ABSOLUTE MEASUREMENT AND IDEAL SYNTHESIS ON AHP

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As in most of the multi-criteria decision analysis (MCDA) methods, the application of the Analytic Hierarchy Process (AHP) runs through three major steps: first, *structuring*; second, *measuring*; and third, *synthesizing*. The ways to conduct these three steps makes MCDA different from the other methods. Originally, AHP application consisted of hierarchical structuring, relative measurement and distributive synthesis (Saaty T. L., 1977; Saaty T. L., 1980). More than any other MCDA method, AHP theory and practice evolved, with different ways to perform the three steps.

Network structuring, for instance, implies a violation of the axiom of independence (Vargas, 1990). That is, instead of hierarchical structuring, criteria and alternatives may depend on or influence each other. As a matter of fact, this generalization of AHP is another MCDA method, the Analytic Network Process (ANP) (Saaty T. L., 1997; Ishizaka & Nemery, 2013). Absolute measurement, also known as “ratings”, and ideal synthesis are different ways to apply AHP (Saaty, 1986; Millet & Saaty, 2000). The combination of ratings with ideal synthesis may bring many advantages for AHP application. Two advantages that this combination yields are the ability to increase the set of alternatives to more than nine and ranking (Saaty, Vargas, & Whitaker, 2009).

In spite of the advantages of the *Absolute measurement/Ideal synthesis* approach, the original *Relative measurement/Normalized synthesis* is the preferred way to apply AHP. In the last two years, in Volume 7 and Volume 8, the *International Journal of the AHP* published 48 papers. Only five of these papers addressed theoretical issues, that is, with no application. From thoe 43 practical papers with application, 31 presented applications of AHP alone, seven papers presented applications of ANP alone, and five papers bring combinations of AHP with other theories, like Fuzzy Sets or Linear Programing. Only three papers presented applications of AHP with ratings and ideal synthesis (Tramarico, Marins, Urbina, & Salomon, 2015; Saaty & Wei, 2016; Bhandari & Nakarmi, 2016). Then, in the overwhelming majority of applications, measurement was relative and synthesis were distributive. Expert Choice and Super Decisions were the most commonly used software. However, both brands of software enable ratings and ideal synthesis.

The purpose of this essay is not to investigate why AHP has not often been applied with absolute measurement and ideal synthesis. My purpose is to call attention to different ways to apply AHP, highlighting three advantages: allowing one set alternatives greater than nine, avoiding rank reversal and providing overall priorities based on ideal priorities.

Absolute measurement implies in alternatives compared with standard levels, instead of pairwise compared to each other, as in AHP’s original relative measurement. The first advantage of absolute measurement is that there is no boundary for the set of alternatives. In relative measurement, the set of alternatives must be less or equal than nine, or else, “seven, plus or minus two” (Saaty & Ozdemir, 2003). Another advantage to using ratings

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is the opportunity to avoid biases. In relative measurement, the pairwise comparisons of alternatives can keep some historical trends. Absolute measurement seems to provide a less partial or unbiased measurement, comparing alternatives with a standard (Salomon, Tramarico, & Marins, 2016).

With ideal synthesis, priorities are not normally distributed. That is, the sum of the priority vectors components will not be equal to one hundred percent. In this way of synthesis, the highest priority regarding each criterion will be equal to one. Normalizing priorities creates a dependency among priorities. However, when deleting an old alternative, or inserting a new one, normalized priorities can lead to illegitimate changes in the rank of alternatives, known as rank reversal (RR). RR was firstly associated with AHP (Belton & Gear, 1983). Nevertheless, the application of other MCDA methods, such as ELECTRE, MAUT, PROMETHEE and TOPSIS, can also lead to RR (Triantaphyllou, 2000).

Combining absolute measurement with ideal synthesis will always preserve ranks (Saaty, Vargas, & Whitaker, 2009). Firstly, the discussion shall be on the legitimacy of RR. That is, RR does happen in real world decision problems (Saaty & Sagir, 2009). In the case RR is not a major concern, then, relative measurement and normal synthesis may be adopted in the AHP application. On the other hand, if the decision-maker is looking for rank preservation, then absolute measurement and ideal synthesis are proper ways to apply AHP.

Let us consider a decision of project selection by a company. The decision criteria are Benefits, Opportunities and Risks; the alternatives are Projects X, Y and Z. Table 1 presents the pairwise comparisons matrix and the priorities of the criteria.

Table 1
Priorities of benefits, opportunities and risks

Criterion	B	O	R	Eigenvector	Priority
Benefits (B)	1	4	9	3.3	0.74
Opportunities (O)	1/4	1	2	0.79	0.18
Risks (R)	1/9	1/2	1	0.38	0.09

Tables 2, 3 and 4 present the pairwise comparisons matrices and the priorities of the alternatives regarding the criteria. All comparisons matrices are consistent.

Table 2
Priorities of projects regarding benefits

Project	X	Y	Z	Eigenvector	Priority
X	1	5/3	5	2.0	0.56
Y	3/5	1	3	1.2	0.33
Z	1/5	1/3	1	0.41	0.11

Table 3
Priorities of projects regarding opportunities

Project	X	Y	Z	Eigenvector	Priority
X	1	1/9	1/3	0.33	0.08
Y	9	1	3	3	0.69
Z	3	1/3	1	1	0.23

Table 4
Priorities of projects regarding risks

Project	X	Y	Z	Eigenvector	Priority
X	1	1/5	4/5	0.54	0.14
Y	5	1	4	2.7	0.69
Z	5/4	1/4	1	0.68	0.17

Table 5 presents the decision matrix (with local priorities regarding each criterion) and the decision vector (with overall priorities) [1]. Due to its highest overall priority, Project X will be selected.

Table 5
Local and overall priorities of Projects X, Y and Z

Project	B (0.74)	O (0.18)	R (0.09)	Overall
X	0.56	0.08	0.14	0.44
Y	0.33	0.69	0.69	0.43
Z	0.11	0.23	0.17	0.14

Now, let us suppose that for some reason (for example, problems with raw material imported from distant countries), Project Z became unfeasible. If the decision were not announced, Tables 6, 7,8 and 9 present updating for Tables 2,3,4 and 5, just deleting Project Z.

Table 6
New priorities of projects regarding benefits

Project	X	Y	Eigenvector	Priority
X	1	5/3	1.29	0.63
Y	3/5	1	0.77	0.38

[1] In the examples in this paper, the lower the R priority, the lower the risks of the project; therefore, the overall priorities can be calculated as a weighted sum.

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Table 7
New priorities of projects regarding opportunities

Project	X	Y	Eigenvector	Priority
X	1	1/9	0.33	0.10
Y	9	1	3	0.90

Table 8
New priorities of projects regarding risks

Project	X	Y	Eigenvector	Priority
X	1	1/5	0.45	0.17
Y	5	1	2.2	0.83

Table 9
New local and overall priorities of Projects X and Y

Project	B (0.74)	O (0.18)	R (0.09)	Overall
X	0.63	0.10	0.17	0.49
Y	0.38	0.90	0.83	0.51

As we can see, in this example a RR occurs. Considering Project Z, Project X has a higher priority than Project Y. However, Project Z has the lowest priority among the three projects. And, surprisingly, after deleting Project Z from the decision, Project Y became the highest priority vector.

Now, let us apply the ideal synthesis with the same data, that is, with the same comparisons. Tables 10,11 and 12 present the local priorities with ideal synthesis. The comparison matrices and the right eigenvectors are the same from Tables 2,3 and 4.

Table 10
Ideal priorities of projects regarding benefits

Project	X	Y	Z	Eigenvector	Priority
X	1	5/3	5	2.0	1
Y	3/5	1	3	1.2	0.6
Z	1/5	1/3	1	0.41	0.2

Table 11
Ideal priorities of projects regarding opportunities

Project	X	Y	Z	Eigenvector	Priority
X	1	1/9	1/3	0.33	0.11
Y	9	1	3	3	1
Z	3	1/3	1	1	0.33

Table 12
Ideal priorities of projects regarding risks

Project	X	Y	Z	Eigenvector	Priority
X	1	1/5	4/5	0.54	0.2
Y	5	1	4	2.7	1
Z	5/4	1/4	1	0.68	0.25

Table 13 presents the decision matrix (with local priorities regarding each criterion) and the decision vector (with overall priorities). Due to its highest overall priority, Project X will be selected, as in Table 5.

Table 13
Local and overall priorities with ideal synthesis of Projects X, Y and Z

Project	B (0.74)	O (0.18)	R (0.09)	Overall
X	1	0.11	0.2	0.77
Y	0.6	1	1	0.71
Z	0.2	0.33	0.25	0.23

Tables 14, 15 and 16 present the new local priorities with ideal synthesis. The comparison matrices are the same from Tables 6, 7 and 8 which are the same of Table 2, 3 and 4, just deleting Project Z.

Table 14
New ideal priorities of projects regarding benefits

Project	X	Y	Eigenvector	Priority
X	1	5/3	1.29	1
Y	3/5	1	0.77	0.6

Table 15
New ideal priorities of projects regarding opportunities

Project	X	Y	Eigenvector	Priority
X	1	1/9	0.33	.11
Y	9	1	3	1

Table 16
New ideal priorities of projects regarding risks

Project	X	Y	Eigenvector	Priority
X	1	1/5	0.45	0.2
Y	5	1	2.2	1

Table 17
New local and overall priorities with ideal synthesis of Projects X and Y

Project	B (0.74)	O (0.18)	R (0.09)	Overall
X	1	.11	0.2	0.77
Y	0.6	1	1	0.71

This numeric example illustrates that, with ideal synthesis, RR can be avoided. That will be important for the decision maker because there are some situations in the real world when RR is unjustifiable, undesired and perhaps unfair.

However, another great advantage from ideal synthesis is the value in the priority. That is the “0.77” for Project X in Tables 13 and 17 represent a concept similar to “utility” (Ishizaka & Nemery, 2013). This is the degree of satisfaction expected by the decision maker with the selection of Project Y. For some decision problems it can makes more sense than the “44%” or “49%” from Tables 5 and 9.

I expect to have made the case in this essay for the convenience of using the absolute measurement/ideal synthesis when applying AHP, in particular by new researchers and users of this MCDA method. Currently, as we can see in the *IJAHP* papers, ideal synthesis has not been applied, as it could or should be. However, the way one applies AHP is still a question of opinion. This essay does not prove, and I do not intended to prove, that the *absolute measurement/ideal synthesis* approach is better than the original, with just one example. Exhaustive experiments may be able to prove or even clearly identify situations in which one way may be better than another way. This is a great subject for a future research.

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