumptions.

The concept behind the weighted variance method is to split the skewed distribution into two segments at its average, then use each segment to generate a new symmetric distribution. These symmetric distributions are used to set up the control chart limits in WV method.

If we know the parameters of the process, then the control limits of $\bar{X}$ control chart are given by

$$UCL = \mu + 3\frac{\sigma}{\sqrt{n}}\sqrt{2P_x},$$

$$LCL = \mu - 3\frac{\sigma}{\sqrt{n}}\sqrt{2(1-P_x)}),$$

where "P_x" represents the probability of a random variable x will be less than or equal to its mean. It is important to note that the weighted variance $\bar{X}$ chart reduces to the standard $\bar{X}$ chart when $P_x = 0.5$. Here, n is the sample size while σ represents the standard deviation of x.

The theoretical control limits of weighted exponential distribution by using its mean and standard deviation are given as

$$UCL = 2\theta_0\left(1 + \frac{k}{\sqrt{2}} \times \sqrt{2P_x}\right), \tag{2.5}$$

$$CL = 2\theta_0, \tag{2.6}$$

$$LCL = 2\theta_0\left(1 - \frac{k}{\sqrt{2}} \times \sqrt{2(1-P_x)}\right), \tag{2.7}$$

where k specifies the width of the control limits, $\theta_0$is the in-control value of scale parameter θ. For out of control (OOC) process, the parameter shifts to another value $\theta_1 = \delta\theta_0$.

The control limits LCL, UCL and CL are the parameters of the proposed Weighted-TBE chart. The charting statistic from the weighted exponential distribution will be plotted against the control limits given eq. (7-9). In case of natural variations if the sample points of charting statistic falling inside the UCL and LCL with random behavior, that the process is said to be a "stable process" otherwise it is known as "shifted process". The ARL and $\Delta\mathrm{ARL}$ performance measures used in proposed study.

3. Results and Discussion

3.1 Simulation Study

Based on R language 4.1.0 [22], the simulation study has been performed and necessary measures have been calculated given in Tables (1-4). The in-control average run length (ARL_0) has been fixed at 200,300 and 370. The corresponding probabilities of first OOCpoint are 0.005,0.0033 and 0.0027, respectively.

The step-by-step algorithm to perform the simulations of proposed Weighted-TBE chart is given by:

3.2 Algorithm

The following steps are used to carry out the simulation study.

Step 1: Control limits of Weighted-TBE chart are derived by utilizing known parameter of weighted exponential distribution.

Step 2: 30000 samples of size *n* = 1 is selected from weighted exponential distribution with pdf given in equation 3.1.

Step 3: Each sample is tested against pre-determined control limits until it exceeds them.

Step 4: If a sample falls outside the control limits, the sample number is recorded as the value of random variable RL and loop goes to step 2 again.

Step 5: The above procedure is repeated 50000 times and the random variable RL contains 50000 values.

Step 6: On the last step mean, median, standard deviation, minimum and maximum of RL are computed. The percentiles (25^{th}, 75^{th} and 99^{th}) of RL and percent decrease in ARL are computed as well.

3.3 Performance assessment of proposed Weighted-TBE chart

To determine the behavior of ARL for the proposed chart in more detail, it is suggested to use some other beneficial measures of RL like standard deviation of run length (SDRL) median of run length (MRL), minimum run length (MinRL), maximum run length (MaxRL) and some percentiles of RL along with RL curves. All measures for recommended chart have been evaluated by using Monte Carlo simulations. The resulting measures for some selecting cases are presented in Table-1. Further From Tables 2–4, the following trend has been observed in $ARLs$ of the proposed Weighted-TBE chart:

1. A decreasing trend has been observed in OOC value of average run length (ARL_1) as the δ increases. This indicates that as the change in mean is large the shifted process can be identified more quickly.
2. A faster decline in trend has been noted in ARL_1 as ARL_0 is set at higher value.
3. The difference between the decreasing trend in ARL_1 is large when δ is set as a lower value. For example, $\mathrm{ARL1}$ decreases to 48.07% when $\mathrm{ARL}_0 = 200$ and $\delta = 1.1$, on the same value of shift parameter the ARL_1 decreases to 53.83% when we set $\mathrm{ARL}_0 = 370$, but as the value of shift parameter increases the decreasing trend get smaller. For example, when the value of shift parameter $\delta = 2.50$ ARL_1 decreases to 97.71% when $ARL_o = 200$, on the same value of shift parameter the ARL_1 decreases to 98.26% and 98.51% when we set $\mathrm{ARL}_0 = 300$ and 370 respectively.

Table 1

RL profile of Weighted-TBE chart for shifted process (ARL_0=370)

			K=3.742	THETA=1	$\mathrm{ARL_0}$=370			
Shift	ARL	MRL	SDRL	MIN RL	MAX RL	P25	P75	P99
1	370.56	254	371.16	1	2994	104	517	1677
1.1	170.84	120	168.12	1	1354	51	235.25	777.01
1.2	95	66	95.75	1	1062	29	130	448.01
1.3	60.23	41	59.86	1	592	18	83	270.01
1.4	40.77	28	40.68	1	381	12	55	191
1.5	29.75	21	29.14	1	366	9	41	133
1.6	22.55	16	22.35	1	221	7	31	102
1.7	17.93	12	17.35	1	165	6	25	80.01
1.8	14.47	10	13.79	1	134	5	20	64
1.9	11.81	8	11.31	1	119	4	16	53
2	10.1	7	9.54	1	83	3	14	45
2.5	5.53	4	5.04	1	51	2	7	24
3	3.7	3	3.17	1	27	1	5	15

The efficiency of any control chart is associated with its detection ability for the shifted process by holding in-control ARL at the fixed point. Lower values of ARL_1 correspond to faster indication about the shifted process. So, any control chart with the smaller values of $\mathrm{ARL_1}$ is considered superior in the detection of shift in the process.

3.3 Comparative Study

For a fixed value of $\mathrm{ARL_0}$, it is desired that the $\mathrm{ARL_1}$ values for a control chart are as small as possible, so that when the process deviates from the targeted value, the suggested control chart spot that as quickly as possible. A comprehensive comparison of the proposed charting method and existing chart with the help of their ARL values is shown in this section.

For the stated value of ARL_o the width of the control limit (k) for the Weighted-TBE chart will be computed, and the $\mathrm{ARL_1}$ associated with various process shift (δ) values will be retrieved.

Table 2 given below represents different values of ARL when the width of control limit k= 3.391 and $\mathrm{ARL_0} =$ 200 $(\text{ risk of type} - \text{I error } 0.005)$ for various values of the deviated process. Table-3.5 and Table-3.6 represents the situation of $\mathrm{ARL_0} = 300$ and 370 respectively. In Tables 2-4, we have included the $ARLs$ for the comparative t-chart.

3.3.1 Advantages of the Weighted-TBE chart over the t-chart

The advantages of the Weighted-TBE chart over the t-chart [9] have been discussed in this section. In the existing chart, the power transformation $X^{1/4}$ has been used and the control limits have been established. The results of simulation study of proposed Weighted-TBE chart and the existing t-chart have been compared at $\mathbf{ARL_0}$ = 200,300 and 370 on the various values of shift parameter δ. The $\mathbf{ARL_1}$ values of t-chart proposed by [9] are reported in the Tables 2–4 for various values of $\mathbf{ARL_0}$. The tables indicate that the Weighted-TBE chart provides smaller values of $\mathbf{ARL_1}$for different shift sizes. It has been indicated that Weighted-TBE chart is more effective than t-chart to identify the scale shift. It can be observed from Table-2 that on the average t-chart noticed δ = 1.1 at 155th sample, whereas the proposed chart spotted it at 102nd sample, which shows about 53 samples fast detection by Weighted-TBE chart. As long as ∆ARL is concerned, at δ = 1.1, ARL has decreased by 22.48% and 48.70% for t-chart and Weighted-TBE chart respectively. Similarly, it can be seen from Table-4 that on average δ = 1.1 has been detected by existing t-chart on the 278th sample, whereas it has been detected by the Weighted-TBE chart at 171st sample, which indicates about 93 samples earlier detection by Weighted-TBE chart. At δ = 1.1, ARL has decreased by 24.99% and 53.83% for t-chart and Weighted-TBE chart, respectively.

Table 2

The ARL when $\mathbf{ARL_0}$ = 200 (k = 3.391)

Shift (δ)	t-chart by Santiago and Smith	Proposed
1.00	200.00	200.00
1.10	155.05 (22.48)	102.61(48.70)
1.20	119.00 (40.50)	61.98 (69.01)
1.30	91.88 (54.06)	41.66 (79.17)
1.40	71.95 (64.03)	28.87 (85.57)
1.50	57.33 (71.34)	21.66 (89.17)
1.60	46.52 (76.74)	16.87 (91.57)
1.70	38.42 (80.79)	13.55 (93.23)
1.80	32.24 (83.88)	11.28 (94.36)
1.90	27.45 (86.28)	9.46 (95.27)
2.00	23.68 (88.16)	7.95 (96.03)
2.50	13.22 (93.39)	4.58 (97.71)
3.00	8.80 (95.6)	3.3 (98.35)

Table 3

The ARL when $\mathbf{ARL_0}$ = 300 (k = 3.629)

Shift (δ)	t-chart by Santiago and Smith	Proposed
1.00	300.00	298.92
1.10	227.57 (24.14)	143.48 (52.17)
1.20	170.82 (43.06)	81.47 (72.84)
1.30	129.11 (56.96)	52.17 (82.61)
1.40	99.14 (66.95)	36.64 (87.79)
1.50	77.62 (74.13)	26.41 (91.20)
1.60	61.99 (79.34)	20.1 (93.30)
1.70	50.45 (83.18)	15.92 (94.69)
1.80	41.79 (86.07)	12.93 (95.69)
1.90	35.17 (88.28)	10.96 (96.35)
2.00	30.02 (89.99)	9.4 (96.87)
2.50	16.07 (94.64)	5.23 (98.26)
3.00	10.39 (96.54)	3.58 (98.81)

Table 4

The ARL when $\mathbf{ARL_0}$ = 370 (k = 3.742)

Shift (δ)	t-chart by Santiago and Smith	Proposed
1.00	370.01	370.56
1.10	277.52 (24.99)	170.84 (53.83)
1.20	205.93 (44.34)	95.00 (74.32)
1.30	153.94 (58.39)	60.23 (83.72)
1.40	117.02 (68.37)	40.77 (88.98)
1.50	90.78 (75.46)	29.75 (91.96)
1.60	71.91 (80.56)	22.55 (93.91)
1.70	58.10 (84.30)	17.93 (95.15)
1.80	47.80 (87.08)	14.47 (96.09)
1.90	39.99 (89.19)	11.81 (96.81)
2.00	33.95 (90.82)	10.10 (97.27)
2.50	17.78 (95.19)	5.53 (98.51)
3.00	11.33 (96.94)	3.700 (99.00)

3.4 Performance comparison using ARL graphs.

The performance of the Weighted-TBE chart is shown in figure 1-3 for $\mathrm{ARL_0} = 200$, $\mathrm{ARL_0}$=300 and $\mathrm{ARL_0}$=370. From Figure 1-3, it has been observed that each ARL curve of Weighted-TBE chart is lower than the existing control chart. That leads to the conclusion that the suggested Weighted-TBE chart identifies the scale shift more rapidly than the existing technique.

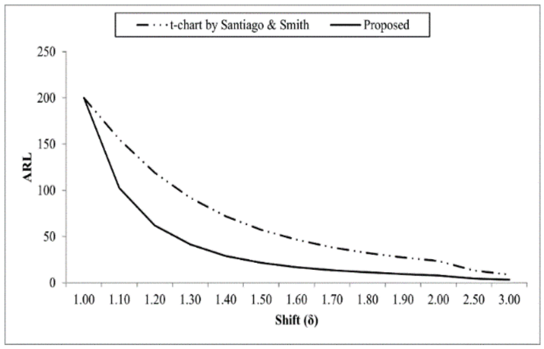

Figure 1: ARL curves of Proposed and Santiago and Smith (2013) charts at $ARL_0 = 200$

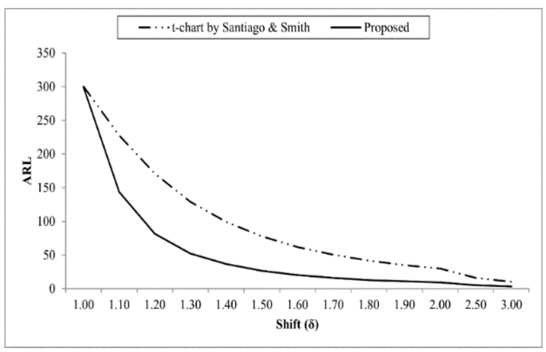

Figure 2: ARL curves of Proposed and Santiago and Smith (2013) charts at $ARL_0 = 300$

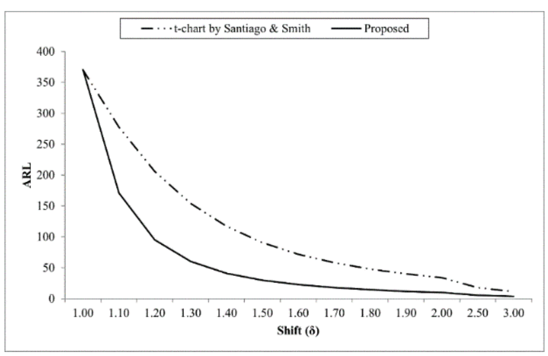

Figure 3: ARL curves of Proposed and Santiago and Smith (2013) charts at $ARL_0 = 370$

It can be comprehended from Figure 1-3 and Table 1-3 that the Weighted-TBE chart outperforms the chart proposed by Santiago and Smith at all shifts levels.

3.5 Applications

In this section, the practical application of Weighted-TBE chart has been presented to the given data sets for the purpose of comparison with the existing control chart to identify process shift.

In section 3.5.1, existing and Weighted-TBE chart has been applied to the data set representing the time between failures for 30 repairable items for showing the performance of the Weighted-TBE chart over the control chart based on unweighted distribution. In section 3.5.2, example of simulated data set has been presented to evaluate the performance of the proposed control chart.

3.5.1 Data Application

In this section, we intend to provide an application example for illustration purposes.

Weighted exponential distribution has wide range of applications in the field of reliability engineering. The proposed Weighted-TBE chart has been applied in this section to a real life data set showing the time between failures for 30 repairable items taken from [23].

The estimation of parameter has been performed in R 4.1.0 software [22]. For stable process, the sample size $n = 30$ has generated the estimated value of scale parameter is $\hat{\theta}_o = 0.771$. To evaluate the detection ability of proposed and existing charts, the shift of size $\delta = 1.5$ has been introduced in the stable value of scale parameter $\hat{\theta}_o = 0.771$ using $\hat{\theta}_1 = \delta\hat{\theta}_o$. The values of charting statistic have been plotted on control limits and the comparison has been made between proposed and existing charts. The process shift has not been noticed at any sample point by the existing control chart based on unweighted exponential distribution. Whereas, the proposed Weighted-TBE chart has detected the same shift at first sample after its occurrence.

This reveals that using the weighted distribution instead of unweighted distribution may enhance the detection probability of control charts when TBE data is required to monitor. Moreover, the production lot may be saved from producing the faulty items caused by unwanted variations.

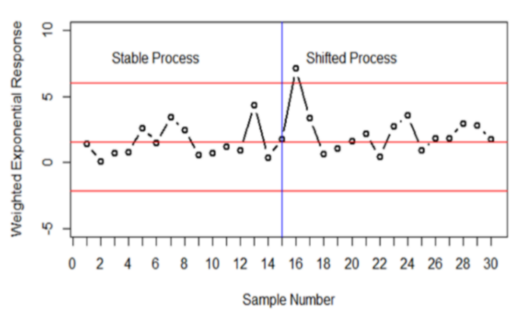

Figure 4: Weighted-TBE chart for time between failures of repairable items

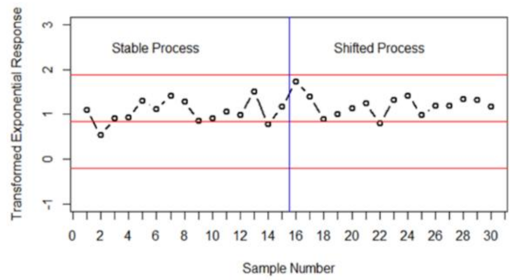

Figure 5: Santiago and Smith control chart for time between failures of repairable items

3.5.2 Simulated Data Example

In this section simulated data have been used to show the practical application of Weighted-TBE chart and the chart based on usual unweighted exponential distribution with scale parameter $\theta = 1$. In this simulated data set, 20 values have been taken from both the weighted and unweighted exponential distribution with in-control process. Twenty (20) more observations taken with shifted parameter using shift size $\delta = 2$. In Figures (6-7), blue vertical line divides the shifted and stable processes and the comparison between the proposed and existing charts has been established. The calculated values of the parameters are as follows:

$$CL = 2, LCL = -2.76867, UCL = 7.76802$$

We have considered the existing control chart of Santiago & Smith (2013) the resulting values of parameters are UCL = 1.84332, LCL = -0.04111, and CL = 0.90111 shown in Figure 7.

It is noticeable that from Figures 6-7 the unweighted-based chart did not detect the shift at any point from 20th to 40th sample. The Weighted-TBE chart outperform the shift overall on 7 out of shifted 20 sample points at 24th, 25th, 27th, 28th, 30^{th,} 35th and 38th samples. From these figures, it can be noted that the existing chart declared the process as in control. So, the chart did not detect the shift in the process.

The Weighted-TBE chart can be used to identify the unwanted variations. It can be helpful in preventing the lot from more defective items and to keep the process in control by taking essential measures.

The above example clearly showed that Weighted-TBE chart is more effective in identifying unwanted variations. It is effective in the process of TBE such as field of food industry, reliability engineering, medicine, and others. The utilization of weighted variance method to proposed chart leads to minimize the effect of skewness on ARL. Therefore, Shewhart type structure of proposed control chart is workable in monitoring industrial processes related to TBE.

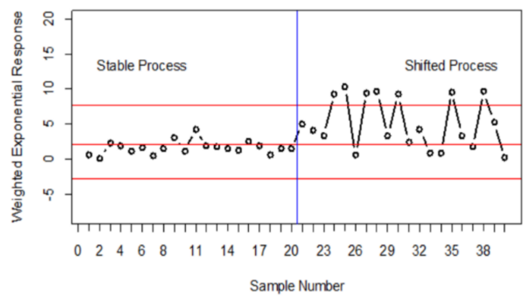

Figure 6: Proposed Weighted-TBE chart applied to simulated data

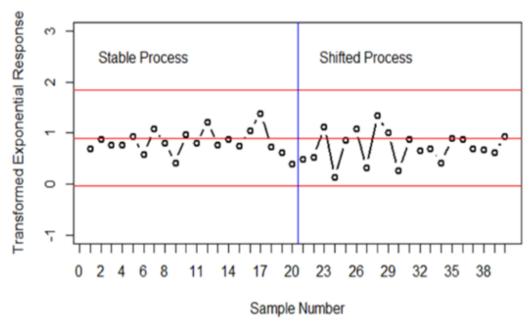

Figure 7: Santiago and Smith control chart applied to simulated data.

4. Summary and Future Works

In the field of quality control, one of the most challenging tasks is to identify the unnatural variations occurred in manufacturing processes. From the last many years number of studies have been conducted on monitoring TBE. Many researchers have used different skewed distributions like exponential [24] gamma [25] Weibull [26] to model the TBE data.

Traditional and existing studies used the unweighted distributions without considering the fact that weather or not the sampling units of production process have equal probability of selection. Often the quality engineers are not able to choose sampling units with the same probability in observational studies. Using unweighted distributions instead of weighted distributions may destroy the originality of the data and lead the process engineers in trouble to identifying process shifts due to misrepresentative results. In such conditions, the observations reported by the process are biased.

Such observations can only be modeled by the suitable choice of weighted distributions where adjustment bias is considered.

In this article, a new Weighted-TBE chart has been presented. Extensive simulations study has been conducted on the different values of control chart parameters. The proposed control charting technique has been proved to be superior to the existing control chart as it provides the smaller values of ARL. The proposed control charting scheme is recommended to extend to other probability distributions, such as Weighted-Weibull, Weighted-Gamma and Weighted-Erlang distribution etc.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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