

A SYNTHESIS OF THEORETICAL RELATIONSHIP BETWEEN SYSTEMATIC RISK AND FINANCIAL AND ACCOUNTING VARIABLES

Edward R. Lawrence* , Suchismita Mishra**, Arun J Prakash**

*University of Nebraska, Lincoln

**Florida International University, Miami

Abstract

In this paper we summarize the theoretical relationship between beta, the measure of relative systematic risk on one hand and financial and accounting variables, such as leverage, size, growth in earnings, capital adequacy etc. The purpose is to bring together a comprehensive treatise of these relationships.

1. Introduction

Over the last half century considerable research has appeared in financial and accounting literature in depicting the theoretical as well as the empirical relationship between the systematic risk and the various finance and accounting variables such as leverage, size, earning variability, dividends, growth in earnings, bid-ask spread, duration etc. The researchers over the years have concentrated on two aspects of this relationship. Beaver, Kettler and Scholes (1970), Pettit and Westerfield (1972), Rosenberg and McKibben (1973), Fewings (1975), Boquist, Racette, Schlarbaum (1975), Hamid, Prakash and Anderson (1994) and Mishra, McCabe, Prakash (2003) etc. have concentrated on the theoretical and empirical examinations of the relationship between beta, and financial and accounting variables, whereas Ball and Brown (1969), Gonedes (1973, 1975), Beaver and Manegold (1975) have concentrated on the relationship between beta and “accounting” beta¹.

In this paper we provide a brief summary of these relationships and suggest some areas of future research. The paper is organized as follows: In section II we provide a description of the market model and the genesis of beta. Section III describes the partitioning of total risk into systematic and unsystematic risk. Section IV presents a synthesis of the relationships between beta and the various accounting and financial variables that have so far appeared in the literature. The paper ends with few concluding remarks and suggestions for future research.

¹ For explanation of “accounting” beta see Gonedes (1973)

2. Genesis of Beta²

The market model, a common specification of the return generating process for assets, is also known as the one factor model, the single index market model, the linear characteristic line, and the diagonal model. While studying the efficient frontier and portfolio analysis, Markowitz (1959) in a footnote, suggested that the stochastic process generating security returns could be expressed as a linear function of the general prosperity of the economy as an index.

Sharpe (1963) built on Markowitz's suggestion and developed what he called the diagonal model, now known as the market model, which theorizes that the expected rate of return on stock *i*, in time period *t*, is a linear function of the expected rate of return on a global market portfolio. To allow for interrelationships, the model assumes securities' rates of return are related only through their individual relations with one overall index of business activity.

In its original ex-ante form, the market model hypothesizes a stochastic process which generates security returns. That is,

$$E(R_{it}) = \alpha_i + \beta_i E(R_{mt}); \quad t = 1, \dots, T \tag{1}$$

where α_i and β_i are the intercepts and the slope, respectively, of the straight line defined on the $[E(R_{mt}), E(R_{it})]$ plane and R_{it} and R_{mt} are the rates of return on the i^{th} security during the t^{th} time period.

In ex-ante/ex-post form, the market model can be written as:

$$E(R_{it} | R_{mt}) = \alpha_i + \beta_i R_{mt} \tag{2}$$

wherein equation (2) the left-hand side means the expected rate of return in period *t* given the market rate of return.

The above two models are not empirically testable because of their ex-ante specifications. Therefore, in its testable ex-post form, the model may be written as:

$$\tilde{R}_{it} = \alpha_i + \beta_i \tilde{R}_{mt} + \tilde{\epsilon}_{it}; \quad t = 1, \dots, T \tag{3}$$

Where $\tilde{\epsilon}_{it}$ is the random error term for the i^{th} security, or the residual portion of R_{it} which was unexplained by the regression of the i^{th} stock during the t^{th} time period.

The only behavioral assumption associated with the market model is that investors are single-period, risk-averse, and expected utility of terminal wealth maximizers who select their holdings of securities on the basis of the mean and variance of the

² The bulk of the discussion in sections II and III has been taken from the book "The Return Generating Models in Global Finance", Prakash et al. (1999), pages 19-21.

distribution of returns. This model is a statistical model which can be given a theoretical base by relating it to the capital asset pricing model.

Sharpe's assumptions for the above model presume that rate of return for any security is generated linearly by exactly one stochastic factor measured by global economic activity. This factor is thought to be the most important single influence on the returns from securities. In order to empirically obtain the estimates of α_i and β_i , expression (3) can be thought of as a type IV regression of \tilde{R}_i on \tilde{R}_m .

Even though equation (3) looks like the simplified linear regression model, in reality, it is not. In the simplified linear regression model, the so-called independent variable is assumed to be known, hence constant (Scheffe, 1959, page 4). Whereas, in equation (3), R_m is not exactly constant, but random, thus making equation (3) a regression equation of Type IV, or error in the variables, rather than of Type I, or functional regression (Press, 1972, pages 189-192).

In the estimation of alpha and beta, it does not make any difference whatsoever which type is specified. For all practical purposes, equation (3) is stated as a simple linear regression model of type I, with attendant wide-sense stationarity assumptions (Reinmuth and Wittink, 1974), which are

- 1) $E(\varepsilon_{it})$ (Zero mean)
- 2) $\text{var}(\varepsilon_{it}) = \varepsilon^2_\varepsilon$ (Homoscedastic)
- 3) $\text{cov}(\varepsilon_{it}, \varepsilon_{i,t+k}) = 0$ for all $k \neq 0$, and
- 4) $\text{cov}(\varepsilon_{it}, R_{mt}) = 0$

Three things must be noted about these assumptions. First, they are not assumed to be independent. They are simply uncorrelated; meaning, thereby, there is no linear relationship between any two error terms. Second, no distributional assumptions on the error terms have been made. Eventually, the error terms will be assumed to be normally-distributed to facilitate hypothesis testing. Finally the last assumption, that the covariance between ε_{it} and R_{mt} equals zero, simply means that the rates of return on the market are observed without error. This, in turn, implies that the conditional expectation of \tilde{R}_i given R_m is $E(R_{it} | R_{mt}) = \alpha_i + \beta_i R_{mt}$, the ex-ante/ex-post specification of the model.

3. Partitioning Risk with the Market Model

In finance, the workable measure of total risk is taken as the variance of returns on the i^{th} asset σ_i^2 . This measure of total risk is the sum of systematic risk and unsystematic risk. Sharpe (1963) first suggested that the total variation of portfolio return may be segregated into two forms: (1) systematic variation, resulting from covariation of the returns on the individual securities with the market return; and (2) unsystematic variation, that portion of variation attributable to the variation of the market return. Evans and Archer (1968) segregated the total variation of a portfolio in this way when they looked at the relationship between the number of securities included in a portfolio and the level of portfolio description. The market model can

be used to partition these components (Francis 1980, pages 363, Ben-Horim and Levy, 1980, or Fama, 1976, page 70) as:³

$$\text{var}(R_{it}) = \text{var}(\alpha_i + \beta_i R_{mt} + \varepsilon_{it}) \quad (4)$$

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{\varepsilon_i}^2$$

Thus, the systematic risk measure is $\beta_i^2 \sigma_m^2$ and the unsystematic risk is $\sigma_{\varepsilon_i}^2$

4. Relationships

1. Leverage: Hamada (1969) obtained the relationship between the levered beta and the beta of the firm, had the firm been unlevered, commonly referred to as “unlevered beta” as:

$$\beta_L = [1 + \frac{(1-\tau)D_L}{S_L}] \beta_U \quad (5)$$

where D_L and S_L are the market values of the firms debt and common stock respectively and τ is the corporate tax rate. The symbols β_L and β_U denote the levered and the unlevered beta. Note that the expression $\frac{(1-\tau)D_L}{S_L}$ is the measure of leverage. Since the expression under the parenthesis of the above equation is always positive and greater than 1, β_L will always be greater than β_U .

2. Accounting Beta: Ball and Brown (1969) were the first one to develop the relationship between the beta of market model and “accounting” beta β_i^A . The relationship is given by:

$$\beta_i = \frac{S_m}{S_i} \beta_i^A \quad (6)$$

where S_p is the market value of the firm and S_m is the market value of the market portfolio of equity securities. Since S_m/S_i will always be positive, the financial beta will be directly proportional to accounting beta.

3. Earnings Variability: Bowman (1979) was the first one to provide a theoretical relationship between beta and the earning variability. His finding is given by the following expression:

³ Strictly speaking β_i is not a measure of systematic risk; rather it is a measure of the relative sensitivity of an asset’s rate of return to the market. However, for practical purposes, beta is taken as a measure of systematic risk, rather than $\beta_i^2 \sigma_m^2$; this is because any decision for a given time period made on the basis of $\beta_i^2 \sigma_m^2$, or beta, will be the same, as long as beta is positive; hence, the name systematic risk for beta.

$$\beta_i = \frac{S_m}{S_i} \cdot \frac{\sigma(X_i)}{\sigma(X_m)} \cdot \rho(X_i, X_m) \tag{7}$$

where X_i is the firm's accounting earnings, X_m is the accounting earnings of the market portfolio, $\sigma(X_i)$ denotes the standard deviation and $\rho(X_i, X_m)$ is the correlation coefficient between X_i and X_m .

As can be seen, the relationship is not very explicit. In the words of Bowman (1979)

“...by making assumptions or observations concerning the RHS, we can determine the effect upon market risk. Using different approaches and assumptions, it is possible to represent the relationship between earnings variability and market risk in a number of additional ways. The original point however remains. There is no direct relationship between earnings variability and market risk”.

4. *Dividends:* As far as we know, there is no definitive expression of the relationship between systematic risk and dividends (more commonly the payout ratio defined as ‘dividends/ earnings’). Furthermore we are not aware of any empirical work related to the relationship between beta and dividend payout ratio.

5. *Size:* Assuming no synergy between the different assets of a firm (a critical assumption which leaves a lot of room for criticism), it can be shown that overall beta of the firm is the weighted average of the betas of individual units, an accepted procedure used to obtain the portfolio beta, one can show the relationship between the beta and the size of the firm. Even in the portfolio the expected returns of different assets are correlated, hence an assumption of no synergy between the assets of the same firm is not tenable. Hence intuitively we can expect that the size will have an impact on the overall beta of the firm, though it is not possible to provide a clear cut theoretical relationship. (Bowman, 1979)

6. *Growth:* Fewings (1975) provided a relationship between growth and the risk which is so complicated that no viable conclusions can be drawn without making a plethora of assumptions about the various variables. Essentially his relationship is given by the following equations

$$\frac{\text{Cov}_{t-1}(R_{jt}, R_{mt})}{\text{Var}_{t-1}(R_{mt})} = \frac{(1 + a_j r_j) \text{Cov}_{t-1} \left[\left(\frac{k_{jt-1} - k_{jt}}{k_{jt-1} - a_j r_j} \right), \left(\frac{k_{mt-1} - k_{mt}}{k_{mt-1} - a_m r_m} \right) \right]}{(1 + a_m r_m) \text{Var}_{t-1} \left[\left(\frac{k_{mt-1} - k_{mt}}{k_{mt-1} - a_m r_m} \right) \right]} \tag{8}$$

where, R_{jt} is the return on the j^{th} asset, R_{mt} is the return outcome on the market portfolio of all risky assets, Cov_{t-1} and Var_{t-1} denote the covariance and variance respectively at the end of period $t-1$ and k_j represents the average capitalization rate. Subscript ‘ j ’ is

used for the j^{th} asset and 'm' is used for the market portfolio. Variable 'a' is defined by the following equation:

$$a = b + s$$

where 'b' is the rate of earnings retention and 's' is the rate of new equity issue as a fraction of total earnings. The symbol 'r' stands for the rate of return on equity investment and the product 'ar' equals the growth rate of total earnings and dividends. Further as, Bowman (1979) points out "empirical studies investigating the association between systematic risk and financial (accounting) variables have generally hypothesized and observed a positive correlation between risk and growth. This has been true for growth measured in earnings or total assets". His derivation is not very convincing in the sense that no explicit conclusion can be drawn as it has been observed in the empirical findings

However, Hamid, Prakash and Anderson (1994) gave a simple and empirically testable expression. Essentially they showed that the relationship between growth in earnings (or dividends) and beta is given by:

$$Cov(g_{it}, \beta_{it}) = \left(1 + \frac{D_{i,t-1}}{P_{i,t-1}}\right)^{-1} Cov(R_{it}, \beta_{it}) \quad (9)$$

where g_{it} is the expected growth in dividends, $D_{i,t-1}$ is the dividend paid during the period $t-1$, $P_{i,t-1}$ is the known price at $t-1$ and R_{it} is one plus the expected rate of return on asset i in period t . Hamid Prakash and Anderson further verified their theoretical relationship empirically and the empirical evidence strongly supports the theoretical relationship (9).

7. *Spread*: Empirically Benston and Hagerman (1974) and Stoll (1978) showed that the total risks, firm specific risks and beta are all related to the bid-ask spread. Later on Mishra, McCabe and Prakash (2003) showed that *variability* in spread is positively related to the systematic risk. Furthermore, they provide a condition under which the relationship between spread (and not the variability of spread) will be positively correlated with the risk. In their own words the argument can be presented as follows:

"Denoting $\bar{S}_{it+1} = \bar{P}_{iat+1} - \bar{P}_{ibt+1}$, i.e. spread at time $t+1$, if $Cov(\bar{S}_{it+1}, \Delta \bar{S}_{it+1}) \geq 0$, then for a small increment in \bar{S}_{it+1} , say $\Delta \bar{S}_{it+1}$, in next adjacent period will increase the variance of the security return (total risk of the security). Thus ceteris paribus, the spread will be positively correlated with the total risk. Thus positive correlation between variability of spread and total risk is ensured given the covariance between spread and an infinitesimal change in spread in the adjacent interval is positive. Since this condition is sufficient for the spread and total risk to be positively correlated hence it will be true for the other components of the risk as well. This condition will also apply to the relationship between other components of risk (i.e. firm-specific risk and systematic risk)"

8. *Duration*: Boquist, Racette and Schlarbaum (1975) provide the relationship between the systematic risk and duration as follows:

$$\beta_i = \frac{D_i [Cov(\tilde{d}g_i, \tilde{R}_m) - Cov(\tilde{d}k_i, \tilde{R}_m)]}{Var(\tilde{R}_m)} \quad (10)$$

where D_i is the duration, dg_i and dk_i are respectively, measures of infinitesimal changes in the perpetual growth rate g_i and k_i discount factor. They further assert that,

“A more general risk measure must therefore incorporate the relationships between each component of stock price volatility and the market return. More important, however, the above equation emphasizes the vital role of duration in the risk assessment of assets characterized by uncertain cash flow expectations. Any change in expected payments is likely to alter duration and therefore beta. In addition, any exogenous change in the structure of equilibrium rates of return will also be accompanied by a new covariance relation that serves to mitigate the necessary alteration of beta. Nonetheless, the prospect for a constant beta in the fact of such change is unlikely”.

9. *Bank's capital Adequacy*: A bank's capital adequacy ratio is defined as:

$$CA = \frac{S_L}{TA_L} = \frac{S_L}{V_L} \quad (11)$$

Where total assets $TA_L = V_L$ and S_L is the market value of the bank's equity.

Assuming that default risk (expected bankruptcy) associated with debt financing as negligible (following Hamada, 1969), so that the value of a levered firm V_L is the sum of its equity (S) and debt (D), i.e. $V_L = S_L + D_L$, Equation (11) can be written as

$$CA = \frac{S_L}{V_L} = \frac{S_L}{S_L + D_L} = \frac{1}{1 + D_L/S_L} \quad (12)$$

Karels, Prakash and Roussakis (1989) theoretically and empirically showed that the relationship between the bank's capital adequacy and beta is negative i.e. $Cov(CA, \beta) \leq 0$.

5. Concluding Remarks

In this paper we provided a brief summary of the theoretical and empirical relationship of beta with the financial and accounting variables which has so far been reported in literature. A few financial variables such as growth in earnings and capital adequacy exhibit an explicit relationship with beta, whereas for some other variables such as size and dividends etc. no explicit theoretical relationship can be obtained. Going through the literature we find that the relationship between beta and growth in

earnings is discussed more often than any other variable. For example Fewings (1975) was the first one who tried to obtain the relationship between the growth and risk but the relationship he gave is so complicated that testing that empirically will be very difficult. Later on Bowman (1979) didn't even obtain the relationship but gave an intuitive explanation that risk and growth will be positively correlated. It was Hamid, Prakash and Anderson (1994), who gave a very simple and empirically testable relationship. The relationship between spread and beta follows essentially the same pattern as the relationship between beta and growth.

It will be an interesting topic to obtain the theoretical relationship of beta with size and dividend payout ratio etc. Furthermore, in the current literature, much attention is being focused on yet another concurrent measure of risk, i.e. the existence of skewness in the rates of returns and investors preference for positive skewness. It will be interesting to see how this measure of risk i.e. the positive skewness, will relate with various financial and accounting variables.

References

- R. Ball and P. Brown, "Portfolio Theory and Accounting", *Journal of Accounting Research* (Autumn 1969)
- W. Beaver, Kettler and Scholes, "The association between Market-Determined and Accounting-Determined Risk Measures", *Accounting Review* (1970)
- W. Beaver and J. Manegold, "The Association between Market-Determined and Accounting-Determined Measures of Systematic Risk: Some Further Evidence", *Journal of Financial and Quantitative Analysis* (1975)
- M. Ben-Horim and L. Haim, "Total Risk, Diversifiable Risk and Nondiversifiable Risk: A Pedagogic Note", *Journal of Financial and Quantitative Analysis* (June 1980)
- G. J. Benston and R. L. Hagerman, "Determination of Bid-Ask Spreads in the Over the Counter Market", *Journal of Financial Economics* (March 1974)
- J. A. Boquist, G. A. Racette and G. Schlarbaum, "Duration and risk Assessment for Bonds and Common Stocks", *The Journal of finance* (December 1975)
- R. G. Bowman, "The Theoretical Relationship between Systematic Risk and Financial (Accounting) variables", *The Journal of Finance* (June 1979)
- J. L. Evans and S. H. Archer, "Diversification and the Reduction of Dispersion: An Empirical Analysis", *The Journal of Finance* (December 1968)
- E. F. Fama, "Foundation of Finance", Basic Books, New York (1976)
- D. R. Fewings, "The impact of Corporate Growth on the Risk of Common stocks", *The Journal of Finance* (May 1975)
- J. C. Francis, "Investments: Analysis and Management", McGraw-Hill Book Company New York (1980)
- N. Gonedes, "Evidence on the Information Content of Accounting Numbers: Accounting-Based and Market-Based Estimates of Systematic Risk", *Journal of Financial and Quantitative Analysis* (June 1973)
- N. Gonedes, "A Note on Accounting-Based and Market-Based Estimates of Systematic Risk", *Journal of Financial and Quantitative Analysis* (June 1975)
- R. Hamada, "Portfolio analysis, Market Equilibrium and Corporation Finance", *Journal of Finance* (March 1969)
- S. Hamid, A. J. Prakash and G. A. Anderson, "A Note on the Relationship between Systematic

- Risk and Growth in Earnings”, *Journal of Business Finance & Accounting* (March 1994)
- G. V. Karels, A. J. Prakash and E. Roussakis, “The Relationship between Bank Capital Adequacy and Market Measures of Risk”, *Journal of Business Finance & Accounting*, (Winter 1989)
- Harry M. Markowitz, “Portfolio Selection: Efficient Diversification of Investments”, Yale University Press, New Haven (1959)
- S. Mishra, G. McCabe and A. J. Prakash, “Bid-Ask spread and Variability, and Measures of risk: A Theoretical perspective”, *The International Journal of Finance*, (Vol. 15, No.2, 2003)
- R. Pettit and R. Westerfield, “A Model of Capital Asset Risk”, *Journal of Financial and Quantitative Analysis* (March 1972)
- J. S. Press, “Applied Multivariate Analysis”, Holt, Rinehard and Winston, New York (1972)
- J. E. Reinmuth, and D. R. Wittink, “Recursive Models for Forecasting Seasonal Processes”, *Journal of Financial and Quantitative Analysis* (Vol.9, No.4, 1974)
- B. Rosenberg and W. McKibben, “The Prediction of Systematic and Specific Risk in Common Stocks”, *Journal of Financial and Quantitative Analysis* (March 1973)
- H. Scheffe, “The Analysis of Variance”, John Wiley & Sons, New York (1959)
- W. F. Sharpe, “A Simplified Model for Portfolio Analysis”, *Management Science* (January 1963)
- H. R. Stoll, “The Pricing of Security Dealer Services: An Empirical Study of Nasdaq Stocks”, *The Journal of Finance*, (September 1978)