LINEAR PROGRAMMING MODELS FOR ESTIMATING WEIGHTS IN ANALYTIC HIERARCHY PROCESS AND FOR OPTIMIZATION OF HUMAN RESOURCE ALLOCATION

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ABSTRACT

The Analytic Hierarchy Process (AHP) provides a way to rank the alternatives by deriving priorities. In this paper we used Linear Programming (LP) models to estimate the weights of a pairwise comparison matrix derived within the frame work of the Analytic Hierarchy Process. The priorities obtained for the alternatives served as the coefficients of the objective function of linear programming to optimize a human resource problem at Bakhresa Food Product Limited (BFPL).

Keywords: AHP; linear programming; resource allocation; element dominance; sensitivity analysis

1. Introduction

The success of any organization lies in its ability to make critical decisions on growth and sustainability. However, decision making is a complex process as it involves multiple stakeholders with different opinions and interests. To avoid making ad-hoc decisions, decision makers are required to evaluate every alternative to the problem. With the Analytical Hierarchy Process, the problem is modeled by the decision maker and is structurally decomposed into a hierarchy consisting of levels of criteria, sub-criteria and alternatives with homogeneous clusters of factors (Saaty, 1977, 2007). Subsequently, an assessment of the usefulness of elements at each hierarchical level is made. AHP is a very suitable multi-criteria decision making tool proving to be effective in different application areas such as planning, optimization, selection of the best alternative and allocation of resources. It is also a reliable tool in resolving conflicts (Saaty& Vargas, 2006). AHP is critical in defining decision making processes taking into consideration decision maker's input, judgments, views and feelings (Vaidya & Kumar, 2006).

Specifying the hierarchy is of crucial importance; the hierarchical structure gives a clear overview of the complex relationships existing in the problem. This is important because it enables the decision maker to take into consideration every aspect in each level of the hierarchy. It also allows a decision maker to take into consideration a set of evaluation criteria, and alternative options from among which the best selection is to be made. The focus of the problem, usually the goal, is the highest level of the hierarchy. There are subsequent levels of criteria further down that include sub criteria, and finally the level of alternatives from which decisions are generated. Elements with a global composition may be included along the top levels of the hierarchy.

In each hierarchical level division, a pairwise comparison matrix is developed with $n\,(n-1)/2$ number of comparisons, where *'n'* is the number of criteria or alternatives in each level (Garg, Rahman, Qureshi & Kumar, 2012). Using a fundamental scale developed by Saaty, decision makers are able to assign the corresponding importance of one criterion relative to the other (Saaty, 1977). Finally, the weights of the elements being compared are estimated. In the end all the pairwise comparison results are synthesized and the decision is made in accordance with the final overall ranking of the alternatives. (Saaty,1999).

In summary, AHP applications in decision making involve four main procedures. These procedures include: 1) decomposition of the problem, 2) making judgments in the pairwise comparison matrices and checking their inconsistency, 3) improving it to derive the priority weights, and 4) the synthesis step where the final global weights for all the elements in the model are determined.

With each comparison matrix, the decision maker commonly uses the eigenvector method (EM) or additive normalization (AN) or logarithmic least square method (LLS) to generate a priority vector. These methods give the estimated relative weights of the elements as a result of the judgments (Saaty, 1999). To produce the final weight for the alternatives, weights generated at different levels of the hierarchy are synthesized according to the principle of hierarchic structure (Saaty, 1980)

In the next section, AN, EM, LLS, and LP methods are explained. The relative weights in a problem have been estimated using all these methods. In the subsequent section, the necessity of using AHP and LP models in human resource selection is covered.

2. Estimating weights

Let $A = (a_{ij})$for all $i, j = 1,2, \dots, n$ denote an $n\ x\ n$ pairwise comparison matrix, where a_{ij} is the importance of element $i$over the jth element. All the entries in matrix A are positive $(a_{ij} > 0)$ and reciprocal $(a_{ij} = 1/a_{ji}$ for all $i, j = 1,2, \dots. n)$. The decision maker wants to compute a vector $w = (w_{1,} w_2, \dots. w_n)$ of weights associated to pairwise comparison matrix A (Saaty, 1999).

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The matrix A is considered to be consistent when $a_{ij} = a_{ik}a_{kj} for all\ i,j,k = 1,2,\dots,n$, which implies that the decision maker is coherent (no error) in his judgments to develop the comparison matrix (Saaty, 1999).

Assuming A contains no error and $w_i$is the weight of thei^{th} element, we then have

$$a_{ij} = \frac{w_i}{w_j}, \quad i,j = 1,2,\dots,n \tag{1}$$

Summing over all j, we obtain

$$\sum_{j=1}^{n} a_{ij}\, w_j = nw_i, \quad i = 1,2,\dots,n \tag{2}$$

Which, in matrix notation, is equivalent to

$$Aw = nw. \tag{3}$$

The vector w is the principal eigenvector of the matrix A corresponding to the eigenvalue n., alternatively, we can say that the matrix A is consistent when $Aw = nw$ (Garg, Rahman, Qureshi & Kumar, 2012).

2.1 Additive normalization (AN)

In obtaining the priority vector w using the AN method, columns are first normalized such that elements of each column of the matrix A is divided by the sum of that column; then in each resulting row, normalized elements are summed up and divided by the number of elements in each row which is arithmetic average of the row (Srdjevic, 2005). The following equations (4) to (6) describe the above process;

$$a'_{ij} = a_{ij} / \sum_{i=1}^{n} a_{ij}\ i,j = 1,2,\dots n \tag{4}$$

$$w_i = \left(\frac{1}{n}\right) \sum_{j=1}^{n} a'_{ij}\,, \quad i = 1,2,\dots.,n \tag{5}$$

It can be observed that

$$\sum_{i=1}^{n} w_i = 1 \tag{6}$$

If A is consistent, then the columns of the normalized matrix $N = (a'_{ij})$ of A are identical. If A is not consistent, then we can write $Aw = \lambda_{max} w$, whereλ_{max} is the principal eigenvalue and given by

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$$\lambda_{max} = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}w_j \tag{7}$$

According to Saaty (1980) the consistency of the method can be checked as follows:
The consistency index (CI) is given by

$$CI = (\lambda_{max} - n)/(n-1) \tag{8}$$

while the consistency ratio (CR) is given by

$$CR = {}^{CI}/_{RI} \tag{9}$$

The random index (RI), which depends on the order of the matrix, is the average CI of a large number of randomly generated matrices. A CR of 0.10 or less is considered acceptable.

2.2 Eigenvector method (EM)

The principal eigenvector λ_{max} of A is determined by solving the determinant,

$$\det(A - \lambda_{max} I) = 0 \tag{10}$$

Then using the value of λ_{max}, the eigenvector $w = (w_{1,} w_2, \ldots . w_n)$ is find out from

$$(A - \lambda_{max} I)w = 0 \tag{11}$$

The consistency of the matrix is checked using Equations (8) and (9), in Section 2.1.

2.3 Logarithmic least square method (LLS)

This method has also been developed to estimate the vector of weights (Srdjevic, 2005). With LLS, the weights $w_{i,} for\ i = 1, \ldots, n$, are chosen to minimize the objective

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\left[\ln a_{ij} - \left(\ln w_i + \ln w_j\right)\right]^2 \tag{12}$$

Given that $a_{ij} = {}^{1}/_{a_{ji}}\ foralli, j = 1,2, \ldots, n,$ (13)

the LLS is quite simple: $w_{i,} for\ i = 1, \ldots, n$ is given by the geometric mean of the row I (Saaty, 1980).

2.4 Linear programming approach (LP)

There are two desirable properties of a pairwise comparison matrix – element dominance (ED) and row dominance (RD).

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ED is said to be preserved if $a_{ij} > 1$ *implies* $w_i \geq w_j$. If a_{ij} is exactly equal to 1, then an argument can be made for either $w_i \geq w_j or w_j \geq w_i$. RD is said to be preserved if $a_{ik} \geq a_{jk} for\ all\ k\ and a_{ik} > a_{jk} for\ some\ k\ implies\ w_i \geq w_j$. If the comparison matrix has cardinal inconsistency, that is, $a_{ij} \geq 1, a_{jk} \geq 1, and a_{ki} \geq 1,$ then the only feasible solution is $w_i = w_j = w_k$. Such a comparison matrix would be highly inconsistent.

EM and LLS both warrant RD (but not the ED). In the LP approach we can incorporate ED and RD as constraints, which have additional benefit of detecting cardinal inconsistency by ED constraints.

The two stage LP approach (Chandran, B., et.al, 2005) is described in sub-sections 2.4.1 to 2.4.3.

2.4.1 First stage: LP to establish the consistency bound

In general, any estimate of relative preference a_{ij} can be written as

$$\frac{w_i}{w_j} = a_{ij}\varepsilon_{ij}, \quad i,j = 1,2,\ldots,n \tag{14}$$

If the decision maker is consistent then ε_{ij} is equal to 1. Defining three transformed decision variables for the model: $x_i = \ln(w_i)$, $y_{ij} = \ln(\varepsilon_{ij})$, and$z_{ij} = |y_{ij}|$

The first stage LP can be written as:

$$Minimize \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} z_{ij} \tag{15}$$

Subject to

$$x_i - x_j - y_{ij} = \ln a_{ij}, \quad i,j = 1,2,\ldots,n;\ i \neq j, \tag{16}$$
$$z_{ij} \geq y_{ij}, \quad i,j = 1,2,\ldots,n;\ i < j, \tag{17}$$
$$z_{ij} \geq y_{ji}, \quad i,j = 1,2,\ldots,n;\ i < j, \tag{18}$$
$$x_1 = 0 \tag{19}$$
$$x_i - x_j \geq 0 \quad i,j = 1,2,\ldots,n;\ \ a_{ij} > 1 \tag{20}$$

$$x_i - x_j \geq 0 \quad i,j = 1,2,\ldots,n;\ \ a_{ij} \geq a_{jk}, \quad forallk;$$
$$a_{ik} > a_{jk} forsomek \tag{21}$$
$$z_{ij} \geq 0 \quad i,j = 1,2,\ldots,n \tag{22}$$
$$x_i, y_{ij},\ \ unrestricted i,j = 1,2,\ldots,n \tag{23}$$

The objective function (15) which is $\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} z_{ij}$, minimizes the sum of logarithms of positive errors in natural log space, whereas the constraint (16) is defining the errors. Equations (17) and (18) are the degree of over estimation, (19) sets one of the weight w_1 to zero, (20) preserves element dominance and (21) for row dominance. For a perfectly consistent comparison matrix, z^* is equal to zero.

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The objective function provides consistency index

$$CI(LP) = \frac{2z^*}{n(n-1)} \quad (24)$$

2.4.2 Second stage: LP to generate a priority vector

The first stage LP minimizes the product of all errors ε_{ij}, but multiple optimal solutions may exist.

In the second stage LP, the solution that minimizes the maximum errors ε_{ij} is selected. The second stage LP can be presented as:

$$Minimze z_{max} \quad (25)$$

Subject to

$$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} z_{ij} = z^* \quad (26)$$

$$z_{max} \geq z_{ij}, \quad i,j = 1,2,\dots,n; \; i < j, \quad (27)$$

and all first stage LP constraints.

$z^*$is the optimal first stage solution value, z_{max} is the maximum value of error z_{ij}.

Constraint (26) ensures that only those solutions that are optimal in the first stage LP are feasible in the second stage model.

2.4.3 Advantages of the LP approach

In AN or EM or LLS, the error from Equation (14) is

$$\varepsilon_{ij} = \frac{w_i}{w_j} a_{ji}, \quad i,j = 1,2,\dots,n \quad (28)$$

and then can be decided which a_{ij} can reduce the inconsistency. In the first stage of the LP model, the decision maker may go for a sensitivity analysis on inputs to know which entry in the pair wise comparison matrix should be changed to reduce inconsistency. The question is, how much should the entry should be changed?

The values of the dual variables (shadow prices) at optimality provide an indication of the incorrect entries in the pair wise comparison matrix. It is a well-known fact in LP that a shadow price with value *k* at optimality has the following interpretation: “if we increase the right hand side of the dual variable’s corresponding constraint by one unit, then the value of the objective function increases by k units”

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The dual variable that corresponds to each constraint in (16) changes the value of the objective function for unit change on the right hand side (that is, $\ln a_{ij}$). Since it is in natural logarithm space, a dual variable with value k at optimality will increase the objective function value by k units when the corresponding a_{ij} increase by a factor of e, the base of natural logarithm, which is a useful information for a decision maker (Chandran, B., et.al, 2005). It is now possible to identify which a_{ij} to change in order to decrease the inconsistency value by the greatest amount. A dual variable with negative value indicates that a_{ij} should increase, while a positive value indicates that a_{ij} should be decreased. The following section describes how the AHP was used to analyze the selection process in BFPL in Tanzania.

2.5 Numerical example

In this section, relative weights (principal eigenvector) of a 4x4 comparison matrix (Table 1) have been estimated using the AN, EM, LLS and LP approaches. The entries in Table 1 are the geometric means of a set of responses from 1/9 to 9. The principal eigenvalue λ_{max}, consistency index *CI,* and consistency ratio *CR* are calculated using Equations (7), (8) and (9) respectively and shown in Table 2. Relative weights obtained using LP shows that it preserves ED.

Table 1
4 x 4 pairwise comparison matrix

1	1	0.34	0.53
1	1	1.58	0.60
1/0.34	1/1.58	1	1.24
1/0.53	1/0.60	1/1.24	1

Table 2
Priority vectors, eigenvalues, consistency index and consistency ratio

	AN	EM	LLS	LP
w_1	0.1567	0.1560	0.1577	0.1583
w_2	0.2450	0.2490	0.2388	0.2716
w_3	0.3035	0.3017	0.2982	0.2716
w_4	0.2948	0.2933	0.3053	0.2985
λ_{max}	4.25	4.2486	4.1328	4.2667
CI	0.0833	0.0829	0.0443	0.0889
CR	0.0842	0.0837	0.0447	0.0898

The consistency index within the LP framework Equation (24) is 0.377. It can be observed that $a_{23} = 1.58$ has been violated in the AN, EM and LLS approaches whereas it has been preserved in the LP approach. Any highly overrated cell can be detected using Equation (28) for all the four methods. It does not directly give how much the CR has improved. But in the LP approach, the highly overrated cell can be detected with the help

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of a dual variable, and how much the improvement in first stage can be known determined. In this numerical example, cell $a_{23} = 1.58$ has been detected as the most overrated cell and the dual value corresponding equation from (16) has reduced the value of z^* from 2.262 to 1.307.

3. Human resource allocation at BFPL

Bakhresa Food Product Limited (BFPL) is a carbonated soft drinks manufacturing company owned and founded by Mr. Said Salim Awadh Bakhresa in the 1970s. The company, among several others, together make up the Azam Group of Companies, a conglomerate that deals with the production and provision of different products and services. It is among the largest family owned businesses in East and Central Africa. BFPL generates revenue of a minimum of three million dollars annually. With its headquarters located in the coastal region of Dar Es Salaam, Tanzania, this multi-national corporation faces different decision making challenges in its day to day operations and it is in the company's best interest to manage risky situations that may come from wrong decision making choices.

Proper decision making choices will ensure the company's sustainability and prosperity. Since a company's activities help to ensure its longevity, managers and decision makers at BFPL understand the need to evaluate weights and priorities of each activity in relation to its consequent outcome. Their aim is to equip the companies with strategic decision making techniques for sustainable growth, and consequently to achieve a competitive upper hand in the market. Decision makers must therefore adopt and apply different business practices, methods, and various tools that prove effective in decision making.

In any organization, selection of employees to fill different departmental posts is a crucial and sensitive matter (Kashi & Friedrich, 2013). Decision makers are well aware of the costs involved in the improper selection of employees on one hand, and the associated benefits of equipping the company with the right employees on the other. It is therefore vital for companies to not only select and hire employees whose skills and capabilities align well with the goals and objectives of the company, but also to achieve optimality in the return of their investment in human resources (Saaty, Peniwati & Shang, 2007). Having limited resources, the need for selection of potential employees must also incorporate the need to provide the company with employees whose values, mission and vision match those of the companies.

An effective tool is required, one that will be able to take into consideration both tangible and intangible aspects of criteria to take into consideration when selecting employees. The key is to find proper measurement of the weights and priorities of different criteria such that error and bias in decision making is minimized. With this tool, the right kind of employees will be chosen to man the right kind of tasks in the organization and ultimately help the organization to achieve the optimum value of its investment in human resources (Saaty, Peniwati, & Shang, 2007). AHP proves to be such a useful tool for selection of employees, and the application of Linear Programming will ensure the company achieves the optimality it requires (Saaty, Peniwati & Shang, 2007; Dye & Forman, 1992). Our purpose here is to illustrate AHP by deriving priorities and applying LP models and to formulate LP models to optimize returns. The company has identified

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the General Management and Quality Assurance Departments to fill different positions for BFPL as listed in Table 3.

With the example given, we seek to obtain the optimal number of posts that need to be filled, and to select only the most qualified potential employees for every post whose contribution will result in the achievement of quality products, increase efficiency, expand capacity, and product development for BFPL (Saaty, Peniwati & Shang, 2007).The selection hierarchy for any human resource allocation problem of BFPL can be represented through Figure 1 with a 5-level of hierarchy. The 5^{th} level of hierarchy of alternatives is not shown, which can be on the bottom.

3.1Illustration of human resource requirement and selection problem

The problem has been elaborated with hierarchical disintegration, creation of the pair-wise comparison matrix, priority weights development and the global weights development. Finally, optimal recruitment satisfying constraints imposed by the company is determined (Saaty, Peniwati & Shang, 2007).The company has many departments including Human Resources, Production, General Management, Quality Assurance, and Finance and Sales. For one of the plants situated near the port of Dar Es Salam, the company wants to employ a technical manager, a driver and assistant(s) within the General Management Department, as well as lab-technicians and micro-biologists within Quality Assurance Department. The company is looking to invest not more than 11 million TSH to be used as monthly salary packages for all the posts. The particulars of the posts including qualifications, salary packages and other details are given in Table 3 (Saaty, Peniwati & Shang, 2007).

To begin with, a hierarchical structure with 5 levels is created after analyzing the problem. The highest level (goal) is the selection of potential candidates. The following level is the department level where in this case, there are two departments with vacanies, the General Management and Quality Assurance departments. The third level is the posts to be filled; level four contains criteria for consideration in each of the posts within the departments. For example, the Technical Manager for the General Management department is evaluated based on experience, educational qualification, technical skill and communication skills. This level establishes different intensities for each selection criteria. The lowest level, the alternatives, consists of potential candidates who have shown interest in the post (see Figure 1).

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Table 3
Personnel requirements

Variable	Post	Department	Number of positions	Criteria of selection	Monthly salary in Tzs (m)
y_1	Technical Manager	General Management	1	Experience Education Technical Skill Communication Skill	6.0
y_2	Driver	General Management	1	Experience Education Mechanical Skill	0.8
y_3	Assistant	General Management	1 - 3	Experience Education Office Management Communication Skill	1.2
y_4	Lab Technician	Quality Assurance	0 - 1	Experience Education Technical Skill Communication Skill	1.0
y_5	Micro Biologist	Quality Assurance	0 - 2	Experience Education Technical Skill Communication Skill	0.9

A pair wise comparison matrix is developed for each hierarchical level using the fundamental scale. Five senior level personnel from BFPL were selected and a questionnaire was given to them. Instead of responding individually, persons responsible for selection helped the authors in providing the ratings in consensus. The priority weights for any level are obtained using the LP approach described in Section 2.4. These weights are mentioned in Figure1.

Level 4 demonstrates the criteria considered under each post. These are usually set by the organization to ensure the right candidates are selected for the right kind of job positions. Each post has its own specific set of selection criteria. For instance, the technical manager's post in the General Management department has four selection criteria namely; experience, education, technical skills and communication skills. To obtain the weight of each of the criterion a pair wise comparison matrix is developed and the LP approach is further applied for local weights. The global weights of criteria are obtained by synthesizing with the global weight of the root level. For example, the global weight of criteria for technical manager is obtained as (0.473, 0.0118, 0.059, 0.067).

3.2 Intensities and scores of criteria

To implement the absolute measurement mode in AHP, each selection criterion for every post is further broken down into levels of intensity. These intensities should be located at level 6 (Saaty, Peniwati & Shang, 2007). For example, the evaluation of the Technical

Manager consists of the following intensities: (i) experience is divided into three intensities of high (corresponds to 3+ years of experience), medium (1-3 years), and low (less than one year); (ii) education is divided into master, degree and diploma; (iii) technical skills are divided into excellent, good and fair; (iv) communication skills into high, medium and low (Saaty, Peniwati & Shang, 2007). The rating for each position is given in Table 4.Using the LP approach, priorities of the intensities are obtained from pair wise comparisons and idealized by dividing each by the highest value so that it becomes 1 and the rest follow proportionally (Table 5).

The evaluation of each candidate is done by a group of experts who conduct multiple interviews relating to required job skills, personality and character, communication skills and ability to work in a group. This evaluation is done according to the criteria required for the successful selection of a potential employee. The experts (not less than 3 in number) generate a pairwise comparison matrix after obtaining the geometric mean of their judgments. The candidate's synthesized score for each post they applied for is calculated according to the corresponding local weights of the selection criteria shown in Table 4.Their results are organized into a list in Table 6. The total scores for each employee in Table 6 have been adjusted multiplying by 3/5 for the General Management department and 2/5 for the Quality Assurance department.

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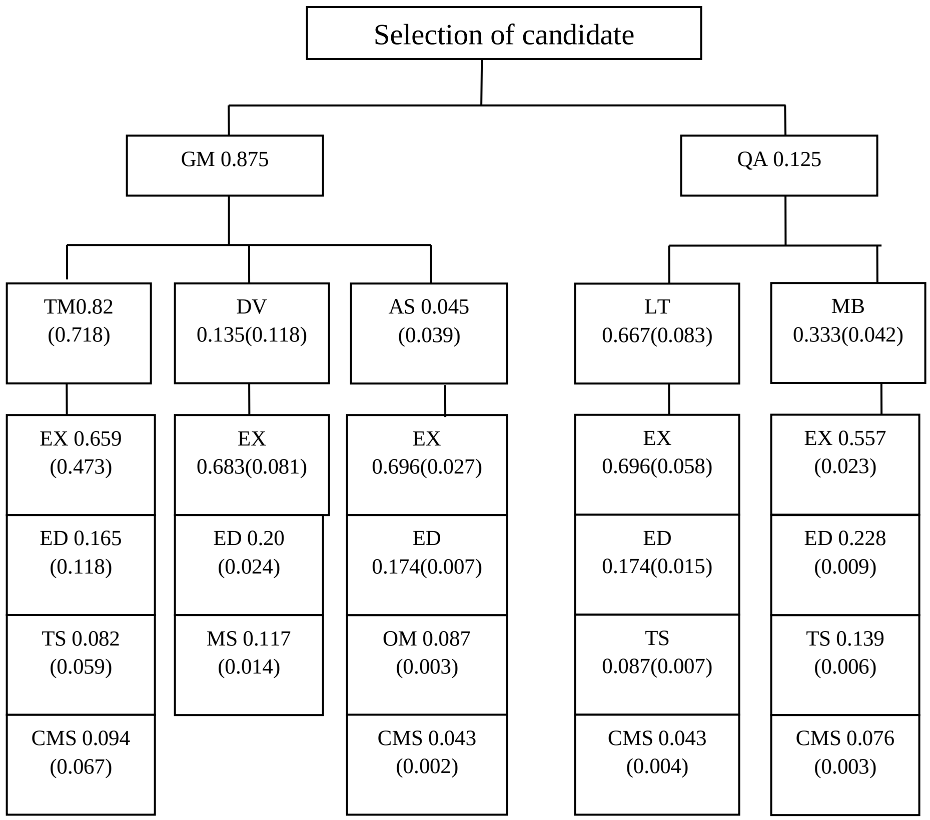

Figure 1.Selection hierarchy for BFPL with local (global) weights

In Table 4, the applicants are rated for quality for each criterion of the position for which they are applying. The standards for quality from which their rating is chosen are given in Table 5.

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Table 4
Criteria for positions to be filled with an example of applicant x_1's ratings for the Tech. Manager position

Applicants				
	Tech. Manager			
	Experience 0.473	Education 0.118	Tech. skill 0.059	Comm. skill 0.067
x_1	*High*	*Master*	*Good*	*High*
x_2				
x_3				
x_4				
	Driver			
	Experience 0.081	Education 0.024	Mech.skill 0.014	
x_5				
x_6				
x_7				
x_8				
	Assistant			
	Experience 0.027	Education 0.007	Office mgmt 0.003	Comm. skill 0.002
x_9				
x_{10}				
x_{11}				
x_{12}				
	Lab-Technician			
	Experience 0.058	Education 0.015	Tech. skill 0.007	Mech. skill 0.004
x_{13}				
x_{14}				
x_{15}				
x_{16}				
	Micro-Biologist			
	Experience 0.023	Education 0.009	Tech. skill 0.006	Mech. skill 0.003
x_{17}				
x_{18}				
x_{19}				
x_{20}				

Table 5
Ideal priorities of the intensities used for rating alternatives in Table 4

Criteria by job (global priorities)	Intensities and idealized priorities
Technical Mangers	
Experience(0.473)	High (1) Medium (0.55) Low (0.302)
Education(0.165)	Masters (1) Degree (0.363) Diploma (0.132)
Technical Skills(0.082)	Excellent (1) Good (0.25) Fair (0.125)
Communication Skills(0.094)	High (1) Medium (0.573) Low (0.219)
Driver	
Experience (0.081)	High (1) Medium (0.55) Low (0.302)
Education (0.024)	Diploma (0.142) Certificate (0.705) Secondary (1)
Mechanical Skills (0.014)	High (1) Medium (0.210) Low (0.088)
Assistant	
Experience (0.027)	High (1) Medium (0.55) Low (0.302)
Education (0.007)	Degree (0.142) Diploma (0.705) Certificate (1)
Office Management Skills (0.003)	Best (1) Good (0.363) Fair (0.132)
Communication Skills (0.002)	High (1) Medium (0.573) Low (0.219)
Lab Technician	
Experience (0.058)	High (1) Medium (0.55) Low (0.302)
Education (0.015)	Masters (1) Degree (0.363) Diploma (0.132)
Technical Skills (0.007)	Excellent (1) Good (0.25) Fair (0.125)
Communication Skills (0.004)	High (1) Medium (0.573) Low (0.219)
Micro Biologist	
Experience (0.023)	High (1) Medium (0.55) Low (0.302)
Education (0.009)	Masters (1) Degree (0.363) Diploma (0.132)
Technical Skills (0.006)	Excellent (1) Good (0.25) Fair (0.125)
Communication Skills (0.003)	High (1) Medium (0.573) Low (0.219)

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Table 6
Applicant's priorities obtained from their rating totals

Number	**Applicants**	**Post applied for**	**Score**
1	x_1	Technical Manager	0.4040
2	x_2	Technical Manager	0.3680
3	x_3	Technical Manager	0.2403
4	x_4	Technical Manager	0.3506
5	x_5	Driver	0.0295
6	x_6	Driver	0.0648
7	x_7	Driver	0.0332
8	x_8	Driver	0.0453
9	x_9	Assistant	0.0225
10	x_{10}	Assistant	0.0069
11	x_{11}	Assistant	0.0145
12	x_{12}	Assistant	0.0084
13	x_{13}	Lab. Technician	0.0169
14	x_{14}	Lab. Technician	0.0150
15	x_{15}	Lab. Technician	0.0271
16	x_{16}	Lab. Technician	0.0085
17	x_{17}	Micro. Biologist	0.0123
18	x_{18}	Micro. Biologist	0.0056
19	x_{19}	Micro. Biologist	0.068
20	x_{20}	Micro. Biologist	0.0046

3.3 Manpower allocation for BFPL

Two comparable linear models are presented in this section for the best human resource allocation for BFPL.

Model 1: Optimization for individual applicants

The objective function coefficients are the scores given in Table 6. The decision variables $x_1, x_2, \dots, x_{20}$are binary, subject to salary constraint, upper and lower bound constraints on the number of people given in Table 3.

$$\begin{aligned} Max\ \ 0.4040x_1 &+ 0.3680x_2 + 0.2403x_3 + 0.3506x_4 + 0.0295x_5 + 0.0648x_6 \\ &+ 0.0332x_7 + 0.0453x_8 + 0.0225x_9 + 0.0069x_{10} + 0.0145x_{11} \\ &+ 0.0084x_{12} + 0.0169x_{13} + 0.0150x_{14} + 0.0271x_{15} \\ &+ 0.0085x_{16} + 0.0123x_{17} + 0.0056x_{18} + 0.0068x_{19} \\ &+ 0.0046x_{20} \leq 11 \end{aligned}$$

Subject to

$$6x_1 + 6x_2 + 6x_3 + 6x_4 + 0.8x_5 + 0.8x_6 + 0.8x_7 + 0.8x_8 + 1.2x_9 + 1.2x_{10} + 1.2x_{11} + 1.2x_{12} + 1.0x_{13} + 1.0x_{14} + 1.0x_{15} + 1.0x_{16} + 0.9x_{17} + 0.9x_{18} + 0.9x_{19} + 0.9x_{20} \leq 11 \qquad \text{(Salary constraint)}$$

$$x_1 + x_2 + x_3 + x_4 = 1 \qquad (TechnicalManager)$$
$$0 \le x_5 + x_6 + x_7 + x_8 \le 1 \qquad (Driver)$$
$$1 \le x_9 + x_{10} + x_{11} + x_{12} \le 3 \qquad (Assistant)$$
$$0 \le x_{13} + x_{14} + x_{15} + x_{16} \le 1 \qquad (LabTechnician)$$
$$0 \le x_{17} + x_{18} + x_{19} + x_{20} \le 3 \qquad (Micro\ Biologist)$$
$$x_j, \ \ \mathrm{j} = 1,2,\dots,20 \quad \text{are binary}$$

Model 1 was solved using Excel Solver to maximize the goal of BFPL and solution is given in Table 7.

Table 7
The optimal solution of LP Model 1

Selected Applicant	Position	Salary in million (Tsh)
x_1	Technical Manager	6
x_6	Driver	0.8
x_9	Assistant	1.2
x_{15}	Lab Technician	1
x_{17}	Micro Biologist	0.9
x_{19}	Micro Biologist	0.9
Total Salaries		10.8

Model 2: Optimizing different positions

This approach consequently provides the priorities of the five positions found in level four of the hierarchy of Figure 1 as coefficients of the objective function. The posts are represented by $y_1 \ to \ y_5$, such that their values integers denote the vacancy of jobs posts. The previous model made the selection of applicants based on their rating taking the relative importance of the post. With these models we are able to obtain the optimal number of jobs and the selection of the best applicants for those job posts. Coefficients of objective function from the fourth level of Figure1 are adjusted by multiplying 3/5 with the General Management department and 2/5 with the Quality Assurance department.

The model is

$$Max\ 0.492y_1 + 0.081y_2 + 0.027y_3 + 0.2668y_4 + 0.1332y_5$$

Subject to

$$y_1 + 0.8y_2 + 1.2y_3 + y_4 + 0.9y_5 \le 11 \qquad salary\ constraint$$
$$y_1 \ = 1 \qquad Technical\ Manager$$
$$y_2 \ = 1 \qquad Driver$$
$$y_3 \ \ge 1 \qquad Assistant, \qquad Lower\ bound$$
$$y_3 \ \le 3 \qquad Assistant, \qquad Upper\ bound$$
$$y_4 \ge 0 \qquad Lab\ Technician, \qquad Lower\ bound$$
$$y_4 \ \le 1 \qquad Lab\ Technician, \qquad Upper\ bound$$
$$y_5 \ \ge 0 \qquad Micro\ Biologist, \qquad Lower\ bound$$
$$y_5 \ \le 2 \qquad Micro\ Biologist, \qquad Upper\ bound$$

y_j are integers

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Table 8
Optimal solution second model

Position	Selection	Position salary	Total salary
Technical Manager	1	6000000 Tzs	6000000 Tzs
Driver	1	800000 Tzs	800000 Tzs
Assistant	1	1200000 Tzs	1200000 Tzs
Lab Technician	1	1000000 Tzs	1000000 Tzs
Micro Biologist	2	900000 Tzs	1800000 Tzs
Total salaries			10800000 Tzs

The optimal solution of model 2 is given in Table 8. The solution of model 2 is consistent with the model 1 solution. All the posts can be filled if the company can spend 13.2 million Tzs in place of 11 million Tzs per month.

4. Summary and conclusions

AHP can measure intangibles and LP proves to be effective in optimizing the resource allocation problem by also considering tangible measurements. This paper has used both tangible and intangible measures. After converting intangibles by using the AHP technique, priority has also been calculated using LP. Element dominance and row dominance have been incorporated as constraints in LP. LP has several advantages over the additive normalization or eigenvectors or LSS methods for determining priorities. In Section 3, the BFPL case has been presented to fill the vacant post. Combined AHP and LP models seem to provide an effective tool.

REFERENCES

Bryson N., (1995). A goal programming method for generating priority vectors. *Journal of Operational Research Society, 46,* 641-648.doi: 10.1057/jors.1995.88

Battistoni, E. et al (2013). Analytic Hierarchy Process for new product development. *International Journal of Engineering Business Management, 5(42),* 1- 6.doi: 10.5772/56816

Bhatt, R. et al. (2013). Best engineering college selection through analytic hierarchy process: a case study. *International Journal of Technology & Engineering Education,10(2),*1- 6.

Chandran, B. et al. (2005).Linear programming models for estimating weights in the analytic hierarchy process. *Computer& Operation Research*, *32,* 2235-2254.doi:10.1016/j.cor.2004.02.010

Cheng, E.,& Heng, L. (2001).Analytic hierarchy process: An approach to determine measures for business performance. *Measuring Business Excellence*, *5(3),*30-37. http://dx.doi.org/10.1108/EUM0000000005864

Cogger, C., & Yu, P. L. (1985). Eigenweight vectors and least distance approximation for revealed preference in pairwise weight ratios. *Journal of Optimization Theory and Application,46,* 483- 491.doi: 10.1007/BF00939153

Crawford, G., & William, C. (1985).A note on the analysis of subjective judgments matrices.*Journal of Mathematical Psychology, 25,* 387- 406. doi:10.1016/0022-2496(85)90002-1

Enea, M., & Piazza, T. (2004). Project selection by constrained fuzzy analytic hierarchy process. *Fuzzy Optimization and Decision Making, 3,* 39-62. doi:10.1023/B:FODM.0000013071.63614.3d

Fisher, C.D., et al. (1992). *Human resources management*. Boston: Houghton Mifflin Company.

Garg, R., Rahman, Z., Qureshi, M., & Kumar, I. (2012). Identifying and ranking critical success factors of customer experience in banks. An Analytic Hierarchy Process (AHP) approach. *Journal of Modeling in Management, 7(2),* 201-220.http://dx.doi.org/10.1108/17465661211242813

Hassan, N. et al. (2013). Selection of mobile network operators using Analytic Hierarchy Process. *Advances in Natural and Applied Sciences*, *7(1),* 1- 5.

Mathis, R.L., & Jackson, H.J. (1991).*Human resources management*. Minneapolis: West Publishing Corp.

Saaty, T. L., (1980). *Analytic Hierarchy Process*, New York: McGraw-Hill.doi:10.1080/00137918308956077

Saaty, T. L., (1999). Basic theory of the Analytic Hierarchy Process: How to make decision. *Revista de la Real Academia Ciencias Exactas Fiscicas y Naturales, 93(4),* 395-453.

Saaty, T. L. (2003). Decision making with the AHP: Why the principal eigenvector is important. *European Journal of Operational Research, 14*, 85-91. doi:10.1016/S0377-2217(02)00227-8

Saaty, T. L., Peniwati, K., & Shang, S. J. (2007).The Analytic Hierarchy Process and the human resource allocation: Half the story. *Mathematical and Computer Modeling, 46,* 1041-1053.doi:10.1016/j.mcm.2007.03.010

Saaty, T. L., (2008). Decision making with the Analytic Hierarchy Process. *International Journal Services Science, 1(1),*83-97.doi: http://dx.doi.org/10.1504/IJSSCI.2008.017590

Saaty, T. L., (2013). On the measurement of intangibles. A principal eigenvector approach to relative measurement derived from paired comparisons, *Notices of the AMS, 60(2),* 192-208.doi: http://dx.doi.org/10.1090/noti944

Sinuary-Stern, Z., Mehrez, A.,& Hadad, Y., (2006).An AHP/DEA methodology for ranking decision making units. *International Transactions in Operational Research, 7*, 109-124.doi: 10.1111/j.1475-3995.2000.tb00189.x

Shuler, R.S., & Huber,V. L. (1993). *Personnel and human resources management*. New York: West Publishing Corp.

Srdjevic, B., (2005). Combining different prioritization methods in the analytic hierarchy process synthesis. *Computer & Operations Research, 32*, 1897-1919. doi:10.1016/j.cor.2003.12.005

Tien-Chin, W., & Ying-Ling, L. (2009).Using a multi-criteria group decision making approach to select merged strategies for commercial banks. *Group Decision Negotiations*, 18, 519-536. doi:10.1007/s10726-008-9112-3

Vaidya, S., & Kumar, S., (2006).The Analytic Hierarchy Process: An overview of application. *European Journal of Operational Research, 169(1),* 1-29.doi:10.1016/j.ejor.2004.04.028