PRIORITIZATION OF ALTERNATIVES BASED ON ANALYTIC HIERARCHY PROCESS USING INTERVAL TYPE-2 FUZZY SETS AND PROBABILITY-THEORETICAL INTERVAL COMPARISON

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ABSTRACT

The Analytic Hierarchy Process (AHP) enables decision-makers to prioritize alternatives. However, when an expert expresses judgments using natural language statements (e.g. words or phrases) inherent vagueness of language constructs can cause the interpretation to be imprecise. The fuzzy Analytic Hierarchy Process (FAHP) can be viewed in the context of the classical AHP expansion. While performing pairwise comparisons domain experts are accustomed to operating with verbal terms in their judgments. Most existing FAHP approaches do not consider a human's confidence in the estimates provided. This paper presents a model that gives weight to the constraints on domains of expert assessments as they are almost always supplied with certain degrees of confidence. Interval type-2 membership functions (IT2MF) along with the probability-theoretical procedure for comparison of intervals can be applied here as suitable modeling options. Empirical comparison of FAHP that makes use of triangular fuzzy numbers and IT2MF-based FAHP is also presented.

Keywords: Analytic Hierarchy Process; expert assessment; degree of confidence; linguistic label; type-1 membership function; fuzzy number; interval type-2 membership function; Fuzzy Synthetic Extents; interval calculations; prioritizing alternatives

1. Introduction

People constantly make decisions starting from utterly trivial cases based on unconscious instantaneous choices and ending with important ones that require a deep grasp of the matter. As a rule, human decision-framing abilities allow for specifying a potential set of domain specific alternatives (options). The rough (shallow) approach to thinking and deciding often relies on basic perceptions prompting that alternative a_i is generally 'better' than alternative a_j, $i, j = \overline{1,n}$, $i \neq j$, $n > 1$, without conducting comparisons with regard to specific criteria or some additional details. This kind of approach does not always lead to a good choice.

The Analytic Hierarchy Process (AHP) proposed by Thomas L. Saaty enables decision-makers to prioritize alternatives and make choices in favor of alternatives that correspond to understanding the problem and requirements imposed on its solution (Saaty, 1980). This goal is achieved by defining a hierarchical model that reflects characteristic properties of the problem, determining criteria and alternatives in reference to the goal as the top element of the hierarchy (model), and prioritizing alternatives and criteria using pairwise comparisons. With reference to the "classical" 3-level hierarchy (goal [L0] – criteria [L1] – alternatives [L2]), the method allows the use of intuitive expert estimates that express the superiority of alternative a_i over alternative a_j, $i, j = \overline{1, n}$, concerning the determined criteria. Results of such comparisons are not always expressed in terms of quantitative characteristics; verbal constructs can be used instead. Usually the intuitive evaluation is transformed by the expert to a corresponding number on Saaty's fundamental scale with integers from 1-9. The AHP calls for a series of pairwise comparisons of alternatives against a specified criteria set, and it provides for the construction of several positive reciprocal matrices *M* (Matrices of Overall Priorities, MOP). Namely, they are criteria comparison matrices relative to a system's objective and alternatives comparison matrices regarding each criterion on the list. The number of such matrices is determined by the number of constituents at levels L1 and L2 as well as the links between layers. For the sake of simplicity, any extra indices in matrix notations are omitted, if possible, in the following text. The maximum eigenvalue of the corresponding matrix, its order, the values of Consistency Index and tabulated Random Index are used to check the consistency of every constructed square matrix (Saaty, 1980; Saaty, 1987).

Once these potentially lengthy steps of AHP realization are over, we switch our attention to calculating weights of alternatives giving a proper account to the weights of criteria obtained earlier. Weights of alternatives make it possible to rank them, and the alternative with the highest priority (rank) value is proclaimed as the most feasible decision option for the problem at hand. An extra valuable step, the sensitivity analysis, allows the slight changes of criteria weights to ranking of alternatives to be considered more fully. This step serves as a base to formulate comments on those details of the model that are worthy of notice in making the ultimate decision when considering results of calculations (Saaty, 1994a, 1994b).

The expert usually follows a complex system of reasoning while making his/her assessment. It conforms to Kahneman's System 2 of thinking that requires mental efforts, deep analysis and attention (Kahneman, 2013). As a rule, estimates are formulated in terms of natural language statements, i.e. words, phrases or short sentences. This outcome can be mapped to some form that is reminiscent of numeric values of the fundamental scale, i.e. numbers ranging from 1 to 9 as an expression of the assessment done. However, when a person expresses judgments in the context of 'a_i versus a_j' comparisons using natural language statements, the latter can often be interpreted imprecisely due to inherent vagueness of the language constructs and specifics of information pieces captured from diverse sources. Vagueness of lingual forms stands for the deficiency of meaning. Its interpretation is usually multiplied by a human's mental capacity, embracing beliefs, thoughts and sensations in the mind or collaborative environment of decision-making problems; it then becomes a matter of degree and leads to variations (Fine, 1975; Keefe, 2007). Hence, fuzzy logic methodology as a modeling base can be suitable in such case(s).

Fuzzy Analytic Hierarchy Process (FAHP) can be considered as a development of the classical AHP that allows experts to operate with customary linguistic terms (e.g. strong predominance, highly preferred etc.) while performing pairwise comparisons which are not immediately converted into plain numeric representation form (Kabir & Hasin, 2011). To provide such opportunities to experts, calculations within FAHP can be done using T1MF (Zadeh, 1965). According to Zadeh (1965), the concept of a fuzzy set is bound up with a way to "treat fuzziness in a quantitative manner". Let U (universal set) be a set of elements denoted generically as x, i.e. $U=\{x\}$; fuzzy set $A \subseteq U$ is a set of ordered pairs $\{(x,\mu_A(x))\}$, and the mapping $\mu_A : x \to [0,1]$ is a (type-1) membership function (abbr. T1MF) of a fuzzy set A. We can simply write $A : U \to [0,1]$ identifying the notion of the set with its representing function μ_A. Fuzzy set is represented by its membership function, and this fact makes two commonly employed abbreviations FS and MF interchangeable (Zadeh, 1965; Klir & Yuan, 1995; Wierman, 1997; Mendel, 2017). Value $\mu_A(x)$ is a degree (or grade) of membership of x in the set A. Convex normal (height equals to 1) fuzzy set defined on the real line $\mathbb{R}$ is called a fuzzy number (abbr. FN). It is a generalization of the usual number commonly called crisp (Wierman, 1997). Type-1 fuzzy sets serve as a basic framework to handle vagueness that is typical of natural language statements (linguistic values) and the context of their use. Thus, the Saaty scale can be modernized in a way that expert(s) choose a linguistic term associated with a corresponding T1MF that best characterizes his/her assessment. As an example (Figure 1), five type-1 symmetrical triangular membership functions that cover domain $U=[1,9]\subset\mathbb{R}$ can be used (Kabir & Hasin, 2011).

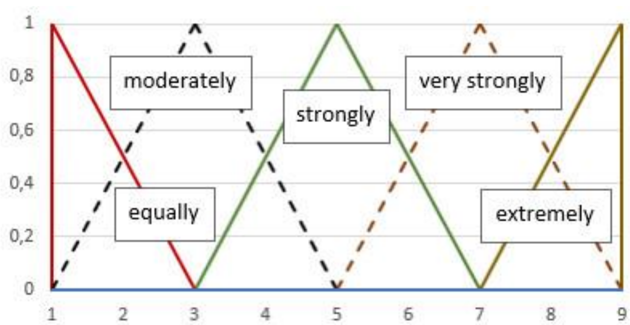

Figure 1 Possible form of modified fundamental (Saaty) scale

However, the fuzzy approach to AHP has its drawbacks (e.g. fuzzy numbers ranking that lacks prevailing view about its realization, disparity between FAHP arithmetic operations and basic principles of AHP, checking validity of results obtained) and still cannot be considered an ideal approach (Zhü, 2014). It's worth noting that Thomas L. Saaty questioned unjustified fuzzification of AHP. In his reasoned opinion, he states that the numeric representation of pairwise comparison judgments embraces the "fuzziness" as it is a subject to uncertainty and posits that the fundamental scale in use is already fuzzy (Saaty & Tran, 2007). It is rather difficult to quarrel with such a stance, since the fuzzification of judgements blindly for the sake of improving consistency or effectiveness of decision-making results, "without giving good reasons for doing it", does not seem sound. Nevertheless, the idea of using a fuzzy logic approach as applied to AHP preserves the consideration of important practical nuances of expert judgements and forms of their expression used while filling in matrices *M*. In the author's opinion, an important serious deficiency of FAHP lies in the fact that the approach does not take into account the expert's confidence in the estimate he/she provides. It means that if one expert is confident about a specified k-th assessment in pairwise comparison (e.g. the opinion may look like a 2-tuple {"moderate importance", "surely"}), while another one expresses certain doubt in the same case, the calculations within the FAHP are made without taking such assurance levels into account.

A vital question must be asked in view of this dilemma. What could be a way to give a proper weight to an expert's confidence (reliability) degree provided along with the main estimate (restriction on values of variable interest) in FAHP? The main objective of the work is to develop the approach, which would make it possible to integrate an expert's confidence degree into the FAHP model. Consequently, the paper is organized as follows: Analytic Hierarchy Process (AHP) and its fuzzy counterparts (transition from AHP to FAHP) are considered in section 2. For each of those methods, drawbacks are identified and the need to develop a new approach is justified. Section 3 discusses the potential of interval type-2 membership functions (IT2MF) usage in FAHP, corresponding calculations based on Fuzzy Synthetic Extents (FSE) and the probability-theoretical interval comparison approach following the ideas of Chang (1996) and Sevastyanov, et al. (2002). Comparison of FAHP utilizing T1MF and FAHP using IT2MF along with interval comparison constitutes the foundations of section 4. The example based on the numerical data from Kabir & Hasim (2011) is discussed here. Necessary computational steps are performed both manually and with the assistance of a developed program (working prototype) ensuring the correctness of results obtained. Final remarks concerning the approach and potential for its further development are drawn in the conclusion section.

2. Transition from AHP to FAHP – brief overview of approaches

If experts express their opinions verbally (words or phrases in natural language), then these judgments are inherently inaccurate (vague); such a situation complicates the conversion of verbal estimates to specific integers ranging from 1 to 9. If we consider odd elements 1,3,5,7 and 9 as core values on the fundamental scale, the intermediate even values 2,4,6 and 8 can be considered by way of auxiliary 'anchors' for experts to adjust the mapping of verbal statement(s) to proper numeric label(s) (Saaty & Tran, 2007). Such an opportunity seems practical, but another challenging problem emerges. Calculations with values that fill in matrices *M* do not consider degrees of confidence that an expert may, and usually does, attach to judgments provided. A specialist normally supplements their opinion with remarks, which can be reduced to short linguistic labels like "almost sure", "beyond a doubt", "unsure at large", etc. Two expert opinions O_1 and O_2 can be identical ($O_1 = O_2 = O_{\text{eq.}}$) in classical AHP, but the appearance of confidence (reliability) degrees C_1 and C_2, $C_1 \neq C_2$, as constituents of expert judgments results in different estimates as 2-tuples $\left(O_{\text{eq.}}, C_1\right)$ and $\left(O_{\text{eq.}}, C_2\right)$ that must be addressed properly.

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2.1 FAHP –Use of type-1 membership functions

If MOP is filled in with fuzzy expert estimates (e.g. triangular T1MF), the basic question is closely connected with processing of fuzzy pairwise comparison matrices. Working out approaches to potential computational schemes of FAHP was not limited to isolated instances in the last few decades. For example, the fuzzy variant of Saaty's pairwise comparison extended by the Graan-Loostma method was proposed by van Laarhoven and Pedrycz (1983) to obtain fuzzy performance scores (and weights) from the matrix of fuzzy ratios. Fuzzy opinions/comparisons were expected in the form of triangular fuzzy numbers (de Graan, 1980; Lootsma, 1980; van Laarhoven & Pedrycz, 1983). It was the first paper to address the use of FAHP and it still attracts the attention of researchers today. According to Google Scholar, it was cited more than 2,660 times as of December 2018.

The same approach to pairwise comparisons of triangular fuzzy numbers (TFN) was used by Chang (1992, 1996). Based on a fuzzy comparison matrix being filled with TFNs, he used an extent analysis method to calculate crisp weights (priority vector was obtained on the base of arithmetic means). The rule to compare fuzzy numbers was considered in the paper providing a way to estimate the degree of possibility of $\mathrm{A_1} \geq \mathrm{A_2}$ ($\mathrm{A_2} \geq \mathrm{A_1}$), where $\mathrm{A_1}$ and $\mathrm{A_2}$ are two TFNs. Within the framework of this approach, a set of sums of elements for each i-th row of matrix $M = \left[m_{\mathrm{ij}}\right]_{\mathrm{n\times n}}$ ($\mathrm{R}S_{\mathrm{i}} = \sum_{\mathrm{j=1}}^{\mathrm{n}} m_{\mathrm{ij}} = \left(\sum_{\mathrm{j=1}}^{\mathrm{n}} l_{\mathrm{ij}}, \sum_{\mathrm{j=1}}^{\mathrm{n}} c_{\mathrm{ij}}, \sum_{\mathrm{j=1}}^{\mathrm{n}} r_{\mathrm{ij}}\right)$, $\mathrm{i} = \overline{1,\mathrm{n}}$; l_{ij}, c_{ij} and r_{ij} are left, center and right points defining (i,j)-th matrix element as piecewise linear function, correspondingly) is obtained first. After that, all calculated 3-tuple values $\mathrm{R}S_{\mathrm{i}}$ are normalized, i.e. Fuzzy Synthetic Extents (FSE) $\overline{S}_{\mathrm{i}} = \mathrm{R}S_{\mathrm{i}} \Big/ \sum_{\mathrm{j=1}}^{\mathrm{n}} \mathrm{R}S_{\mathrm{i}}$, $\mathrm{i} = \overline{1,\mathrm{n}}$, as fuzzy variant of arithmetic means are calculated for each i-th row of M matrix (Chang, 1996; Wang et al., 2008; Shapiro & Koissi, 2017). Normalized values are then used to determine degrees of possibility that one FSE is greater than another one as follows (both $\overline{S}_{\mathrm{i}}$ and $\overline{S}_{\mathrm{j}}$ are in the form of TFN, indices i and j allow to distinguish between them in the formula, $\mathrm{i,j} = \overline{1,\mathrm{n}}$):

$$\mathrm{V}\left(\overline{S}_{\mathrm{i}} \geq \overline{S}_{\mathrm{j}}\right) = \begin{cases} 1, & \text{if } c_{\mathrm{i}} \geq c_{\mathrm{j}} \\ (r_{\mathrm{i}} - l_{\mathrm{j}}) / ((r_{\mathrm{i}} - c_{\mathrm{i}}) + (c_{\mathrm{j}} - l_{\mathrm{j}})), & \text{if } l_{\mathrm{j}} \leq r_{\mathrm{i}} \\ 0, & \text{otherwise} \end{cases}$$

Then, these degrees are compared with the purpose of obtaining for each $\overline{S}_{\mathrm{i}}$ the smallest value $\mathrm{mv_i} = \mathrm{V}\left(\overline{S}_{\mathrm{i}} \geq \overline{S}_{\mathrm{j}} \mid \mathrm{j} \neq \mathrm{i},\ \mathrm{j} = \overline{1,\mathrm{n}}\right)$ utilized in calculating components of priority vector W (crisp weights $w_{\mathrm{i}} = \mathrm{mv_i} \Big/ \sum_{\mathrm{k=1}}^{\mathrm{n}} \mathrm{mv_k}$) for criteria (alternatives) under consideration. Further material is built on the ideas presented by Chang (1996). This paper also attracts a lot of attention (according to Google Scholar, it has been cited more than 3,390 times). However, certain issues concerning the application of extent analysis on fuzzy AHP were highlighted by Wang et al. (2008) He stresses weights determined by the extent analysis method. He showed that in some cases calculated weights do not convey relative

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importance of criteria (alternatives). Besides, the potentially possible appearance of zero weights associated with some criteria or alternatives leads to the exclusion of the latter from consideration in decision analysis. These aspects must definitely be placed among the limitations of Chang's model (Shapiro & Koissi, 2017).

2.2 FAHP – Gradual switch to interval type-2 membership functions

The application potential of IT2MF in both the AHP and ANP have received the active attention of researchers during the last decade both in theory and in relation to solving applied problems (transportation, healthcare, vendor selection, infrastructure management, etc.). On the one hand, publications touch upon vital issues of uncertainty measures – "each of them is an interval, and the length of the interval is an indicator of uncertainty" – for IT2FS, transformation of linguistic perceptions (words, phrases, etc.) into IT2FS, and schemes of aggregation of subjective judgments (pre-ranked linguistic measures) in decision-making problems to name a few (Wu & Mendel, 2007b; Mendel & Wu, 2006; Wu & Mendel, 2007a). All those topics are of current importance when the matter concerns data preparation to fill in matrices *M*. We mean here elicitation of verbal opinions from *n* experts, their representation by way of IT2MF and further aggregation to obtain the elements for processing under FAHP. On the other hand, other authors carried out research in the broad field of IT2MF-based fuzzy decision-making approaches that extend the classical Bellman-Zadeh (1970) model and take into account risk attitude, and utilize various ranking methods of IT2FS in the decision making process (Runkler et al., 2017; Chen & Lee, 2010; Chen et al., 2012; Abdullah & Najib, 2014; Chiao, 2016). It makes it clear that IT2FS as an object of research gives ample scope for their use in different computational methods and their modifications aimed at producing decision priorities. For stakeholders involved in a given problem, the latter serves as a reference point and a topic for discussion when the ultimate decision is made.

One important milestone in the development of FAHP is the extension of hierarchical analysis to the use of fuzzy ratios in preference matrices that was considered by Buckley (1985). The proposed model made it possible to capture the vague (imprecise) responses of experts in the form of fuzzy numbers, and it broadens the scope of Saaty's AHP while comparing pairs of alternatives. Fuzzy weights of those alternatives are calculated based on the geometric mean method followed by a regular combination procedure that takes the hierarchical structure under consideration. It was the first time that Burkley's fuzzy AHP model was subsequently used and further extended by Kahraman, Öztayşi et al. (2014) to the case of IT2FS filling in MOP. Their approach makes provision for the transition to a linguistic scale and the use of new defuzzification methods (DTriT and DTraT) for triangular and trapezoidal IT2FS at certain computational steps. These methods relate to calculating defuzzified values (DVs) of fuzzy sets followed by ranking of the latter with respect to the DVs obtained. The ranking approach yields results that may differ from those ranks procured, for example, by means of likelihood method and arithmetic operations on IT2FS (Kahraman et al., 2014; Chen & Lee, 2010). It's normal, since the ranking outcome depends on particularities of the approach in use. Some of the approaches and their brief comparison can be found in Chen & Lee (2010) and Chen and Yang at al. (2012).

The computational scheme proposed by Kahraman et al. (2014) relates to direct manipulations with IT2FS (formal representation of linguistic terms) that are elements of pairwise comparison matrices, use of defuzzified values to check the consistency of the matrix constructed and calculation of geometric means for each row of the comparison matrix with obtaining priority weights of criteria (alternatives). The global weights of alternatives are first calculated in the form of IT2MF, then they are defuzzified to get normalized weights (crisp numbers) determining the final ranking of the alternatives (best option). The core word as applied to manipulations with IT2FS under this method is “direct”. In the present paper, IT2FS as aggregated expert estimates determined by lower and upper type-1 MFs are used in Chang’s model (1992, 1996) which is expanded with the computationally efficient probability-theoretical approach to compare calculated Fuzzy Synthetic Extents (FSE). The results of comparison come to interval-based priorities (ranking) of alternatives.

2.3 FAHP – Most influential papers and the choice of Chang’s model

Several models of fuzzy logic versions of AHP that underlie the previously mentioned approaches are discussed by Shapiro and Koissi (2017). These authors emphasize Lootsma (1980), van Laarhoven & Pedrycz (1983), Burkley (1985) and Chang (1992, 1996) as the most influential FAHP papers that laid the basis for publications that appeared from the 1990s until now. They advance arguments for the appropriateness of fuzzy versions of AHP incorporating the conclusions and observations drawn by several cited authors. In particular, they touch on issues of natural language statements used by decision makers, vagueness of human thoughts that cannot be modeled adequately by crisp numbers, inherent imprecision of pairwise comparisons, casting doubt on precise estimate values. Moreover, each of publications by van Laarhoven & Pedrycz (1983), Burkley (1985) and Chang (1996) received practically 200 or more citations per year from 2012 - 2017 (Ahmed & Kılıç, 2018).

As a result of the previous discussion, it is justified to ask whether or not it is advisable to choose the Chang model as a basis for computations in the proposed approach. The choice can be defended for several reasons. First, the Fuzzy Extent Analysis is one of the most commonly used and cited models in the last 5-6 years when considering the most influential FAHP papers (Ahmed & Kılıç, 2015). In spite of its existing weak points, the citations of Chang’s work totals 300-350 per year for this period of time which perceptibly exceeds indices of “competitors” (Ahmed & Kılıç, 2018). The Fuzzy Extent Analysis is attractive because of its possible modifications that affect the effectiveness of decision making. Ranking (pairwise comparison) of fuzzy sets can be done differently, alternative approaches to defuzzification as a significant constituent of Chang’s method may also be studied in detail to contribute to such changes (Ahmed & Kılıç, 2015; Chang, 1996). These reasons support the use of the given method now as well as support it as a subject of further consideration and improvement. Secondly, the computational steps of the model are simple, and this fact can support its use by way of illustration of the method in the paper. Also, Chang’s method as any other FAHP models, can be extended by type-2 fuzzy sets in different ways (here we mean processing steps as such). Thus, it makes space for potentially extensive studies in the field of comparing results of various FAHP methods enriched with interesting and wholesome T2FS formalism (Kahraman et al., 2014).

2.4 FAHP – Z-numbers as a new trend in CWW

Another attempt to add flexibility and an approach to language-oriented representation of expert estimates to the original AHP is the Z-AHP approach. It is grounded on the concept of Z-number proposed by Zadeh as a new theoretic trend in the field of

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Computing with Words (CWW) (Zadeh, 2011). Z-AHP is worth mentioning here as a topic of independent arresting research in the field of fuzzy logic; however, it is not covered in the present paper. The Z-number concept is based on the important aspect of information's (human assessment) reliability coupled with constraint on values of uncertain variables. A reliable summary (fuzzy granule) formalizes inherent uncertainty of information human's deal with routinely (Zadeh, 2011). Z-number is viewed as a pair $\left(A, B\right)$, where the first component A in the form of T1MF is a constraint on values of uncertain variable $X \subset U$ (□). The second component (B) expresses reliability of A, i.e. the degree of confidence that A is exactly as specified. Thus, Z-number "absorbs" the ideological foundations of fuzzy logic, basics of natural language modeling and reliability of composed information granule(s).

Publications in the field of multicriteria decision making (MCDM) responded to the potential of Z-number application in well-known methods (e.g. AHP, TOPSIS, VIKOR, TODIM); however, they basically provide for conversion of Z-numbers to classical fuzzy numbers (Azadeh et al., 2013; Kang et al., 2012; Zadeh, 2011; Yaakob & Gegov, 2016; Kang et al., 2018; Krohling et al., 2018). For example, Azadeh et al. (2013) considered the Z-AHP approach built upon initial conversion of Z-numbers to regular FN characterized by T1MF. Such conversion was originally proposed by Kang and Wei, et al. (2012) under preserving fuzzy expectation of fuzzy set in view. The conversion simplifies the processing phases, but it also alters fuzzy granules summarized in initial expert estimates. Currently, the name Z-AHP virtually implies FAHP because of the conversion stage that 'adds' the weight of the second part (B) of the Z-number to its first part (A). In that sense, Z-AHP as it is presented in many publications endeavors to escape at the early stages from its ′Z-′ prefix in name and aims at being considered as one of the FAHP approaches that makes use of T1FS.

3. IT2MF in FAHP – how do we come to interval comparison?

We pursue an object to develop a model that implicitly takes into account constraints on an expert's assessments as they can be supplied with certain degrees of confidence. Type-2 membership functions (T2MF) can be considered an option here; however, the lack of available data needed to construct them can be a problem. Despite the conceptual appeal, T2MF requires more lengthy computations in comparison with type-1 fuzzy sets. Therefore, IT2MF does not have the processing complexity associated with generalized T2MF and still provides a sufficient degree of flexibility. They are widely used in practical problems and make it possible to reflect (model) linguistic uncertainty adequately (Mendel et al., 2006; Gimaletdinova & Degtiarev, 2017). IT2MF are characterized by constant secondary membership functions for every parameter $x \in U$ (x denotes a *primary* variable). The matter concerns type-2 function $\mu(x,u)$, where $x \in U$, $u \in J_x \subseteq [0,1]$ (u as a *secondary* variable has domain J_x as a primary membership of x), $0 \le \mu(x,u) \le 1$, and all $\mu(x,u)$, secondary grades of IT2MF, are equal to 1 (Mendel & John, 2002). More details and comments on IT2MF can be found in Mendel, John & Liu (2006), Wu & Mendel (2007b), Mendel (2017), and Mendel et al. (n.d.).

The statement $J_x \subseteq [0,1]$ merits special attention as it is considered at length in Bustince, Fernandez, Hagras et al. (2015) in the context of four special interpretative cases, i.e. multisets, combination of numbers and intervals, interval-valued fuzzy set (IVFS) and multi-IVFS. The theory of IT2FS and their formal processing (operational base) proceed from the interpretation of J_x under identity IT2FS = IVFS (Mendel, Hagras et al., 2016). The intrinsic potential of the relationship between IVFS and IT2FS requires close attention and further study. In the present paper, the definition of J_x as IVFS, i.e. $\left\{(x,u) \mid u \in [\underline{\mu}(x), \overline{\mu}(x)]\right\}$, where $\underline{\mu}(x)$ and $\overline{\mu}(x)$ are lower and upper membership functions of FS, correspondingly, is used without any alterations. Therefore, the abbreviation 'IT2FS' in the sense of IVFS prevails throughout the text.

3.1 IT2MF - can we link LMF and UMF with confidence of estimates?

Lower and upper type-1 membership functions (LMF and UMF) of some IT2MF A play a role of delimiters of the footprint of uncertainty (FOU) area as a union of all vertical slices of type-2 function representing memberships (intervals) for all $x \in U$, $FOU(A) = \bigcup_{x \in U} J_x$ is the region enclosed by LMF and UMF of A. These intervals, or more specifically, their alterable widths, may represent degrees of an expert's confidence in the estimates provided. The wider the corresponding interval is, the lesser degree of confidence it manifests. More narrow intervals correspond to a greater (proportionally) degree of confidence. Based on these degrees, the prototype of the original FAHP approach can be developed. At each point $x \in U$, values of LMF and UMF functions serve in the capacity of expert's estimate. The width of the vertical interval bounded by upper and lower MF values can be used to express the expert's confidence level. We accept the premise that MOP *M* is filled in with expert estimates in the form of IT2MFs (results of aggregation of individual measures obtained from expert group). These functions must be compared, and the possibility degree that one of the functions is bigger than the other one steps aside from ordinary number towards an interval owing to the presence of LMF and UMF. To ensure computational efficiency, the preference is given to piecewise linear functions (triangular shape) as upper and lower MFs of IT2MF.

Following Chang's model (1992, 1996) and performing calculations of Fuzzy Synthetic Extents (FSE) for matrix *M* (generalized notation introduced earlier) filled in with IT2MFs with further comparison of the latter, we should realize that degrees of possibility that $\overline{S}_i$ is greater than $\overline{S}_j$ expressed by ordinary numbers will turn into intervals due to the fact that IT2MF is bounded by upper and lower T1MFs. We must consider the intersections of upper and lower membership functions (UMF and LMF) separately to obtain resultant intervals. Operations of addition, subtraction, multiplication and division as applied to two intervals denoted as $A = [a_1, a_2]$ and $B = [b_1, b_2]$ can be expressed as follows:

1. $A + B = [a_1, a_2] + [b_1, b_2] = [a_1 + b_1, a_2 + b_2]$
2. $A - B = [a_1, a_2] - [b_1, b_2] = [a_1 - b_2, a_2 - b_1]$
3. $A * B = [\min(a_1 * b_1, a_1 * b_2, a_2 * b_1, a_2 * b_2), \max(a_1 * b_1, a_1 * b_2, a_2 * b_1, a_2 * b_2)]$
4. $A / B = [\min(a_1 / b_1, a_1 / b_2, a_2 / b_1, a_2 / b_2), \max(a_1 / b_1, a_1 / b_2, a_2 / b_1, a_2 / b_2)]$,

where a_1 and a_2 are left and right boundaries of the first interval, b_1 and b_2 – left and right boundaries of the 2nd interval at hand.

3.2 Can comparison of intervals be used in IT2MF-based AHP?

To derive the weight vector by the algorithm proposed by Chang (1996), we need to find the minimal possibility degree reflecting the fact that one FSE is greater than another one for each row of matrix *M*. Since these degrees are represented by intervals, it requires some effort to find the smallest of them. In the case of two intervals, the smallest one can easily be uncovered only if the right boundary of one interval is less than the left boundary of the second interval. In all other cases, different approaches to comparison can be utilized; in the framework of the present paper, the algorithm proposed by Sevastyanov et al. (2002) is considered (Sevastianov, 2007).

To summarize the ideas that can be put into practice, assume that there are two independent intervals $A = [a_1, a_2]$, $B = [b_1, b_2]$, and independent random variables $a \in A$ and $b \in B$ having uniform distribution over intervals A and B. If corresponding intervals intersect, it leads to the appearance of subintervals being a subject of further study. Let the event H_k be such that $a \in \overline{A}$, $b \in \overline{B}$, where $\overline{A}$ and $\overline{B}$ are subintervals of A and B, correspondingly. All $\overline{A}$ constitutes interval A, and the same can be said about the totality of subintervals $\overline{B}$ with reference to B. Probability $P(H_k)$ can be calculated based on the reasoning from geometric considerations due to the aforesaid distribution of a and b. The following formula was proposed by Sevastyanov et al. (2002) to compare intervals, i.e.

$$P(B > A) = \sum_{k=1}^{n} P(H_k) \cdot P(B > A/H_k),$$

where A and B are two independent intervals, H_k is the event related to variables a and b falling into corresponding subintervals, n (upper parameter in summation) is the number of such events. For example, consider the following events H_k, $k = \overline{1,4}$, for intersecting intervals A and B (Figure 2):

H_1 - $a \in [a_1, b_1] \cap b \in [b_1, a_2]$ $\quad$ H_2 - $a \in [a_1, b_1] \cap b \in [a_2, b_2]$
H_3 - $a \in [b_1, a_2] \cap b \in [b_1, a_2]$ $\quad$ H_4 - $a \in [b_1, a_2] \cap b \in [a_2, b_2]$

Thus, P(·) values can be expressed as follows:

$P(H_1) = ((b_1 - a_1)/(a_2 - a_1)) * ((a_2 - b_1)/(b_2 - b_1))$,
$P(H_2) = ((b_1 - a_1)/(a_2 - a_1)) * ((b_2 - a_2)/(b_2 - b_1))$,
$P(H_3) = ((a_2 - b_1)/(a_2 - a_1)) * ((a_2 - b_1)/(b_2 - b_1))$ and
$P(H_4) = ((a_2 - b_1)/(a_2 - a_1)) * ((b_2 - a_2)/(b_2 - b_1))$, i.e. $\forall P(B > A/H_k) = 1$, $k \neq 3$; if $k = 3$, corresponding $P(\cdot) = 0.5$.

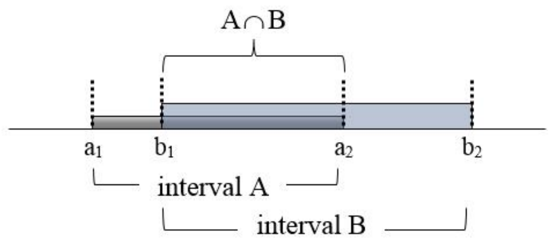

Figure 2 Schematic representation of intersection of intervals A and B

The result of calculations is the probability value to be interpreted so that if it is greater than a certain threshold value θ, one interval is considered as being bigger than the other one (Sevastyanov et al., 2002). The choice of this threshold value depends on the problem being solved, and in certain situations it makes sense to take a value close to 0.95 (as an example of a rather big value that is approaching 1). If no special restrictions are imposed, it is suggested to take a value greater than 0.5 (Sevastyanov et al., 2002). Is it possible to unequivocally accept this advice while dealing with modeling fuzziness in FAHP? Most likely, the value of 0.5 can be accepted as a primary general reference point, but the answer to the question raised relates, in the first place, to peculiarities of natural language statements in use and the task (problem) per se. Responses obtained by way of such constructs (words, phrases, short sentences) bear the tinge of vagueness and inaccuracy. It is almost impossible to specify exactly the "optimal" deviation from the 0.5 landmark (apparently towards bigger values) to derive the threshold value applicable to the case at hand. It means that such a value can be uncovered based on empirical studies only. Results of several conducted experiments (attainment of identical outcomes by means of FAHP based on using T1MF and the proposed approach covered by the paper) allowed the threshold to be fixed to a value θ on average at the level of appr. 0.83; several tests ensured values in the range $\theta \in [0.64, 0.96]$. Thus, it meets the previously mentioned suggestion (the value exceeding 0.5) and does not look like an excessively big one. Naturally, we may consider such an estimate as tentative (rough) and a purely empirical trial that may require subsequent refinement, but now it entirely suits us as a landmark to use in the following steps of the approach. On the other hand, the value of 0.83 corresponds to the case of coincidence of results assured by basic FAHP utilizing T1MF and the proposed approach. Coincidence is not just *idee fixe* here, it simply shows that under a certain value of threshold we may get the same results (in terms of final ranking) when different approaches are used. The choice of threshold at intermediate computational steps adds extra flexibility in terms of perception of attained results allowing the human factor to show itself not only in fuzzification of linguistic data.

Having at our disposal the method for comparing FSE, calculations over matrix M can be performed in the context of FAHP with recourse to IT2MF. However, one important question remains. How do we fill in cells of matrix M? In the case of T1MF-based FAHP, a fuzzy scale based on T1MF (extension of the fundamental scale) is used, and it does not consider the degree of confidence of assessments experts provide. The latter seems applicable in our case, although it requires some changes of the scale subject to expressed degrees of confidence. For instance, as was mentioned above, depending on the degree of confidence expressed by experts, we may transform the upper and lower boundaries into intervals proceeding from the fact that the higher the degree of confidence is, the narrower the interval becomes. Thus, we may introduce a fuzzy expert's confidence scale that bears a stark analogy with the scale shown in Figure 1. Membership functions $A_1 \div A_5$ shown can be associated from left to right with linguistic labels "not sure at all" (A_1), "not very sure" (A_2), "surely" (confident/certain, A_3), "very sure" (A_4) and "absolutely sure" (A_5). A symmetrical case of membership functions is easier in the view of processing. However, it is not a mandatory requirement, and it can be tempered at any time.

The starting point for T1MF-to-IT2MF conversion can be presented as follows: as a natural conviction, the estimate from the base scale corresponds to the average degree of confidence (linguistic term "surely"). To do this, we also need to choose the rule to convert T1MF to IT2MF. For this purpose, a series of experiments was carried out to recognize that it is appropriate to convert upper and lower boundaries to interval $\Delta = 0.03$ units long being symmetrically located relative to original upper and lower bounds. For example, consider the estimate (2.02, 3.08, 4.64) representing T1MF in the context of basic FAHP. To convert it to the expert's estimate (opinion) with a confidence degree "surely" ("I'm sure"), we perform the transformation described above to obtain realistic enough albeit empirically specified boundaries ((2.005, 2.035), 3.08, (4.625, 4.655)) of IT2MF. On the condition that the threshold value θ equals 0.83, experimental studies reveal almost identical results obtained by IT2MF-based FAHP and 'original' FAHP that uses T1MF. However, it should be emphasized that the actual choice of "conversion" value(s) is a topic for rapt attention and further study because it is closely bound up with distinctive features of the problem under consideration, preferred methods to aggregate expert opinions to obtain IT2MF and other substantial factors. Essentially, such value(s) must be determined, but not just chosen to minimize the impact of the human factor and possible biases in favor of various inexplicable reasons.

To use other linguistic terms as degrees of expert's confidence (e.g. $A_1 \div A_p$, where 'magic' number p equals 7 ± 2), defuzzification of fuzzy numbers representing linguistic terms on a fuzzy confidence scale can be done to calculate the ratio of these numbers. Depending on the degree of confidence, the interval covered by IT2MF should be either expanded or grow narrower in accord with the ratio of defuzzified values of FNs on the confidence scale.

4. Comparison of FAHP based on T1MF and FAHP using IT2MF

To compare the proposed modification of FAHP (pmFAHP) with the already existing version(s), we consider the example used by Kabir and Hasim (2011). The authors compare results of AHP and FAHP. By the same example, we will in turn examine the results provided by basic FAHP (the one that uses T1MF (TFN) as expert estimates) and pmFAHP. The following data table (matrix $M = [m_{ij}]$ that is filled in with triangular FNs represents aggregated results as provided by fourteen domain experts and corresponding reciprocals) is used in the example (Kabir & Hasim, 2011).

Table 1
Fuzzy matrix *M* of attribute comparison

Attributes	Unit Price	Annual Demand	Criticality	Last Use Date	Durability
Unit Price	**1.0, 1.0, 1.0**	0.89, 1.6, 2.25	0.65, 1.07, 1.88	0.82, 1.47, 2.76	0.8, 1.37, 3.19
Annual Demand	0.44, 0.62, 1.12	**1.0, 1.0, 1.0**	2.02, 3.08, 4.64	0.8, 1, 1.47	1.17, 2.36, 4.53
Criticality	0.53, 0.93, 1.53	0.22, 0.34, 0.5	**1.0, 1.0, 1.0**	0.68, 1.11, 1.66	0.8, 1, 1.72
Last Use Date	0.36, 0.68, 1.21	0.68, 1, 1.26	0.6, 0.9, 1.47	**1.0, 1.0, 1.0**	0.76, 0.93, 1.25
Durability	0.31, 0.73, 1.26	0.22, 0.42, 0.86	0.58, 1, 1.26	0.8, 1.08, 1.32	**1.0, 1.0, 1.0**

Fuzzy numbers m_{ij} summarized in Table 1 can be represented as $(\alpha_{\text{ij}}/\beta_{\text{ij}},\beta_{\text{ij}}/\delta_{\text{ij}})$, where $0<\alpha_{\text{ij}}\le\beta_{\text{ij}}\le\delta_{\text{ij}}$ (Buckley, 1985). For example, TFN $m_{12}=(0.89,1.6,2.25)$ may be written by way of pair $(0.89/1.6,1.6/2.25)$, and m_{12}^{-1} becomes approximately $(\delta_{12}^{-1}/\beta_{12}^{-1},\beta_{12}^{-1}/\alpha_{12}^{-1})$, i.e. $m_{21}\approx(0.44/0.62,0.62/1.12)$. The fuzzy matrix M can be expressed as a totality of three crisp matrices M_{l} (left), M_{c} (center) and M_{r} (right) by the number of parameters defining each TFN. It is shown by Buckley (1985) that a fuzzy positive reciprocal matrix M is consistent if and only if M_{c} is consistent (reduction to classical hierarchical analysis). With reference to M_{c} matrix, the principal eigenvalue $\lambda_{\max}\approx 5.27$, consistency index $\text{CI}\approx 0.068$ and consistency ratio $\text{CR}=\text{CI}/\text{RI}\approx 0.06$ (under $\text{RI}_{(n=5)}=1.12$) point to the conclusion that matrix M_{c} (the same as matrix M) is acceptably consistent (Saaty, 1980; Saaty, 1987).

Based on the matrix *M* shown above, FSE are calculated first, i.e.

$$S_{\text{U}}=(4.16,6.51,11.08)\times(1/42.14,1/27.68,1/19.13)=\left(0.09,0.235,0.58\right),$$
$$S_{\text{A}}=(5.43,8.06,12.76)\times(1/42.14,1/27.68,1/19.13)=\left(0.13,0.291,0.67\right),$$
$$S_{\text{C}}=(3.23,4.38,6.41)\times(1/42.14,1/27.68,1/19.13)=\left(0.077,0.158,0.34\right),$$
$$S_{\text{L}}=(3.4,4.51,6.19)\times(1/42.14,1/27.68,1/19.13)=\left(0.08,0.163,0.32\right),$$
$$S_{\text{D}}=(2.91,4.23,5.7)\times(1/42.14,1/27.68,1/19.13)=\left(0.07,0.153,0.3\right).$$

Thereafter, probabilities that one FSE is greater than another one for each FSE concerned are as follows:

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$$
\begin{array}{llll}
\mathrm{P}(S_{\mathrm{U}} \geq S_{\mathrm{A}}) = 0.9 & \mathrm{P}(S_{\mathrm{U}} \geq S_{\mathrm{C}}) = 1 & \mathrm{P}(S_{\mathrm{A}} \geq S_{\mathrm{U}}) = 1 & \mathrm{P}(S_{\mathrm{A}} \geq S_{\mathrm{C}}) = 1 \\
\mathrm{P}(S_{\mathrm{U}} \geq S_{\mathrm{L}}) = 1 & \mathrm{P}(S_{\mathrm{U}} \geq S_{\mathrm{D}}) = 1 & \mathrm{P}(S_{\mathrm{A}} \geq S_{\mathrm{L}}) = 1 & \mathrm{P}(S_{\mathrm{A}} \geq S_{\mathrm{D}}) = 1 \\
& \mathrm{P}(S_{\mathrm{C}} \geq S_{\mathrm{U}}) = 0.75 & \mathrm{P}(S_{\mathrm{C}} \geq S_{\mathrm{A}}) = 0.61 & \\
& \mathrm{P}(S_{\mathrm{C}} \geq S_{\mathrm{L}}) = 0.98 & \mathrm{P}(S_{\mathrm{C}} \geq S_{\mathrm{D}}) = 1 & \\
& \mathrm{P}(S_{\mathrm{L}} \geq S_{\mathrm{U}}) = 0.75 & \mathrm{P}(S_{\mathrm{L}} \geq S_{\mathrm{A}}) = 0.6 & \\
& \mathrm{P}(S_{\mathrm{L}} \geq S_{\mathrm{C}}) = 1.0 & \mathrm{P}(S_{\mathrm{L}} \geq S_{\mathrm{D}}) = 1 & \\
& \mathrm{P}(S_{\mathrm{D}} \geq S_{\mathrm{U}}) = 0.7 & \mathrm{P}(S_{\mathrm{D}} \geq S_{\mathrm{A}}) = 0.55 & \\
& \mathrm{P}(S_{\mathrm{D}} \geq S_{\mathrm{C}}) = 0.98 & \mathrm{P}(S_{\mathrm{D}} \geq S_{\mathrm{L}}) = 0.96 &
\end{array}
\tag{1}
$$

For each $\mathrm{S}_{(\square)}$ in the results of comparisons (1), we choose the smallest value and placed it into the vector of weights *W*. For the example under view, a 5-component (by the number of attributes used in matrix *M*) vector $W = (w_1, \ldots, w_5)^{\mathrm{T}} = (0.9, 1.0, 0.61, 0.6, 0.55)^{\mathrm{T}}$ is obtained. After normalization it takes the form of

$$
W^{\text{норм}} = \left(w_1^{\text{н}}, \ldots, w_5^{\text{н}}\right)^{\mathrm{T}} \approx (0.246, 0.273, 0.167, 0.164, 0.15)^{\mathrm{T}} \tag{2}
$$

Vector (2) expresses preferences of experts summarized in the assessment provided. In this case, attributes can be enumerated in descending order of preference (priorities). Annual Demand (**A**), Unit Price (**U**), Criticality (**C**), Last Use Date (**L**) and Durability (**D**) constitute a chain $\mathbf{A} \succ \mathbf{U} \succ \mathbf{C} \succ \mathbf{L} \succ \mathbf{D}$ with **A** having the highest preference (27.3%). Weights of alternatives **C** and **L** are barely discernible, and this issue should be addressed by analysts separately. The names of attributes used here are the same as those used by Kabir and Hasim (2011).

What can be said about results of the proposed approach? As we use the same reference Table 1 and since we have no information on the degree of certainty (confidence), to which HE evaluations are associated with, we may naturally assume that these estimates imply an average degree of certainty. It seems that it is a natural conviction, i.e. a group of experts confirm the assessment (aggregated IT2MF) with the label/degree "surely". Thus, matrix M_{MOD} takes the following form:

Table 2
Modified fuzzy matrix M_{MOD} of attribute comparison

Attributes	Unit Price	Annual Demand	Criticality	Last Use Date	Durability
Unit Price	**(1.0, 1.0), 1.0, (1.0, 1.0)**	(0.875, 0.905), 1.6, (2.235, 2.265)	(0.635, 0.665), 1.07, (1.865, 1.895)	(0.805, 0.835), 1.47, (2.745, 2.775)	(0.785, 0.815), 1.37, (3.175, 3.205)
Annual Demand	(0.441, 0.447), 0.625, (1.104, 1.142)	**(1.0, 1.0), 1.0, (1.0, 1.0)**	(2.005, 2.035), 3.08, (4.625, 4.655)	(0.785, 0.815), 1.0, (1.455, 1.485)	(1.155, 1.185), 2.36, (4.515, 4.545)
Criticality	(0.527, 0.536), 0.934, (1.503, 1.574)	(0.214, 0.216), 0.324, (0.491, 0.498)	**(1.0, 1.0), 1.0, (1.0, 1.0)**	(0.665, 0.695), 1.11, (1.645, 1.675)	(0.785, 0.815), 1.0, (1.705, 1.735)
Last Use Date	(0.36, 0.364), 0.68, (1.197, 1.242)	(0.673, 0.687), 1.0, (1.226, 1.273)	(0.597, 0.607), 0.9, (1.438, 1.503)	**(1.0, 1.0), 1.0, (1.0, 1.0)**	(0.745, 0.775), 0.93, (1.235, 1.265)
Durability	(0.307, 0.314), 0.729, (1.226, 1.273)	(0.22, 0.221), 0.423, (0.843, 0.865)	(0.576, 0.586), 1.0, (1.226, 1.273)	(0.79, 0.803), 1.075, (1.29, 1.342)	**(1.0, 1.0), 1.0, (1.0, 1.0)**

To check the consistency of the matrix M_{MOD}, the Defuzzified Triangular Type-2 Fuzzy Set (DTriT) approach can be applied without concern for the threshold value θ (Kahraman et al., 2014). For all elements of the modified matrix the maximum membership degree α of LMF equals one, and, for example, the defuzzified value (1.58) of m_{12} is calculated as $\left(\left(2.265-0.875+1.6-0.875\right)/3+0.875+\left(2.235-0.905+1.6-0.905\right)/3+0.905\right)/2$. The principal eigenvalue $\lambda_{\max}$ with reference to the crisp matrix with values defuzzified in such a way equals 5.57, $\mathrm{CI}\approx 0.142$, thus bringing CR to exceed the 0.1 guiding line (appr. 0.127). Type-2 defuzzification methods lead to obtaining non-fuzzy (crisp) values, and alternatively interval type-2 defuzzification approaches can be used to convert LMF and UMF of IT2MF to a type-1 membership function. Such transformation is called a type reduction technique (Mendel, 2017; Runkler et al., 2018). In particular, computationally efficient defuzzication that uses the vertical slice representation of IT2MF can be suggested for use (Nie & Tan, 2008). For instance, the element m_{21} of M_{MOD} matrix results in T1MF $(0.444, 0.625, 1.123)$, i.e. for each point $x \in U$ the method involves calculation of the mean of LMF and UMF. The subsequent computational steps are related to consistency checking on the basis of derived TFN. Proceeding from the close resemblance between defuzzified TFN from IT2MF (Table 2) and T1MF of the first model (fuzzy matrix M, Table 1) as well as the use of θ threshold, matrix M_{MOD} is rated as acceptably consistent ($\text{CR} \approx 0.061$).

Based on values summarized in Table 2, FSE are calculated on the assumption that in this case IT2MF constitute elements of the modified matrix:

$$S_{\text{U}} = (0.096, 0.101),\ 0.235,\ (0.57, 0.59) \qquad S_{\text{A}} = (0.126, 0.131),\ 0.291,\ (0.656, 0.676)$$
$$S_{\text{C}} = (0.075, 0.078),\ 0.157,\ (0.328, 0.342) \quad S_{\text{L}} = (0.079, 0.082),\ 0.162,\ (0.315, 0.331)$$
$$S_{\text{D}} = (0.068, 0.07),\ 0.152,\ (0.289, 0.303)$$

Also, probabilities that one FSE is greater than another one for each FSE become as

$$\begin{aligned}
&\text{P}\left(S_{\text{U}} \geq S_{\text{A}}\right) = [0.886, 0.891] \quad \text{P}\left(S_{\text{U}} \geq S_{\text{C}}\right) = [1.0, 1.0] \\
&\text{P}\left(S_{\text{U}} \geq S_{\text{L}}\right) = [1.0, 1.0] \quad \text{P}\left(S_{\text{U}} \geq S_{\text{D}}\right) = [1.0, 1.0] \\
&\text{P}\left(S_{\text{A}} \geq S_{\text{U}}\right) = [1.0, 1.0] \quad \text{P}\left(S_{\text{A}} \geq S_{\text{C}}\right) = [1.0, 1.0] \\
&\text{P}\left(S_{\text{A}} \geq S_{\text{L}}\right) = [1.0, 1.0] \quad \text{P}\left(S_{\text{A}} \geq S_{\text{D}}\right) = [1.0, 1.0] \\
&\text{P}\left(S_{\text{C}} \geq S_{\text{U}}\right) = [0.746, 0.76] \quad \text{P}\left(S_{\text{C}} \geq S_{\text{A}}\right) = [0.595, 0.617] \\
&\text{P}\left(S_{\text{C}} \geq S_{\text{L}}\right) = [0.979, 0.98] \quad \text{P}\left(S_{\text{C}} \geq S_{\text{D}}\right) = [1.0, 1.0] \\
&\text{P}\left(S_{\text{L}} \geq S_{\text{U}}\right) = [0.748, 0.765] \quad \text{P}\left(S_{\text{L}} \geq S_{\text{A}}\right) = [0.589, 0.614] \\
&\text{P}\left(S_{\text{L}} \geq S_{\text{C}}\right) = [1.0, 1.0] \quad \text{P}\left(S_{\text{L}} \geq S_{\text{D}}\right) = [1.0, 1.0] \\
&\text{P}\left(S_{\text{D}} \geq S_{\text{U}}\right) = [0.695, 0.715] \quad \text{P}\left(S_{\text{D}} \geq S_{\text{A}}\right) = [0.532, 0.561] \\
&\text{P}\left(S_{\text{D}} \geq S_{\text{C}}\right) = [0.976, 0.978] \quad \text{P}\left(S_{\text{D}} \geq S_{\text{L}}\right) = [0.953, 0.956]
\end{aligned} \tag{3}$$

In each case of intervals (3) we choose the lowest possible calculated values which define the bounds of intervals (Sevastyanov, Rog & Venberg, 2002; Sevastianov, 2007). Thus, the procedure results in the interval-based vector

$W = \left(w_1, ..., w_5\right)^{\text{T}} = \left([0.886, 0.891], [1.0, 1.0], [0.595, 0.617], [0.589, 0.614], [0.532, 0.561]\right)^{\text{T}}$.

After normalization of upper and lower bounds separately it leads to obtaining vector with five interval-based elements, i.e.

$$W^{\text{норм}} = \left(w_1^{\text{н}}, ..., w_5^{\text{н}}\right)^{\text{T}} = \begin{pmatrix} [0.242, 0, 245], [0.271, 0.277], [0.165, 0.167], \\ [0.163, 0.166], [0.147, 0.152] \end{pmatrix}^{\text{T}} \tag{4}$$

As it is seen from Equation 4, the attributes form a descending order preference list **A** $\succ$ **U** $\succ$ **C** $\succ$ **L** $\succ$ **D** that under the threshold value of 0.83 is completely equal to the same list (chain) obtained earlier. The weights-intervals of **C** and **L** intersect, but the one for alternative **C** is located slightly to the right of $w_4^{\text{н}}$ (it corresponds to **L**). It can be noted that the number of operations needed to perform the later version of the computational

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procedure as well as complexity of realization are not strongly affected. However, the flexibility of the approach becomes more apparent due to taking the degree of confidence of the expert(s) into account. Besides, ranking in the form of intervals gives an extra degree of freedom to decision-makers. Discussions with stakeholders can be rather tough in many practical cases, many arguments that are disregarded in the model for objective reasons (complexity of situation, information deficiency, diverse views, etc.) can be considered during discussions and consultations in the context of possible deviations from crisp values of priorities. The width of intervals makes it possible to add more conscious flexibility to the process of final decision's elaboration.

The prototype of the program implemented in Java 1.8 using JavaFX graphics packages and JFoenix (JavaFX material design library) gives an opportunity to verify the computational aspects of the proposed approach and to perform several experiments on both different and similar, although slightly altered, sets of data. When the program runs, the user as a decision-maker (expert) can choose his/her assessments and degrees of confidence from pre-defined lists (the latter can be modified in case of need) while filling in cells of matrices M at both levels L1 and L2 (criteria and alternatives). Besides, parameters of T1MFs associated with corresponding verbal labels can also be tuned as needed. The program converts given T1MFs to IT2MFs, performs calculations described above to generate final ranking of alternatives and displays results in a handy form of a bar graph.

5. Conclusion

The Analytic Hierarchy Process (AHP) is a widespread approach in decision making based on pairwise comparisons that implicitly consider both quantitative and qualitative characteristics of compared elements of the system's hierarchical model. In general, fuzzification of AHP makes it possible to deal with expert's imprecise estimates; however, in practice degrees of confidence that are normally attached to almost all judgments expressed by humans are those features that differentiate results of comparisons in matrices M. In other words, the same hypothetical estimate (opinion) O_1 with different degrees of confidence C_1 and C_2 must be perceived as two different information granules (O_1, C_1) and (O_1, C_2) to be considered and processed accordingly.

Such a view gives an impetus to the development of the FAHP approach (its prototype), which allows the mentioned trait to be taken into account. In the framework of the present paper, the original approach to FAHP is discussed. It handles not only fuzzy expert estimates, but also fuzzy degrees of confidence associated with them. Fuzzy logic creates grounds for modeling estimates expressed by vague words and statements in natural language. Ideas summarized in the paper are also implemented in a Java program whose interface (in Russian) allows one to choose a working project, specify/modify criteria and alternatives used in the project and graphically display the results (ranking of alternatives) of the algorithm. The relative transparency of primordial estimates specified in natural language constructs, ease of their use, cognitive clearness of possible changes in matrices M as well as graphical representation form of prioritization results become highly imperative during group discussions with stakeholders held while solving "soft" problems, i.e. problems, in which the human factor and verbal descriptive gestalts are the core dominants.

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The proposed approach also allows expert assessments to be handled (O_i, C_j) represented by IT2MFs under different threshold values θ. On the one hand, there is a possibility to tweak it neatly to the peculiarities of the task at hand. On the other hand, the choice of varying θ values permits the iterative realization of FAHP for the purpose of thorough understanding of the resultant prioritization of alternatives (tendencies observed, agility of interval's numerical bounds, etc.) within the all-around system analysis required in any sound decision-making problem. Further research into the role of θ and Δ parameters as well as fine tuning of their values also merits a detailed consideration. FAHP based on using IT2MF and a probability-theoretical approach to interval comparison does not seem like an unduly overburdened variant of AHP with no practical purport. Finally, experts get an opportunity to express their estimates (opinions) in customary and handy verbal forms that are transparently modeled for use at further computational steps.

We are fully aware of the fact that this fuzzy approach to AHP in the present state can be seen only as a prelude to its onward development. It refers mainly to formal representation of individual expert assessments by way of IT2MF (judgements as such jointly with confidence degrees) and computations that operate on such fuzzy information granules. Aggregation of individual IT2MF as expert's estimates (opinions), the use of different defuzzification approaches, modifications of Chang's model or the choice in favor of alternative computational schemes in the capacity of the core of IT2MF-based FAHP are those aspects and topics that bring specificity to further development of the method covered by the paper. In our opinion, interval-based computations also have a good expansion potential here. Along with the use of threshold values they can both be elaborated to arrive at a FAHP computational method that is comprehensible by all stakeholders and customizable.

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