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GENERALIZED IDENTITIES INVOLVING COMMON FACTORS OF GENERALIZED FIBONACCI, JACOBSTHAL AND JACOBSTHAL-LUCAS NUMBERS

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Abstract. In this paper, we present generalized identities involving common factors of generalized Fibonacci, Jacobsthal and jacobsthal-Lucas numbers. Binet's formula will employ to obtain the identities.

1. Introduction

It is well-known that the Fibonacci sequence is most prominent examples of recursive sequence. The Fibonacci sequence is famous for possessing wonderful and amazing properties. Fibonacci numbers are a popular topic for mathematical enrichment and popularization. The Fibonacci appear in numerous mathematical problems. The Fibonacci numbers F_n are terms of the sequence $\{0,1,1,2,3,5,...\}$ wherein each term is the sum of the two previous terms, beginning with the values $F_0 = 0$ and $F_1 = 1$.

There are a lot of identities of Fibonacci and Lucas numbers described in [7]. M. Thongmoon [6], defined various identities of Fibonacci and Lucas numbers. Singh, Bhadouria and Sikhwal [2], present some generalized identities involving common factors of Fibonacci and Lucas numbers. Gupta

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and Panwar [8], present identities involving common factors of generalized Fibonacci, Jacobsthal and jacobsthal-Lucas numbers. In this paper, we present generalized identities involving common factors of generalized Fibonacci, Jacobsthal and jacobsthal-Lucas numbers.

2. Preliminaries

Before presenting our main theorems, we will need to introduce some known results and notations.

Generalized Fibonacci sequence [9], is defined as

$$F_k = pF_{k-1} + qF_{k-2},\ k \geq 2\ with\ F_0 = a,\ F_1 = b, \qquad (2.1)$$

where p, q, a & b are positive integers.

For different values of p, q, a & b many sequences can be determined.

If $p = 1,\ q = a = b = 2$, we get

$$V_k = V_{k-1} + 2V_{k-2}\quad for\ k \geq 2\ with\ V_0 = 2,\ V_1 = 2 \qquad (2.2)$$

The first few terms of $\{V_k\}_{k \geq 0}$ are 2, 2, 6, 10, 22, 42 and so on.

Its Binet forms is defined by

$$V_k = 2\frac{\Re_1^{k+1} - \Re_2^{k+1}}{\Re_1 - \Re_2} \qquad (2.3)$$

The Jacobsthal sequence [1], is defined by the recurrence relation

$$J_k = J_{k-1} + 2j_{k-2},\ k \geq 2\ with\ J_0 = 0,\ J_1 = 1 \qquad (2.4)$$

Its Binet's formula is defined by

$$J_k = \frac{\Re_1^k - \Re_2^k}{\Re_1 - \Re_2} \qquad (2.5)$$

The Jacobsthal-Lucas sequence [1], is defined by the recurrence relation

$$j_k = j_{k-1} + 2\,j_{k-2}\;,\; k \geq 2 \;\; with \;\; j_0 = 2\,,\; j_1 = 1 \qquad (2.6)$$

Its Binet's formula is defined by

$$j_k = \Re_1^k + \Re_2^k \qquad (2.7)$$

where $\Re_1$ & $\Re_2$ are the roots of the characteristic equation $x^2 - x - 2 = 0$.

3. Main results

Generalized Fibonacci sequence ([9], [10]), similar to the other second order classical sequences. In this section we present generalized identities involving common factors of generalized Fibonacci, Jacobsthal and Jacobsthal-Lucas numbers. We shall use the Binet's formula for derivation.

Theorem 1: *If V_k is the generalized Fibonacci numbers and j_k is Jacobsthal-Lucas numbers, then,*

$$V_{2k+p}\,j_{2k+1} = V_{4k+p+1} - 2^{2k+1}V_{p-1} \quad , \; where \; k \geq 0 \; \& \; p \geq 0 \qquad (3.1)$$

Proof: $V_{2k+p}\,j_{2k+1} = 2\left(\frac{\Re_1^{2k+p+1} - \Re_2^{2k+p+1}}{\Re_1 - \Re_2}\right)\left(\Re_1^{2k+1} + \Re_2^{2k+1}\right)$

$$= 2\left(\frac{\Re_1^{4k+p+2} - \Re_2^{4k+p+2}}{\Re_1 - \Re_2}\right) + \frac{2}{\left(\Re_1 - \Re_2\right)}\left(\Re_1\Re_2\right)^{2k+1}\left(\Re_1^p - \Re_2^p\right)$$

$$= V_{4k+p+1} + 2\left(\Re_1\Re_2\right)^{2k+1}\left(\frac{\Re_1^p - \Re_2^p}{\Re_1 - \Re_2}\right)$$

$$= V_{4k+p+1} - 2^{2k+1} V_{p-1}$$

This completes the proof.

Corollary 1.1:

(i) $If\ \ p = 0,\ then:\ V_{2k} j_{2k+1} = V_{4k+1}$

(ii) $If\ \ p = 1,\ then:\ V_{2k+1} j_{2k+1} = V_{4k+2} - 4^{k+1}$

(iii) $If\ \ p = 2,\ then:\ V_{2k+2} j_{2k+1} = V_{4k+3} - 4^{k+1}$

Corollary 1.2: $V_{2k+p} j_{2k+1} = 2J_{4k+p+2} - 4^{k+1} J_p \quad ,\ where\ k \geq 0\ \&\ p \geq 0$ (3.2)

Following theorems can be solved by Binet's formulae (2.3), (2.5) and (2.7)

Theorem 2: $V_{2k+p} j_{2k+2} = V_{4k+p+2} + 4^{k+1} V_{p-2} \quad ,\ where\ k \geq 0\ \&\ p \geq 0$ (3.3)

Corollary 2.1:

(i) $If\ \ p = 0,\ then:\ V_{2k} j_{2k+2} = V_{4k+2} + 2^{2k+1}$

(ii) $If\ \ p = 1,\ then:\ V_{2k+1} j_{2k+2} = V_{4k+3}$

(iii) $If\ \ p = 2,\ then:\ V_{2k+2} j_{2k+2} = V_{4k+4} + 2^{2k+3}$

Corollary 2.2: $V_{2k+p} j_{2k+2} = 2\left\{ J_{4k+p+2} + 4^{k+1} J_{p-1} \right\},\ where\ k \geq 0\ \&\ p \geq 0$ (3.4)

Theorem 3: $V_{2k+p} j_{2k} = V_{4k+p} + 4^k V_p \quad ,\ where\ k \geq 0\ \&\ p \geq 0$ (3.5)

Corollary 3.1:

(i) $If\ \ p = 0,\ then:\ V_{2k} j_{2k} = V_{4k} + 2^{2k+1}$

(ii) $If\ \ p = 1,\ then:\ V_{2k+1} j_{2k} = V_{4k+1} + 2^{2k+1}$

(iii) $If\ \ p = 2,\ then:\ V_{2k+2} j_{2k} = V_{4k+2} + 3\left(2^{2k+1}\right)$

Corollary 3.2: $V_{2k+p} j_{2k} = 2\left\{ J_{4k+p+1} + 4^k J_{p+1} \right\} \quad ,\ where\ k \geq 0\ \&\ p \geq 0$ (3.6)

Theorem 4: $V_{2k-p}\,j_{2k+1} = V_{4k+1-p} - 2^{2k+1}V_{-p-1}$, where $k \geq 0$ & $p \geq 0$ (3.7)

Corollary 4.1:

(i) If $p = 0$, then: $V_{2k}\,j_{2k+1} = V_{4k+1}$

(ii) If $p = 1$, then: $V_{2k-1}\,j_{2k+1} = V_{4k} - 2^{2k+1}$

(iii) If $p = 2$, then: $V_{2k-2}\,j_{2k+1} = V_{4k-1} + 4^{k}$

Corollary 4.2: $V_{2k-p}\,j_{2k+1} = 2\left\{J_{4k-p+2} - 2^{2k+1}J_{-p}\right\}$, where $k \geq 0$ & $p \geq 0$ (3.8)

Theorem 5: $V_{2k-p}\,j_{2k-1} = V_{4k-1-p} - 2^{2k-1}V_{1-p}$, where $k \geq 0$ & $p \geq 0$ (3.9)

Corollary 5.1:

(i) If $p = 0$, then: $V_{2k}\,j_{2k-1} = V_{4k-1} - 4^{k}$

(ii) If $p = 1$, then: $V_{2k-1}\,j_{2k-1} = V_{4k-2} - 4^{k}$

(iii) If $p = 2$, then: $V_{2k-2}\,j_{2k-1} = V_{4k-3}$

Corollary 5.2: $V_{2k-p}\,j_{2k-1} = 2\left\{J_{4k-p} - 2^{2k-1}J_{2-p}\right\}$, where $k \geq 0$ & $p \geq 0$ (3.10)

Theorem 6: $V_{2k-p}\,j_{2k} = V_{4k-p} + 4^{k}V_{-p}$, where $k \geq 0$ & $p \geq 0$ (3.11)

Corollary 6.1:

(i) If $p = 0$, then: $V_{2k}\,j_{2k} = V_{4k} + 2^{2k+1}$

(ii) If $p = 1$, then: $V_{2k-1}\,j_{2k} = V_{4k-1}$

(iii) If $p = 2$, then: $V_{2k-2}\,j_{2k} = V_{4k-2} + 4^{k}$

Corollary 6.2: $V_{2k-p}\,j_{2k} = 2\left\{J_{4k-p+1} + 4^{k}J_{1-p}\right\}$, where $k \geq 0$ & $p \geq 0$ (3.12)

Theorem 7: $V_{2k}\,j_{2k+p} = V_{4k+p} + 2^{2k+1}V_{p-2}$, where $k \geq 0$ & $p \geq 0$ (3.13)

Corollary 7.1:

(i) $If \quad p = 0, \; then: \; V_{2k} j_{2k} = V_{4k} + 2^{2k+1}$

(ii) $If \quad p = 1, \; then: \; V_{2k} j_{2k+1} = V_{4k+1}$

(iii) $If \quad p = 2, \; then: \; V_{2k} j_{2k+2} = V_{4k+2} + 2^{2k+3}$

Corollary 7.2: $V_{2k} j_{2k+p} = 2\left\{ J_{4k+p+1} + 2^{2k+1} J_{p-1} \right\} \quad , \; where \; k \geq 0 \; \& \; p \geq 0$ (3.14)

4. Conclusion

In this paper we have derived many identities of generalized common factors of generalized Fibonacci, Jacobsthal and jacobsthal-Lucas numbers with the help of their Binet's formula. The concept can be executed for Fibonacci-Like sequences as well as polynomials.

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