

THE CONTAGION FROM THE 2007-09 US STOCK MARKET CRASH

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Abstract

The global financial crisis that took place during the period 2007-09 had its most prominent manifestation in the general stock market crash. This could be studied from the perspective of financial contagion, using a mathematical tool known as wavelets. This paper aims to assess the impact of the US stock market crash on other stock markets all over the world. As an initial point the assumption that the former was the epicenter of the global financial crisis stands out. In order to determine the existence of differentiated impacts that show the presence of inertial factors in different stock exchange markets, a filtering technique is used on stock market indexes to assess such impacts. The data series are worked out on different time scales in order to identify short and long term effects.

Key Words: Contagion, Global stock market crash, United States, Wavelets

JEL Classification: G01, G15, G12, F36.

1. Introduction

In the summer of 2007 the first global financial panic of the 21st century was sparked: see time-map of events in Ariff (2012). In January 2009, asset losses of financial institutions were estimated at \$700 billion. The global stock market losses exceeded thirty billion dollars. The stock market index of the five hundred largest companies in the world in the S&P500 had fallen 39 percent in 2008: the NASDAQ stocks fell by 42 percent while that of Dow Jones by 35 percent.

The financing capacity of global financial system had declined significantly in 2008 relative to 2007. The issuance of securities in 2008 for debt funding fell 37.7 percent compared to the previous year, indeed an unprecedented decline in the last twelve years. The stock issuance was dramatically reduced by 73 percent for the same period. This implied a reduction in bond issues amounting to \$269.86 billion and 148.1 billion dollars decline in stock

financing. The aim of this study is to evaluate the impact of this US-origin stock market crash on other major stock exchanges in the world. The main hypothesis is that stock market crash in the US was the epicenter of the global stock market crash. The analysis has been conducted by a technique of filtering the data using wavelets.

The paper is organized as follows. Previous works that have been written about the stock market using wavelets are briefly summarized in the next section. The filtering process of stock series using a discrete transformation of wavelet coefficients is outlined in section 3. The results of regression and Pearson correlation coefficients are summarized and inferences drawn in section 4. Some concluding remarks are given in the final section.

2. Previous Studies Using Wavelets

Capobianco (2004) uses search algorithms with wavelet shaped dictionaries to break down the scale of the dynamics of stock returns. The wavelet analysis is used to identify intraday periodicities of both one and five minutes, in the timeline. In order to find side effects out of returns in the stock markets at different time scales using wavelet analysis, Fernandez (2004) worked out with stock markets in North America and found that there were side effects to Latin America, emerging Asia, Far East countries and Pacific markets. She also identifies side effects from Europe and Latin America to North American markets.

To study the relationship between the markets of South Korea and US, Lee (2004) worked with wavelets. He used a multi resolution technique at different scales. A strong evidence of side effects and price volatility from the stock markets of developed countries to those of developing nations is reported in that study.

Vuorenmaa (2004) studied the stock share volatility of Nokia using wavelet multi resolution analysis to find out that wavelet variance and covariance revealed a considerable amount of stock market activity in intraday levels. Moreover, applying a rule of local scale and long memory to volatility, he found that the variation in long memory was supported in the medium term (months).

With high-frequency financial data and a Markov tree model, Gençay and Whitcher (2005) used wavelets to establish a new stylized fact about volatility: the low volatility of longer maturities is more likely to be followed by short period low volatility and it would not be the

case for a high volatility of shorter time horizons. This phenomenon is called asymmetric vertical dependence.

3. What are Wavelets?

One way to introduce wavelets is from the Fourier analysis, which is a process to analyze components in a function. Such an analysis could represent a sound, an electrical signal, a light beam, and so on. By considering time dependent functions, as might be the case of a measuring device that connects one side of the power meter from one's house to record voltage transients to find that varies with time. The Fourier transform is a mathematical procedure that follows the shape:

$$F(\nu) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(\tau) e^{i2\pi\nu\tau} d\tau \quad (1)$$

It is a change of one function, f , to be named the signal, whose variable τ is the time, to another function, F , whose variable is the frequency of oscillations ν . For instance, in the case of a light signal, this procedure allows us to extract the frequencies (patterns) that make up the signal, and its importance. However, if the signal is localized, Fourier transform begins losing quality because it does not help much to locate in what time interval a signal was allocated, in such a way that tends to be blind to the details located at specific interval spaces.

One way to address the solution of this problem is by modifying the Fourier transform to get an expression like the following:

$$F(\nu, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} g(\tau - t) f(\tau) e^{i2\pi\nu\tau} d\tau \quad (2)$$

In the previous equation a function that places the signal in a time interval around the instant t is added. Such an addition was introduced by Dennis Gabor in the 1940s. An improvement developed nearly forty years later is due to Morlet and Grossman, who modified the Gabor transformation adding up the frequency as a factor multiplying the time difference, which resulted as the following equation:

$$F(v, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} g[v(\tau - t)] f(\tau) e^{i2\pi v \tau} d\tau \quad (3)$$

These kinds of transformations are called wavelet transformations and allow for a great variety of them, depending on the g function chosen.

The above approach is useful when the ability to observe an event at any given time is feasible. In addition, such a process permits to manage time as a continuous variable. In contrast, when the signals are collections of data taken at specific moments, whose modification is not convenient or when it cannot be changed, it is better to use discrete wavelet transforms. In these cases the time variable is a collection of moments that are usually equally spaced, as is the case of the data used in economics.

For discrete wavelet transforms several propositions are put forward. The first one is due to the Hungarian mathematician Alfred Haar, who in 1909 outlined a system of square waves to pass or bypass certain portions of a signal. The Haar wavelet is defined as follows:

$$\phi(t) = \begin{cases} 1 & \text{if } 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

In order to place the square at different time intervals the following algebraic expression is used:

$$\psi_{n,k}(t) = (2^n t - k), \quad (5)$$

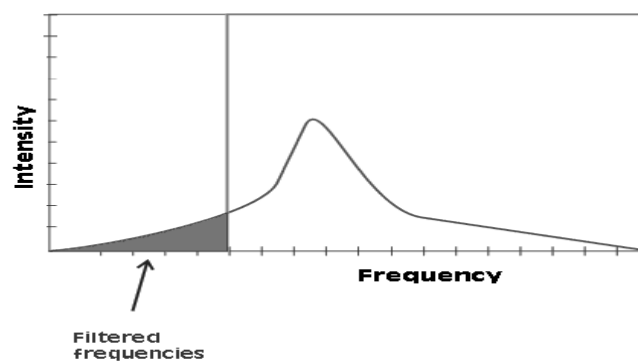
$$\text{with } n = 1, 2, \dots, y \quad 0 \leq k < 2^n$$

which provides the following set of formula: If $n = 1$ the only option is $\{-1 + 2t\}$, which provides the instant $t = 0.5$ and it occurs around the square. If $n = 2$ options are $\{-1 + 4t, -2 + 4t, -3 + 4t\}$, which provide the instants $t = 0.25, t = 0.5, t = 0.75$. If $n = 3$ options are $\{-1 + 8t, -2 + 8t, -3 + 8t, -4 + 8t, -5 + 8t, -6 + 8t, -7 + 8t\}$, the moments $t = 0.125, t = 0.25, t = 0.375, t = 0.5, t = 0.625, t = 0.75, t = 0.875$ are provided.

The allocation of squares at different moments to accept or delete frequencies at different instants is performed by the simple method of multiplying by 1 to pass up, or by 0 to remove them out. The graphs below give a rough idea of how to carry out this procedure.

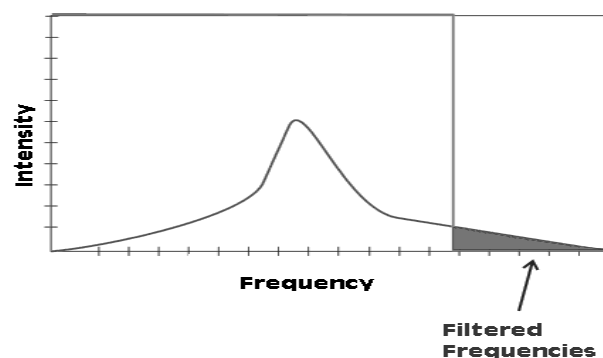
To sum up, a set of simple functions that take one and zero as value, can be arranged to pass a section of frequencies and to eliminate others. For example, the following figure shows how a set of frequencies counted on the horizontal axis, with weights computed on the vertical axis, can be canceled to filter them, i.e. do not pass up. Firstly, a low-frequency filter is presented in Figure 1. The shaded area on the left indicates that lower frequencies are eliminated when multiplied by zero.

Figure 1: Low Frequency Filter



Subsequently, a high frequency graph is included as shown in the shaded area on the right of Figure 2.

Figure 2: High Frequency Filter



A combination of both filters allows the researcher to pass a defined band of frequencies. Therefore, given that the filters have different sets of moments, wherein one can be selected under certain frequency band, it is possible to have the most important frequencies in different points on time.

It is possible to find details of frequencies of oscillation in time, what is difficult with the Fourier transform. To understand the importance of this development, it is worth posing the following question: Is it feasible to have a computer program such that a musical signal is received by a microphone, so that once you send an electrical signal to a system you can allocate the frequency to identify musical notes issued?

If one wants to pass it on to writing musical language, it is also necessary to determine how long the note lasted to be automatically written on the score. The wavelets are designed to solve that problem, because due to its definition, includes a location in a time interval. Whichever signal, the filtered process can provide frequencies on specific time intervals in the same way as a musician that listen to or imagine a musical piece and writes the musical score down indicating which note (frequency), which tune (time location) and an intensity (strong or soft sounds) are introduced.

However, it is not possible to achieve full precision simultaneously both in time and frequency. As a result of a theorem called uncertainty principle, it happens that as long as more precision on frequency is achieved less accuracy in time location is reached and the reverse is true. For this reason, one of the results of wavelets uses, which is called spectrogram, locate frequencies on instants when they occurred. It has a property, when many time periods are involved, the frequency knowledge decline. Conversely, when the number of periods decreases, frequency knowledge accuracy increases. Likewise, an analogous situation is faced when looking at a digitalized picture in a computer, in a zoom out, some details are faded.

4. Pearson Parameter

The Pearson parameter is useful to compute as the correlation degree between two statistical variables. Assuming n measures of a variable X and n measures of any other variable Y , the question to answer is: what is the correlation between both? Such a parameter is symbolized as r and it is defined as

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}} \quad (6)$$

where \bar{x} and \bar{y} are the averages.

By definition r is a parameter that takes values in between -1 and 1 , in such a way that those two extremes indicate total correlation between two variables. While the minus sign shows that if one of the variables increases the other decreases, a positive sign points out that when one of the variables increases, the same occurs with the other. Absence of correlation corresponds to $r = 0$. However, there exist intermediate values which provide with a number for a visual image as follows: If each pair of values of the random variables (x_i, y_i) is matched with a dot in the Cartesian plane, a scatter of n points will show up. Depending on the value of the Pearson parameter, the scatter will look as follows:

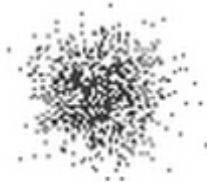
For $r=1$, the dots resemble a positive oriented line.



For $r=-1$, results in a negative oriented line.



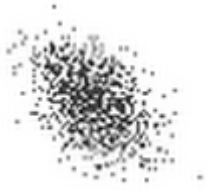
For $r=0$, a cloud of unorganized dots shows up as follows:



For $r=0.4$, a quite organized scatter is observed



Meanwhile with an $r = -0.4$, the following image is perceived:



In this paper is shown that if the values of X variable are the main oscillation frequencies of the New York Stock Exchange (independent variable) and Y represents the oscillation frequencies of any overseas stock exchange, such frequencies are randomly distributed, with a correlation coefficient close to zero that look like spherical distributions. On the contrary, when observation periods are longer than daily, for instance, weekly, bimonthly or monthly, the distributions appear much more organized. The Pearson parameter takes values around 0.9.

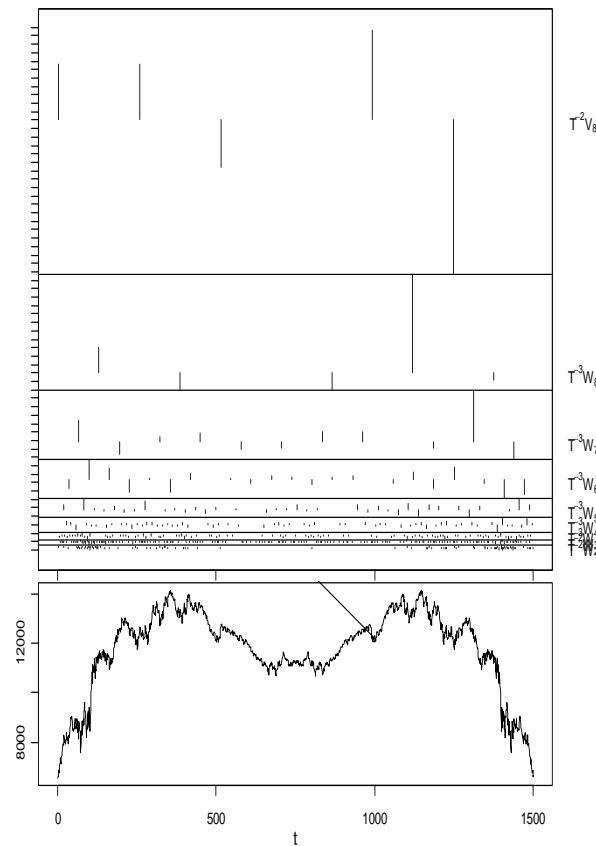
5. Data and Results

Comparison of stock exchange indexes encompasses the period of March the 15th, 2006 till March the 9th, 2009. In order to synchronize physical time, the data for continents different from America was lagged one day with respect to New York Stock Exchange (NYSE) values.

The coefficients of discrete transform wavelets were obtained for each stock exchange. Here a graphic example of different levels of such coefficients for the American case is presented. The bottom part of graph no. 3 shows the Dow Jones Index evolution. Then, eight levels of wavelets coefficients are displayed, identified as W_n . Once the discrete transform wavelet coefficients were obtained, the computations were carried out in order to calculate the Pearson correlation coefficients, so as to measure the synchrony, if there was any.

The applied method allows for a selection of periods to measure frequencies. Measurement was performed in six periods, each of which is called level. The studied period roughly includes: each working day as the first level, every two working days, on the second level, every four working days, for the third level, every eight working days, on the fourth level (two weeks approximately), every sixteen working days, on the fifth level (one month approximately) and every thirty two working days, on the sixth level (two months approximately).

Figure 3: Discrete transform wavelets (W_n) obtained from Dow Jones Index (Period: March the 15th, 2006 to March the 9th, 2009)



The results were used to calculate the oscillation frequency correlation of each stock exchange to the NYSE values. With this purpose a linear regression was run. The method used was to approximate a straight line with ordinary least squares. Then the Pearson parameter was calculated so that to search dispersion around the approximated line.

Based on the previous output, a qualitative concept called tune up, that shows the synchrony of the stock exchanges with the NYSE was introduced. The following results were found: in the first level, calculated frequencies weakly resemble a straight line, which means that the Pearson parameter, in absolute value, is greater than one tenth only for a limited number of cases, such as Mexico, Chile, Peru, Canada, Germany, Hong Kong, Czech Republic and Austria. The remaining countries show such a high dispersion that Pearson parameters lies below one tenth.

On the second level, correlations between frequencies are similar. As a result, the tune up for every two days between the overseas stock exchanges and NYSE has not grown significantly. The tune up tends to grow for longer periods. For instance, in the third level there are 21 stock exchanges whose Pearson parameter is over one tenth in absolute value.

Table 1: Regression coefficients (Wnb) with the main stock exchanges and Pearson correlation coefficients ($WnCP$) for every level of discrete transform wavelet coefficient (Wn), of world stock exchanges with respect to the Dow Jones Index

Country	Code	W1b	W1CP	W2b	W2CP	W3b	W3CP	W4b	W4CP	W5b	W5CP	W6b	W6CP
Egypt	CCSI	0.038	-0.026	0.034	0.031	0.022	0.037	-0.010	-0.016	0.102	0.236	0.020	0.076
Argentina	MVAL	0.012	0.062	0.032	0.162	0.016	0.065	-0.014	-0.050	-0.025	-0.102	0.113	0.494
Mexico	IPC	0.432	0.183	0.581	0.189	0.573	0.171	1.200	0.374	1.943	0.558	2.370	0.860
Brazil	VBPA	0.209	0.032	2.130	0.285	-0.544	0.067	-0.775	-0.108	0.200	0.029	2.174	0.358
Chile	IPSA	0.026	0.140	0.003	0.011	0.057	0.170	0.005	0.014	-0.019	-0.060	0.110	0.485
Peru	IGVL	0.148	-0.106	0.186	0.107	-0.093	0.038	-0.363	-0.143	0.093	0.039	1.316	0.555
Canada	GTSE	0.132	0.124	0.074	0.064	0.225	0.180	-0.150	-0.104	23.865	0.221	0.301	0.341
Germany	DAX	0.058	0.107	0.035	0.057	-0.101	0.160	0.021	0.027	-0.312	-0.452	0.478	0.861
Venezuela	IBC	0.009	0.002	0.180	0.037	-0.430	0.083	0.230	0.033	-0.121	-0.026	1.767	0.351
Japan	N225	0.034	0.026	0.150	0.096	0.242	0.141	0.105	0.078	0.429	0.245	0.244	-0.173
Indonesia	JKSE	0.004	-0.019	0.019	0.082	0.032	0.094	0.030	0.123	0.058	0.218	0.001	-0.005
Hong Kong	H S I	0.254	-0.102	0.065	0.024	0.822	0.262	1.308	0.440	1.954	0.656	2.662	0.830
India	BSN	0.009	0.006	0.097	0.047	-0.021	0.009	0.054	0.021	-0.892	-0.355	0.100	-0.050
Taiwan	TW	0.026	-0.042	0.009	0.012	0.028	0.029	0.147	0.218	0.001	0.001	0.179	-0.191
Singapore	STI	0.006	-0.025	0.019	0.077	-0.012	0.037	0.042	0.144	0.005	0.014	0.166	0.559
China	SSE	0.006	-0.013	0.002	0.004	-0.059	0.102	0.072	0.131	0.019	0.035	0.075	0.118
Filipinas	PSEI	0.000	0.000	0.008	0.026	0.042	0.106	0.007	0.019	0.036	0.102	0.143	-0.396
Pakistan	KSE	0.007	-0.008	0.002	0.001	-0.073	0.049	0.028	0.023	-0.156	-0.096	0.471	-0.246
Malaysia	KLSE	0.001	0.020	0.003	0.032	-0.021	0.188	0.018	0.180	0.021	0.181	0.031	0.342
Sri Lanka	CSE	0.001	-0.011	0.019	0.113	0.020	0.104	0.018	0.082	-0.008	-0.034	0.080	0.354
South Korea	KS11	0.009	0.065	0.036	0.197	-0.024	0.138	-0.010	-0.045	-0.011	-0.059	0.068	0.418
Denmark	KFMX	0.091	-0.041	0.007	0.018	0.029	0.070	-0.014	-0.036	-0.059	-0.141	0.051	-0.265
Greece	ASE	0.203	0.050	0.001	0.002	0.052	0.112	-0.006	-0.016	-0.018	-0.057	0.046	0.249
Switzerland	SSMI	0.025	0.043	0.147	0.204	0.025	0.034	0.444	0.567	0.591	0.773	0.550	0.909
Slovakia	SAX	7.232	-0.037	6.890	0.028	40.323	0.154	66.493	0.247	78.385	0.229	9.879	0.046
Russia	RTS	0.016	-0.083	0.006	0.024	0.058	0.184	0.043	0.131	0.060	0.184	0.134	0.613
Czech Rep.	PX	0.014	0.106	0.043	0.257	0.012	0.070	0.067	0.424	0.152	0.729	0.171	0.902
Portugal	PS20	0.002	-0.003	0.021	0.029	-0.019	0.023	0.127	0.140	-0.179	-0.202	0.071	-0.074
Norway	OSEAX	0.002	-0.038	0.002	0.028	0.014	0.203	-0.013	-0.223	0.016	0.238	0.019	0.357
Sweden	OMPI	0.000	-0.008	0.002	0.048	0.012	0.357	0.000	0.001	0.001	0.018	0.014	0.457
N. Zealand	NZ50	0.009	0.052	0.009	0.042	0.073	0.313	0.141	0.550	0.210	0.768	0.256	0.919
Italy	MTEL	0.016	0.009	0.018	0.009	-0.641	0.278	0.162	0.065	-1.286	-0.508	1.731	0.815
Spain	IBEX	0.051	0.049	0.032	0.030	0.030	0.024	-0.051	-0.048	-0.055	-0.071	0.225	-0.197
United Kingdom	FTSE	0.001	-0.001	0.107	0.194	0.055	0.102	-0.111	-0.182	-0.204	-0.384	0.484	0.925

Belgium	BFX	-	-	-	-	-	-	-	-	-	-	-	-
		0.026	-0.096	0.074	0.219	0.113	0.324	0.105	0.280	0.064	0.160	0.055	0.151
Austria	ATX	-	-	-	-	-	-	-	-	-	-	-	-
		0.039	-0.105	0.000	0.001	0.006	0.012	0.174	0.369	0.226	0.379	0.407	0.883
Netherlands	AEX	-	-	-	-	-	-	-	-	-	-	-	-
		0.003	-0.095	0.006	0.152	-0.001	0.029	0.000	-0.003	-0.021	-0.352	0.030	0.608
France	FCHI	-	-	-	-	-	-	-	-	-	-	-	-
		0.031	-0.073	0.100	0.209	-0.038	0.081	-0.018	-0.037	-0.174	-0.342	0.260	0.640
Israel	TA	-	-	-	-	-	-	-	-	-	-	-	-
		0.003	0.038	0.006	0.075	0.009	0.088	0.005	0.045	-0.043	-0.418	0.058	0.572
Australia	AORD	-	-	-	-	-	-	-	-	-	-	-	-
		0.022	0.054	0.078	0.161	0.181	0.367	0.127	0.245	-0.198	-0.407	0.272	0.577

Essentially, the number of countries is threefold as much as for the first level. Moreover, the value of the Pearson parameter tends to grow. There are stock exchanges that do not show any tune up in the first two levels. However, there is some tune up in the third level.

The stock market is synchronized all over the world in an approximated period of two months. In effect, there are only five stock exchanges whose Pearson parameter lies below one tenth in absolute value. In the opposite case, for instance, we have that the Mexican stock exchange has 0.860, the Czech Republic 0.902, and the United Kingdom 0.925.


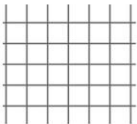
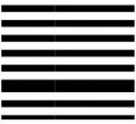

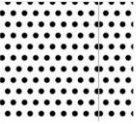


In the previous pages, it was found the average frequencies of oscillation of the world's stock market became alike. This is a clear trend towards homogenization of the frequencies. The phenomenon could be clearer if a frequency transformation were made, so that each economic frequency ν_e were associated with a light frequency, ν_v . The procedure is similar to the one used to change Fahrenheit to Celsius degrees and vice versa. The transformation is:

$$\nu_v = A\nu_e + \nu_r \quad (7)$$

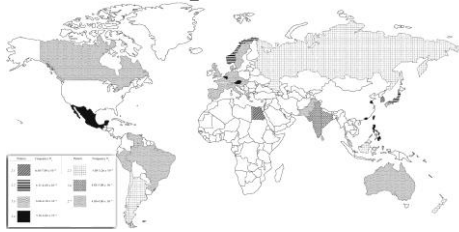
Where $A = 61199.2$ is a number that is calculated as follows: calculate the difference between the maximum frequency (pattern 2.1) visible and low frequency visible (pattern 2.7).

Next, the difference between the maximum and minimum economic frequency is calculated. Patterns are allocated based on the following Table 2. The standard deviations of the regressions of various indices of stock exchanges in the world against the U.S. Dow Jones converted as variations in pattern range are shown. It is observed that the longer the time span, the larger the homogeneity degree of oscillations, which is clearly reflected in the predominance of one pattern.

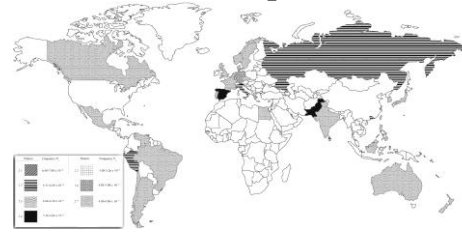
Table 2: Allocation of Patterns

Pattern	Frequency V_v	Pattern	Frequency V_v
2.1 	$6.68-7.89 \times 10^{14}$	2.5 	$5.08-5.26 \times 10^{14}$
2.2 	$6.31-6.68 \times 10^{14}$	2.6 	$4.84-5.08 \times 10^{14}$
2.3 	$6.06-6.30 \times 10^{14}$	2.7 	$4.00-4.80 \times 10^{14}$
2.4 	$5.26-6.06 \times 10^{14}$		

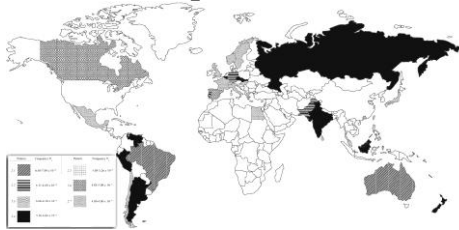
Map No. 1.



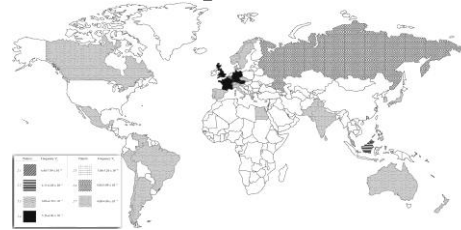
Map No. 2



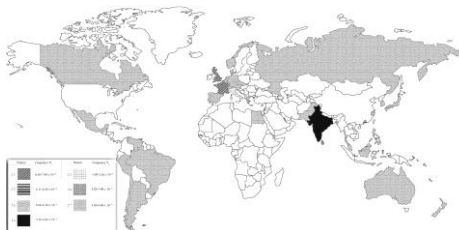
Map No. 3



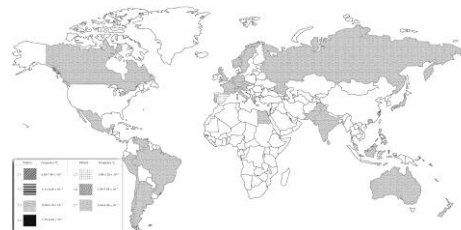
Map No. 4



Map No. 5



Map No. 6



6. Concluding remarks

The first financial crises of the 21st century that took place during 2007-09 disclosed several drawbacks. The global stock market underwent considerable losses. Capital generation diminished to unprecedented levels. The main hypothesis of this paper is that US stock market was the epicenter of the world market crash.

NYSE movements on other stock markets is studied using the wavelets mathematical tool to assess the effects. Frequencies of oscillation for the NYSE as well as stock exchanges of 40 other countries were calculated and filtered. Such frequencies were correlated between US and other stock markets.

It was found that NYSE movements exerted a great influence, during the first two working days, in a limited group of countries such as Mexico, Chile, Peru, Canada, Germany, Hong Kong, Czech Republic and Austria. After eight working days, it is observed that 21 stock exchanges (see map of the world) were significantly affected.

The most important concluding remark is that the stock market is synchronized all over the world in an approximated period of two months.

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