

Investigating the Potential of Integrating Augmented Reality into the 6E Instructional 3D Geometry Model in Fostering Students' 3D Geometric Thinking Processes

<https://doi.org/10.3991/ijim.v16i06.27819>

Sudirman^{1,2}(✉), Yaya S. Kusumah¹, Bambang Avip Priatna Martadiputra¹

¹Department of Mathematics Education, Universitas Pendidikan Indonesia, Bandung, Indonesia

²Department of Mathematics Education, Universitas Wiralodra, Indramayu, Indonesia

sudirmanstudent@upi.edu

Abstract—Schools as a source of knowledge need to facilitate the 3D geometric thinking process involving a series of cognitive actions. Therefore, this research investigates the potential of integrating augmented reality (AR) into the 6E instructional 3D geometric model (6E I3DGM). This is exploratory research carried out with a teacher and 28 junior high school students in Indramayu District, Indonesia. The data were collected through observations and interviews and examined, selected, extracted, and coded based on the criteria determined. The data collected were further interpreted, concluded, and descriptively analyzed using content analysis. The result showed that learning that integrates AR into 6E I3DGM could raise 3D geometric thinking processes in the dimensions of representation, spatial structuring, and measurement. This is because, characteristically, AR can facilitate students to represent and visualize 3D shapes directly. This will increase students' ability to measure the surface area and volume of 3D geometric shapes. Therefore, this learning deserves to be used as an alternative to 3D geometry learning in the post-COVID 19 pandemics.

Keywords—augmented reality, 3D geometric thinking, junior high school students, 6E instructional 3D geometric models

1 Introduction

Geometry is one of the oldest formal mathematical domains [1] implemented in various fields of life such as architecture, art, astronomy, geography, music, etc. [2]–[5]. This makes it relevant in the school curriculum [6]–[8]. Therefore, the purpose of teaching this topic is not only to develop knowledge of concepts, properties, and theorems; instead, it also includes geometric thinking skills, intuition and proof, spatial awareness, visualization capabilities, problem-solving, estimation, deductive reasoning, and logical argument [9]. Even [8] considers that studying geometry aids one to organize, predict, and possess the ability to represent physical objects and experiences.

Moreover, as a place for formal learning, the school needs to facilitate knowledge development from in-depth thinking processes. Schools must also be able

to accommodate the technological developments of the twenty-first century [10]. One of such procedures applied in learning this topic is 3D geometric thinking [11]. It requires complex cognitive actions and three components, including the external material world, internal spatial abilities, and communication or signs [12]. This approach starts with the individual's ability to perceive and interpret geometric objects by using their spatial skills through communication [10]. Meanwhile, [13] stated that three dimensions are involved in the 3D geometric thinking process: representation, spatial structuring, and measurement.

Several preliminary studies have been carried out on these processes. For instance, [13] analyzed the structure of 3D geometric thinking by testing its relationship with spatial abilities. Moreover, [11] developed a three-dimensional geometric thinking test for elementary school students. Then [14] designed an assessment framework of its thinking representation for high school students and analyzed the difficulties encountered. Meanwhile, [15] studied 3D geometric thinking skills in children after participating in the 3D program in Early Childhood (3DinEC). Furthermore, [16] investigated this process in children using the Dynamic Transformation Context program and [17] tested the effect of AR on students' 3D geometric thinking skills during learning.

Although several studies have been carried out on 3D geometric thinking processes, only a few have investigated the learning potential that integrates AR technology into I3DGM 6E, a modification of the 5E Instructional Model to foster this approach in junior high school students. It consists of elicit, engage, explore, explain, elaborate, and evaluate phases. Furthermore, elicit and engage aims to activate prior knowledge and new concepts learned by the students. Meanwhile, the exploration phase aims to facilitate a thought process related to developing new concepts, and these need to be improvised and confirmed through activities in the explanation stage. Moreover, confirmed new concepts must be internalized and assimilated through elaborate and evaluation activities. In addition, AR technology facilitates the thinking process of 3D geometry. This is because digital technology can change how lessons are taught in the classroom and strengthen didactic situations to bridge teaching mathematics and mathematics [18]. AR technology is integrated into the exploration and evaluation phases in 3D geometry in research. In addition, in this study, the use of AR is also integrated into 3D geometry textbooks.

Therefore, in general, this study investigates the potential of implementing learning that integrates AR technology into 6E I3DGM in fostering junior high school students' 3D geometric thinking process. The specific research questions studied in this study are as follows:

- The Interactive 3D Geometry Book (I3DGB) design form with AR used in the I3DGM 6E learning?
- What learning potential integrates AR in 6E I3DGM in developing 3D geometric thinking skills?

2 Literature review

2.1 3D geometric thinking ability

The 3D geometric thinking ability was first introduced [13] to distinguish it from spatial skills. [13] Stated that its reasoning types need to be modeled as different constructs. Moreover, [13] further explained that these are related to three dimensions, namely representation, spatial structuring, and measurement. The representation of geometric objects is divided into 2D and 3D [19]. [20] Considered that 2D geometry is an external representational mode that is most often used to illustrate 3D geometric objects in textbooks. Then, [21] generally stated that these are represented using two concepts, namely decoding and coding. Decoding refers to interpreting structural elements and properties of 3D geometric shapes, while coding refers to constructing plane representations and 3D shape nets and translating from one representational mode to another [21]. The decoding construction consists of 2 factors, the student's ability to (a) interpret the representation of 3D shapes, especially in terms of recognizing structural elements (corner points, plane surfaces, edges) in various modes, such as perspective (opaque or transparent) and orthogonal, (b) describe geometric properties of 3D planes [21]. Meanwhile, that coding consists of 3 factors, namely (a) manipulating and constructing 3D nets, (b) converting 2D images into 3D shapes, and (c) translating one representational mode such as orthogonal views of 3D shapes to 2D images [21]. The construction process in creating 3D geometric webs requires the ability to translate objects into 2D by focusing and studying their parts in both representational modes [20]. This is because the transformation process is a mental operation performed by manipulating images [22]. Moreover, the transition from 3D objects to their webs requires the activation of appropriate mental actions that coordinate the various perspectives [20]. In particular, [23] assumed that its construction process requires coordination between the mental representation of the object as a whole and the decomposition of its parts.

The dimensions of 3D geometry measurement are a relevant aspect. The importance of measuring both surface area and volume is evident in daily activities and school mathematics [24], [25]. [26] stated that five strategies are involved in determining the arrangement of 3D geometric objects, namely (1) conceptualize a set of cubes arranged in rectangular form into several layers, (2) visualize them as a space that needs to be filled without taking advantage of the layered arrangement, (3) conceptualize these cubes from their front view, (4) use the formula $L \times W \times H$, (5) apply strategies other than those described in numbers 1 to 4. According to [27], students that used the first two categories [26] showed that they are aware of the spatial structure of 3D objects, including hidden parts [28]. Meanwhile, those that applied the third strategy expressed a lack of understanding concerning the volume aspect. They failed to integrate different views of 3D objects [28]. Related to the fourth strategy, it is not always clear whether the students that apply it have a conceptual understanding of volume and only use the formula as a shortcut, or mechanically, without really comprehending the structure [26]. In this context, the ability to calculate the surface area and volume of 3D geometric shapes is related to the student's capacity to perform mathematical calculations.

2.2 Augmented reality

Since it was first introduced in 1968 [29], AR technology has been widely applied in both commercial and research projects [30]–[32]. This is due to the prevalence of connected hardware and software in mobile phones, tablets, etc. [36], and [37] tend to use text, video, and audio components to describe existing images or spaces [33], which is utilized to assist the learning processes both in and outside the classroom [34]–[37]. This is due to the characteristics of AR that creates new realities [38], which provides information, knowledge, experience from something abstract or difficult to understand and observe [39], thereby enabling users to comprehend mathematical concept [40]. The AR system is functional when the user can discern an image created from real (marker) and virtual objects [32]. The most important aspect is that virtual objects represent real ones with extraordinary and valuable information [32]. It consists of a camera, computer unit, and screen [40]. The camera captures the image; then, the system adds a virtual object on top of it and displays the result [40]. The computer unit further captures the environment, detects markers and infers the location and orientation, and adds a virtual object on top of the image displayed on the screen [40].

2.3 3D geometry model (6E I3DGM) 6E instructional

6E I3DGM is a learning development of the 5E Instructional Model, which Rodger W Bybee first developed in 1997 [41]. It is primarily implemented in science, although currently, it is widely used and studied in mathematics education [42]. This is understandable because it is considered one of the best approaches recommended for teaching in a constructivist learning environment [43], which facilitates how knowledge is obtained through engaging, exploring, explaining, elaborating, and evaluating [44]. The 5E instructional cycle model is called because each stage starts with “E” [42]. In this study, the 6E I3DGM phases are designed to help students carry out the construction process of 3D geometric thinking. In the 3D geometry elicit phase, students’ prior knowledge is triggered. This stage is essential because it is necessary to carry out the 3D geometric thinking process. They can think adequately, supposing they possess good retentive memory. Furthermore, the subsequent phase is engaged, which aids the students’ understanding of new concepts. In this phase, prior knowledge is connected to the one recently acquired during learning. The next phase is the exploration stage, which facilitates constructing new concepts into their memories. Next is the explanation phase, where the students discuss the process results. It is assumed that the knowledge is internalized correctly, supposing they can explain the topic learned. Subsequently, in the elaborate phase, the students analyze the knowledge acquired from the 3D geometry obtained in other contexts, aiming to help them memorize certain information for a long time. The last phase is evaluation, where they are made to conclude the topic learned, thereby storing the acquired knowledge in cognitive structures for a lengthy period.

2.4 Learning framework that integrates AR into 6E I3DGM

The learning framework integrating AR into 6E I3DGM starts with the students' already prepared application (Figure 1). Afterward, they study the menu and the instructions provided. In the subsequent process, they learn the concept of 3D geometry in textbooks which has adopted the 6E I3DGM phases. Meanwhile, AR integration is in the exploration and evaluation phases. In addition, two exploration stages are carried out, namely the contextual and concept phase with AR. Students are asked to use AR in solving 3D geometry problems in the evaluation phase. This is carried out to minimize their incorrectness in visualizing these shapes.

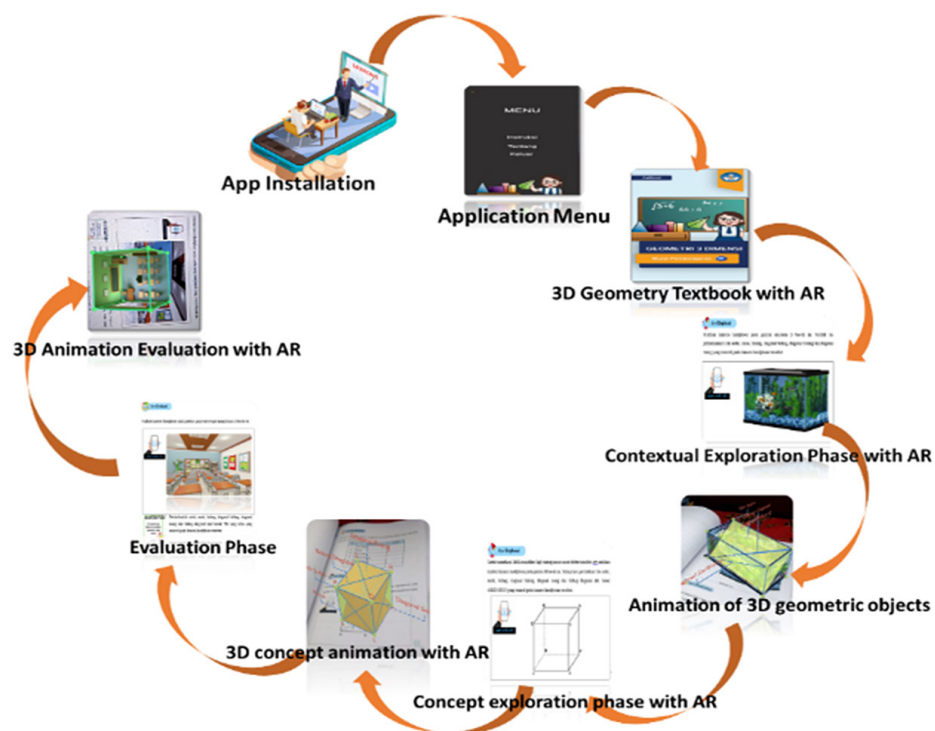


Fig. 1. Framework for integrating AR into the 6E I3DGM

3 Methods

3.1 Design

An exploratory case study design was used to investigate the learning potentials related to the integration of AR into 6E I3DGM to foster 3D geometric thinking processes in junior high school students. This was realized by carefully observing the

process and the 3D geometric thinking ability test. [45] stated that this approach is used to investigate a particular event, situation, or condition to provide information about its occurrence. Furthermore, [46] stated that the case study design also reveals the impact of using technology in the classroom.

In the context of this study, the research process begins with the selection of schools. This was done considering that since the beginning of the COVID-19 pandemic, many school managers have carried out the teaching and learning process via laptops or smart-phones [47]. Besides, the selected school needs to have good national exam results on geometry materials. After negotiating with several principals, one of them was taken and followed by a discussion held with two mathematics teachers that taught 3D geometry in class VIII. It was concluded that they agreed to implement learning approaches that integrate AR into 6E I3DGM. The selected class also learned other subjects online.

By the lesson plan in the school curriculum, the study was carried out during ten meetings, namely 9 for the learning process and 1 for the final evaluation. The distribution of material at each meeting is presented in Table 1.

Table 1. Material distribution

Meeting	Learning Objectives
1st meeting	Students were able to identify the elements and properties of 3D geometry
2nd meeting	Students were able to draw 3D geometry
3rd meeting	Students tend to make nets of prisms, blocks, and cubes
4th meeting	Students possess the ability to calculate the surface area of a prism
5th meeting	Students were able to calculate the surface area of blocks and cube
6th meeting	Students were able to calculate the volume of a prism
7th meeting	Students tend to calculate the volume of the block
8th meeting	Students calculated the volume of a cube
9th meeting	Students were able to compare 3D geometric volumes based on their properties
10th meeting	Evaluation

3.2 Participants

The participants were one male mathematics teacher aged 37 years that possessed good technological literacy and 28 Class VIII students from a school in Indramayu Regency, Indonesia. Meanwhile, before the treatment was started, while still observing the health protocols, the teacher sorted for the students' permission to participate in this study. Afterward, the technical implementation of the research was explained. An agreement was reached with the parents and students regarding their participation, intended to minimize misunderstandings between all parties.

3.3 Data collection

Observation, test, and interview investigated the learning potential that integrates AR into 6E I3DGM. It aimed to understand the implementation process involving teachers

and assistants, carried out during nine meetings. Documentation was made to observe the active and passive students. The data obtained from the first observation session were then reflected on to improve the process at the next meeting. Furthermore, the 3D geometric thinking test was adopted from the indicator framework designed by [13]. The first and second questions were used to measure the dimensional representation, and then the third and fourth were on the spatial dimension of structuring.

Meanwhile, the fifth test question was aimed to analyze the students' 3D geometric thinking on the measurement dimensions. In this study, the preparation of the items was validated by three experts. The average results of the validators' assessments show that the five test items are in an outstanding category and are suitable to measure the ability to think 3D geometry. After the experts assessed the items, the students conducted a test to determine the validity, reliability, level of difficulty, and discriminatory power. The calculation results show that all items are valid to measure KBG3D.

Furthermore, for the reliability calculation, it is obtained that the items are reliable in the medium category. As for the size level, all items are in the medium category. In addition, for the difficulty level in item numbers 1, 2, 3, 4, and 5 in the medium category. Meanwhile, the level of discriminatory power shows that item number 1 is in a suitable category, item number 3 and 5 is in the medium category, and item number 2 and 4 is in the weak category. An interview was carried out to obtain more information about 3D geometric thinking, and it was carried out in 4 sessions. After studying dimensional representation, spatial structuring, and measurement, the first, second, and third sessions were conducted on four students (2 boys and girls). Conversely, the fourth session was performed after they were given test questions to measure their 3D geometric thinking abilities.

3.4 Data analysis

This study analyzed the data from observations, interviews, and tests qualitatively. The process was started by acquiring all necessary information obtained from the first to the fourth observations and interviews. Furthermore, it was examined, selected, extracted, and coded based on the determined criteria. Next, it was interpreted, and conclusions were drawn using content analysis [48] to describe the students' 3D geometric thinking. Also, the test results were analyzed by adopting the steps proposed by [49], which started with the assessment keys. This was continued by creating a framework based on 3D geometric thinking dimensions. The students' answers were evaluated and graded based on the assessment keys. Subsequently, the data from the previous stage were analyzed descriptively.

4 Results

4.1 Interactive 3D geometry book (I3DGB) with AR

The outer part of an I3DGB consists of a cover page, foreword, table of contents, instructions for using the book, concept maps, learning objectives, and introduction to geometric figures, as shown in Figure 2.

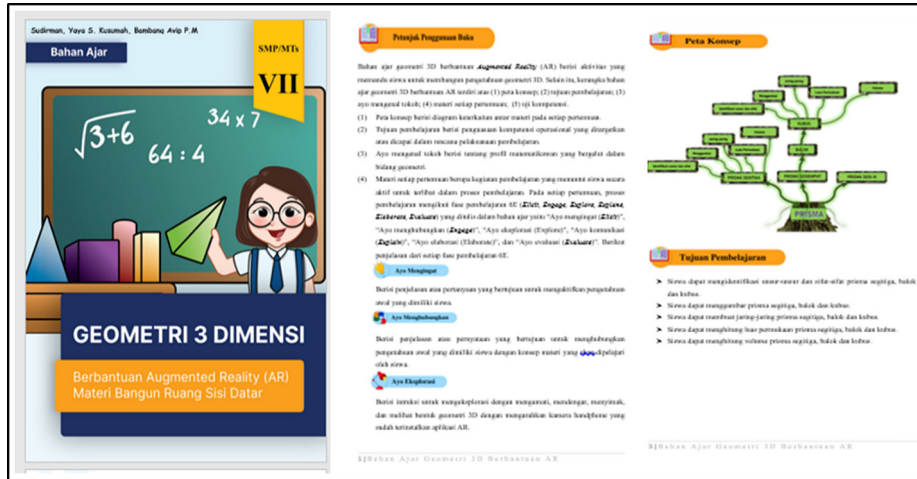


Fig. 2. Cover of I3DGB with AR

The I3DGB user manual contains activities that guide students to develop knowledge of 3D geometry. At each meeting, the learning process follows the 6E I3DGM phases (elicit, engage, explore, explain, elaborate, and evaluate) which is represented in the textbook, as “let’s remember (elicit)”, “let’s connect (engage)”, “let’s explore (exploration)”, “let’s communicate (explanation)”, “let’s elaborate (elaboration)”, and “let’s evaluate (evaluation)”. Meanwhile, the concept map section contains the flow of 3D geometry concepts in the school curriculum.

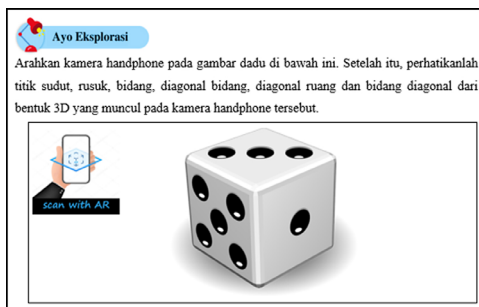


Fig. 3. I3DGB display in exploration phase

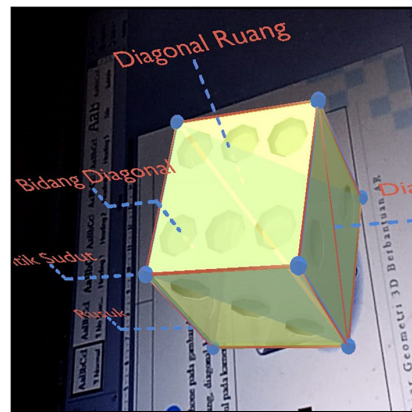


Fig. 4. 3D animation display in the exploration phase

I3DGB contains five materials, namely (1) identifying the elements and properties of 3D geometry, (2) drawing the shapes, (3) creating nets, (4) calculating the surface area, and (5) the volume. Each material is equipped with 6E I3DGM phases integrated with AR technology. This is shown in the exploration (Figures 3 and 4) and evaluation

phases (Figures 5 and 6). In the exploration stage, the students were asked to point their cellphone cameras at images (markers) of objects in the form of 3D geometry. After that, they were made to pay attention to the shape of the 3D animation that appeared on the screen. Subsequently, students were asked to write down the results of their exploration in the explanation phase. Problems related to materials or topics in the evaluation phase were then given to reinforce the memorized new concept.



Fig. 5. Evaluation phase display

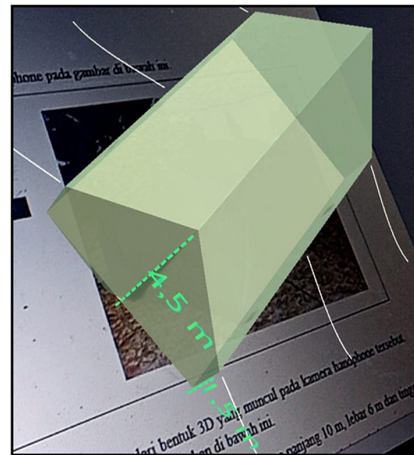


Fig. 6. Evaluation phase display

4.2 Potential in growing 3D geometric thinking skills

In this research, five questions were tested; 2 of them measured representation dimensions and spatial structuring, while one was centered on 3D geometric measurements. Empirically, the results of descriptive statistical analysis are shown in Table 2.

Table 2. 3D geometric thinking ability test results

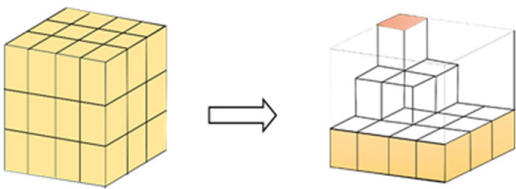
Dimensions	N	Minimum	Maximum	Mean	Variant
Representation Dimension	28	36	93	68	217
Structural Spatial	28	43	100	72	316
Measurement Dimension	28	29	100	70	462
Whole	28	42	95	70	211

The maximum value obtained from each dimension or as a whole is 100. Based on this result, it was realized that the minimum value of each dimension is 36, 43, 29, and as a whole is 42. Furthermore, the maximum value for the representation is 93, and the spatial structuring and measurement dimensions are 100, while the overall is 95. Furthermore, the mean value for each dimension is 68, 72, and 70, and the total is 70. This shows that, on average, students who answered questions related to the spatial structuring dimension correctly are more than the other dimensions.

Furthermore, 217, 316, and 462 were obtained for the variance value in each dimension, and these indicate that they obtained a diverse outcome for the measurement dimension. The table shows that AR, which is integrated into 6EI3DGM, can grow 3D geometric thinking skills.

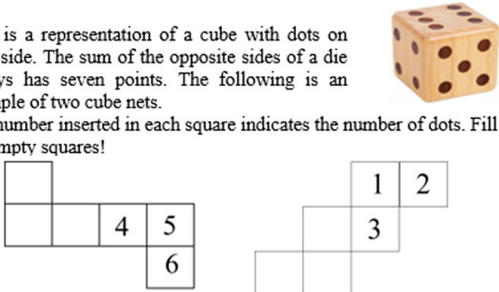
Furthermore, if we take a more in-depth portrait of students' 3D geometric thinking processes on the dimensions of representation, spatial structuring, and measurement, it shows the diversity of students in solving problems given by the teacher.

1. A 36 unit cubes are arranged into a large block, which each side is colored. After that, several unit cubes are drawn so that the 3D geometric shape appears in the image below.



a. How many unit cubes are left in the figure?
 b. Explain how to determine it!
 c. How many unit cubes are left with uncolored sides?
 d. Explain how to determine it!
 e. How many residual unit cubes have one side colored?
 f. Explain how to determine it!

2. Dice is a representation of a cube with dots on each side. The sum of the opposite sides of a die always has seven points. The following is an example of two cube nets.



The number inserted in each square indicates the number of dots. Fill in the appropriate numbers in the empty squares!

Fig. 7. Question form on the dimension of representation

In the representation dimension, the questions tested determine the number of unit cubes in a large one. The 3D geometric thinking process was analyzed based on correctly and incorrectly answered. However, from the results of the first questions, which were answered correctly by the majority, it was seen that they were able to represent the unit cubes, including the invisible ones. They already know ways to determine it by counting the number of unit edges in the first layer both in length and width. Then multiply the two results, representing the number of unit cubes in the first layer. They repeated the process to determine the amount in the second one.

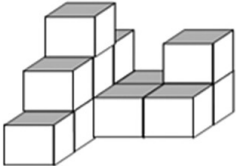
Meanwhile, students who did not answer the questions correctly did not seem to visualize the plane or side of the unit cube that was invisible. They had difficulty identifying the number of unit sides on a large cube. Furthermore, the students were only able to represent the visible side of the unit cube.

Students who answered correctly assumed that the opposite fields were not close in the second question. Therefore, the opposite plane needs to be identified to generate a 3D geometric shape from a 2D grid. The students were asked to fill in the numbers in the opposite field in this context. Based on the interview, they paired the field with

number six with that designated with one because it was assumed to be the farthest. This pair formed the front and back lines.

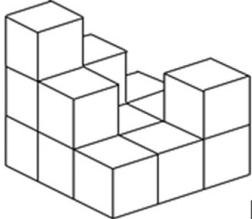
Furthermore, the field with the number 5 was paired with the one with 2. They assumed its position is adjacent to the field with number 4. The opposite ones produce a field positioned both on the right and left sides. Furthermore, the field with the number 4 is opposite the one designated with three, which is in the middle. The numbers 4 and 2 were used to produce the bottom and top fields. The process was repeated to answer the following question.

3. Look at the 3D geometry image below!



Draw the front view, side view and top view of the unit cube arrangement!

4. Look at the 3D geometry image below!



The arrangement of the unit cubes above is a representation of a multi-storey building.

- How many floors are there in the building?
- Explain how it is calculated!
- How many rooms are there on each level?
- Explain how it is calculated!
- Write down the total number of rooms in the building.
- Explain how it is calculated!

Fig. 8. Question form on the spatial dimension of structuring

Related to the students' thinking process in solving problem number 3, they needed to understand the meaning of drawing the front, side, and top views. The data from the interviews showed that they could comprehend the meaning of the front view, which depicts drawing two dimensions from the anterior point of a 3D geometric image. However, most of those that answered correctly arranged the parts of the 2D image. Therefore, producing a complete 3D geometric shape is inseparable. This is similar to drawing from the side or top view. It shows that the students could translate 2D geometric parts formed from 3D. Furthermore, those that failed to answer correctly drew the front, side, and top views separately without combining them into a two-dimensional image.

In the fourth question, the students that answered correctly counted the parts at the lower level and so on. First, those in width multiplied the number of rib structures in the lengthy section. The second and third levels were realized by mere observation. Those whose thinking processes were not precise could not see the unit cubes that were not visible at every level. This includes determining the structure of its edge on the base or invisible side.

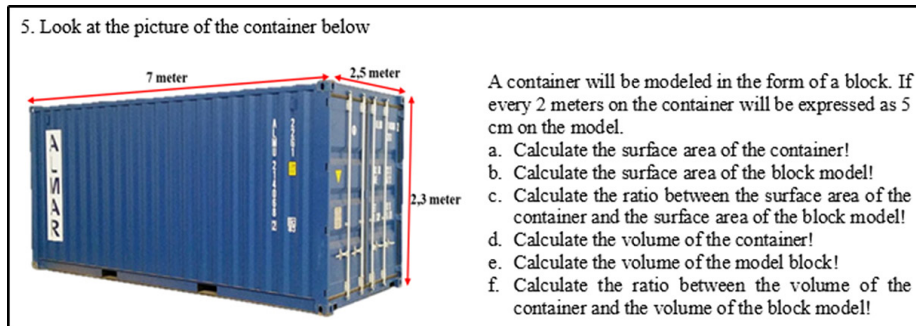


Fig. 9. Question form on measurement dimensions

The students that answered correctly applied the formula for calculating surface area and volume. They understood the concept of geometric models with actual shapes. The majority carried out the comparison to determine their length, width, and height. Besides, determining the actual geometric shape was obtained by using the formula for surface area and volume. Although they could solve the problem, they did not understand the concept correctly.

On the contrary, those who answered incorrectly did not clearly understand the concepts involved in solving the problem. They had difficulty in conceptualizing the formula for surface area and volume. Furthermore, some students did not understand the comparison concept in determining the length, width, and height of the 3D geometric model. Besides, some errors were made when calculating the surface area and volume. Some were able to determine their values but mistakenly failed to compare the volume of the model with the actual one. This is because they were not careful in determining the surface area and calculating the volume of the model and its actual shape.

Based on the analysis of students' 3D geometric thinking processes, it is evident that the average majority could answer questions about representational dimensions, spatial structuring, and measurement. Most of them solved problems related to determining the elements and construction of 3D geometric shapes from 2D grids visible to the students, its translation, counting the number of cubes, and calculating surface area and volume. They started to visualize invisible elements. The ability to represent 3D geometric shapes correctly helps students acquire knowledge. Furthermore, in structuring the spatial dimensions, they can also analyze the side structures and plane shapes that are not directly visible. The majority could calculate and compare the surface area and volume when measuring dimensions. This shows that using learning approaches that integrate AR technology into 6E I3DGM can help students represent 3D geometric objects correctly.

5 Discussion

Schools as places for formal learning need to facilitate knowledge development. One of the processes applied in this research context is geometric thinking, especially 3D. Ontologically, it is built on dimensional representations, spatial structuring, and

measurements [13]. Meanwhile, theoretically, the 3D geometric thinking ability in the representation aspect is based on Duval's theory (visualization, construction, and reasoning) [50]–[52], Fischbein's Figural Concepts [53], [54], and Van Hiele's levels of geometrical thought [55]. Meanwhile, spatial structuring is developed from the theory of mental model [56] and imagery [57], including visuospatial processing [58]. Measurement is built from the child's conception of space, a theory proposed by Piaget [59]. Furthermore, [60] Harel formulated the term way of thinking, defined as a series of cognitive mental actions that produce mathematical understandings. On this basis, one's understanding of mathematics continues to grow through the process of assimilating new knowledge [60]. Moreover, how an individual thinks and gains knowledge or understanding of mathematics are also accommodated in APOS theory, actions, processes, objects, and schemas [61]. Newly memorized schemas realized through assimilation, accommodation, and equilibration become prior knowledge, known as Piaget's cognitive development stage [62]. 3D geometric thinking is defined as cognitive processes involving objects, memory, senses, and prior knowledge. These are interrelated, starting from 3D geometric objects visualized with the five senses through the representation process [21]. Additionally, these are memorized through the perception procedure [63]. Memory in the form of knowledge is stored either for a long or short term depending on the internalization process [64]–[66]. Newly acquired knowledge forms a prior schema. Furthermore, it tends to be reactivated through the connection procedure [67]. This rotational process leads to the acquisition of knowledge by others.

The learning design that facilitates this process is the 6E instructional 3D geometry model. Furthermore, technological aspects significantly augmented reality, also assisted in representing and constructing 3D geometric shapes from 2D nets seen by the students, translating and calculating the number of cubes and surface area and volume. The findings confirm that the phases in 6E I3DGM, such as to elicit, engage, explore, explain, elaborate, and evaluate, facilitate students' 3D geometric thinking. In the elicit phase, they are instructed to perform cognitive actions by activating their prior knowledge. This is done by asking questions to stimulate students' understanding of 3D geometry [68] and foster their motivation and interest [69].

Furthermore, in the elicit phase, the students are advised to participate in the engage phase, and both parties connect prior knowledge with the newly acquired ones in this stage. Moreover, the teacher must create an interactive relationship between the students and the subject matter in the engage phase. This is inconsistent with the study [70], which stated that real engagement in the mathematical ideas learned needs to be meaningfully related to past experiences (knowledge and mathematical identity). This is achieved through exploration, conjecture, logical reasoning, and communication processes [70].

Furthermore, several strategies are adopted in the exploration phase, such as using challenging tasks [71] to carry out various technological products [72]. This study applied AR technological product because it has been widely used in 3D geometric learning [17], [73]–[79] and helped students in understanding the representation [17], visual-spatial [82], [83], and measurement dimensions [17]. In addition, the use of AR can facilitate interaction between teachers and students to learn certain concepts [84], [85]. This finding shows that most could answer the 3D geometric thinking ability test questions. This is understandable because the AR characteristics help students visualize

3D geometric objects directly. They can discern the shapes from various angles, including back, front, and even rotate them. The continuous visualization process molds the students' cognitive mental actions in understanding the structure of 3D geometric shapes, which leads to better test results. The explaining and elaborate phases follow the exploration stage using AR. The explain stage aims to internalize and confirm answers from the exploration results [80] through active communication between teachers and students [81]. In addition, the learners can describe the material they have been taught while the teacher provides formal definitions and academic explanations [81]. Furthermore, in the elaborate phase, the students are asked to work individually to internalize the knowledge gained from the previous stage [81]. Additionally, they are presented with new situations (e.g., real-life related problems) that are challenging [81] for them to apply the acquired knowledge and relate it to existing ones [80].

6 Conclusion

In conclusion, this study showed that learning potentials that integrate AR into 6E I3DGM facilitate the thinking process of 3D geometry in representation, spatial structuring, and measurement dimensions. This is because, first, characteristically, AR integrated interactive 3D geometry books can help students represent, visualize, construct, and form awareness of the size of 3D geometric shapes directly. In addition, the use of AR in 6E I3DGM learning can bridge students to interact directly with 3D geometry objects and minimize students' incorrectness in understanding questions and solving 3D geometry problems. The phases of 6E I3DGM (elicit, engage, explore, explain, elaborate, evaluate) also can help them develop their 3D geometric thinking abilities. In the representation dimension, the majority could represent the elements and determine the number of unit cubes that were not directly visible. They have also been able to construct 3D geometric objects from 2D nets by first identifying opposite planes and believing they are not adjacent. Meanwhile, in the spatial structuring dimension, students already understand the meaning of drawing from the front, side, and top views. They can compare the model's length, width, and height with the actual shape based on the measurement dimension. They can also determine its surface area; although some have difficulties carrying out a series of 3D geometric thinking processes, it does not mean that the learning design factors cause it. This is influenced by many other attributes, such as prior knowledge, learning, and cognitive styles, including design. The results of this study are also in line with the results of other international studies which conclude that the use of new technology in 2D and 3D geometry learning is effective for improving understanding of geometry at both the school and university level [86]–[89].

7 References

- [1] W. Roth and J.S. Thom, "The emergence of 3D geometry from children's (teacher-guided) classification tasks," *Journal Learning Science*, vol 18, no 1, pp 44–99, 2009. <https://doi.org/10.1080/10508400802581692>
- [2] C.A.M. King, "Mathematics in geography," *International Journal of Mathematical Education in Science and Technology*, vol 1, no 2, pp 185–205, 1970. <https://doi.org/10.1080/0020739700010208>

- [3] J.V. Field, “Renaissance mathematics: diagrams for geometry, astronomy, and music,” *Interdisciplinary*, vol 29, no 3, pp 259–277, 2004. <https://doi.org/10.1179/030801804225018873>
- [4] A. Wares and I. Elstak, “Origami, geometry, and art,” *International Journal of Mathematical Education in Science and Technology*, vol 48, no 2, pp 317–324, 2017. <https://doi.org/10.1080/0020739X.2016.1238521>
- [5] H. Pottmann, M. Eigensatz, A. Vaxman and J. Wallner, “Architectural geometry,” *Computers and Graphics*, vol 47, pp 145–164. <https://doi.org/10.1016/j.cag.2014.11.002>
- [6] H. Serin, “Perspectives on the teaching of geometry: teaching and learning methods,” *Journal of Educational and Training*, vol 5, no 1, pp 131–137, 2017. <https://doi.org/10.5296/jet.v5i1.12115>
- [7] I. Marchis, “Preservice primary school teachers’ elementary geometry knowledge,” *Acta Didactica Napocensia*, vol 5, no 2, pp 33–40, 2012. <https://files.eric.ed.gov/fulltext/EJ1054293.pdf>
- [8] P. Herbst, T. Fujita, S. Halverscheid and M. Weiss, “The learning and teaching of geometry in secondary schools: A modeling perspective,” 2017. <https://doi.org/10.4324/9781315267593>
- [9] K. Jones, “Issues in the teaching and learning of geometry,” In *Aspects of Teaching Secondary Mathematics: Perspectives on Practice*, pp 121–139, 2002. <https://doi.org/10.4324/9780203165874>
- [10] S. Papadakis, “Advances in mobile learning educational research (AMLER): Mobile learning as an educational reform,” *Advances in Mobile Learning Educational Research*, vol 1, no 1, pp 1–4. <https://doi.org/10.25082/AMLER.2021.01.001>
- [11] Z.A. Denizli and A. Erdoğan, “Development of a three-dimensional geometric thinking test for early graders,” *Journal on Mathematics Education*, vol 9, no 2, pp 213–226, 2018. <https://doi.org/10.22342/jme.9.2.5741.213-226>
- [12] A. Yeh and R. Nason. “Toward a semiotic framework for using technology in mathematics education: The case of learning 3D geometry,” International Conference on Computers in Education 2004. A Conference of the Asia-Pacific Society for Computers in Education (APSCE). Acquiring and Constructing Knowledge through Human-Computer Interaction Creating New Visions, pp 1191–1199, 2004. <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.148.2313&rep=rep1&type=pdf>
- [13] M. Pittalis and C. Christou, “Types of reasoning in 3D geometry thinking and their relation with spatial ability,” *Educational Studies in Mathematics*, vol 75, no 2, pp 191–212, 2010. <https://doi.org/10.1007/s10649-010-9251-8>
- [14] T. Fujita, Y. Kondo, H. Kamakura and S. Kunimune, “Students’ geometric thinking with cube representations: Assessment framework and empirical evidence,” *Journal of Mathematical Behavior*, vol 46, pp 96–111, 2017. <https://doi.org/10.1016/j.jmathb.2017.03.003>
- [15] T. Ocal and M. Halmatov, “3D geometric thinking skills of preschool children,” *International Journal of Curriculum and Instruction*, vol 13, no 2, pp 1508–1526, 2021. <http://ijci.wcci-international.org/index.php/IJCI/article/view/404/313>
- [16] C. Markopoulos, D. Potari, W. Boyd, K. Petta and M. Chaseling, “The development of primary school students’ 3D geometrical thinking within a dynamic transformation context,” *Crea*, vol 6, pp 1508–1522, 2015. <https://doi.org/10.4236/ce.2015.614151>
- [17] E. İbili, M. Çat, D. Resnyansky, S. Şahin and M. Billinghamurst, “An assessment of geometry teaching supported with augmented reality teaching materials to enhance students’ 3D geometry thinking skills,” *International Journal of Mathematical Education in Science and Technology*, vol 50, no 5, pp 1464–5211, 2019. <https://doi.org/10.1080/0020739X.2019.1583382>
- [18] S. Papadakis, M. Kalogiannakis and N. Zaranis, “Teaching mathematics with mobile devices and the Realistic Mathematical Education (RME) approach in kindergarten,” *Advances in Mobile Learning Educational Research*, vol 1, no 1, pp 5–18, 2021. <https://doi.org/10.25082/AMLER.2021.01.002>

- [19] B. Parzysz, “Knowing” vs “Seeing”. “Problems of the plane representation of space geometry figures,” *Educational Studies in Mathematics*, vol 19, no 1, pp 79–92, 1988. <https://doi.org/10.1007/BF00428386>
- [20] M. Pittalis, N. Mousoulides, and C. Christou, “Students’ 3D geometry thinking profiles,” Proceedings of CERME6, pp 816–825, 2010. <https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.1074.8742&rep=rep1&type=pdf>
- [21] M. Pittalis and C. Christou, “Coding and decoding representations of 3D shapes,” *Journal of Mathematical Behavior*, vol 32, no 3, pp 673–689, 2013. <https://doi.org/10.1016/j.jmathb.2013.08.004>
- [22] N. Cohen and G. Ben, “Curved solids nets,” *International Group for the Psychology of Mathematics Education*, vol 2, pp 229–236, 2003.
- [23] M. A. Mariotii, “Mental images: some problems related to the development of solids,” *Proceedings of the 13th International Conference for the Psychology of Mathematics Education*, vol 2, pp. 258–265 1989. <https://doi.org/10.1158/1535-7163.MCT-16-0142>
- [24] M. Kara, C.L. Eames, A.L. Cullen, C. Cullen, and J. Barret, “Developing an understanding of area measurement concepts with triangular units,” *Proceedings of the 33rd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, 1015–1023, 2011.
- [25] D.S. Hong and C. Runnalls, “Understanding length \times width \times height with modified tasks,” *International Journal of Mathematical Education in Science and Technology*, vol 51, no 4, pp 1–12, 2019. <https://doi.org/10.1080/0020739X.2019.1583383>
- [26] M.T. Battista and D.H. Clements, “Students’ understanding of three-dimensional rectangular arrays of cubes,” *Journal for Research in Mathematics Education*, vol 27, no 3, pp 258–292, 1996. <https://doi.org/10.2307/749365>
- [27] H.-M.E. Huang and H.Y. Wu, “Supporting children’s understanding of volume measurement and ability to solve volume problems: Teaching and learning,” *EURASIA Journal of Mathematics, Science and Technology Education*, vol 15, no 12, pp 1–36, 2019. <https://doi.org/10.29333/ejmste/109531>
- [28] M. Vasilyeva, C.M. Ganley, B.M. Casey, A. Dulaney, M. Tillinger, and K. Anderson, “How children determine the size of 3D structures: Investigating factors influencing strategy choice,” *Cognition and Instruction*, vol 31, no 1, pp 29–61, 2012. <https://doi.org/10.1080/07370008.2012.742086>
- [29] R.T. Azuma, “A survey of augmented reality,” *Presence: Teleoperators and Virtual Environments* 6, vol 6, no 4, pp 355–385, 1997. <https://doi.org/10.1561/1100000049>
- [30] J. Challenor and M. Ma, “A review of augmented reality applications for history education and heritage visualization,” *Multimodal Technologies and Interaction*, vol 3, no 2, 2019. <https://doi.org/10.3390/mti3020039>
- [31] J. Bacca, S. Baldiris, R. Fabregat, S. Graf, and Kinshuk. “Augmented reality trends in education: A systematic review of research and applications,” *Educational Technology and Society*, vol 17, no 4, pp 133–149, 2014.
- [32] I.A. Chicchi Giglioli, F. Pallavicini, E. Pedroli, S. Serino, and G. Riva, “Augmented reality: A brand-new challenge for the assessment and treatment of psychological disorders,” *In Computational and Mathematical Methods in Medicine*. 2015. <https://doi.org/10.1155/2015/862942>
- [33] M. Antonioli, C. Blake, and K. Sparks, “Augmented reality applications in education,” *The Journal of Technology Studies*, pp 96–107, 2014. <https://doi.org/10.4018/978-1-5225-2110-5.ch010>
- [34] S. Cuendet, Q. Bonnard, S. Do-Lenh, and P. Dillenbourg, “Designing augmented reality for the classroom,” *Computers and Education*, vol 68, pp 557–569, 2013. <https://doi.org/10.1016/j.compedu.2013.02.015>

- [35] M. Sun, X. Wu, Z. Fan, and L. Dong, “Augmented reality-based educational design for children,” *International Journal of Emerging Technologies in Learning*, vol 14, no 3, pp 51–60, 2019. <https://doi.org/10.3991/ijet.v14i03.9757>
- [36] F. Ozdamli, and C. Hursen, “An emerging technology: Augmented reality to promote learning,” *International Journal of Emerging Technologies in Learning*, vol 12, no 11, pp 121–137, 2017. <https://doi.org/10.3991/ijet.v12i11.7354>
- [37] M.T. Coimbra, T. Cardoso, and A. Mateus, “Augmented reality: An enhancer for higher education students in math is learning?,” *Procedia Computer Science*, pp 332–339, 2015. <https://doi.org/10.1016/j.procs.2015.09.277>
- [38] S. Sudirman, R.P. Yaniawati, M. Melawaty, and R. Indrawan, “Integrating ethnomathematics into augmented reality technology: Exploration, design, and implementation in geometry learning,” *In Journal of Physics: Conference Series*, vol 1521, no 3, p 032006, 2020. IOP Publishing. <https://doi.org/10.1088/1742-6596/1521/3/032006>
- [39] M. Bower, C. Howe, N. McCredie, A. Robinson, and D. Grover, “Augmented reality in education—cases, places, and potentials,” *Educational Media International*, vol 51, no 1, pp 1–15, 2014. <https://doi.org/10.1080/09523987.2014.889400>
- [40] S. Siltanen, “Theory and applications of marker-based augmented reality,” *In Julkaisija—Utgivare-Publisher*. 2012. Retrieved from <http://www.vtt.fi/publications/index.jsp>
- [41] S.M. Ramaligela, U.I. Ogbonnaya, and A. Mji, “Comparing Pre-Service Teachers’ PCK Through 9E Instructional Practice: A Case of Mathematics and Technology Pre-Service Teachers Comparing Pre-Service Teachers’ PCK Through 9E Instructional Practice: A Case of Mathematics and Technology Pre-Service,” *Te. Africa Education Review*, vol 16, no 3, pp 101–116, 2019. <https://doi.org/10.1080/18146627.2016.1241668>
- [42] M. Tezer and M. Cumhur, “Mathematics through the 5E instructional model and mathematical modelling: The geometrical objects,” *EURASIA Journal of Mathematics Science and Technology Education*, vol 13, no 8, pp 4789–4804, 2019. <https://doi.org/10.12973/eurasia.2017.00965a>
- [43] S.A. Omotayo and J. Adeleke, “The 5E instructional model: A constructivist approach for enhancing students’ learning outcomes in mathematics,” *Journal of the International Society for Teacher Education*, vol 21, no 2, pp 15–26, 2017.
- [44] A. Eisenkraft, “Expanding the 5E model,” *The Science Teacher*, vol 70, pp 5–8, 2003.
- [45] P. Swanborn, “Case study research,” *London: British Library*, 2010.
- [46] J.A. Delello, “Insights from pre-service teachers using science-based augmented reality,” *Journal of Computers in Education*, vol 1, no 4, pp 295–311, 2014. <https://doi.org/10.1007/s40692-014-0021-y>
- [47] T. Karakose, R. Yirci, and S. Papadakis, “Exploring the interrelationship between covid-19 phobia, work-family conflict, family-work conflict, and life satisfaction among school administrators for advancing sustainable management,” *Sustainability*, vol 13, no 15, pp 8654, 2021. <https://doi.org/10.3390/su13158654>
- [48] M. Akçayir, G. Akçayir, H.M. Pektaş, and M.A. Ocak, “Augmented reality in science laboratories: The effects of augmented reality on university students’ laboratory skills and attitudes toward science laboratories” *Computers in Human Behavior*, vol 57, pp 334–342, 2016. <https://doi.org/10.1016/j.chb.2015.12.054>
- [49] G. Tan-sisman and M. Aksu, “A study on sixth-grade students’ misconceptions and errors in spatial measurement: Length, area, and volume,” *International Journal of Science and Mathematics Education*, vol 14, no 7, pp 1293–1319, 2016. <https://doi.org/10.1007/s10763-015-9642-5>
- [50] R. Duval, “Geometrical pictures: Kinds of representation and specific processings,” *In Exploiting Mental Imagery with Computers in Mathematics Education*. Berlin: Springer Berlin Heidelberg. 1995. https://doi.org/10.1007/978-3-642-57771-0_10

- [51] R. Duval, "A cognitive analysis of problems of comprehension in a learning of mathematics," *Educational Studies in Mathematics*, vol 61, pp 103–131, 2006. <https://doi.org/10.1007/s10649-006-0400-z>
- [52] R. Duval, "Understanding the mathematical way of thinking—the registers of semiotic representations," 2017. <https://doi.org/10.1007/978-3-319-56910-9>
- [53] E. Fischbein, "The theory of figural concepts," *Educational Studies in Mathematics*, vol 24, no 2, pp 139–162, 1993. <https://doi.org/10.1007/BF01273689>
- [54] E. Fischbein and T. Nachlieli, "Concepts and figures in geometrical reasoning," *International Journal of Science Education*, vol 20, no 10, pp 1193–1211, 1998. <https://doi.org/10.1080/0950069980201003>
- [55] K. Jones, "Theoretical frameworks for the learning of geometrical reasoning," In *Proceedings of the British Society for Research into Learning Mathematics*, vol 18, 1998.
- [56] P.N. Johnson-Laird, "Mental models and deduction," *Trends in Cognitive Sciences*, vol 5, no 10, pp 434–442, 2001. <https://doi.org/10.1093/jos/1.3-4.371>
- [57] S.M. Kosslyn and W.L. Thompson, "When is early visual cortex activated during visual mental imagery?," *Psychological Bulletin*, vol 129, no 5, pp 723–746, 2003. <https://doi.org/10.1037/0033-2909.129.5.723>
- [58] W.X. Schneider, "Visual-spatial working memory, attention, and scene representation: A neuro-cognitive theory," *Psychological Research*, vol 62, no 2–3, pp 220–236, 1999. <https://doi.org/10.1007/s004260050052>
- [59] A. Ninio, "Piaget's theory of space perception in infancy," *Cognition*, vol 7, no 2, pp 125–144, 1979. [https://doi.org/10.1016/0010-0277\(79\)90017-9](https://doi.org/10.1016/0010-0277(79)90017-9)
- [60] G. Harel, "What is Mathematics? A pedagogical answer to a philosophical question" In *Proof and Other Dilemmas: Mathematics and Philosophy*, pp 285–290, 2008.
- [61] E. Dubinsky and M.A. McDonald, "In the Teaching and Learning of Mathematics at the University Level, Springer, Dordrecht," *APOS: A Constructivist Theory of Learning in Undergraduate Mathematics Education Research*, pp 275–282, 2001. https://doi.org/10.1007/0-306-47231-7_25
- [62] J. Block, "Assimilation, Accommodation, and the Dynamics of Personality Development," *Child Development*, vol 53, no 2, pp 281–295, 1982. <https://doi.org/10.2307/1128971>
- [63] S. Ullman, "Against direct perception," *Behavioral and Brain Sciences*, vol 3, no 3, pp 373–381, 1980. <https://doi.org/10.1017/S0140525X0000546X>
- [64] J. Kimmerle, J. Moskaliuk, A. Oeberst, and U. Cress, "Learning and collective knowledge construction with social media: A process-oriented perspective," *Educational Psychologist*, vol 50, no 2, pp 120–137, 2015. <https://doi.org/10.1080/00461520.2015.1036273>
- [65] M. Tsai and K. Lee, "A study of knowledge internalization: From the perspective of learning cycle theory," *Journal of Knowledge Management*, vol 10, no 3, pp 57–71, 2006. <https://doi.org/10.1108/13673270610670858>
- [66] K. Wipawayangkool and J.T.C. Teng, "Assessing tacit knowledge and sharing intention: A knowledge internalization perspective," *Knowledge and Process Management*, vol 23, no 3, pp 194–206, 2016. <https://doi.org/10.1002/kpm.1505>
- [67] P.G. Sidney and M.W. Alibali, "Making connections in math: activating prior knowledge analogue matters for learning," *Journal of Cognition and Development*, vol 16, no 1, pp 160–185, 2015. <https://doi.org/10.1080/15248372.2013.792091>
- [68] L. Anwar, I.K. Budayasa, S.M. Amin, and D. De Haan, "Eliciting mathematical thinking of students through realistic mathematics education," *Journal on Mathematics Education*, vol 3, no 1, pp 55–70, 2012. <https://doi.org/10.22342/jme.3.1.620.55-70>
- [69] J.D. Bostic, "Model-Eliciting activities for teaching mathematics," *Mathematics Teaching in the Middle School*, vol 18, no 5, pp 262–266, 2012. <https://doi.org/10.5951/mathteacmidscho.18.5.0262>

- [70] M. Klein, "How active involvement in learning mathematics can preclude meaningful engagement: Contributions foucault," *Pedagogy, Culture, and Society*, vol 8, no 1, pp 69–83, 2000. <https://doi.org/10.1080/14681360000200079>
- [71] N. Ingram, M. Holmes, C. Linsell, S. Livy, M. McCormick, and P. Sullivan, "Exploring an innovative approach to teaching mathematics through the use of challenging tasks: A New Zealand perspective," *Mathematics Education Research Journal*, vol 32, no 3, pp 497–522, 2020. <https://doi.org/10.1007/s13394-019-00266-1>
- [72] J.S. Bowers and B. Stephens, "Using technology to explore mathematical relationships: A framework for orienting mathematics courses for prospective teachers," *Journal of Mathematics Teacher Education*, vol 14, no 4, pp 285–304, 2011. <https://doi.org/10.1007/s10857-011-9168-x>
- [73] S. Sudirman, M. Mellawaty, P. Yaniwati, and R. Indrawan, "Integrating local wisdom forms in augmented reality application: Impact attitudes, motivations, and understanding of the geometry of pre-service mathematics teachers," *International Journal of Interactive Mobile Technologies*, vol 14, no 11, pp 91–106, 2020. <https://doi.org/10.3991/ijim.v14i11.12183>
- [74] D. Rohendi and Y. Wihardi, "Learning three-dimensional shapes in geometry using mobile-based augmented reality," *International Journal of Interactive Mobile Technologies*, vol 14, no 9, pp 48–60, 2020. <https://doi.org/10.3991/ijim.v14i09.13035>
- [75] K. Olalde, B. García, and A. Seco, "The importance of geometry combined with new techniques for augmented reality," *Procedia Computer Science*, 25, 136–143, 2013. <https://doi.org/10.1016/j.procs.2013.11.017>
- [76] P. Sarkar, K. Kadam, and J.S. Pillai, "Learners' approaches, motivation, and patterns of problem solving on lines and angles in geometry using augmented reality," *Smart Learning Environments*, vol 7, no 1, pp 1–23, 2020. <https://doi.org/10.1186/s40561-020-00124-9>
- [77] H.C.K. Lin, M.C. Chen, and C.K. Chang, "Assessing the effectiveness of learning solid geometry by using an augmented reality-assisted learning system," *Interactive Learning Environments*, vol 23, no 6, pp 799–810, 2015. <https://doi.org/10.1080/10494820.2013.817435>
- [78] N.A.A. González, "How to include augmented reality in descriptive geometry teaching," *Procedia Computer Science*, vol 75, pp 250–256, 2015. <https://doi.org/10.1016/j.procs.2015.12.245>
- [79] M.V. Voronina, Z.O. Tretyakova, E.G. Krivonozhkina, S.I. Buslaev, and G.G. Sidorenko, "Augmented reality in teaching descriptive geometry, engineering, and computer graphics-systematic review and results of the Russian teachers' experience," *Eurasia Journal of Mathematics, Science and Technology Education*, vol 15, no 12, pp 1–17, 2019. <https://doi.org/10.29333/ejmste/113503>
- [80] S.M.S. Şahin and M.H. Baturay, "The effect of the 5E-learning model supported with WebQuest media on students' achievement and satisfaction," *E-Learning and Digital Media*, vol 13, no 3–4, pp 158–175, 2016. <https://doi.org/10.1177/2042753016672903>
- [81] S. Turan and S.M. Matteson, "Middle school mathematics classrooms practice based on the 5E instructional model. *International Journal of Education in Mathematics, Science and Technology*, vol 9, no 1, pp 22–39, 2021. <https://doi.org/10.46328/ijemst.1041>
- [82] K. Saundarajan, S. Osman, J. Kumar, M. Daud, M. Abu, and M. Pairan, "Learning algebra using augmented reality: A preliminary investigation on the application of photomath for lower secondary education," *International Journal of Emerging Technologies in Learning*, vol 15, no 16, pp 123–133, 2020. <https://doi.org/10.3991/ijet.v15i16.10540>
- [83] O. Aldalalah, Z. Ababneh, A. Bawaneh, and W. Alzubi, "Effect of augmented reality and simulation on the achievement of mathematics and visual thinking among students," *International Journal of Emerging Technologies in Learning*, vol 14, no 18, pp 164–185, 2019. <https://doi.org/10.3991/ijet.v14i18.10748>

- [84] N. Rashevskaya, S. Semerikov, N. Zinonos, V. Tkachuk, and M. Shyshkina, “Using augmented reality tools in the teaching of two-dimensional plane geometry,” *CEUR Workshop Proceedings*. 2020.
- [85] S. Poultasakis, S. Papadakis, M. Kalogiannakis, and S. Psycharis, “The management of digital learning objects of natural sciences and digital experiment simulation tools by teachers,” *Advances in Mobile Learning Educational Research*, vol 1, no 2, pp 58–71, 2021. <https://doi.org/10.25082/AMLER.2021.02.002>
- [86] Z.H. Putra, N. Hermita, J.A Alim, Dahnilyah, R. Hidayat, “GeoGebra integration in elementary initial teacher training: The case of 3-D shapes,” *International Journal of Interactive Mobile Technologies*, vol 15, no 19, pp 21–32, 2021. <https://doi.org/10.3991/ijim.v15i19.23773>
- [87] S. Sumarwati, H. Fitriyani, F.M. Azhar Setiaji, M.H. Amiruddin and S.A. Jalil, “Developing mathematics learning media based on E-learning using moodle on geometry subject to improve students’ higher order thinking skills,” *International Journal of Interactive Mobile Technologies*, vol 14, no 4, pp 182–191, 2020. <https://doi.org/10.3991/ijim.v14i04.12731>
- [88] D. Juandi, Y.S. Kusumah, M. Tamur, K.S. Perbowo, M.D. Siagian, R. Sulastrri, and H.R.P. Negara, “The effectiveness of dynamic geometry software applications in learning mathematics: A meta-analysis study,” *International Journal of Interactive Mobile Technologies*, vol 15, no 2, pp 18–37, 2021. <https://doi.org/10.3991/ijim.v15i02.18853>
- [89] M.A. Al khateeb and A.M. Alduwairi, “Effect of teaching geometry by slow-motion videos on the 8th graders’ achievement,” *International Journal of Interactive Mobile Technologies*, vol 14, no 18, pp 57–67, 2020. <https://doi.org/10.3991/ijim.v14i18.12985>

8 Authors

Sudirman is a Ph.D. student at the Department of Mathematics Education, Universitas Pendidikan Indonesia. He serves as an assistant professor at the Department of Mathematics Education, Universitas Wiralodra, Indonesia. Mr. Sudirman’s primary research interests include digital learning, mobile learning, mathematics education (sudirmanstudent@upi.edu).

Yaya S. Kusumah is a Professor at the Department of Mathematics Education, Universitas Pendidikan Indonesia. Mr. Kusumah’s primary research interests include digital learning, mathematics education (yskusumah@upi.edu).

Bambang Avip Priatna Martadiputra is an Associate Professor at the Department of Mathematics Education, Universitas Pendidikan Indonesia. Mr. Martadiputra’s primary research interests include statistics, mathematics education (bambangavip@upi.edu).

Article submitted 2021-10-24. Resubmitted 2022-01-05. Final acceptance 2022-01-05. Final version published as submitted by the authors.