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PRESENT AND FUTURE VALUE FORMULAE FOR UNEVEN CASH FLOW BASED ON PERFORMANCE OF A BUSINESS

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Abstract

Computational methods for present and future value calculations are difficult when the firm's cash flow income is uneven. The firm’s decision to invest or borrow based on uneven cash flows needs a simple method to arrive at the present value of uneven cash flows. This paper is an attempt to simplify the current tedious calculations. A methodology to quickly estimate the present/future value of uneven cash flows is invaluable to practitioners in the banking and financial industries.

Keywords: Uneven cash flow, Present value formula, Future value formula, Performance rate, Rate of growth, Uneven cash flow income, Economic value added

JEL Classification: C02

1. Introduction

Money has different values at different time periods, which is the foundation of time value of money calculations. The literature on this topic is by now historic. Among the reasons often cited are: time preference, inflation premium, risk premium, and productivity.

Compounding and discounting techniques are commonly employed to calculate the time value of money. The present and future annuity formulas developed through such technique are, by definition, for equal amount of periodic cash flows. They have the following shortcomings, particularly in light of a banking institution:

- They do not easily allow for the calculation of cash flows that grow according to the project’s performance, particularly when such performance is variable.

- They assume constant cash flows throughout the project's life, which is unrealistic. A project whose performance does not change according to the market situations throughout the project life may be out of tune with the market.

- They do not consider future inflation rates, which could also be variable. Inflation is assumed to occur at a constant rate. Since inflation reduces the purchasing power of money, considering the future inflation effect on project cash flows will reduce the firm's unexpected loss.

- They do not consider flexibility of deposits in banks or flexibility of payments by creditors to the banks. Again, the high possibility of uneven cash flows.

This paper attempts to resolve the above problems through offering a simpler method to compute uneven cash flows.

2. Business Aspects of Uneven Cash Flows

We link the sum of the uneven cash flows, either for investment or for repayments of borrowed loans, to the performance of the firm. A new business has a small amount of cash at initial stage. Eventually, after covering the establishment cost, the liquidity amount on hand grows accordingly. The growth of the firm's cash flow will then depend on its performance. When performance improves, its profits and cash flows increase. As result, the depositor can deposit the excess amount above his/her business consumption. This excess may increase from period to period according to the business performance at a later stage of profitability.

Traditional finance measures of firm's performance are profits, earnings per share (EPS), return on investment (ROI), free cash flows (FCF), capital productivity (KP), labor productivity (LP), and return on capital employed (ROCE). Each ignores cash and cost of capital so as to generate the target profit (Brealey and Myers, 2003: 414). A more complete measure of performance of a business firm is economic value added (EVA) which provides the money value created for investor in a given period of time by weighting the profit generated against the cost of capital employed (Firer, Ross, Westerfield and Jordan, 2007).

Since EVA considers the amount of capital invested, the return earned on capital and the cost of capital (WACC) – reflecting the risk adjusted required rate of return - it is thought to have all the characteristics of a complete measure though it is valid only for short period of time. Furthermore, since EVA is a measure of both performance and value, it is assumed to be a method to determine the value created, above the required return, for the shareholders. The firm creates wealth for the shareholders when the revenue of the firm exceeds the cost of doing business and the cost of capital. A business firm creates value for its shareholders on the basis of positive EVA rather than simply making accounting profits. The positive magnitude of EVA indicates the business firm is improving its net cash return on invested capital. The incremental EVA from year to year will result in a change in the market value of the firm (Damodaran, 2001).

Accounting information about the firm's performance for a single accounting period is valuable. A manager with good knowledge about the firm's performance may predict future performance of the firm on the basis of the past financial statements with some degree of accuracy. If records of past trend are available, performance rates can be determined to project the cash flows of the firm investment and of the firm bank loan repayments (Alexander Hamilton Institute, 1998: 64-65 and 331-332). Since the firm's performance rate is often assumed to be progressive, the cash flow of the firm can be assumed to grow from period to period. Performance rate is a percentage by which the current performance of a firm (in this case, EVA) grows from the previous period.

3.Methodology

Assume a business firm decides to borrow money from bank or creditor. This often involves receiving cash now in exchange for a promise to make payments in the future periods. The loan repayment amounts usually are set to include interest and a portion of the principal. Based on this notion, each of the next period's repayment amount is calculated by considering the preceding repayment, the project performance, and the interest rate.

At initial repayment period, the repayment amount is assumed to be small and fixed without performance consideration. However, the second period repayment amount grows according to the business performance. A business performance measuring tool, EVA, can be calculated as follows:

EVA= [adjusted net operating profit after taxes (ANOPAT)] – [capital], or

EVA = (ROCE – WACC) K

where, ROCE is a return on capital employed, WACC is the weighted average cost of capital, and K is capital employed.

From the above, a business firm can increase its EVA by increasing the NOPAT generated by existing capital, reducing the WACC, and investing in new projects where the return on capital exceeds the WACC. Assume that the first small repayment amount to be a_1 the following repayments can be calculated as:

$$a_1(P_1+1+i)(P_2+1+i)....(P_{n-2}+1+i)P_{n-1} \quad (1)$$

Such that

Repayment Period Repayment Amount

1	a_1
2	$a_1(P_1+1+i)$
3	$a_1(P_1+1+i)P_2$
..	
..	
n	$a_1(P_1+1+i)(P_2+1+i)....(P_{n-2}+1+i)P_{n-1}$

Where n = loan period, P_n= Performance rate, any arbitrary figure greater than zero, which can be measured as the percentage by which the $(n+1)^{th}$ period EVA grows from the n^{th} period EVA. That is:

$$P_n = \frac{EVA_{n+1} - EVA_n}{EVA_n} \qquad (2)$$

The reason for the choice of the performance rate as is expressed in relation (2) is to prevent the cash flows from exaggerated changes. EVA_n is projected net income for the n- period loan; and i is the bank interest rate which is added to the performance rate since the n^{th} period repayment should contain a portion of the principal amount and the interest amount for the period. It is in points, i.e., between 0 and 1.

The present value of the future cash flows of the loan repayments can be expressed as:

$$\left[\prod_{r=1}^{n-1}(P_r+1+i)+(i+1)^{n-1}\right]\left(\frac{a_1}{(1+i)^n}\right)=\frac{a_1}{(i+1)}+\frac{a_1(P_1+1+i)}{(1+i)^2}+\frac{a_1(P_1+1+i)P_2}{(1+i)^3}+\frac{a_1(P_1+1+i)(P_2+1+i)P_3}{(1+i)^4}+\dots\dots\dots\dots$$
$$\dots\dots\dots\dots+\frac{a_1(P_1+1+i)(P_2+1+i)(P_3+1+i)\dots(P_{n-2}+1+i)P_{n-1}}{(1+i)^n} \qquad (3)$$

Another aspect of a firm is a decision to invest cash now in order to receive cash, goods or services in the future. Let the periodic cash investment be as:

Period	**Investing Cash**
1	a_1
2	$a_1\,(W_1-T_1)$
3	$a_1\,((p_1+1))\,(W_2-T_2)$
.	. .
n	$a_1\,(p_1+1)(p_2+1)\dots(p_{n-3}+1)(p_{n-2}+1)\,(W_{n-1}-T_{n-1})$

where $W_n = P_n + 1$, $T_n = 1-(P_n+1)(i)$ for $(P_n+1)(i)$ between 0 and 1 (i.e., $0<(P_n+1)(i)<1$). T_n = is the firm performance excluding bank's deposit interest. The deposit interest rate should not be included into the project performance since it is always less than the bank lending interest rate. i = bank interest rate which is between 0 and 1. n = investing period . $W_{n-1} - T_{n-1} =$ A portion of performance rate by which the excess above consumption will be saved in a bank account. Therefore the future value of investment cash flows is:

$$a_1(i+1)\left[\prod_{r=1}^{n-1}(p_r+1)(1+i)^n - i\right] = a_1(i+1) + a_1(W_1 - T_1)(i+1)^2 + a_1(p_1+1)(W_2 - T_2)(i+1)^3 + \ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots + a_1(p_1+1)(p_2+1)\ldots(\mathrm{p}_{\mathrm{n-2}}+1)(W_{n-1} - T_{n-1})(i+1)^n \qquad (4)$$

where:

$$a_1 = a_1,\; a_2 = p_1 a_1, \mathrm{a}_3 = p_2(a_1 + a_2),\; \mathrm{a}_4 = p_3(a_1 + a_2 + a_3), \ldots\ldots\ldots, \mathrm{a}_{\mathrm{n}} = p_{n-1}(a_1 + a_2 + \ldots . + a_{n-1})$$

such that;

$$a_1 = a_1,\; a_2 = p_1(a_1), \mathrm{a}_3 = p_2(a_1)(p_1+1),\; \mathrm{a}_4 = p_3(a_1)(p_1+1)(p_2+1), \ldots..$$
$$\ldots., \mathrm{a}_{\mathrm{n}} = p_{n-1}(a_1)(p_1+1)(p_2+1)\ldots(p_{n-2}+1) \qquad (5)$$

Let the n^{th} period cash flow (a_n), performance rate (p_n), and the discount rate (x) belong to the domain of any numbers that can mathematically be expressed as elements of the real numbers of set of R, i.e., $a_n, x, p \in R.$ ($where \in$ stands for elements). Then, the sum of cash flow for n periods can be explained as follow:

$$\sum_{m=1}^{n+1} a_m = a_1 + a_2 + \ldots\ldots\ldots\ldots + a_{n+1}$$

This can be expressed as

$$\sum_{m=1}^{n+1} a_m = \frac{a_1 x}{x} + \frac{a_2 x^2}{x^2} + \ldots\ldots\ldots. + \frac{a_{n+1} x^{n+1}}{x^{n+1}},\ \text{for } x \neq 0$$

Splitting the terms as:

$$= \frac{1}{x}(a_1 x + 0) + \frac{1}{x^2}((a_2 x^2 + a_1 x) - (a_1 x)) + \frac{1}{x^3}((a_3 x^3 + a_2 x^2 + a_1 x)) - (a_2 x^2 + a_1 x)) + \ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots\ldots + \frac{1}{x^{n+1}}((\mathrm{a}_{\mathrm{n+1}} x^{n+1} + a_n x^n + \ldots.. + a_1 x) - (a_n x^n + a_{n-1} x^{n-1} + \ldots.. + a_1 x)) \qquad (6)$$

Collecting the terms in the brackets yields:

$$= a_1 x\left(\frac{1}{x}-\frac{1}{x^2}\right)+\left(a_2 x^2+a_1 x\right)\left(\frac{1}{x^2}-\frac{1}{x^3}\right)+.............+\left(a_n x^n+a_{n-1}x^{n-1}+.....+a_1 x\right)\left(\frac{1}{x^n}-\frac{1}{x^{n+1}}\right)+$$
$$\frac{1}{\mathrm{x}^{\mathrm{n+1}}}\left(a_{n+1}x^{n+1}+a_n x^n+.....+a_1 x\right)$$

$$=\left(a_1 x\right)\left(\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x^2}-\frac{1}{x^3}+...+\frac{1}{x^n}-\frac{1}{x^{n+1}}\right)+\left(a_2 x^2\right)\left(\frac{1}{x^2}-\frac{1}{x^3}....+\frac{1}{x^n}-\frac{1}{x^{n+1}}\right)+.............+\left(a_n x^n\right)\left(\frac{1}{x^n}-\frac{1}{x^{n+1}}\right)+$$
$$\frac{1}{\mathrm{x}^{\mathrm{n+1}}}\left(a_{n+1}x^{n+1}+a_n x^n+.....+a_1 x\right)$$

$$= a_1 x\left(\frac{1}{x}-\frac{1}{x^{n+1}}\right)+a_2 x^2\left(\frac{1}{x^2}-\frac{1}{x^{n+1}}\right)+.........+a_n x^n\left(\frac{1}{x^n}-\frac{1}{x^{n+1}}\right)+\frac{1}{x^{n+1}}\left(a_{n+1}x^{n+1}+a_n x^n+....+a_1 x\right)$$

$$= a_1 x\left(\frac{x^n-1}{x^{n+1}}\right)+a_2 x^2\left(\frac{x^{n-1}-1}{x^{n+1}}\right)+..................+a_n x^n\left(\frac{x-1}{x^{n+1}}\right)+\frac{1}{x^{n+1}}\left(a_{n+1}x^{n+1}+a_n x^n+....+a_1 x\right) \quad (7)$$

Multiplying both sides by $\frac{x^{n+1}}{x-1}$, for $\mathrm{x}\neq 1$, gives :

$$\left(\sum_{m=1}^{n+1}a_m\right)\left(\frac{\mathrm{x}^{\mathrm{n+1}}}{x-1}\right)= a_1 x\frac{\left(x^n-1\right)}{x-1}+a_2 x^2\,\frac{\left(x^{n-1}-1\right)}{x-1}+.......+a_n x^n\,\frac{\left(x-1\right)}{x-1}+\left(\frac{1}{x-1}\right)\left(\mathrm{a}_{\mathrm{n+1}}x^{n+1}+a_n x^n+.....+a_1 x\right)$$

Since the formula of geometric progression at ratio $= x$ can be expressed as:

$\frac{x^n-1}{x-1}=1+x+....+x^{n-1}$ (Sobel and Lerner,1995:515-535) , it follows that:

$$= a_1\left(x+x^2+.....+x^n\right)+a_2\left(x^2+x^3+.......+x^n\right)+......+a_n\left(x^n\right)+\left(\frac{1}{x-1}\right)\left(a_1 x+a_2 x^2+....+a_{n+1}x^{n+1}\right)$$

Equivalently this can be expressed as:

$$=\left(a_1\right)x+\left(a_1+a_2\right)x^2+\left(a_1+a_2+a_3\right)x^3+.......+\left(a_1+a_2+....+a_n\right)x^n+\left(a_1 x+a_2 x^2+....+a_{n+1}x^{n+1}\right)\left(\frac{1}{x-1}\right) \quad (8)$$

Let $a_{n+1} = 0$, then it follows that:

$$\left(\sum_{z=1}^{n} a_z\right)(x)^n\left(\frac{x}{x-1}\right) = \left(a_1 \frac{x}{x-1}\right)(\mathrm{x}) + \left(a_1 + a_2 \frac{x}{x-1}\right)(\mathrm{x})^2 + \left(a_1 + a_2 + a_3 \frac{x}{x-1}\right)(\mathrm{x})^3 + ... + \left(a_1 + a_2 + ... + a_n \frac{x}{x-1}\right)(\mathrm{x})^n \quad .....(9)$$

From this expression let us assume that the cash flow of a project for each period can be defined as $a_n = p_{n-1}(a_1 + a_2 + a_3 + ... + a_{n-1})$ which can be written as

$$a_1 = a_1,\ a_2 = p_1 a_1, \mathrm{a}_3 = p_2(a_1 + a_2),\ \mathrm{a}_4 = p_3(a_1 + a_2 + a_3), ..., \mathrm{a}_\mathrm{n} = p_{n-1}(a_1 + a_2 + ... + a_{n-1})$$

Or:

$$a_1 = a_1,\ a_2 = p_1(a_1), \mathrm{a}_3 = p_2(a_1)(p_1 + 1),\ \mathrm{a}_4 = p_3(a_1)(p_1 + 1)(p_2 + 1), ...$$
$$...., \mathrm{a}_\mathrm{n} = p_{n-1}(a_1)(p_1 + 1)(p_2 + 1)...(p_{n-2} + 1)$$

such that $P_{n+1} \leq$ or $\geq P_n$ and $P_0 = 1$. Substituting this, we have:

$$= \left(a_1 \frac{x}{x-1}\right)x + \left(a_1 + a_1 P_1 \frac{x}{x-1}\right)x^2 + \left((a_1 + a_2) + (a_1 + a_2)P_2 \frac{x}{x-1}\right)x^3 +$$
$$\left((a_1 + a_2 + a_3) + (a_1 + a_2 + a_3)P_3 \frac{x}{x-1}\right)x^4 + ... + \left((a_1 + a_2 + a_3 + ... + a_{n-1}) + (a_1 + a_2 + ... + a_{n-1})P_{n-1} \frac{x}{x-1}\right)x^n$$

$$= a_1 \frac{x^2}{x-1} + (a_1)((p_1 + 1)x - 1)\frac{x^2}{x-1} + (a_1 + a_2)((P_2 + 1)x - 1)\frac{x^3}{x-1} + (a_1 + a_2 + a_3)((P_3 + 1)x - 1)\frac{x^4}{x-1} + ...$$
$$............. + (a_1 + a_2 + a_3 + ... + a_{n-1})((P_{n-1} + 1)x - 1)\frac{x^n}{x-1}$$

multiplying both sides of the equation by x-1, we obtain:

$$\left(\sum_{m=1}^{n} a_m\right)(x)^{n+1} = a_1 x^2 + a_1((p_1 + 1)x - 1)\mathrm{x}^2 + (a_1 + a_2)((\mathrm{p}_2 + 1)x - 1)x^3 + (a_1 + a_2 + a_3)((p_3 + 1)x - 1)x^4 + ...$$
$$..................... + (a_1 + a_2 + ... + a_{n-1})((\mathrm{p}_{\text{n-1}} + 1)\mathrm{x} - 1)x^n$$

$$\left(\sum_{m=1}^{n} a_m\right)(x)^{n+1} = a_1x^2 + a_1\left((p_1+1)x^3 - x^2\right) + \left(a_1+a_2\right)\left((\mathrm{p}_2+1)x^4 - x^3\right) + \left(a_1+a_2+a_3\right)\left((p_3+1)x^5 - x^4\right) + \ldots\ldots$$

$$\ldots\ldots\ldots\ldots + \left(a_1+a_2+\ldots+a_{n-1}\right)\left((\mathrm{p}_{n-1}+1)\mathrm{x}^{n+1} - x^n\right) \qquad \ldots\ldots\ldots\ldots \quad (10)$$

If the bank interest rate (i) is expressed as $x = \frac{1}{i+1}$, by substituting it in relation (2) and multiplying both sides of the equation by (i+1), we have the following:

$$\left[\sum_{m=1}^{n} a_m\right]\left(\frac{1}{i+1}\right)^n = \frac{a_1}{(i+1)} + \frac{a_1(p_1-i)}{(i+1)^2} + \frac{(a_1+a_2)(p_2-i)}{(i+1)^3} + \frac{(a_1+a_2+a_3)(p_3-i)}{(i+1)^4} + \ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots + \frac{(\mathrm{a}_1+a_2+\ldots\ldots+a_{n-1})(p_{n-1}-i)}{(i+1)^n} \qquad \ldots\ldots$$

…………(11)

Suppose that in the above equation $P_n = P_n + i$, and adding both sides of the equation $\frac{a_1}{(i+1)}$ we obtain the following present value formula:

$$\left[\prod_{r=1}^{n-1}(P_r+1+i) + (i+1)^{n-1}\right]\left(\frac{a_1}{(1+i)^n}\right) = \frac{a_1}{(i+1)} + \frac{a_1(P_1+1+i)}{(1+i)^2} + \frac{a_1(P_1+1+i)P_2}{(1+i)^3} + \frac{a_1(P_1+1+i)(P_2+1+i)P_3}{(1+i)^4} + \ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots + \frac{a_1(P_1+1+i)(P_2+1+i)(P_3+1+i)\ldots(P_{n-2}+1+i)P_{n-1}}{(1+i)^n} \qquad (12)$$

The initial cash flow (a_1) in the above formula grows progressively from one period to another by multiplying progressive performance rates of the periods. Each of the periodic cash flows contains an interest rate that embodies inflation rate and risk which might happen in the future. Furthermore each of the periodic cash flow contains the period performance rate that can move along with the strength of the business.

Now, let us consider equation (10). Setting x = (i+1), deducting $a_1(i+1)^2$ from both sides, and then adding both side of the equation $a_1(i+1)$, we have the following future value formula: it is correct

$$\left[\sum_{m=1}^{n} a_m\right](i+1)^{n+1} - a_1(i+1)i = a_1(i+1) + a_1(W_1 - T_1)(i+1)^2 + (a_1 + a_2)(W_2 - T_2)(i+1)^3 + \ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots + (\mathrm{a}_1 + a_2 + \ldots\ldots + a_{n-1})(W_{n-1} - T_{n-1})(i+1)^n$$

This can be written as:

$$a_1(i+1)\left[\prod_{r=1}^{n-1}(p_r+1)(1+i)^n - i\right] = a_1(i+1) + a_1(W_1 - T_1)(i+1)^2 + a_1(p_1+1)(W_2 - T_2)(i+1)^3 + \ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots + a_1(p_1+1)(p_2+1)\ldots(\mathrm{p}_{\mathrm{n-2}}+1)(W_{n-1} - T_{n-1})(i+1)^n \qquad (13)$$

where $W_n = P_n + 1, \mathrm{T_n} = 1 - (P_n + 1)(i)$ for $(\mathrm{P_n} + 1)(\mathrm{i})$ between 0 and 1 (i.e., $0 < (\mathrm{P_n}+1)(\mathrm{i}) < 1$) ; $\mathrm{P_n}$ = performance rate such that $p_{n+1} \leq$ or $\geq$ p_n , and $\mathrm{T_n}$ = is the firm performance excluding the bank's deposit interest. The rest of the notations are as defined before (see the segment above relation (4)).

In sum, we have focused on how business firms determine their cash inflow or outflow based on their economic profit (or economic value added). As shown by the formulae above, the first payment, which is excess above consumption, for payment of debt or for saving into bank account is relatively small. The next after the first amount progresses or grows along with the business firm's performance rate. Since this performance rate shows the relative level of growth of one's firm, current to last economic profit, it embraces all activities of the firm. As EVA fluctuates from period to period, the net cash left to the firm also fluctuates from period to period. These fluctuations are reflected by the rate (P_n).

Financing organs, such as banks, use ordinary annuity formula to determine the loan capacity as well as the loan repayment of the borrower. The formula does not contain any measure of the performance of the borrower. Some loans become nonperforming. To avoid this situation, banks resort to their own rules and regulations to minimize nonperforming loans.

The present value formula stated in this paper calculates the projected fluctuating repayment amounts along with the performance of the borrowing organ based on the real cash on hand, which is the excess above the consumption. The present value formula enable the borrower, who has no excess cash on hand for investment, to plan to save a portion of income according to the earnings growth. This also may prompt plans for saving by those who have low income and those who are salaried.

4. Application

In Ethiopia, most business units use EVA as a measure of both value and performance. Notable examples are construction firms and banks (Colin et al., 2007). The following applications are based upon real past data of an Ethiopian bank for illustration. Table 1 contains present value and Table 2 and the rest future value calculations.

Table 1: Computation using the formula developed in this paper, millions

Years	Base (Year=0)	2003 (year=1)	2004 (year=3)	2005 (year=4)	2006 (year=5)	2007 (year=6)	2008 (year=7)	2009 (year=8)
Net operating profit after tax	12	15	17	19	63	71	81	74
Less: Cost of capital	1	2	3	3	3	5	7	8
Economic value added	11	13	14	16	60	66	74	66
Performance rate		0.18	0.08	0.14	2.75	0.10	0.12	-0.11
PV of EVA	10	11.6364	0.8463	1.5887	35.1784	4.4772	5.8611	-5.9588

As we have discussed above, performance rate of each years is calculated on the basis of the following formula:

$$P_n = \frac{EVA_{n+1} - EVA_n}{EVA_n}$$

Yielding:

$$P_1 = \frac{(13-11)}{11} = 0.18 \qquad P_2 = \frac{(14-13)}{13} = 0.08 \quad P_3 = \frac{(16-14)}{14} = 0.14$$

$$P_4 = \frac{(60-16)}{16} = 2.75 \;\; P_5 = \frac{(66-60)}{60} = 0.10 \;\; P_6 = \frac{(74-66)}{66} = 0.12 \;\; P_7 = \frac{(66-74)}{74} = -0.11$$

Assume that $EVA_0 = a_1$ is the base year economic value added which can be assumed as an initial amount. Interest rate (i) =10%. Present value of each year's EVA is calculated as:

First year PV of EVA

$$\frac{a_1}{(i+1)^1} = \frac{11}{(0.10+1)^1} = 10$$

Second year PV of EVA

$$\frac{a_1(p_1+1+i)}{(i+1)^2}=\frac{11(0.18+1+0.10)}{(0.10+1)^2}=\frac{14.08}{(1.1)^2}=11.6364$$

Third year PV of EVA

$$\frac{a_1(p_1+1+i)p_2}{(i+1)^3}=\frac{11(0.18+1+0.10)(0.08)}{(0.10+1)^3}=\frac{1.1264}{(1.1)^3}=0.8463$$

Fourth year PV of EVA

$$\frac{a_1(p_1+1+i)(p_2+1+i)p_3}{(i+1)^4}=\frac{11(0.18+1+0.10)(0.08+1+0.10)(0.14)}{(0.10+1)^4}=\frac{2.3260}{(1.1)^4}=1.5887$$

Fifth year PV of EVA

$$\frac{a_1(p_1+1+i)(p_2+1+i)(p_3+1+i)p_4}{(i+1)^5}$$
$$=\frac{11(0.18+1+0.10)(0.08+1+0.10)(0.14+1+0.10)2.75}{(0.10+1)^5}$$
$$=\frac{56.6551}{(1.1)^5}=35.1784$$

Sixth year PV of EVA

$$\frac{a_1(p_1+1+i)(p_2+1+i)(p_3+1+i)(p_4+1+i)p_5}{(i+1)^6}$$
$$\frac{11(0.18+1+0.10)(0.08+1+0.10)(0.14+1+0.10)(2.75+1+0.10)(0.10)}{(0.10+1)^6}$$
$$=\frac{7.9317}{(1.1)^6}=4.4772$$

Seventh year PV of EVA

$$\frac{a_1(p_1+1+i)(p_2+1+i)(p_3+1+i)(p_4+1+i)(p_5+1+i)p_6}{(i+1)^7}$$
$$=\frac{11(0.18+1+0.10)(0.08+1+0.10)(0.14+1+0.10)(2.75+1+0.10)(0.10+1+0.10)0.12}{(0.10+1)^7}$$

$$=\frac{11.4217}{(1.1)^7}=5.8611$$

Eighth year PV of EVA

$$\frac{a_1(p_1+1+i)(p_2+1+i)(p_3+1+i)(p_4+1+i)(p_5+1+i)(p_6+1+i)p_7}{(i+1)^8}$$

$$=\frac{11(0.18+1+0.10)(0.08+1+0.10)(0.14+1+0.10)(2.75+1+0.10)(0.10+1+0.10)(0.12+1+0.10)(-0.11)}{(0.10+1)^8}$$

$$=\frac{-12.7732}{(1.1)^8}=-5.9588$$

Using present value formula for uneven cash flow, the present value of each period's EVA is easily summed by the formula as follows:

Present value of EVA

=10+11.6364+0.8463+1.5887+35.1784+4.4772+5.8611+(5.9588) = **63.6293**

The accuracy of this result can be checked by the following present value formula

$$\left[\prod_{r=1}^{n-1}(P_r+1+i)+(i+1)^{n-1}\right]\left(\frac{a_1}{(1+i)^n}\right)$$

=[(0.18+1+0.10)(0.08+1+0.10)(0.14+1+0.10)(2.75+1+0.10)(0.10+1+0.10)(0.12+1+0.10)(-0.11+1+0.10)+ $(0.10+1)^{(8-1)}$][$\dfrac{11}{(0.10+1)^8}$] = **63.6293** (which is the surplus dollar amount on a project).

If the bank had projected a cash inflow or outflow standing on a base year, the value of those cash amounts within the given life time would have been the above calculated result. This helps the bank to decide, by comparing with the initial investment, whether the investment is feasible or not. Each of the above calculated PV of EVA is positive except the last term for 2009, when the bank shows the lowest performance. Positive EVA increases the value of the firm whereas negative EVA reduces the value. As per this paper's recommendation, the bank should have shown progressive performances, otherwise the formulae detect as the firm has not been performed well in that given period.

It is very strong part of the formulae. Take now the case of the bank deposits. The main objective of a bank is accepting deposits and lending them to those with liquidity shortage. Under the annuity formulas, the loan repayments disregard inflation and risk in interest fluctuations. These may create hardships for the borrowers, including defaults. The future value formula of this paper helps the bank in setting its reserve requirements while considering these factors. Starting with the relatively small initial amount that will progress according to the performance of the bank, EVA is calculated. Using again the above data and based on Table 1 above, if the bank had projected its cash reserve deposits by the end of 2009, it would have had the following:

Initial amount (EVA_0) = 11

Deposit interest rate (i)=10%

Table 2: Computational data using the formula developed in this paper, millions

Year	Base (Year=0)	2003 (Year=1)	2004 (year=2)	2005 (year=3)	2006 (year=4)	2007 (Year=5)	2008 (Year=6)	2009 (Year=7)
Performance rate (p_n)		0.18	0.08	0.14	2.75	0.10	0.12	-0.11
$W_n = p_n + 1$		1.18	1.08	1.14	3.75	1.10	1.12	0.89
$T_n = 1 - (p_n + 1)i$		0.882	0.892	0.886	0.625	0.89	0.888	0.911
$W_n - T_n$		0.298	0.188	0.254	3.125	0.21	0.232	-0.021

Accordingly, the calculation of the current cash flow to get the future value up to year 2008 is shown below:

Year **cash flow**

1 $a_1(i+1) = 11(1.10) = 12.10$

2 $a_1(W_1 - T_1)(i+1)^2 = 11(0.298)(1.10)^2 = 3.9664$

3 $a_1(p_1+1)(W_2 - T_2)(i+1)^3 = 11(1.18)(0.188)(1.10)^3 = 3.2480$

4

$a_1(p_1+1)(p_2+1)(W_3 - T_3)(i+1)^4 = 11(1.18)(1.08)(0.254)(1.10)^4 = 5.2132$

5 $a_1(p_1+1)(p_2+1)(p_3+1)(W_4 - T_4)(i+1)^5$
$= 11(1.18)(1.08)(1.14)(3.125)(1.10)^5 = 80.4297$

6 $a_1(p_1+1)(p_2+1)(p_3+1)(p_4+1)(W_5 - T_5)(i+1)^6$
$= 11(1.18)(1.08)(1.14)(3.75)(0.21)(1.10)^6 = 22.2951$

7 $a_1(p_1+1)(p_2+1)(p_3+1)(p_4+1)(p_5+1)(W_6 - T_6)(i+1)^7$
$= 11(1.18)(1.08)(1.14)(3.75)(1.10)(0.232)(1.10)^7 = 29.8033$

Summing the cash investment flows, we have =12.10 +3.9664 +3.2480 +5.2132 +80.4297 +22.2951 +29.8033 = **157.0557**. All the current cash investments of each period are positive in magnitude except the last year. The negative sign indicates that the bank performance in this period is less than the prior year.

As can be seen above, the steps to arrive at the final result is very tedious and time consuming. However, the future value formula of this paper calculates the above cash flow as easily as follow

$a_1(i+1)\left[\prod_{r=1}^{n-1}(p_r+1)(1+i)^n - i\right]$ =11(1.10)[(1.18)(1.08)(1.14)(3.75)(1.10)(1.12)(1.10^7)-(0.10)]

=157.0557

5. Conclusion

The performance of a firm can be projected from its income statement. However, in order to project a firm net income, there should be a record of past trend regarding the firm's performance. Using periodic economic profit within an economic value added (EVA) approach, performance rate of each period is measured as the ratio of $(n+1)^{th}$ period's projected net EVA increment to the n^{th} period's EVA. This will result in uneven cash flows. Further, it implies that each projected net economic profit and loan repayment occur in the same direction, which in turn implies that projected net income, loan repayment and projected net cash flow will have the same direction. As such, the borrower will be expected not to incur cash shortages since net income, loan repayment, and net cash flow are in the same direction. Thus, the borrower is guided to meet his/her loan repayment obligation properly.

In most instances the firm's cash flows are uneven and as such one cannot project the future cash receipts or payments. A good experienced manager may forecast flexible cash flows based on the past data such as income statement and balance sheet of the business firm. As the performance rate of the business moves up or down, the projected cash flow moves in the same direction.

In this paper, the performance of a business firm is assumed to be progressive, i.e., accelerated growth. The firm can achieve its growth objective either by expanding its existing market or entering into new market. As the performance rate moves from one period to another, the loan repayments move along with the direction of this fluctuation. Hence, we have uneven cash flows. We have shown how the present and future values of such uneven cash flows may be calculated.

The method that we have presented may also help investors in projecting their cash inflow or outflow according to their business progressive performance rate. The formulas may also facilitate the creditors or investors decision making process in arriving at realistic measures of firm's performance.

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