ON THE EQUIVALENCY OF PROFIT MAXIMIZATION AND COST MINIMIZATION: A NOTE ON FACTOR DEMANDS

by

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Abstract

Students are often confused when introduced to the two conceptually different factor demands of the firm: those demands for inputs which originate from the firm's desire to maximize profits, and those that are constructed to minimize the cost of producing a target level of output. These two demands for inputs are, of course, equivalent when the firm is optimizing. This note demonstrates that an often used method of teaching these concepts makes it impossible for students to visually compare these different input demands because different technologies are assumed. I provide a graphical proof of the equivalency of these factor demands by assuming the same technology in both the cost-minimization and profit-maximization problem. By employing this new analysis, it is hoped that students will gain a better understanding and instructors will gain a better teaching tool regardless of whether the course is calculus or graphically based. (JEL. A22, D21)

I. Introduction

A common problem students face in an intermediate microeconomics course is learning the different applications of *observable* (profit-maximizing) and *conditional* (cost-minimizing) factor demands.[1] Part of the problem is probably poor terminology.[2] More significant, in my opinion, is the fact that the average student has difficulty understanding why a firm would have two conceptually different demands for the same input. Of course, when a firm is optimizing, the two demands are equivalent. The common argument is that if a firm is profit maximizing then it must be cost minimizing, or there would exist a cheaper (more profitable) way to produce. This point is easily proved analytically.[3] To my knowledge, however, no student-friendly graphical depiction of this equivalency exists. This note points out the shortcomings of a popular treatment of this issue and provides a graphical proof of the equivalency of each type of factor demand under the assumption of profit maximization. The following presentation will benefit both instructors who utilize calculus techniques and those who employ only graphical analysis.

II. The Traditional Approach

Observable factor demands, constructed from the firm's profit-maximization problem, are the firm's optimal choices of input quantities and are a function of input prices and the price of the output good. Conditional factor demands, on the other hand, are the firm's optimal choices of input quantities which are a function of input prices and the output level of the firm. These factor demands represent the least-cost method of producing a *conditional* level of output and are constructed from the firm's cost-minimization problem. The most common application of observable factor demands is the construction of factor demand curves and the calculation of comparative statics with respect to input and output prices. The most common application of conditional factor demands is the construction of the firm's cost function.

Many textbook authors present the firm's cost-minimization problem in the following manner.[4] Assume that a perfectly-competitive firm has a monotonic and strictly convex technology represented by the production function

(1) $$y = f(x_1, x_2),$$

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where x_i represents the amount of input i used per period, and y represents the resulting output per period.[5] The firm's conditional factor demands, x_i^C, are the choices of input quantities which minimize the cost of producing some target level of output, y_0. In Figure 1, all of the input combinations used to produce exactly y_0 units of output are shown by the isoquant with the equation

(2) $$f(x_1, x_2) = y_0 .$$

The firm takes input prices (w_1,w_2) as given. Thus, costs to the firm can be written

(3) $$C = w_1 x_1 + w_2 x_2 .$$

Graphically, the firm's cost-minimization problem reduces to finding the lowest (closest to the origin) isocost line which allows y_0 units of production. In the input-input space, isocost lines have the equation

(4) $$x_2 = \frac{C}{w_2} - \frac{w_1}{w_2} x_1 .$$

In Figure 1, the lowest isocost line is drawn. Equality of the slopes of both the isoquant and isocost functions (tangency) is a sufficient condition for cost minimization. This, of course, implies that the ratio of marginal products (slope of the isoquant) equals the ratio of input prices (slope of the isocost). Rearrangement of this equality yields the familiar result that in order to minimize costs, firms use inputs until the marginal products per dollar spent are equal across inputs. This result in shown in Figure 1.

When the discussion changes to profit maximization and observable factor demands, however, some students begin to question the difference between these two conceptually different factor demands. One of the problems with the traditional presentation, in my opinion, is that when the discussion shifts to profit maximization and observable factor demands, instructors often change the technology from a 2-input/1-output technology to a 1-input/1-output technology. Figure 2 demonstrates this traditional treatment. The firm's technology is now represented by the production function

(5) $$y = f(x).$$[6]

The graphical analysis is no longer presented in input-input space, but rather input-output space. It is therefore impossible for the student to visually compare the input choices from the previous cost-minimization problem with those from the firm's profit-maximization problem.

Define the firm's profit as

(6) $$p = py - wx .$$

Conceptually, the firm's problem reduces to finding the highest (away from the origin) isoprofit line for which a portion still lies in the feasible set of production. In input-output space, isoprofit lines have the equation

(7) $$y = \frac{p}{p} + \frac{w}{p} x .$$

The highest isoprofit line is depicted in Figure 2. Given the needed assumptions about technology, a sufficient condition for profit maximization is the equality of the slopes (tangency) of the production and isoprofit functions. Thus, the observable factor demand for the input, x^O, is found by setting the marginal product of the input (slope of the production function) equal to the input price divided by the output price (slope of the isoprofit line). Rearrangement of this condition yields the familiar result that the competitive firm will employ the resource until its marginal revenue product equals its input price. This result is shown in Figure 2.

III. A Graphical Proof

To allow students to visually compare optimal choices of inputs in both the cost-minimization and profit-maximization context, I will present the analysis using the same technology for both problems. Students will be able to see the equivalency of both types of factor demands under the condition of profit maximization.

Assume that a competitive firm employs a 2-input/1-output monotonic and strictly convex technology represented by the production function in (1). This production function is shown in Figure 3. Our first task is to determine the profit maximizing choices of inputs, the observable factor demands x_1^O and x_2^O. Profit for the firm can be written

(8) $$p = py - w_1 x_1 - w_2 x_2 .$$

Much like the traditional 1-input/1-output case examined earlier, the firm's profit maximization problem here reduces to finding the highest isoprofit *plane* for which a portion still lies in the feasible set of production. Isoprofit planes in output-input-input space have the equation

(9) $$y = \frac{p}{p} + \frac{w_1}{p} x_1 + \frac{w_2}{p} x_2 .$$

The highest isoprofit plane along with the optimal choices of inputs and resulting optimal output are shown in Figure 3.

The next step is to move the graphical analysis into 2-diminsional input-input space by projecting a level set of the isoprofit plane and production function at the optimal level of output y^* onto the input space. The corresponding level sets are shown in Figure 4. The curved line is the familiar isoquant with the equation

(10) $$f(x_1, x_2) = y^* ,$$

and the straight line is the level set of the isoprofit function with the equation

(11) $$x_2 = \frac{p}{w_2} y^* - \frac{p}{w_2} - \frac{w_1}{w_2} x_1 .$$

Figure 4 looks very similar to Figure 1. In fact, to convince students that observable and conditional factor demands are equivalent when the firm is optimizing, the next step is to show that Figures 1 and 4 are *exactly* the same.[7]

Note that the slope of the isoprofit level set in Figure 4 is $-w_1/w_2$. This ratio of input prices is also the slope of the isocost line drawn in Figure 1. Thus, to prove that the two lines are coincident, students must show that their vertical (or horizontal) intercepts are the same. The vertical intercept of the isocost line in Figure 1 is C/w_2. The vertical intercept of the isoprofit level set in Figure 4 is $(p/w_2)y^* - p/w_2$. Equating these two expressions yields

(12) $$p = py^* - C ,$$

which is the correct expression for the firm's profit.[8] This analysis has shown, therefore, the equivalency of observable and conditional factor demands under the assumption of profit maximization.

IV. Conclusion

A traditional textbook treatment of observable and conditional factor demands presents the analysis in a way that obscures the potential insights of students into the equivalency of these two conceptually different demands under the assumption of profit maximization. This note presents a graphical analysis that enhances the prospect for students' understanding of this important relationship. By assuming the same technology for both the profit-maximization and cost-minimization problems, the analysis is reduced to a single two-dimensional graph. This type of analysis should assist students who find the concept of two different factor demands troubling. Instructors in courses that employ either calculus or graphical analysis may find this presentation helpful in convincing students that profit maximization necessarily implies cost minimization.

IV. Notes

1. I have taken some liberty with terminology here. Demands for inputs from the firm's profit-maximization problem are generally called *factor demands* and are generally *observable* in practice. On the other hand, the firm's *conditional factor demands*, those demands for inputs which minimize the cost of producing a target level of output, are a theoretical construct and generally not observed in practice.

2. I consider this terminology to be in the same ignominious league as "changes in *demand*" versus "changes in *quantity demanded*."

3. To find the least-cost method of producing y units of output per period, the firm equates the marginal product per dollar spent across all inputs. If the firm employs a monotonic and strictly convex technology represented by the production function $y = f(x_1, x_2, ..., x_n)$, where x_i represents the amount of the ith input used per period, then the unique conditional factor demands of the firm, $x_i^C(w_1, w_2, ..., w_n, y)$, are implicitly defined by the equations $\frac{\partial f/\partial x_1}{w_1} = \frac{\partial f/\partial x_2}{w_2} = ... = \frac{\partial f/\partial x_n}{w_n}$ and $y = f(x_1, x_2, ..., x_n)$, where the w_i represent exogenous input prices. Next, define the firm's profit as $p = pf(x_1, x_2, ..., x_n) - \sum_{i=1}^{n} w_i x_i$, where p represents the exogenous output price. The (observable) factor demands of the firm, $x_i^O(w_1, w_2, ..., w_n, p)$, are implicitly defined by the first-order conditions for profit maximization, $\partial p/\partial x_i = p\,\partial f/\partial x_i - w_i = 0$, $i = 1,...,n$. Rearrangement of these n equations also shows that profit maximization leads the firm to employ each input until marginal products per dollar spent are equal across all inputs. Thus, when the firm is optimizing, observable and conditional factor demands are equivalent.

4. This presentation and my later discussion of the firm's profit-maximizing problem are shown in many intermediate microeconomics texts, including Varian (1996).

5. A 2-input/1-output technology is assumed to make graphical analysis possible. Of course, all results can be generalized to an *n*-input technology.

6. Decreasing returns to scale technology is needed over some range of inputs and corresponding output for the firm's profit-maximization problem to be well-defined.

7. The isoquant in Figure 1 is drawn for an arbitrary target output level, y_0. Conceptually, we now replace this isoquant with the isoquant associated with producing y^* units of output, the profit-maximizing level of output from Figure 3.

8. Equating the horizontal intercepts will yield the same result.

References

Varian, Hal R. *Intermediate Microeconomics: A Modern Approach*. New York: W.W. Norton & Co. 1996.

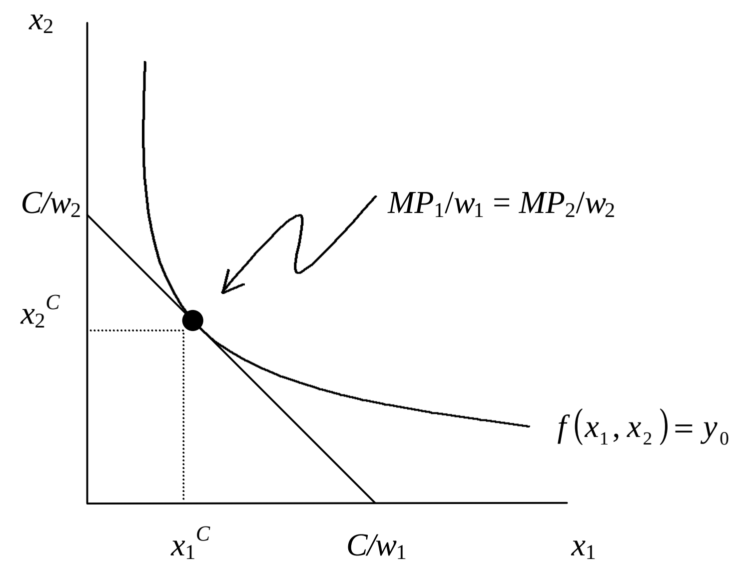

Figure 1
Cost Minimization With 2-Input/1-Output Technology

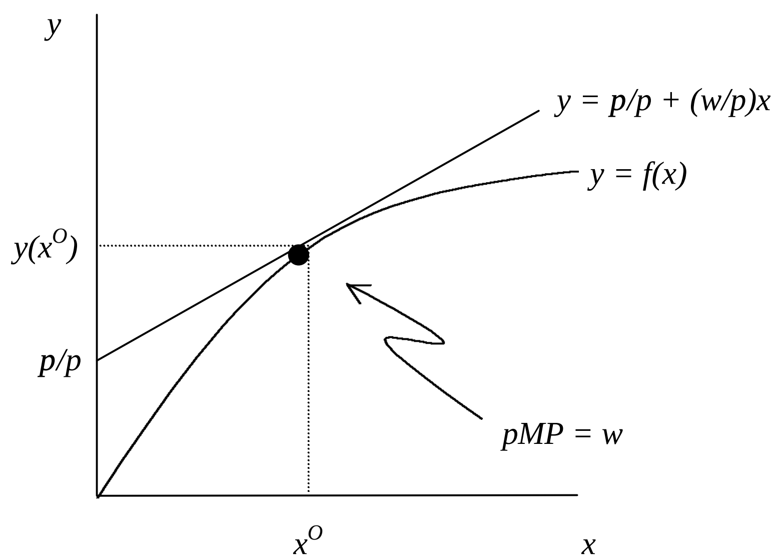

Figure 2
Profit Maximization with 1-Input/1-Output Technology

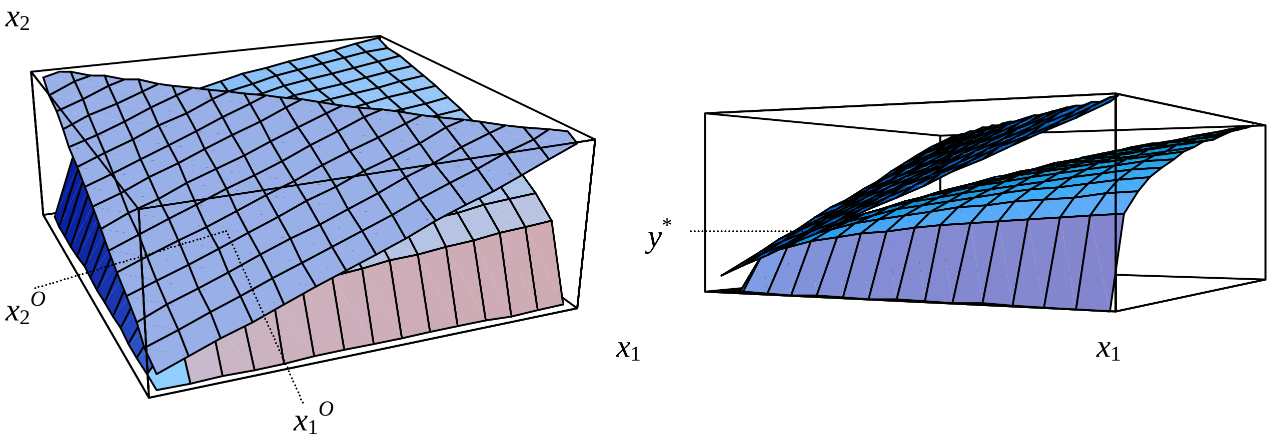

Figure 3
Profit Maximization With 2-Input/1-Ouput Technology

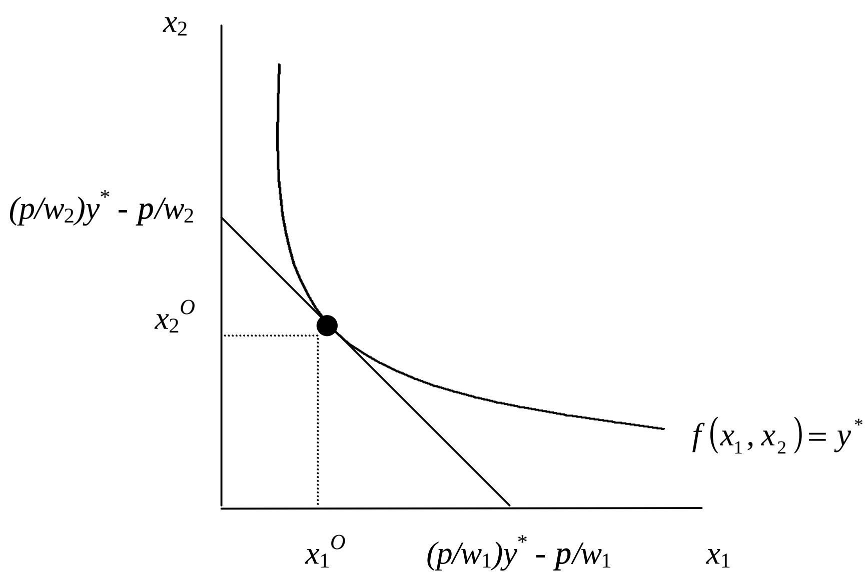

Figure 4
Level Sets of the Production and Isoprofit Functions