Baran Bulut, D.& Özçevik Kal, M. (2022). The assessment of the dynamic learning environment designed for the construction of basic geometric concepts. *International Online Journal of Education and Teaching (IOJET), 10*(1). 321-347.

Received	: 12.08.2022
Revised version received	: 28.10.2022
Accepted	: 03.11.2022

THE ASSESSMENT OF THE DYNAMIC LEARNING ENVIRONMENT DESIGNED FOR THE CONSTRUCTION OF BASIC GEOMETRIC CONCEPTS

Research article

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THE ASSESSMENT OF THE DYNAMIC LEARNING ENVIRONMENT DESIGNED FOR THE CONSTRUCTION OF BASIC GEOMETRIC CONCEPTS*

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Abstract

In this study, it is aimed to evaluate the dynamic learning environment designed for 5th grade students to draw geometric objects and shapes. In this study, case study design was used. In this case study, 10 worksheets prepared by the researchers and were used to examine the construction processes of basic geometric objects and shapes in dynamic environments. In this learning environment, construction processes were carried out with three 5th grade students. One-to-one interviews, were conducted with the students to determine the changes in students' conceptual learning before and after the process which contains and basic geometry constructions tests (Pre-Post) were applied to determine the development of the students in the construction processes. As a result, it has been determined that the learning environment designed with dynamic support has a positive effect on the process of constructing basic geometric concepts and the development of their knowledge about them.

Keywords: Dynamic learning environment, Basic geometric concepts, GeoGebra, Construction process

1. Introduction

The learning outcomes of Geometry are included in all grades of the mathematics curriculum. In the curriculum (Head Council of Education and Morality [HCEM], 2018), regarding construction and object creation; it is aimed that the 5th grade students explain, show and draw basic geometric concepts such as line, line segment and ray. Another aim was that 5th grade students calculate the area of the rectangle in square centimeters and square meters, recognize the rectangular prism, determine its basic properties, draw the surface expansion and calculate the surface area. However, drawing the views of objects from different directions is included in the 7th grade. By the 8th grade, the sub-learning area of triangles is covered in depth and students are expected to understand the Pythagorean theorem and solve related problems. At this grade, the translation and reflection transformations are given in the sub-learning area of transformation geometry. Also, the concepts of equality and similarity in polygons are examined, and students are expected to identify and construct congruent and similar polygons. In addition, the right prism, right cylinder, right pyramid, and cone are present geometric objects (Ministry of National Education [MoNE], 2018). As can be seen, learning concepts and

* This study was produced from the master thesis study conducted by the second author under the supervision of the first author.

object drawings are heavily involved in new programs (Çekiç, 2018). Especially in the 5th grade learning outcomes, the basic geometric concepts are discussed in detail. According to the secondary school mathematics curriculum, the increase in the learned and taught concepts affects the significance of conceptual learning (Köprücü, 2020).

Accordingly, secondary school students have many misconceptions, and incomplete or incorrect information about the basic concepts of the line segment and ray in geometry (Kiriş, 2008; Doyuran, 2014; Kılıç, Temel & Şenol, 2015). Also, they confuse those concepts with each other, have difficulties in making sense of them and have trouble understanding them (Dane & Başkurt, 2011). Moreover, the students could not establish the relationship between geometric concepts and had misconceptions about basic geometry in every grade (Doyuran, 2014). In a study examining student opinions, students misunderstood the concept of the ray by thinking that it would go from both sides to infinity as the line and that both ends of the ray were limited (Başkurt, 2011). In addition, students confuse basic concepts such as the line segment and ray with each other (Altıntaş & İlgün, 2018).

While the geometry subjects of the 5th grade mathematics textbooks, which have a predominant place in geometry teaching, mostly focus on problems involving the transition of "construction-geometric shapes", "geometric shapes-geometric shapes", problems involving the transition between "construction-construction" and "geometric shapes-construction" were given less attention (Khalidova & Tapan-Broutin, 2017). One of the up-front aims and principles in the mathematics curriculum is the effective use of technology. Therefore, based on the developments in science and technology, innovations in mathematics teaching and integrating technology into education are given high importance today (Okumuş, 2011). In the section called "Competence" in the mathematics curriculum (HCEM, 2018), in other words, the part which explains the items to be given in the program and expected to be completed, two items appear that are "Technological Competence" and "Digital Competence". In addition, the learning outcomes that need to be supported by technology are explained in the "objectives" section of the curriculum.

The concretization of mathematics lessons, which require high-level cognitive skills and contain abstract concepts, with technological tools facilitates the learning of concepts (Baki, 2002). With the active use of technological tools in teaching, students can learn concepts efficiently, explore mathematics and geometry by providing learning environments through mathematics software and appropriate tools, and even create their mathematical thinking. The numerous mathematics software and tools will have an impact on students' concretization of the concepts included in the "achievements" section, as well as the development of their problem-solving skills, which will increase the cognitive levels of students and will also contribute to the formation of their mathematical structures. (MoNE, 2013).

To increase the quality and efficiency of education, the use of technology and the development of educational activities in this direction are crucial in Türkiye; in addition, the use of technology at all levels, particularly the dynamic geometry software, is prioritized (MoNE, 2005). Also, when the mathematics curriculum is examined, expressions such as "the dynamic geometry software is available for use" in the explanations section confirm this idea. Besides, The National Council of Teachers of Mathematics (NCTM, 2000) stated that concrete materials, constructions, and dynamic geometry software are necessary to learn geometry in school.

There are cases for the use of Dynamic Geometry Software (DGS) in the mathematics curriculum of 2018. In the mathematics curriculum of 2018, there are statements regarding the use of DGS in all grades from the 2nd grade to the 8th in the learning outcomes explanations.

For example, the 5th grade achievements include the following statements; “M.5.2.2.3. Can identify and draw the basic elements of rectangle, parallelogram, rhombus and trapezoid.” “b) In-class studies on dynamic analysis of special quadrilaterals can be included with dynamic geometry software as well as square and isometric papers.” (MoNE, 2018). With these software, students can experience discovery activities, draw their conclusions, and most importantly, learn by doing and experiencing (Bintaş, Ceylan & Dönmez, 2006). In addition, DGYs enable students to develop positive attitudes by providing sufficient experience in this context (Güven & Karataş, 2003). Also, the use of DGY is crucial in the educational environment and is more advantageous for the comprehensive learning of mathematics (Adelabu, Makgato & Ramaligela, 2019). With DGY, students can draw geometric shapes and easily discover geometric relationships. In addition, the dragging and changing feature improves students' higher-order thinking skills, and thanks to these programs, students can easily make generalizations and assumptions. (Genç & Öksüz, 2016). It is seen that DGY increases success in the lessons when it is used, increases motivation, arouses interest and curiosity, and facilitates and visualizes information (Tatar, Akkaya & Kağızmanlı, 2011; Thambi & Eu, 2013; Zengin & Tatar, 2015). In addition, according to some studies, computer-assisted teaching is more effective than the constructivist teaching method (Zengin, Furkan & Kutluca, 2012).

In the literature, it has been emphasized that dynamic geometry software has features such as correct construction, visualization, exploration, experience, transformation, feedback and geometric location (King & Schattschneider, 1997; Arcavi & Hadas, 2000; Okumuş, 2011). In the study conducted by Öçal and Şimşek (2016), it was aimed to examine the processes of solving basic geometric construction problems of teachers using different tools (compass-straightedge constructions [PCI] and GeoGebra constructions [GI] as dynamic geometry software) and their views on this subject. It is a qualitative case study, and its participants consist of four secondary school maths teachers. When the compass-straightedge and GeoGebra constructions were compared, teachers were able to do some drawings with GeoGebra that they could not do with compass-straightedge because GeoGebra allows trial and error.

This study aims to assess the learning environment designed for the construction processes of basic geometric concepts in dynamic geometry environments of secondary school 5th grade students. Also, the construction processes in the worksheets prepared in this context, the achievements of the construction process before and after the process, and their conceptual developments were examined.

The sub-problems of this study, which aims to evaluate the learning environment designed for the construction of basic geometric concepts in a dynamic environment of secondary school 5th grade students, were determined as follows:

- What is the effect of the designed learning environment on the development of students in their construction processes in dynamic environments?
- What is the effect of the designed learning environment on students' learning basic geometric concepts of construction processes in dynamic environments?

In this context, the students were provided to construct the basic geometric concepts in the 5th grade "Basic Geometric Concepts and Drawings" sub-learning area in dynamic geometry environments. In addition, the effect of students' construction processes in dynamic environments on their conceptual learning was also examined.

2. Method

2.1. Research Model

In this study, which aims to evaluate the learning environment designed for the construction processes of basic geometric concepts in dynamic geometry environments of secondary school 5th grade students, the case study, which is one of the qualitative research designs, was used. A case study is a research design made in many fields, in which the researcher usually performs an in-depth analysis of a program, event, activity, or process, and researchers gather detailed information over a long time using various data collection processes (Stake, 1995; Yin, 2014).

2.2 Study Group

The study group consists of three students, two boys and a girl, studying in the 5th grade of a secondary school. Purposive sampling method was used for student selection in this study. The selected students are interested in mathematics and academically successful. The reason for making such a selection is because of the thought that the construction activities to be carried out in a dynamic environment can be identified better with moderately successful students. In addition, attention was paid to whether students were prone to technology. According to Yıldırım and Şimşek (2016), while qualitative research has a structure that offers the opportunity to examine in detail and in-depth, a small-sized study group was preferred. For this reason, there was a limited number of participants. Also, an opportunity was provided for an in-depth examination of each student's work in the process. Furthermore, the codes S1, S2, and S3 are used for the students to protect the confidentiality of the students.

2.3. Data Collection Tools

Interview Form

An interview form consisting of questions containing basic geometric concepts was prepared to determine the changes in students' conceptual learning before and after the process. The questions in this form were directed to the students before and after the process. Also, the nine questions in the interview form were prepared in a way to include the basic geometric concepts (line, line segment, ray, line segments of equal length, acute angle, right angle, obtuse angle, line segment parallel to a line segment and perpendicular to a line). For taking the students' answers to the questions in the interview form, an interview was made according to the semi-structured interview technique and recorded. Thus, as a result of the process, the conceptual development of students regarding basic geometric concepts was determined.

Basic Geometry Constructions Test

The Basic Geometry Constructions Pre-Test (BGCPreT) and Basic Geometry Constructions Post-Tests (BGCPostT) were prepared to be applied before and after the process to determine the development of the students in the construction processes. While preparing these tests, attention was paid to ensuring that the questions were equally demanding. Each test consists of four questions containing geometric concepts. Students were asked to do the construction task of each question in a dynamic software environment. Also, the construction processes of the students were recorded through the screen recording program to prevent data loss.

2.4. Data Analysis

The following holistic rubric was used to analyze the answers given to the questions in the interview form, which consisted of questions including BGCPreT, BGCPostT and basic geometric concepts.

Table 1. *Holistic rubric*

Scores	Criteria
0	The answer is completely wrong, irrelevant or inconsistent.
1	The answer is incomplete and contains many errors, yet not entirely wrong. Mathematical concepts and steps are reflected in a limited way.
2	The answer is partially correct. Mathematical concepts and steps were partially reflected.
3	The answer is complete and correct. Mathematical concepts and steps are fully reflected.

The screen recordings of students' construction processes in BGCPreT and BGCPostT were examined, and the responses to each question were evaluated and tabulated through rubrics.

The answers of the students were transcribed and similarly evaluated and tabulated via rubric.

2.5. The Process

To examine the flaws of GeoGebra activities, one of the materials to be used in the research, the clarity of the questions and instructions, and the comprehensibility of the first and last interview questions were tested with a pilot study. After the necessary arrangements were made as a result of the data obtained from the pilot study, the main study was applied. Also, the table of the process is given below.

Table 2. *The distribution of the process by weeks*

Week	Duration (Lesson)	Content
Week 1	1	Information on the Content and the Research Semi-Structured Interview (First Interview)
Week 2	2	GeoGebra Software Installation and Definition Activity Basic Geometry Constructions Pre-Test (BGCPreT)
Week 3	2	Activity 1: If Colored Points Move Activity 2: 3 Color Points Against Line Activity 3: Swap Points
Week 4	2	Activity 4: Line Segment Fraternity Activity 5: Creating Angles in GeoGebra Activity 6: Types of Angles Activity
Week 5	2	Activity 7: Vertical Lines Activity Activity 8: Let's Discover the Straight Lines
Week 6	2	Activity 9: Parallel Lines Activity Activity 10: Let’s Discover the Parallel Lines
Week 7	2	Basic Geometry Constructions Post-Test (BGCPostT) Semi-Structured Interview (Last Interview)

First of all, necessary explanations were made about the process and research to the three selected students. Afterwards, the students were given an interview form and asked to answer them. While the answers to the questions in the interview form were taken, the meetings were made individually according to the semi-structured interview technique and recorded. The interview consists of 9 questions, which include the achievements of the research, were sent to the opinions of 2 different experts before the pilot study and were applied after their approval. According to the data obtained in the pilot study, the interview questions were crystal clear for the students to understand. Therefore, no change was made, and the same questions were applied in the original study.

After the first interviews were completed, the introduction activities of the GeoGebra software, were carried out. The basic tabs (move, point, line, line segment, perpendicular line, angle, slider, etc.) to be used in the study, were introduced via the GeoGebra software. In the introduction, the other tabs that the students were curious about, although they would not be used in the study, were also briefly mentioned. However, this part was mainly focused on the basic tools of GeoGebra. After the introduction of the basic tools of GeoGebra, the BGCPreT, which is the pre-test phase of the study, was started. In addition, the audio, video and screen recordings of each student were taken during the BGCPreT process. After the BGCPreT, the process of GeoGebra activities was started. There are 10 GeoGebra activities in this study. During the application of the activities, the audio, screen and, video recordings of each student were taken. After the activities, the BGCPostT was applied to the students as a post-test. Also, during the BGCPostT, the audio, video, and screen recordings of each student were taken. The final interviews with three students were also made as one-on-one interviews.

2.6. The Validity and Reliability of the Research

According to some studies, it is recommended to use multiple validity processes to ensure validity in qualitative studies based on determining whether the results are correct from the perspective of the researcher, participant, and reader (Creswell & Miller, 2000). The first one is to ensure that the participants check whether their answers are correctly interpreted. Hence, the results obtained in this study were shared with the participants resulting in participant control assurance.

The presentation of data that provides contradictory or negatory information in the results also contributes to validity. In this context, the aspects of the designed learning environment that support the students' construction processes and conceptual learning processes are presented; however, data showing situations that cannot support this process in some cases are also presented and interpreted.

It is crucial for the researcher to spend more time with the participants to gain an in-depth understanding of the phenomenon studied and to strengthen the narrative. In this context, the researcher personally followed the work with the participants in all processes in the designed learning environment and experienced it.

In qualitative studies, peer inquiry is required to check the accuracy of the interpretations. Therefore, in this study, expert opinion was sought in the interpretation of the results, and the process was completed by reaching a common consensus.

Yin (2009) states that qualitative researchers should explain the steps of case studies in detail and put them down in writing as much as possible. So this study has presented a rich perspective by making detailed definitions of the learning environment. In particular, the work done by the students in the worksheets in their dynamic environments is included to prevent data loss. Another validation process is to make rich and full portraying to explain the results.

4. Findings

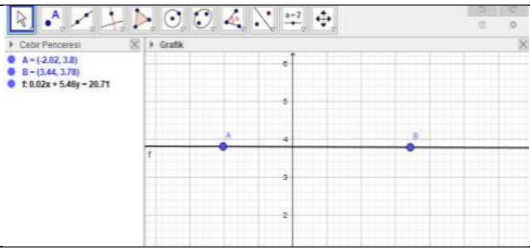
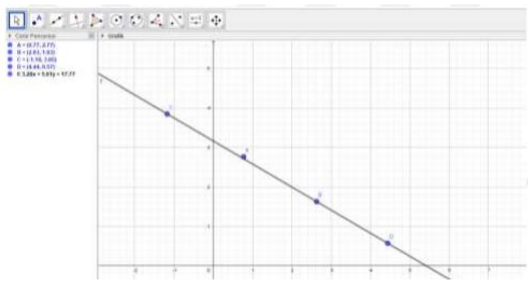
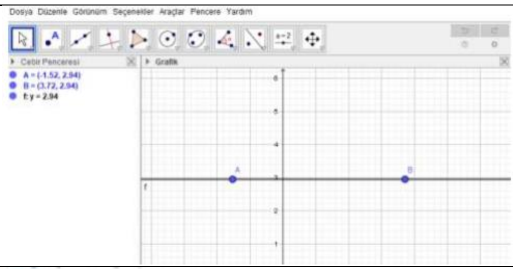
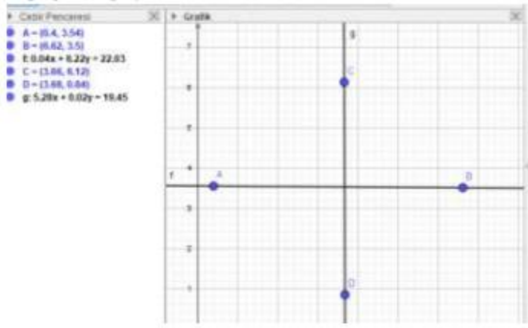
In this section, the results as part of the research problems are presented.

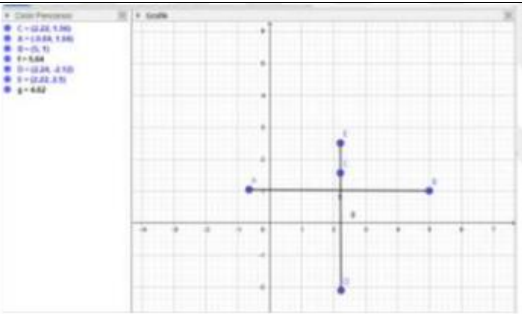
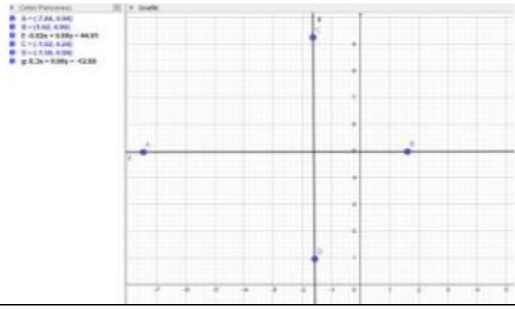
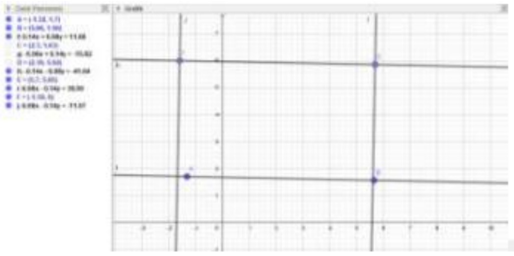
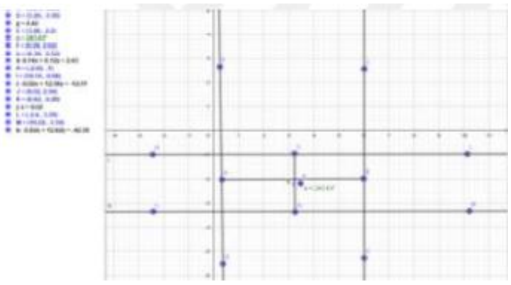
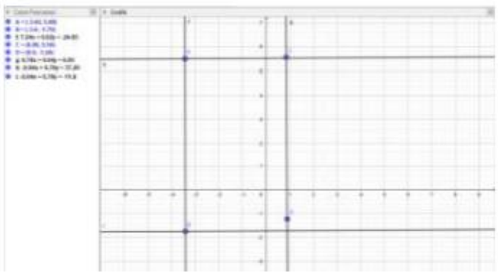
The Results Regarding the Effect of Construction Processes in Dynamic Environments on Students' Learning of Basic Geometric Concepts

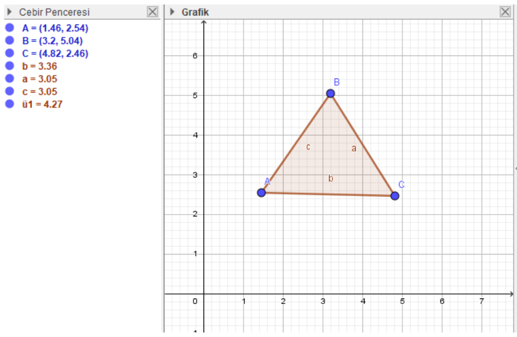
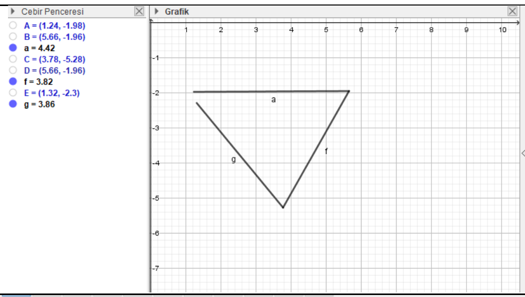
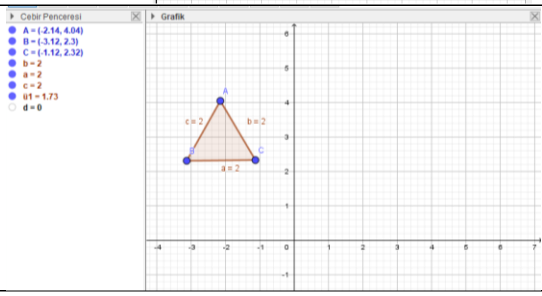
In this section, firstly, the results before and after the process of the construction processes of the students will be given. Afterwards, the effect of this process on students' learning of basic geometric concepts will be presented.

The screenshots and explanations of the students' construction processes in BGCPreT are presented in the table below.

Table 3. *Screenshots and explanations of the students' construction processes in BGCPreT*

Construction Process	Student	Screenshots from Geogebra	Explanation
Line construction	S1		S1 constructed the line correctly.
	S2		There are four different points in the formation of S2. When the C and D points from these points are held and dragged as desired, the line also moves, but since the redundant points A and B are points that are not located on the line, the A and B points remain idle when the line is moved from the C or D point.
	S3		S3 constructed the line correctly.
The construction of the perpendicular line	S1		When the points on the screen were moved in the construction of S1, it was determined that the lines formed by the student were not always a perpendicular dynamic formation.

	S2		When the points on the screen were moved in the construction of S2, it was determined that the lines formed by the student were not always a perpendicular dynamic formation.
	S3		When the points on the screen were moved in the construction of S3, it was determined that the lines formed by the student were not always a perpendicular dynamic formation.
Rectangle construction	S1		When the rectangle built by S1 is moved from A and B points, the rectangle moves without breaking, but when the points E and F on the screen are moved, rectangles with different widths are formed.
	S2		When we look at the formation of S2, it is seen that a completely different construction was done than other students. When the GeoGebra formation of the student was examined, it was determined that the rectangle did not form with the movement of the points where each line and point moved independently.
	S3		It is seen that the construction of S3 forms a shape similar to a rectangle, but when the GeoGebra formation is examined, the shape of the rectangle is lost with the movement of any selected point.

<table>
<tr><td rowspan="3">The construction of the equilateral triangle</td><td>S1</td><td>
</td><td>When the side lengths of the triangle in the construction of S1 are measured, it is seen that each side has a different length. In addition, when it is held from a corner point, the side lengths of the triangle change resulting in a different shape other than an equilateral triangle.</td></tr>
<tr><td>S2</td><td>
</td><td>In the construction of the S2, there is no formation of a triangle and one side is short.</td></tr>
<tr><td>S3</td><td>
</td><td>The construction of the S3 formed an equilateral triangle with sides of a length of 2 units. However, the triangle formed cannot provide the dynamism feature.</td></tr>
</table>

The table of the scores obtained by the students from BGCPreT is given below.

Table 4. *The BGCPreT scores of the students*

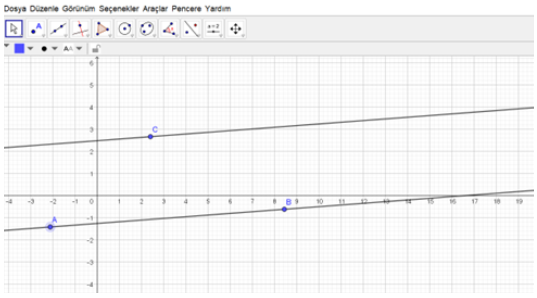
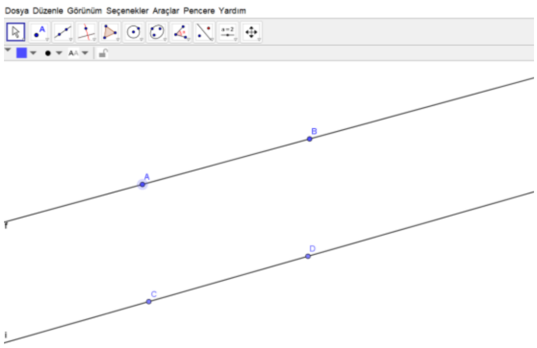
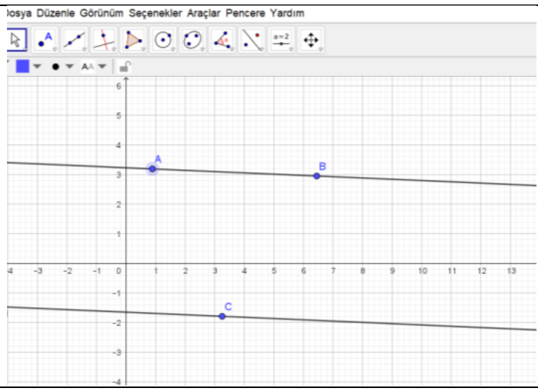
	Question 1	Question 2	Question 3	Question 4	Total
S1	3	1	2	2	8
S2	2	1	1	1	5
S3	3	1	2	2	8

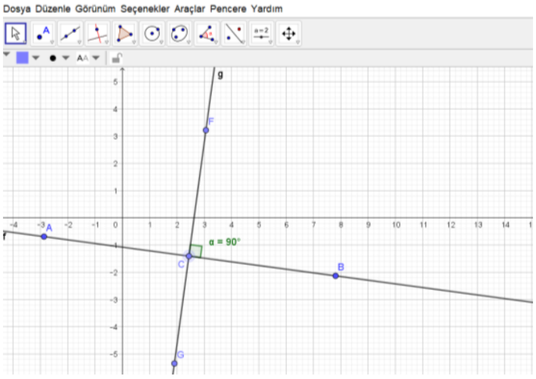
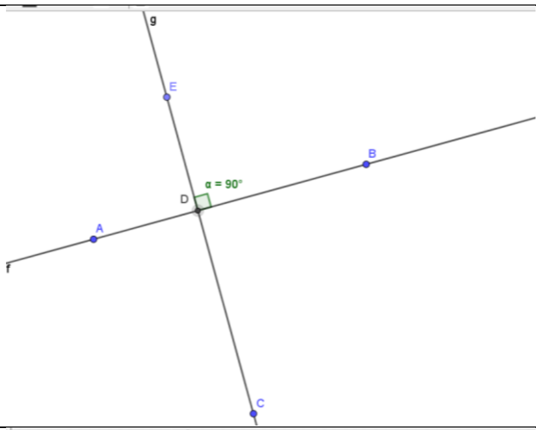
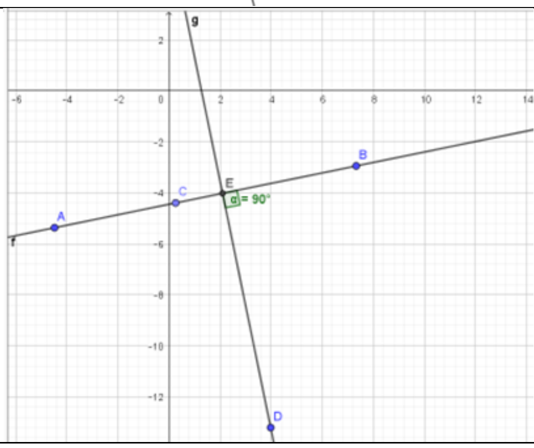
In the evaluation of the GeoGebra formations made by the students according to the holistic rubric; when the students' scores for each question and the total scores they received were examined, it was determined that the highest-scored question was the first question. On the other hand, they did not give complete and correct answers to other questions. In addition, during the GeoGebra formations in the process of the test, students had difficulties with other questions and had problems and made mistakes in their constructions.

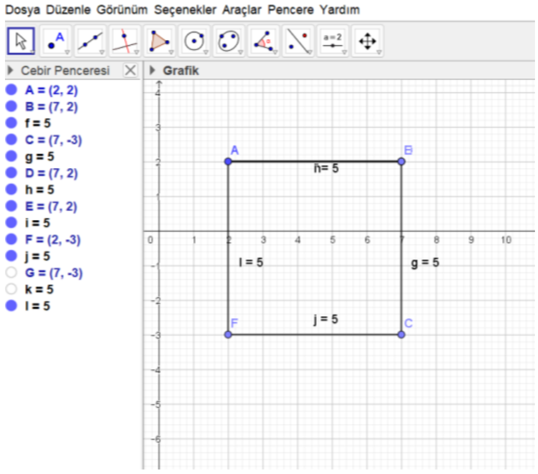
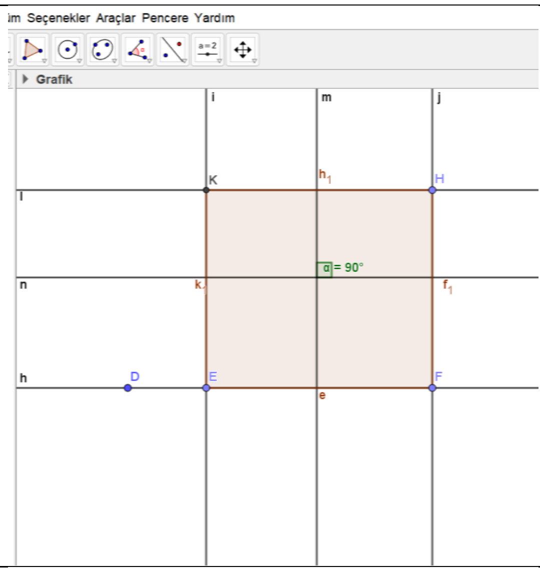
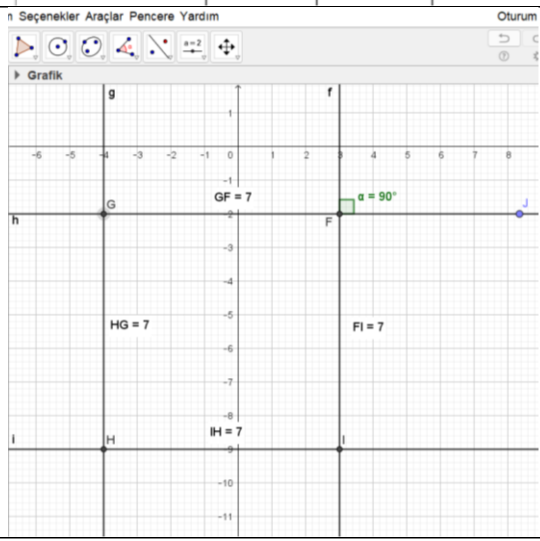
The screenshots and explanations of the students' construction processes in BGCPostT are presented in the table below.

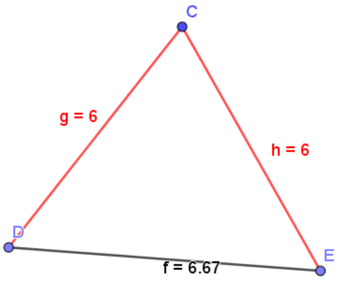
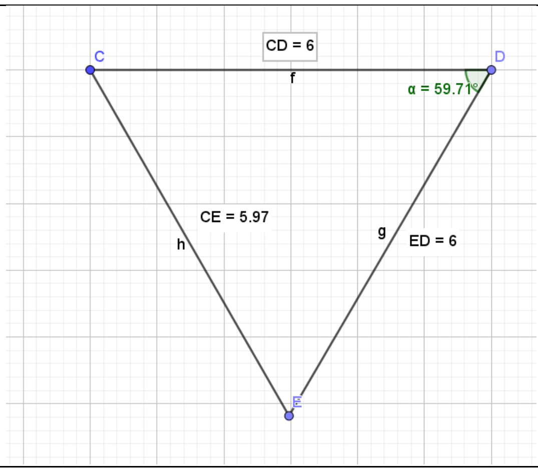
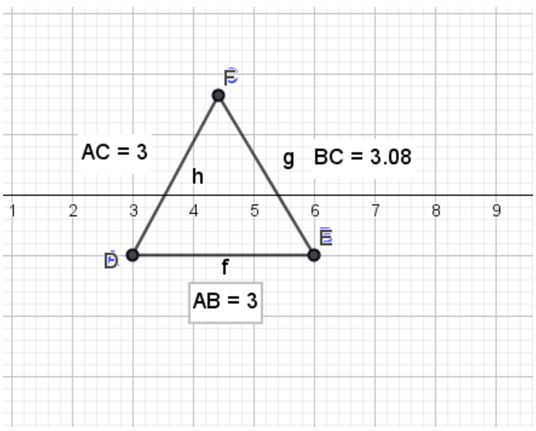
Table 5. *Screenshots and explanations of the students' construction processes in* BGCPostT

Construction process	Student	Screenshot of Geogebra	Explanation

The construction of parallel lines	S1		When the points A, B and C that appear on the screen in the student's construction are moved, both lines move without losing their parallelism. Also, when point C is moved, the distance between parallel lines can be changed. To conclude, the student has completed the construction correctly.
	S2		When the student's construction is examined, points A and B on the screen provide movement, yet the other two points on the screen, points C and D, do not move the lines. The distance between parallel lines could not be changed. Therefore, the construction can be accepted as correct even if there is a deficiency.
	S3		Accordingly, the construction of the student was correct, and the parallels did not deteriorate when the points were moved.

The construction of the perpendicular line	S1		In the construction of the student, the points A, B and C move and provide the movement of the lines without deteriorating the perpendicularity. Hence, the construction made by the student is correct. The F and G points, which are unnecessary on the screen, are the points that should be hidden and do not need to be seen.
	S2		When we look at the construction of the student, the points A, B and C can be moved without deteriorating the perpendicularity. Therefore, the construction of the student has been correct and dynamic.
	S3		Point C is unnecessary and does not need to be visible on the screen. When we look at the formation in general, as with other students, the construction of the student has been correct and dynamic.

	S1		A square formation is seen in the student's construction. By moving point C, the square turns into a parallelogram, that is, a rhombus, without deteriorating the side lengths. Although different objects have been created by moving these points in the construction of the student, the square can be harmlessly moved with the movement of point A, which means the student is able construct.
Square costruction	S2		Although there is a partial error in the construction of the student, since the side lengths can be changed; he made a much more technical construction than other students.
	S3		When the algebra window in the student's construction is examined, it is seen that the student constructs the square by hiding the points that may distort the side length and angle of the square.

<table>
<tr><td rowspan="3">The construction of isosceles triangle</td><td>S1</td><td>
</td><td>It is seen that the student has done the construction completely and correctly. In addition, the student can now use GeoGebra actively and make colourings by making arrangements as he wishes, although no intervention is made during the construction.</td></tr>
<tr><td>S2</td><td>
</td><td>S2 added the side lengths to the sides as text in his construction, which he did correctly, and also added the vertex angle measure of the isosceles triangle to his construction.</td></tr>
<tr><td>S3</td><td>
</td><td>When the construction of S3 was examined, it was observed that there were problems in the formation. Although an isosceles triangle appears, it is seen that the isosceles property is broken when the vertices of the triangle are moved.</td></tr>
</table>

The table of the scores obtained by the students from BGCPostT is given below.

Table 6. *The BGCPostT scores of the students*

	Question 1	Question 2	Question 3	Question 4	Total
S1	3	3	2	3	11
S2	2	3	2	3	10
S3	3	3	2	2	10

In the table above, the constructions of the students in BGCPostT were evaluated based on questions according to the holistic rubric. Accordingly, the constructions made by the students in the questions in the post-construction test were correct or almost correct. This indicates that the development of the students in the construction processes in dynamic environments after the designed learning environment is in a positive direction.

The results regarding the effect of the designed learning environment on students' learning of basic geometric concepts

The interview forms consisting of a total of nine questions were applied to the students as the first and last interview forms. The questions in the interview forms also include definitions of basic geometric concepts. To better see the answers given by the students in the first and last interviews and their development towards basic geometric concepts, a student answer for each question is given in the table below as an example.

Table 7. *Sample student answers for the first and last interview*

Concepts	Student	First Interview	Score	Last Interview	Score
Line	Ö2	"An arrow that goes to infinity on both sides."	2	"A set of points that goes to infinity on both sides."	3
Line segment	Ö3	"Line segment… Something that only goes in one direction. Line segment…"	0	"An infinite set of points that start at a point and end at a point, sorry limited."	3
Ray	Ö1	"I think the ray, my teacher, was the ray that goes to infinity… It starts from a point and goes like this."	1	"My teacher, it starts at one point and goes to infinity."	2
Acute angle	Ö2	"Angle less than 90 degrees, between 90 and 0..."	3	"Acute angle is between 90 and 0…"	3
Right angle	Ö1	"Right angle my teacher, 90 degrees is just 90 degrees like that (shows perpendicularity with his hand)."	3	"Right angle is a 90 degree angle."	3
Obtuse angle	Ö1	"The obtuse angle is between 90 to 180"	2	"From 91 degrees to 179, that angle is 180 degrees right angle."	3
Congruent segments	Ö3	"Two equal line segments..."	1	"Parallel lines… congruent line segments."	1
Parallel line segments	Ö3	"Teacher, I don't have much in mind about that."	0	"Same length… The distance between them is the same."	2

Perpendicular line segments	Ö2	"Teacher, there is a right angle, so perpendicularity…"	2	"So a line and 90 degree angles intersect."	3

When the table is examined, it is seen that the students generally made more accurate concept definitions and explanations in the last interview compared to the first interview.

The basic geometric concept definitions made by the students before and after the process were evaluated according to the holistic rubric, and the total scores they received are given in the graph below.

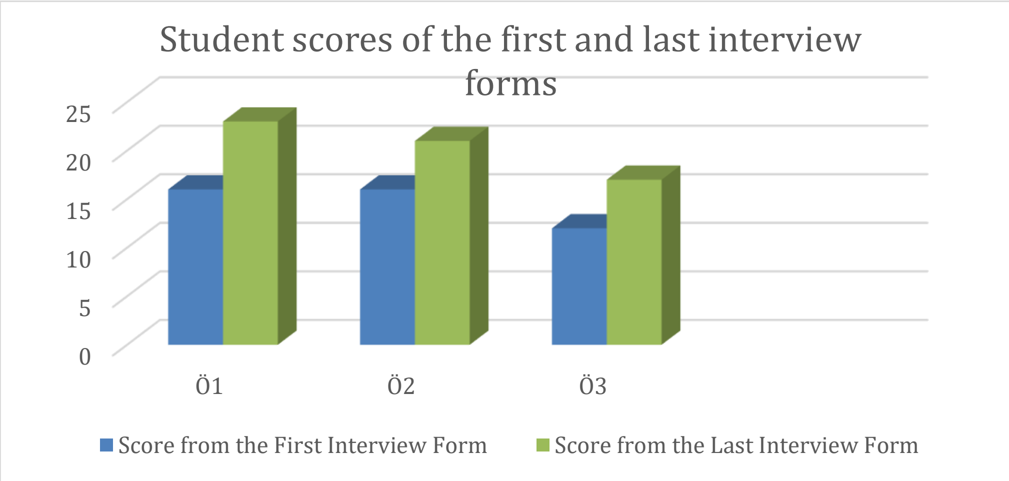


Figure 1. *Students' scores of the first and last interview form*

When the graph was examined, it was seen that there was an increase in the scores of the students in favour of the last interview. This shows that the designed learning environment has a positive effect on students' conceptual development of basic geometric concepts.

5. Conclusion and Discussion

In this part, the study results were discussed in line with the sub-problems of the research.

In the first sub-problem of the study, the effect of the designed learning environment on the development of students' construction processes in dynamic environments was examined. In this direction, BGCPreT and BGCPostT were applied to the students before and after the designed learning environment, respectively, in a way that they would practice in dynamic environments and create their answers.

In the pre-test, students generally could not construct the line following the dynamic environment, according to the question directed to test line construction in the dynamic environment. Although the students drew a shape similar to line, it was observed that the dynamic feature was not achieved. This may be due to the limited previous experience of students in such dynamic environments. After the designed learning environment, in the post-test, students were asked to construct parallel lines similarly. Here, the parallel lines created by the students in the dynamic environment, were completed both conceptually and dynamically right. The Geogebra software allows students to make trial and error in their construction (Kabaca et al., 2010) and as Öçal and Şimşek (2017) stated, the construction processes are a sequential chain of operations and Aydın (2021)'s statement that the construction process must be done in order. As a result of the students' frequent experiences of these sequential processes through the designed learning environment, it was seen that they showed positive development and completed the construction process.

Constructing a perpendicular line is another construction process in the pre-test and post-test. In the pre-test, it was requested from students to construct a perpendicular line from a point outside the line. When the construction processes were examined, the students had difficulties and made mistakes while trying to construct a perpendicular from a point other than the line. When the formations of the students were moved from the points, it was observed that the perpendicularity was disturbed and even the lines were lost because of these mistakes. In this multi-step activity, the rate of making mistakes increased since the students were in a dynamic environment while performing the construction process. Similarly, it was stated in other studies that students have difficulties in multi-step activities in dynamic environments (Deniz, 2006; Aydın, 2021). In addition, this situation once again confirms the necessity of sequential comprehension in the comprehension of geometric shapes mentioned by Duval (1994; cited in Tapan-Broutin, 2016). In the process about constructing a straight line from a point on the line in the post-test, all students completed the construction process correctly. Thus, the learning environment designed through the dynamic environment positively affected the constructions of the students. In addition, it was determined that the students overcame the difficulties they experienced during the construction process in the pre-test, thanks to the activities in the designed learning environment. Also, Öçal and Şimşek (2017) concluded in their study that GeoGebra positively affected the construction processes. On the other hand, it was seen that this environment facilitates the students to reach the answer as they have the opportunity to make changes in dynamic environments in the process. Again, studies supporting the positive contributions of this feature to the construction processes in dynamic environments are available in the literature (Kondratieva, 2013; Köe et al., 2012; Köse, Uygan & Özen, 2012; Stylianides and Stylianides, 2005)

According to the results of the process "constructing the rectangle", none of the students completed the construction steps correctly before the process. As per the observations, students completed the process visually rather than their geometric features while constructing the rectangle and made their drawings without paying attention to the rectangular features. This situation coincides with the conclusion made by Doğan et al. (2012), which states that geometric concepts are interpreted according to the image rather than the characteristics of the shape. Similarly, it is similar to the results obtained from the study of Aydın (2021). In the final construction test, students were asked to form a square. According to the observations, all three students completed their construction correctly, although there were some minor mistakes. Again, here we can conclude that GeoGebra activities in the designed learning environment had a positive effect on students' learning and construction of the concept.

In the process of equilateral triangle construction in the BGCPreT, when the side lengths of the triangles of the students were measured, it was determined that each side was of a different length, one side was short, and that the triangle formed was not a dynamic triangle although it was an equilateral triangle with sides of 2 units. In addition, it was observed that the triangle side lengths were distorted when the constructed triangles were held and dragged from any corner point. In general, students can use line segments but cannot use the slider feature. Also, students stated that thanks to the drag tool in GeoGebra, they encountered many types of functions through various drags, thus making it easier for them to establish relationships between symbolic and visual representations. According to studies conducted with other dynamic software, it has been observed that students can discover new mathematical relationships with the help of the drag tool (Santos-Trigo & Cristóbal-Escalante, 2008; Gonzàlez & Herbst, 2009; Ceylan, 2012). As stated in Şengün's (2017) study, knowing the necessary and sufficient information to construct the triangle, the required elements and whether the elements with which values will indicate a triangle, has been understood after the processes in the software environment. This finding is similar to the current study. The last process in BGCPostT is to form an isosceles triangle. While the construction of one student is partially correct, the constructions of two students are considered correct. As can be seen from here, students made progress in the construction process of the triangle. With the effect of the activities in the designed learning environment, students were able to interpret the tasks they were working on and discovered the triangular relationships. In this sense, this study coincides with the studies of Güven and Karataş (2005) and Yavuzsoy-Köse et al. (2012). Also, Güven and Karataş (2005) state that dynamic geometry software creates favourable environments for students to prove their ideas. In the triangle construction processes in the GeoGebra environment, it was observed that students' motivation to construct the triangles given in tasks was high. Therefore, this study shows parallelism with the construction studies in the literature (Erduran & Yeşildere, 2010; Cheung, 2011; Karakuş, 2014; Gür & Kobak-Demir, 2017; Öçal & Şimşek, 2017).

When the scores obtained according to the holistic rubric used for the development of the students in the pre-construction test (BGCPreT) and the post-construction test (BGCPostT) processes were compared, it was seen that all of the students had an increase in their construction level scores. Also, it is remarkable that the students made random formations in the pre-test or did not make any formations. This can be explained by the lack of geometric skills of students in geometry lessons in secondary education (Napitupulu, 2001). Correlatively, the study of Köse et al. (2012) has similar results.

In the second sub-problem of the study, the effect of the designed learning environment on the students' learning of basic geometric concepts of construction processes in dynamic environments was examined. Therefore, the interview form was applied to the students before and after the designed learning environment.

The first question in the interview form is about "the definition of the line". Accordingly, students had difficulties expressing the correct definition of "line" before the process. Instead of using the formal definition of the concept, "An arrow that goes to infinity on both sides." answers appear to be given. During the process, students discovered that a line is a set of points thanks to the GeoGebra activities and used it in their definitions. So, they were successful in giving the formal definition of "line" right after the process. Similarly, in Öksüz (2010)'s study, students defined the line as "a set of points", which is correct. In the study of Çekiç (2018), on the other hand, he concluded that the students did not realize that the line is formed by the union of infinite points and made mistakes in the definition. In this case, it differs from the results of this study.

Considering the progress achieved at the end of the process of GeoGebra activities for defining the line segment; students completed each step correctly according to all the stages of discovering the line segment, and they gave answers that could be considered correct even if they used some incomplete expressions while making the definitions. While reviewing the approaches to this process, a student made a completely wrong definition in the first interview form, yet the software had a positive effect on recognizing and defining the line segment, resulting in giving a fully and correctly answer in the last interview form. However, since two students gave answers that would be considered partially correct in both forms, it was seen that there was no improvement. The findings of Dane and Başkurt (2011), Doyuran (2014), and Çekiç (2018) are similar to this result. Doyuran (2014) presented the word "segment" in the word group "line segment" and the expression "segment of the line" that teachers use while explaining the line segment in the lesson as reasons for this. Explaining "line segment" as "any segment of the line " may facilitate the teaching of the concept.

In the process of defining the ray, it was determined that the students completed the steps correctly and gave answers that could be considered partially correct, even if they were incomplete while making the definitions. According to the answers in the first and last interview forms, GeoGebra activities had a positive effect on the development of students' ray comprehension. Similarly, the use of worksheets accompanied by GeoGebra activities had a positive effect on increasing the level of students' basic geometric concepts. In addition, when similar studies are examined regarding students' recognition and definition of concepts, it is emphasized that teaching using worksheets has positive effects (Atasoy & Akdeniz, 2006; Işık & Çelik, 2017; Kaleli-Yılmaz & Yüksel, 2019).

The answers of the students in the interview questions related to the angles and the answers of the students in the first and last interview forms were similar. Therefore, the definitions given were accepted as correct. Since the students generally knew the concept of angle types sufficiently before the process, they gave similar answers after the process. The reason for this is that angle types are an easily understandable concept. According to the study of Çekiç (2018), similar to the current study, students did not have any misconceptions about forming acute angles, right angles and obtuse angles.

Considering the answers to the concept of congruent line segments, two students could not show any improvement in this concept and remain at the same level. One student, on the other hand, showed a negative development and gave a completely wrong answer in the final interview. The reason for this is thought to be the fact that a GeoGebra activity that specifically includes the concept of congruent segments has not been given to the students. According to some studies carried out using dynamic geometry software in the literature, using dynamic geometry software positively affect the achievements of the subject studied, so it is beneficial to include dynamic geometry software (Sulak, 2002; Baki et al., 2004; Clark, 2004; Aydoğan, 2007; Vatansever, 2007; Güven & Kösa, 2008; Filiz, 2009; Güven & Karataş, 2009; İçel 2011; Selçik & Bilgici, 2011; Başaran-Şimşek, 2012).

While defining the concept of parallel line segments before and after the process, the students stated that parallel line segments should be aligned. According to the observations, they hesitate to explain the concept of parallelism in cases where the line segments are not aligned. Ulusoy (2014) determined that students have the misconception that lines have lengths, so they think that lines that are not the same length will not be parallel. The study results of Bayram & Duatepe-Paksu (2019) have supported this finding. Also, Kiriş (2008) stated in his study that there is a misconception that lines must be consecutive and reciprocal to be parallel. Additionally, Mansfield & Happs (1992) have stated that in determining parallelism, students intuitively decide according to the appearance of the drawings, such as whether the line segments are side by side or end-to-end.

In their answers to the question about "perpendicular line segments" in the first interview form, students mentioned perpendicularity and 90-degree intersection. In the last interview form, they gave a complete and correct answer to this question with straight lines intersecting clearly. As can be seen, although the students knew orthogonality before the process, they improved afterwards and developed their knowledge by making exact and correct definitions. Thanks to its visuality, the GeoGebra-supported learning environment increased the students' success by enabling them to visualize and comprehend events more quickly. This result is consisting with the findings of Emlek (2007), Filiz (2009), Reisa (2010), Saha et al. (2010), Şataf (2010), and Canevi (2019).

5.1 Recommendations

As a result, it has been determined that the learning environment designed with dynamic support has a positive effect on the process of constructing basic geometric concepts and the development of their knowledge about them. Following the results, an environment where the construction process will be carried out using GeoGebra, one of the dynamic software, was designed; also, it was foreseen that including compass-straightedge studies in some activities used in the learning environment could support the construction process. For this reason, it is recommended that new researchers do mixed studies that combine GeoGebra's dynamic software and compass-straightedge studies. Also, the study was conducted by evaluating the individual works and construction processes of 3 students who participated in the study. In the developmental processes, there have been cases where the desired level could not be reached due to the individual differences of the students. Therefore, it is recommended to design construction environments that can be done with group work. Besides, it has been seen that when students are provided with sufficient time, they can do construction work appropriate to their level. Thus, the curriculum intensity should be reviewed, and the students should be allowed to establish their mental structures to work with dynamic geometry software that is included in the school curriculum or prescribed by the teachers.

5.2 Ethical Text

"In this article, the journal writing rules, publication principles, research and publication ethics, and journal ethical rules were followed. The responsibility belongs to Mediha Özçevik Kal for any violations that may arise regarding the article. " Ethics committee approval within the scope of the research The Assessment of the Learning Environment Designed for the Construction of Basic Geometric Concepts in a Dynamic Environment. It has been taken from the ethics committee with the decision numbered 2021/177 on 29.06.2021. There is no conflict of interest between the authors. The contribution rate of the first author to the article is 50%, and the contribution rate of the second author to the article is 50%.

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