Energy system optimization including carbon-negative technologies for a high-density mixed-use development

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ABSTRACT

In this paper, we use the 'energy hub' optimization model to perform a multi-objective analysis on a high-density mixed-use development (termed the 'mothership') under different scenarios and compare these results to appropriate base cases. These scenarios explore how the optimal energy system changes under different assumptions, including a high carbon tax, net metering, net-zero emissions and negative emissions, as well as two different electrical grid carbon intensities. We also include 'carbon negative' technologies involving biochar production, to explore the role that such processes can play in reducing the net emissions of energy systems,

The annualized cost and total emissions of the mothership with a simple energy system are 4 and 8.7 times lower respectively than a base case using single detached homes housing the same population, due to the more efficient form and hence lower energy demand. Of the scenarios examined, it is notable that the case with the lowest annualized cost was one with a net-zero carbon emissions restriction. This gave an annualized cost of CAD 2.98M, which 36% lower than the base case annualized cost of CAD 4.66M. This relied upon the carbon negative production and sale of biochar. All scenarios examined had lower annualized costs than the base cases with many of the cases having negative operating costs (generating profit) due to the sale of renewable energy or carbon credits. This illustrates that the integration of renewable energy technologies is not only beneficial for reducing emissions but can also provide an income stream. These results give hope that suitably optimized urban developments may be able to implement low cost solutions that have zero net emissions.

1. Introduction

1.1. Background

Urban populations around the world are growing, so cities must expand or densify [1]. In North America, much of this growth is in the form of urban sprawl. Urban sprawl is characterized by single use type developments, typically single detached homes, where transportation is dominated by personal vehicle use [2]. Single detached homes are less energy efficient than other denser forms of housing, due to higher surface area to volume ratio, meaning more area for heat transfer, as well as the greater overall floor area, number of appliances etc. Single dwellings also use more resources to build than higher density residential buildings to house the same number of residents.

In Bowley et al. [3] we propose a potential solution: a high-density mixed-use building that we term a Mothership, designed to contain all amenities of a typical suburb for 10,000 residents in one large building. Advantages of this style of building includes reduced surface area for heat transfer, more practical use of high-performance building envelope. There are also many advantages in terms of reduced emissions from

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Nomenclature

P	Energy input, kW
P	Price, CAD /kWh
J	converters, n/a
T	timestep, hours
K	Storage capacity, kWh
AEC	Annual equivalent cost, CAD
F	Emissions factor, $kgCO_{2e}$/kWh
L	Load to be met, kW
Θ	Converter efficiency, %
Q^-	Discharge (flow rate), kW
Q^+	Charge (flow rate), kW
ε^-	Discharge efficiency, %
ε^+	Charge efficiency, %
I	Time series representing the availability of an energy source
E	Total energy stored in a storage technology, kWh
η	Decay loss of energy in a storage technology, kW
SUF	Storage Utilization Factor

transportation: co-location of amenities eliminates many trips, and a public transportation hub and an electric vehicle car share fleet reduce the use of personal vehicles.

The emissions sources of an urban area are largely from building operation, the emissions embodied in the materials of the buildings, and transportation emissions. There are numerous ways to reduce the emissions from these sources. High performance building envelopes can reduce heating and cooling loads, which could then be met with renewable energy and heat pumps. The embodied emissions in buildings can be reduced through minimizing the use of cement, either through reducing concrete use, or using supplementary cementitious materials such as fly-ash instead of cement. Transportation emissions could be lowered through numerous ways including public transportation measures, eliminating vehicle trips by creating walkable neighbourhoods, or using electric vehicles powered with clean energy.

It is rare however, to reduce these energy demands to zero, especially in colder climates with high heating demand. Therefore, it is important that these remaining minimized loads be satisfied in the most efficient, cleanest, and cost-effective manner. There are many potential technologies to choose from, each with advantages and disadvantages, from simple gas boilers and heat pumps, to more complex combined heat and power systems. There is potential to implement promising emerging technologies, and even negative emissions technologies that sequester more carbon than they emit.

One such technology is char optimized pyrolysis, which can be used for boilers or combined heat and power plants. Using biomass as a feedstock, it heats it up in the absence of oxygen, which thermally decomposes the volatile organic compounds, leaving behind the structure of almost pure carbon or char. Depending on the conditions of the pyrolysis, about 50% of the carbon of the feedstock is converted to char[4]. This can be used in agriculture [5] [6] , water filtration, and other applications.

The carbon in this char form is recalcitrant, meaning it is stable and will stay in that form for potentially hundreds to thousands of years depending on conditions [7]. As a result, biochar (so called when char is applied to soils) producing systems is considered a negative emissions technology by the IPCC if the carbon is sequestered and not subsequently burnt [8] [9]. The other 50% of the carbon is released as pyrolysis oils and gases that can be combusted for energy and to provide the process heat to perpetuate the pyrolysis.

There is also the potential to integrate renewable energy generation technologies and storage systems with the building. There is a significant amount of roof area for solar collectors, either solar photovoltaic or solar thermal collectors. Different storage technologies such as hot water thermal energy storage, traditional lead acid and lithium ion batteries, compressed air, and hydrogen. Some technologies like hydrogen, do have a higher cost, but have the additional advantage that you can also sell the hydrogen as well as store it, providing an additional income stream.

1.2. Literature review

Multi-objective optimization applied to energy-related aspects of building design is becoming more common as a process to lower costs, energy use and emissions [10]. This can be used to vary many properties of the buildings themselves, for example envelope properties, massing and glazing areas. However, often such decisions are taken for aesthetic or practical reasons, which are hard to incorporate into a computational analysis.

Complex buildings with a mix of uses, complex energy systems or finite renewable sources of energy require an optimization process that can balance demands and supplies of energy at each moment. One method for doing this is the 'energy hub' model originally proposed by [11]. This uses mixed integer linear programming (MILP) to find combinations of technologies (renewable generation, storage, energy converters, etc.) that best

meet a specified design goal defined by the objective function. More recent formulations [12] have extended the model formulation.

Energy hubs, or similar models have been used many times before. Krause et al [13] discuss how energy hubs can be used to optimize energy systems in a variety of scenarios with multiple energy carriers. They also discuss some of the benefits of using this model's framework. Brahman et al [14] apply an energy hub to a residential building, integrating electric vehicle charging and other types of demands. Best et al [15] models and optimizes the energy systems for an urban area using a similar model to the energy hub.

Orehounig et al [16] use the energy hub model to decentralized energy system at neighbourhood scale. Zhang et al. [17] use MILP to determine optimal integrated energy system configurations and simulate operation in a Swedish building. Niu et al. [18] use MILP to optimize the use of thermal and electrical energy storage and how it interacts with the energy grid. Setlhaolo et al. [19] model the interaction between co-generation, solar PV, and energy storage interact with the electricity grid using an energy hub framework to lower CO_2 emissions for residential building.

Raza et al. [20] use an energy hub model to assess costs and operation of a biogas supported energy system using particle swarm optimization. Farshidian et al. [21] models a multi-hub configuration considering the competition between hubs and the planning implications thereof.

This work focuses on applying an energy hub model to a large mixed-use building which combines load patterns from residential, retail, and office spaces together. It also introduces a material flow, rather than only energy flows, to the model, which has not been done before to the best of the authors knowledge. Additionally, the breadth of technologies considered in this analysis is significantly larger than is usually considered in the above papers. Potential combinations of these technologies are evaluated for different economical and environmental constraints, optimized for lowest cost, and emissions.

1.3. Contributions and structure of this paper

In this paper, we explore the benefits of high-density mixed-use development related to the energy systems that provide power and heat, with the mothership serving as an example of any form of high-density mixed-use development. The size of the loads and the range of different demand profiles present can enable district-scale

energy systems that aid renewable energy integration, without the expense and complexity of traditional district heating networks. Because one energy system can serve the development, combinations of multiple technologies can be used, whereas for individual smaller buildings this would be impractical.

This makes it more challenging to find the correct combination and sizes of technologies that provide a balance between the most cost-effective option and the option with the lowest carbon emissions. This cannot be determined in advance without examining the hour-by-hour requirements and availability of many different energy streams. The 'energy hub' model formulation is used to achieve this, by optimizing a proposed energy system for the predicted loads of the mothership. This is conducted as a multi-objective optimisation that can explore the balance between the lowest overall cost and low carbon emissions for a variety of options.

In addition to finding the optimal energy system design for a general scenario, additional scenarios will be explored to see how this optimum changes in response to these additional constraints. These scenarios will be created to answer the following research questions:

- What is the most cost-effective energy system to meet the required loads?
- What is the optimal capacity of solar PV or solar thermal? Is the rooftop area sufficient or would more space be desirable?
- Does seasonal storage at this scale make sense? Would the storage size be too large to be practical?
- What is the impact of hydrogen production and storage? Is it used for storage or for export?
- Do biochar technologies get used? What is the impact of carbon negative power and heat production?
- What is the impact of a strict carbon budget, such as being net-zero carbon? What if a negative carbon budget was enforced, meaning that carbon is sequestered each year?
- What is the effect of carbon credits and carbon taxes? What is the threshold for fossil fuels to be avoided?

The core argument of this paper is that the energy system of a high-density mixed-use development can be much more efficient, cheaper and have fewer emissions than the base case of single detached homes housing the same population. This paper presents a comprehensive analysis of the energy systems options available for a

large high-density mixed-use development, and propose new developments to the energy hub model formulation to facilitate this. The new developments are the formulation of a storage utilization factor, to describe how much a storage technology is used in the system, and the use of materials streams alongside energy streams, to capture the benefit of carbon-negative technologies. These are detailed in the methodology section.

Next, we first establish a reference case based on a standard expansive single-dwelling development, then compare this to various high-density cases using the mothership concept as an example. We examine the impact of many different exogenous factors such as carbon taxes and technology availability that affect the optimal system configuration, assessing the differences in cost and emissions. Finally, conclusions are drawn regarding the performance of different energy systems options for a high-density mixed-use development.

2. Methods

This analysis uses an energy hub model to explore the design goals of low costs but also low carbon emissions. The analysis process is outlined in Figure 1. First, heating, cooling, appliance, lighting, and hot water loads for proposed designs are calculated using the building energy simulation tool called the Urban Modeling Interface (UMI) [22]. This calculates loads based on building geometry created using Grasshopper [23], a parametric extension of the Rhinoceros 5 [24] computer aided design software. These hourly-resolution annual time series (summarized in Table 1) are then used as loads that need to be satisfied in energy hub models.

The buildings modelled are sized to house 10,000 residents at 40 m^2 floor area per resident, as well as 50,000 m^2 each of office and commercial space. Data for the technologies was gathered from a variety of sources including papers cited in the literature review, manufacturer websites, and discussions with industry professionals. The breadth of scenarios explored as part of the analysis was used to understand the sensitivity of the model to different parameters and inputs.

2.1. Energy hub models

This paper uses the energy hub model formulation of Evins et al [12], a summary of which is given in this section. For more information, readers are referred to the paper. The general summary of the model is that there are energy demands that need to be met at each time step. There are energy sources such as grid electricity, natural gas, solar radiation, etc. In between there are technologies which convert one type of energy stream into one or more other streams. There are also storage technologies which can store certain energy streams for later use. The model then creates a system of linear equations made up of constraints which it attempts to solve.

The key equations and constraints are outlined below (with slightly updated nomenclature).

$$\text{Cost} = \sum_{t,j} p_j P_j(t) + AEC \tag{1a}$$

$$\left(\sum_j C_j P_j^{capacity} + \sum_k C_k E_k^{capacity} \right)$$

Table 1: The annual sum and peak loads for the different load types for the base case buildings and the mothership.

		Heating	Cooling	Hot Water	Lighting	Equipment
Individual Single Detached	Sum [kWh]	13,609	8,186	8,099	2,708	3,278
	Peak [kW]	8	37	4	1	1
Retail	Sum [kWh]	249,245	1,412,942	287,988	3,442,240	1,290,840
	Peak [kW]	975	1,767	82	800	300
Office	Sum [kWh]	1,290,247	8,817	762,187	1,945,200	1,348,800
	Peak [kW]	923	786	285	600	400
All Single detached with Retail and Office	Sum [kWh]	58,154,750	35,476,825	34,741,709	16,651,056	16,277,784
	Peak [kW]	34,804	153,616	18,678	5,319	3,628
Mothership	Sum [kWh]	4,315,693	1,295,663	17,493,837	10,943,972	9,321,953
	Peak [kW]	4,161	5,201	9,223	2,940	1,927
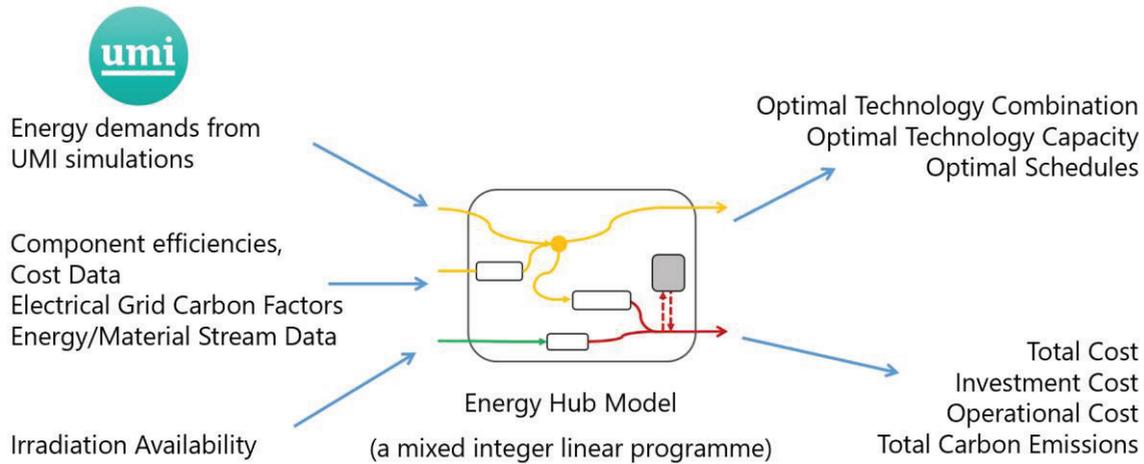
Figure 1: Analysis flow chart, inputs on the left of central figure, and outputs on the right.

$$\text{Emissions} = \sum_{t,j} F_j P_j(t) \tag{1b}$$

$$L_i(t) = \theta_{i,j} P_j(t) + \epsilon_k^- Q_k^-(t) - \varepsilon_k^+ Q_k^+(t) \tag{2}$$

$$\frac{P_j(t)}{P_j^{capacity}} \leq I_j^{max}(t) \tag{3}$$

$$E_k(t+1) = (1-\eta_k)E_k(t) + Q_k^+(t) - Q_k^-(t) \tag{4}$$

$$0 \leq P_j(t) \leq P_j^{capacity} \tag{5}$$

$$0 \leq E_k(t) \leq E_k^{capacity} \tag{6}$$

$$0 \leq Q_k^-(t) \leq Q_k^{-max} \tag{7}$$

$$0 \leq Q_k^+(t) \leq Q_k^{+max} \tag{8}$$

$$0 \leq P_j^{capacity} \leq P_j^{capacity\ limit} \tag{9}$$

Equations 1a and 1b define the two possible objective functions of the optimization problem, to minimize costs (in Canadian dollars) and carbon emissions respectively. In 1a the operating cost is the energy input P times price p, summed over all converters j in the system and all time steps t, plus annual equivalent cost (AEC) of the capital costs, which multiply capacities by costs C for all converters j and storages k. In 1b the total carbon emissions are calculated from the energy

inputs and the emissions factor F associated with that energy stream.

Equation 2 is the core energy balance, stating that the load L to be met must equal the output from each converter (input energy P times the efficiency θ), energy from storage (discharge Q^- times discharge efficiency ε^-) minus the energy used to charge the storage (charge Q^+ times charging efficiency ε^+). The availability of energy is sometimes limited, for example irradiation to PV panels, which is defined as a time series I in equation 3.

Equation 4 enforces the storage continuity: the state of the storage E is equal to the state at the last time step (minus the decay loss η) plus any charge minus any discharge. Equations 5 and 6 ensure that converters and storages operate below their capacities, and equations 7 and 8 do the same for storage charging and discharging rates. Finally, Equation 9 turns the capacities of converters into optimization variables themselves, which can be varied up to a fixed capacity limit.

Minimum loads were not included, as the model formulation required for this increases the model runtime dramatically (see [12]). Fixed capital costs and maintenance costs were also not included, though could be easily incorporated in Equation 1a. Storage capacities are fixed rather than optimized. Ideally, the capacity of the storage technologies would be optimized along with the converter capacities. However, the computational time of the model goes up dramatically with the addition of more storage technologies. This is because the storage equations mean that the energy flows at each time step are dependent on storage state at the previous and next steps, so the model takes a very long time assessing

whether it is better to store the energy for later use or not.

Giving wide capacity ranges for multiple storages with different efficiencies and costs makes this problem much more convoluted. The run time for the hard-coded storage capacity models are many orders of magnitude shorter. The cost of the unused portion of each storage technology is subtracted from the total cost after the optimization is completed. This is not a true replacement for an optimization in which the storage capacity is a variable to be optimized, but it is a reasonable approximation that retains a reasonable run time.

The energy hub models in this paper are implemented in PyEHub[1]. PyEHub is an energy hub modelling library written in Python that forms part of the Building Energy Simulation, and Optimization and Surrogate (BESOS) modeling platform[2]. PyEHub performs MILP optimization using IBM CPlex via intermediate python libraries (PyLP and PULP).

2.2. Storage utilization factor

In order to evaluate the utility of storage technologies in the energy system, including how much they were used, we define a 'storage utilization factor' (SUF) as the sum of the discharge from the storage (kWh) for each hour of the year, divided by the capacity of the storage technology (kWh). This is shown in Equation 11.

$$SUF_i = \frac{\sum_t Q_-^i}{E_{max}^i} \qquad (11)$$

This factor, which is analogous to the capacity factor used for renewable generation technologies, gives an indication of how much the storage is used. For example, SUF=100 means that overall the storage discharges fully 100 times per year, or cycles from full to 50% and back 200 times per year. Larger values indicate that the storage is being utilized more, however it does not indicate the manner in which it is used (lots of short charging and discharging cycles vs. fewer larger ones), nor the effectiveness of this utilization at reducing costs.

2.3. Materials streams

This paper extends the energy balancing and conversion performed in the energy hub model to include a material stream for a carbon-negative material called *char*. Carbonization uses the same underlying pyrolysis process as gasification, but is optimized for different purposes, with gasification producing mostly gas and carbonization producing a charcoal-like product called char.

The advantage of gasification is that nearly all the biomass is consumed in the process and converted to energy, meaning solid waste is low and energy per unit feedstock is relatively high. However, there are still carbon emissions associated with this process, even though many would consider it carbon neutral. Carbonization, depending on the feedstock and the process parameters, converts about 50% of the carbon from the biomass into the char; the other half is eventually converted into carbon dioxide. As a result, the energy produced per unit of feedstock is lower, but the carbon in the char is recalcitrant, meaning it is stable and won't be released into the atmosphere over time. This provides interesting opportunities to get carbon credits as part of the revenue stream as well as selling the char itself.

Carbonization does have the downside that it requires more feedstock than gasification to produce the same amount of energy because it doesn't utilize feedstock entirely for energy. Both gasification and carbonization systems are included in the potential technologies. Char can be sold as an expert for money and carbon credits in the model.

3. Analysis Cases

In this paper, we compare a standard low-rise expansive development without advanced energy systems with the energy systems options available for a high-density mixed-use case, using the mothership as an example of the latter. Both cases consist of residential space for 10,000 people, plus 50,000m² each of retail and office space.

Each of these building types will have individual energy hub models, and in the single detached case, the results will be scaled based on the number of homes that are required. For the mothership case, there will be one model for the combined residential, retail and office spaces, since they are all in the same building. The retail and office floor area in the base case and the mothership are the same. The residential floor area is not, because the floor area per resident ratio for single detached homes is much higher than that for apartment style residential spaces.
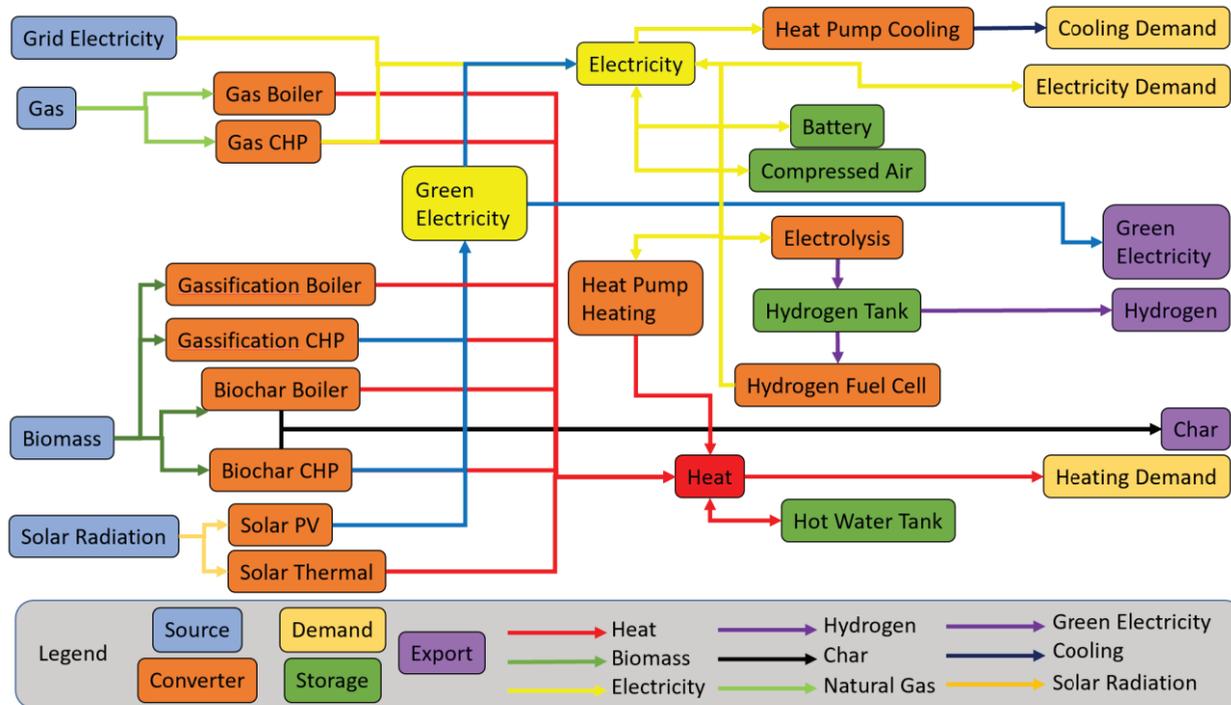
Figure 2: The configuration of the overall system to be optimized using the energy hub model, showing all possible storage and conversion technologies, as well as the different energy and material streams and how they are connected. Blue boxes on the left of the figure indicate input energy streams that are converted (orange boxes) and stored (green boxes), eventually to supply the demands in the tan coloured boxes on the right side of the figure. The purple boxes indicate exports that can be sold to provide income and carbon credits. The lines indicate energy or material flows. The technologies shown are all those that are available for the model to choose from and aren't necessarily used in the optimal solutions.

The configuration of the energy system to be optimized for the mothership is shown in Figure 2, giving all possible converters (orange) and storage technologies (green) along with the energy and material streams that connect them. This configuration is defined by the inputs to the energy hub model that govern the input and output streams of each converter and storage, which are discussed in more detail in the following sections.

3.1. Converters
Converters are technologies that change energy (or in this case also materials) from one form to another. Table 2 gives the properties of the converters included in the model. Many typical technologies are provided, including heat pumps, gas boilers, gas-powered combined heat and power (CHP) systems, photovoltaic (PV) panels and solar thermal collectors. These are relatively common and mature technologies. Other technologies that are less mature include biomass gasification (for a boiler or CHP) and hydrogen electrolyzer and fuel cell components. Finally, the highly novel carbonization

technologies are included to generate heat for a boiler or CHP system as well as making carbon-negative char as an output.

Table 2 shows the capital cost per kW capacity of each technology (C in Equation 1a), the efficiency (θ in Equation 2), the lifetime used to calculate the Annual Equivalent Cost, the input energy stream, the output energy stream(s), and the maximum capacity ($P^{capacity-limit}$ in Equation 9). If more than one output stream is produced by the converter, the ratio is given in brackets, for example the CHP produces 1.73 units of heat for every unit of electricity. Max capacity for technologies is unlimited, except for PV and solar thermal capacity which is limited by roof area depending the scenario.

It may be noted that small scale wind generation is not included as a potential generating technology. This is because small scale wind turbines are not as cost effective as large scale wind, or other renewable technologies. This is especially true in urban environments where building/turbine height is limited, and wind is often blocked by surrounding buildings and trees.

Table 2: Converter technology properties. If more than one output stream is produced by the converter, the ratio is given in brackets, for example the CHP produces 1.73 units of heat for every unit of electricity.

	Capital cost (CAD/kW)	Efficiency	Lifetime (years)	Input	Output(s) (output ratio in brackets)
Grid connection	0.1	1	1000	Grid purchase	Elec
Air-source Heat Pump	1400	3.2	20	Elec	Heat
Chiller	1500	3.2	20	Elec	Cooling
Gas Boiler	500	0.94	30	Gas	Heat
MicroCHP	3400	0.7	20	Gas	Heat (1), Elec (0.16)
PV panels	2000	1[1]	20	Irradiation	Green Elec
Solar thermal panels	2000	1.5	35	Irradiation	Heat
CHP	2275	0.3	20	Gas	Elec (1), Heat (1.73)
Ground-source Heat Pump	2777	6	50	Elec	Heat
Biomass CHP	6227	0.3	20	Biomass (Gasification)	Green Elec (1), Heat (1.2)
Biomass Boiler	4567	0.85	30	Biomass (Gasification)	Heat
Biochar Boiler	5023	0.75	30	Biomass (Pyrolysis)	Heat (1), Char (0.07)
Biochar CHP	6850	0.29	20	Biomass (Pyrolysis)	Green Elec (1), Heat (2.3), Char (0.2)
Electrolyser	5902	0.92	15	Elec	Hydrogen
Hydrogen Fuel Cell	4719	0.4	15	Hydrogen	Elec

[1] = 1.038 kW$_{peak}$/m^2 * 0.20 panel efficiency*0.9 system efficiency*5.56 m^2/kwh installed capacity

Table 3: Storage technology properties.

	Lead-Acid battery	Li-Ion battery	Hot water	Compressed air	Hydrogen
Energy Stream	Elec	Elec	Heat	Elec	Hydrogen
Capacity (MWh)	10	10	26900	10	10
Capital cost (CAD/kWh)	390	272	1.33	78	20
Lifetime (years)	20	10	20	30	20
Charging efficiency	0.99	0.8	0.99	0.8	0.75
Discharging efficiency	0.99	1	0.99	1	1
Decay efficiency	0.001	0.001	0.001	0.001	0
Max charging rate	0.3	0.3	0.3	0.5	1
Max discharging rate	0.3	0.3	0.3	0.5	1

3.2. Storage technologies

The storage technologies that could be used in the model are shown in Table 3. The five options used standard lead-acid and lithium-ion batteries, a hot water tank, and more novel options like compressed air storage and a hydrogen storage tank. The table gives the stream that the technology can store, capital cost per kWh capacity of each technology (*C* in Equation 1a), the lifetime used to calculate the Annual Equivalent Cost, the efficiencies (ε^+, ε^- and η in Equations 2 and 4), and the maximum charge and discharge rates (Q$^{- max}$ and Q$^{+ max}$ in

Equations 7 and 8). As discussed in the previous section, costs are updated after the optimization to remove the cost of any unused storage capacity.

3.3. Energy and Material Streams

The streams that are used in this analysis are show in Table 4. Streams are flows of energy or materials that are converted or stored by one of the converters or storages respectively. They can also be imported or exported, as indicated by the presence of purchase price / carbon factor values and export price / carbon credit values respectively.

Table 4: Purchase price, export price, carbon factor and carbon credit of each energy and material stream..

Name	Grid purchase	Gas	Green Elec	Biomass (Gasification)	Biomass (Pyrolysis)	Char	Hydrogen
Purchase price (CAD/kWh)	0.14	0.038		0.04	0.04		
Export price (CAD/kWh)			0.14			1.266	0.469
Carbon factor (kg CO2/kWh)	0.14	0.21		0	0		
Carbon credit (kg CO2/kWh)			0.14			2.6	0.14

The grid carbon factor for the simulations was the Canadian average, which is still relatively low at 0.14 kg CO_2/kWh. Electricity produced by PV panels, biomass CHP or biochar CHP is denoted 'Green Elec', meaning that if it is exported it receives a carbon credit. Hydrogen can also be exported for hydrogen powered vehicles and receives a carbon credit equal to the carbon intensity of natural gas. Units are calculated in kWh, so all streams are assessed in terms of energy content rather than for example by weight.

3.4. Scenarios

Base cases

There are three base cases to provide a baseline to compare the other mothership cases to. Base Case A and B are modelled with single detached home models and are meant to be the base cases that the motherships are compared to, as busines as usual cases. This shows the benefits on the different urban form as well as the energy systems. Base Case C uses the mothership building loads, but uses the same energy systems as Base Case A. This case is meant to isolate the effect of urban form and energy systems, ignoring the effect of building form. The details of each case are as follows:

A. This case takes the peak and total heat, electrical, and cooling loads and sizes a gas boiler, grid, and cooling heat pump to those loads and calculates the costs and emissions. The loads for a single house are scaled by 4160 to get the loads for all the houses, and this is added to the loads for the retail and office base case buildings. There is no PV or storages installed, the Canadian grid factor is used, and there is no carbon tax or credits.

B. This case uses the same loads as Case A, however it runs separate optimization models for each of the single detached, office and retail buildings. Like Case A, the single detached loads are scaled and added to the retail and office loads. Storages are installed with sizes of 1000kWh for each, and PV is also allowed.

C. This case does the same scenario as Case A, but uses the mothership's loads, satisfying them with gas boilers, grid electricity and cooling heat pump. No PV or storages are installed.

Mothership cases

Below we outline the main scenarios to be explored in addition to the base case, in order to address the questions posed in the introduction:

1. **Small storages:** 1,000 kWh each; Roof area PV capacity of 16,000 kW. PV capacity determined by dividing roof area of 50,000 m^2 by area of each panel (1.6m^2/panel), multiplied by the wattage of the panel (300W).

2. **Big storages:** Same as Case 1, but with the storage capacities listed in *Table 1*.

3. **Net-zero:** Same as Case 2 with maximum emissions of 0 kgCO$_2$/a, i.e. net-zero in operational emissions.

4. **Carbon negative:** Same as Case 2 with maximum emissions of -10,000,000 kgCO$_2$/a, i.e. sequestering or offsetting one ton of CO_2 per resident per year.

5. **Carbon neutral, net metering:** Same as Case 3, but with the constraint that yearly electricity exports must be equal to or less than grid imports.

6. **Carbon tax:** Same as Case 2 but with a carbon tax of CAD 200/t CO_2.

7. **BC grid factor:** Same as Case 2, with a grid carbon factor of 0.009

8. **BC grid factor, carbon tax:** Same as Case 7 but with a CAD 200/t CO_2 carbon tax.

9. **BC grid factor, hydrogen export:** Same as Case 7, but with hydrogen exportable at CAD0.2/kWh
10. **BC grid factor, net metering:** Same as Case 7, but with the constraint that exported electricity can't be higher than grid imports.
11. **BC grid factor, carbon neutral:** Same as Case 7, but with maximum emissions of 0 kgCO$_2$/a.
12. **Unlimited PV:** Same as Case 2, but unlimited PV capacity (capped at 999,999,999 kW due to model limits).

4. Results

Table 5 shows the results of the energy system optimization giving the metrics of cost and emissions and the optimal converter capacities, as well as the important input parameters that change between each case. The colours show a red to green gradient in each column separately to visually show differences in the results and variable inputs for each of the scenarios. The colours generally show more red being negative in impact, such

Table 5: Shows the results of the energy system optimization giving the metrics of cost and emissions and the optimal converter capacities, as well as the important input parameters that change between each case. The Retail, Office and Single detached cases are the optimization results for individual building loads. Base Cases A, B, and C and cases 1 through 11 are the results for scenarios described previously. The results for Case 12 are not shown due to the unlimited solar capacity giving very unreasonable values.

	Total Cost, $M	Total Carbon Emissions, Mkg CO2	Operating Cost, $M/a	Investment Cost, $M	Lead-Acid SUF	Li-Ion SUF	Compressed air SUF	Hot water SUF	BioCharCHP Capacity, kW	Grid Capacity, kW	HP cooling Capacity, kW	GSHP Capacity, kW	MicroCHP Capacity, kW	Gas Boiler	PV Capacity, kW	GreenElec Carbon Credit, kgCO2/kWh	Hydrogen Carbon Credit, kgCO2/kWh	Gas Cost, $/kWh	Grid Cost, $/kWh	Hydrogen Export Price, $/kWh
Retail	0.75	0.71	0.18	0.61	334	10	21	1,203	-	1,127	332	192	350	-	1,500	0.14	0.14	0.04	0.14	-
Office	0.57	0.52	0.01	0.57	226	51	77	231	-	835	151	102	360	-	1,500	0.14	0.14	0.04	0.14	-
Single detached	0.00 -	0.00 -	0.00	0.09	2.6	-	-	5.4	-	3.2	6.8	0.8	1.8	-	74,880	0.14	0.14	0.04	0.14	-
Base Case A	19.78	26.92	9.92	9.86					-	580	48,005	-	-	56,896	77,880	0.14	0.14	0.04	0.14	-
Base Case B	21.37 -	0.96 -	13.54	34.91					-	15,269	28,757	3,418	8,220	-	######	0.14	0.14	0.04	0.14	-
Base Case C	4.66	7.77	3.78	0.88					-	6,492	1,625	-	-	14,238	-	0.14	0.14	0.04	0.14	-
1 Small storages	3.53	2.10 -	1.38	4.91	273	230	239	831	-	4,210	1,237	2,061	2,610	-	16,000	0.14	0.14	0.04	0.14	-
2 Big storage	3.45	3.59 -	2.18	9.42	362	17	21	0.47	-	2,783	1,237	552	3,765	-	16,000	0.14	0.14	0.04	0.14	-
3 Net zero	3.57	- -	3.55	10.91	359	21	19	0.47	2,173	2,794	1,237	533	3,721	-	16,000	0.14	0.14	0.04	0.14	-
4 Carbon negative	3.96 -	10.00 -	7.40	15.12	352	25	24	0.47	-	2,791	1,237	478	3,648	-	16,000	0.14	0.14	0.04	0.14	-
5 Net zero, net metering	3.65	- -	0.17	8.11	145	-	-	0.40	3,479	3,726	1,237	542	2,405	-	-	0.14	0.14	0.04	0.14	-
6 Carbon tax	4.22 -	0.04	0.20	8.13	285	-	-	0.42	-	5,496	1,237	629	-	-	-	0.14	0.14	0.08	0.17	-
7 BC grid factor	3.37	6.64 -	2.18	9.40	361	15	20	0.46	-	2,811	1,237	551	3,724	-	16,000	0.01	0.21	0.04	0.12	-
8 BC grid factor, carbon tax	3.46 -	3.21 -	0.16	8.13	-	-	-	0.40	-	5,490	1,237	623	-	-	16,000	0.01	0.21	0.08	0.14	-
9 BC grid factor, hydrogen export	3.36	6.64 -	2.18	9.40	362	15	19	0.46	-	2,811	1,237	551	3,724	-	16,000	0.01	0.21	0.04	0.12	0.20
10 BC grid factor, net metering	3.38	5.94	0.98	6.48	290	-	-	0.42	-	3,164	1,237	558	3,230	-	2,502	0.01	0.21	0.04	0.12	-
11 BC grid factor, net zero	2.98	- -	2.70	10.18	-	-	-	0.40	2,176	4,330	1,237	570	1,582	-	16,000	0.01	0.21	0.04	0.12	-

as higher cost or CO_2 emissions, whereas green shows lower cost or emissions. Each row shows a model run scenario, and each column shows an output or input parameter. The input parameters that remain static throughout all simulations are given in the analysis cases and scenario descriptions in the previous section. The total cost values account for the cost for unused storage capacity, since these had to be set manually for each run, and the full capacity may not have been used. The results for Case 12 are not shown, due to the unlimited solar capacity giving unreasonable values.

The base case of single detached homes and separate retail and office buildings are given individually and in combination to give a basis for comparison for the mothership scenarios. The combined loads of the base case buildings are much higher than the mothership: 13.4, 1.6, and 27 times higher for heating, electrical and cooling loads respectively. Therefore, the investment costs and the emissions are much higher.

For Base Case B, the one advantage that the base case has over the mothership is the greater total roof surface area available, permitting a total solar PV capacity of 78,000 kW as opposed to 16,000 kW for the mothership, resulting in much more power sold to the grid and reduced operating costs. The total cost of the energy systems in single detached homes scaled to 10,000 residents (4,160 homes) is almost CAD 21 million (of which almost CAD 15.8 million is for PV), which is much higher than any of the mothership cases. However, this case has negative carbon emissions, due to the large amount of green electricity from solar PV that is sold to the grid and the associated carbon credits received.

The retail and office base cases also made good use of solar PV, however they did not achieve negative emissions, due to their heavy use of natural gas.

It should be noted that it may be impractical to install very large PV systems in urban areas in British Columbia, where the utility restricts the export of solar electricity in order to maintain the integrity of the electricity grid. This makes it more difficult to build a system for a building that produces more power than it uses in a typical year. For the same reason, results are not presented for the mothership case in which the PV capacity was unlimited, as this model attempts to install an infinite capacity of PV to generate a profit even though there is not the roof space to do so. The impact of specific PV limits is investigated in the net-metering case (scenario 10).

In the simple cases of base case A and C, comparing the mothership to the single detached homes case, the mothership has much lower costs, simply due to the smaller magnitude of its energy demands and economy of scale in it's systems. Case A costs over four times as much and emits 3.5 times as much carbon dioxide as Case C.

In the following sections we discuss the answers to the research questions posed in the introduction.
- What is the most cost-effective energy system to meet the required loads?

The most cost effective option, other than the unlimited solar PV case which is unrealistic, is Case 11, which is a net zero carbon emissions case, with a total annual equivalent cost of just under 3 million. One reason for this is the use of the biochar CHP and the sale of the char and PV electricity. The most expensive scenario is unsurprisingly the case with the high carbon tax at CAD4.2 million. It is interesting to note however, that the yearly operating cost is negative for most of the cases that do not restrict the selling of green electricity and char. So although the investment costs are high, the building can make a profit from the sale of energy and carbon sequestration.

Case 10 with net metering has relatively low total costs, likely due to the limited allowable solar capacity installed, reducing capital costs. However it also doesn't benefit from the lase of the electricity and has positive operating costs.

Base case C, the simple mothership energy system that doesn't allow pv or storage, has a higher cost and higher emissions compared to the other mothrship cases. Additionally it has no form of income, so its operational costs are much higher. This illustrates that integrating renewable energy technologies is not only helpful for reducing emissions, but can have significant financial advantages.
- What is the optimal capacity of solar PV or solar thermal? Is the rooftop area sufficient or would more space be desirable?

The model never selects solar thermal in any of the runs. This is potentially due to solar PV being more versatile, in that the system can use the electricity to create heat or cooling through heat pumps, use it directly, or sell it and potentially earn export income and carbon credits.

The model uses the maximum PV capacity permitted in all simulations except for cases 10 and 5 due to net metering, and case 6 with the carbon tax. When size is limited to that of the mothership roof area, the maximum permitted capacity is installed. In Case 10 with net

metering, the optimal PV capacity is found to be 2,582 kW, due to the restrictions on how much power can be sold to the grid. Interestingly the model decided to not install PV in case 5 or 6, possibly due to the already high costs of the biochar tech needed for reducing emissions. As noted above, results are not shown for Case 12, where PV size was not limited, since this attempts to install an infinite capacity.

- Does seasonal storage at this scale make sense? Would the storage size be too large to be practical?

The models showed that certain types of storage are useful, namely the batteries and the hot water storage. Battery storage was typically used for short term storage to provide load shifting and peak shaving. Hot water was also used to store heat and has the potential to store large quantities for use during the winter, however the storage size needed is very large. The maximum permitted hot water tank in the model forms a disk with the diameter of the mothership (214m), and a height of three meters giving a potential storage of 26.9 million kWh, which is more than enough for the annual heating demand. The volume of the tank would be over one third of the building volume (due to the hollow ring shape of the building) and would cost an estimated $35M. The hot water SUF for this large storage was between 0.4 and 0.47, meaning in a year it fills and empties about half way, implying that a tank of approximately half this size would be optimal. It is notable that for a much smaller storage size of 1000kWh, the SUF is 865, meaning it fills and empties more than twice a day on average.

Compressed air is also used; however, this technology is only applicable at large scales which can only be implemented in certain areas. The model uses it minimally with a SUF of around 20 for the larger storage sizes, but quite a lot for the smaller storage size (SUF of 211). Hydrogen storage was also included as an option but is not used by the model.

- What is the impact of hydrogen production and storage? Is it used for storage or for export?

Hydrogen production and storage was included in the model so that it would be used as longer term/seasonal electricity storage, with the additional versatility of being sold to local consumers such as hydrogen fuel cell vehicles and public transit. The results show that when the sale of hydrogen is allowed, it isn't used until a certain threshold in export price is reached, whereby the model maximizes production and uses all available energy (solar PV, biochar and gas CHP and grid) to

produce and sell as much as possible. When the export price is lowered to CAD 0.2 per kWh, the model does not make any hydrogen.

While this shows that it could be cost effective to do so, it may not be practical or desirable to co-locate a hydrogen production facility with a residential development. An interesting question for future research is whether there is a viable local market for hydrogen in large volumes, which may be unlikely without a power to gas operation where the hydrogen is pumped into the natural gas grid.

- Do the biochar technologies get used? What is the impact of carbon negative power and heat production?

The usage of the biochar technologies was not as prevalent as expected. The model did not choose to build biochar boilers at all, and only built biochar CHP when there were carbon limits imposed on the model in Cases 4, 5, and 7. In these cases, it was mainly used to offset the carbon released by the natural gas CHP or boiler that was also implemented.

Having both a natural gas and biochar CHP plant is impractical and complex, and likely would not happen if the building were built. The low cost of natural gas makes it difficult for other technologies to compete. Even when carbon credits are implemented, only case 6 where the tax is CAD 200/ton does it stop using natural gas and chooses biochar CHP and heat pumps instead.

There is some promise with biochar systems in the sequestration aspect and receiving carbon credits for producing the char, as well as then having a marketable product that can then be sold or used on site for its numerous benefits to agriculture. Biochar and its benefits are not widely known, nor is there a widespread carbon marketplace where the carbon credits can be sold. Once these factors change in the future then the situation could change dramatically.

- What is the impact of a strict carbon budget, such as being net-zero carbon? What if a negative carbon budget was enforced, meaning that carbon is sequestered each year?

There are several effects that occur with the implementation of emissions restrictions. The main one is that biochar technology, typically the CHP plant type, is installed so that it's sequestration can counteract the emissions from using the grid, or natural gas.

Troublingly it seems that when the negative emissions requirement is implemented, instead of cutting sources of emissions, it builds more capacity of biochar CHP to produce more char to counter the emissions. Instead of

cutting gas use, building heat pumps and biochar CHP along with maximum solar PV installed, the model continues to use gas CHP in addition to the biochar. It is unlikely however that such a practice would occur in reality, as it is more likely that a larger system consisting of just one of the technologies would be built, to reduce complexity and redundancy. These constraints should be added to the model in future.

The only case to eliminate natural gas use was Cases 6 and 8, both of which have carbon taxes. The sale of biochar does provide a good source of income for the building and could have numerous indirect benefits in the community depending on how the char gets used, as discussed in the material stream section above.

- What is the effect of carbon credits and carbon taxes? What is the threshold for fossil fuels to be avoided?

The implementation of a carbon tax had numerous effects. The total cost generally increased compared with similar cases without the tax. Emissions were also reduced for both cases. Interestingly, the utilization of storage was also reduced slightly. However, this could potentially be accounted for by the higher use of grid imports to power heat pumps, and therefor less need for storing intermittent renewable energy.

5. Discussion

The analysis performed in this paper optimizes the energy system of a mixed-use high-density development under different scenarios and compares this to base cases consisting of single detached homes and office and retail buildings scaled to house the equivalent number of people. The different scenarios modeled are designed to explore the changes to the systems under different conditions such as more or less storage, a carbon tax of CAD 200/tCO2, a net metering scheme, and hydrogen export. Additionally, the effect of imposing a net-zero emissions constraint and negative (1-ton CO_2 per resident) emissions requirement was explored.

When a carbon tax was implemented, less natural gas was used, instead using more grid power and heat pumps to meet the heating demand. Natural gas use was only eliminated when the carbon tax was implemented. Carbon sequestration was provided by a biochar producing combined heat and power plant which under the right conditions can produce carbon negative heat and power.

The mothership cases consistently had better performance than the base cases in terms of total cost. Base case B had the advantage of much greater roof surface area, so energy produced was sold to the grid to offset costs. Base case A had much higher costs and emissions relative to the mothership under the same conditions due to the magnitude of its loads being 13.4 and 1.6 times higher for heating and electricity respectively.

Base case C which used mothership loads but no renewable energy or storage technologies performed relatively poorly compared to the other mothership cases, with higher costs, more emissions, and no income (and higher operating costs) than most of the other mothership cases. This indicates that it is advantageous to implement renewable energy technologies not just because they reduce emissions, but because they offer significant financial rewards for doing so. The most cost-effective case in terms of total cost was a carbon neutral requirement. This shows that it may be possible to have a cost-effective energy system, while also achieving net zero emissions.

6. Conclusions

Some limitations with this analysis include the requirement of the MILP algorithm to maintain linearity in the system of equations. This can somewhat limit the parameters that can be analysed since it could cause the system to become nonlinear. Additionally, some variables, such as storage, could not be optimized for as it exponentially increases computation time, and as a result, had to be manually iterated and the excess storage capacity cost accounted for.

This paper illustrates how the energy hub model can be used to optimize energy systems for buildings, choosing from numerous technology options that would be impractical to determine manually, all operating in multiple costing scenarios imposing taxes and emissions restrictions. Results indicate that implementing renewable energy systems such as solar PV and hydrogen production and storage, as well as emerging carbon sequestration technologies such as biochar CHP can not only be carbon negative, but can be more cost effective than using fossil fuels. This is due to primarily to creating material streams that can be sold for profit, such as hydrogen, carbon negative electricity, and carbon credits. The tool can be easily adjusted to a specific scenario where a potential building will be built in order to help determine the best energy system mix for the project.

Future study opportunities include expanding the analysis with additional technologies and scenarios.

Including more detailed costing information would also be of benefit. Additionally, being able to have the carbon tax be a variable to solve for would be interesting to see at what level it needs to be to remove fossil fuels from the energy mix.

Acknowledgements

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