Agricultural Sciences, 1 (1996) 57-64

A Computer Model for the Hydraulic Analysis of Open Channel Cross Sections

W. H. Shayya*, R. H. Mohtar#, and M. S. Baasiri+

**Department of Bioresource Engineering, College of Agriculture, Sultan Qaboos University, P.O.Box 34 Al-Khod 123, Sultanate of Oman; #Department of Agricultural Engineering, Penn. State University, U.S.A.; and +Department of Soils, Irrigation, and Mechanization, American University of Beirut, Lebanon.*

خلاصة : يواجه مهندسو الري والهايدروليكا كثيراً بصعوبات الحلول المملة لمعادلة مانق لتحديد عناصر الأشكال الهندسية المختلفة للقنوات المفتوحة. هذه الورقة تعنى بتطوير نموذج على الحاسب الآلي لتصميم أكثر أشكال القنوات استعمالاً، بقصد الاستفادة منه كأداة تعليمية. ويمكن استعمال النموذج للتصميم الهايدروليكي لأشكال شبه المنحرف، المستطيل، المثلث، مكافئية المقطع، المستطيل ذو الزوايا المستديرة، والدائرة. ولقد تم إستخدام طريقتين لحل المعادلات الضمنية المجابهة وهما: طريقة نيوتن - رافسون وطريقة ريقيولا - فالسي. وليدء* عملية الحل تحتاج الطريقتان لإفتراض أولي أو إفتراضين أوليين على التوالي. أظهرت النتائج أن طريقة ريقيولا - فالسي تحتاج الى تكرار أكثر للتقارب من الحل مقارنة بطريقة نيوتن - رافسون، بصرف النظر عن قرب الإفتراض الأولي للحل النهائي. متوسط عدد التكرار في طريقة ريقيولا - فالسي كانت ثلاث أضعاف طريقة نيوتن - رافسون.

ABSTRACT: Irrigation and hydraulic engineers are often faced with the difficulty of tedious trial solutions of the Manning equation to determine the various geometric elements of open channels. This paper addresses the development of a computer model for the design of the most commonly used channel-sections. The developed model is intended as an educational tool. It may be applied to the hydraulic design of trapezoidal, rectangular, triangular, parabolic, round-cornered rectangular, and circular cross sections. Two procedures were utilized for the solution of the encountered implicit equations: the Newton-Raphson and the Regula-Falsi methods. In order to initiate the solution process, these methods require one and two initial guesses, respectively. The results revealed that the Regula-Falsi method required more iterations to converge to the solution compared to the Newton-Raphson method, irrespective of the nearness of the initial guess to the actual solution. The average number of iterations for the Regula-Falsi method was approximately three times that of the Newton-Raphson method.

The history of channels is as old as human civilization. Their proper design is of utmost importance due to their widespread use. This includes drainage ditches, through spillways, flood ways, log chutes, roadside gutters, culverts, siphons, flumes, and irrigation canals. Channel sections involve a wide variety of geometric shapes depending on the type and the specific application of the channel, as well as field conditions.

Many procedures have been developed over the years for the hydraulic design of open channel sections. The complexity of these procedures vary according to flow conditions as well as the level of assumption implied while developing the given equation. The Chezy equation is one of the procedures that was developed by a French engineer in 1768 (Henderson, 1966). The development of this equation was based on the dimensional analysis of the friction equation under the assumption that the condition of flow is uniform. A more practical procedure was presented in 1889 by the Irish engineer Robert Manning (Chow, 1959). The Manning equation has proved to be very reliable in practice. In addition, it has well documented friction factors that were developed over the years. This makes the Manning equation more desirable for the design of open channels. Both the Chezy and Manning equations have been discussed in detail in many references. For more information, the reader is referred to the U.S. Soil Conservation Service (1954), Chow (1959), and Henderson (1966).

Other design procedures for open-channel flow have been developed in recent years. These procedures were aimed at the solution of both non-uniform and unsteady flow conditions in open channels that are complex in nature. Hager (1985) used the Bernulli (energy) equation to derive a formulation for estimating the discharge rate from open channels.

*Corresponding Author.

Hager and Volkart (1986) and DeLong (1989) utilized the mass and momentum equations to solve for flow in channels. Such approaches have the potential to become widely used in the future design of open channels. However, the development of computer models based on these techniques is very cumbersome. It requires the determination of a number of parameters which are variable and difficult to assess. For this reason, users of such techniques would have to rely on approximations for these parameters. This undermines the higher accuracy normally associated with these solution procedures and limits their practical use. In comparison, simplified procedures such as the Manning equation require fewer parameters which can be readily determined. Henderson (1966) stated that the Manning formula has proven to be practically reliable and extremely popular for designing open channels in most Western countries.

The Manning equation invokes the determination of flow velocity based on the slope of channel bed, surface roughness of the channel, cross-sectional area of flow, and wetted perimeter of flow. Using this equation, the solution procedures are direct for flow velocity, slope of channel bed, and surface roughness. However, the solution for any unknown related to the cross-sectional area of flow and wetted perimeter involves the implementation of an implicit recursive solution procedure which can not be achieved analytically. Many implicit solution procedures have been developed in the literature. Among these techniques are the Newton-Raphson, Regula-Falsi (false position), secant, and the Van Wijngaarden-Dekker-Brent methods (Press et al., 1986).

This paper addresses the development of a computer model for the hydraulic design of the most commonly used channel sections using the Manning equation. The specific objectives include the development of a general educational tool for the analysis of various geometric sections of open channels; the application of existing numerical procedures to the solution of the resultant implicit equations; and a comparison between the utilized implicit solution procedures in terms of speed of convergence, accuracy, and the effect of initial guess on convergence.

Theoretical Development

The Manning equation, which can be used to describe both uniform and non-uniform flow conditions in open channels, takes the form (Henderson, 1966)

$$V = \frac{R^{\frac{2}{3}} S^{\frac{1}{2}}}{n} \tag{1}$$

where n is the coefficient of surface roughness of the channel, R is the mean hydraulic radius in meters, S is the slope of the energy gradient which is equivalent to the slope of channel bed under uniform flow conditions, and V is the mean velocity in *m/sec*. The hydraulic radius represents the ratio of cross-sectional area of flow to the wetted perimeter. Equation (1) can be converted to feet-second units by inserting the factor $(3.281 \text{ ft/m})^{1/3}$ or *1.486* to become (Henderson, 1966)

$$V = \frac{1.486\ R^{\frac{2}{3}} S^{\frac{1}{2}}}{n} \tag{2}$$

Equations (1) and (2) can be written in the general form

$$Q = \frac{c A^{\frac{5}{3}} S^{\frac{1}{2}}}{n P^{\frac{2}{3}}} \tag{3}$$

where c is a constant dependent on the system of units which takes a value of *1* for the m^3/sec units and a value of *1.486* for the ft^3/sec units, Q is the flow rate in m^3/sec or ft^3/sec, A is the cross-sectional area normal to the direction of flow in m^2 or ft^2, and P is the wetted parameter in m or ft.

The solution for the parameter S, Q, and n in Equation (3) is explicit for any channel section. However, the parameters which relate to A and P require an implicit solution procedure when such parameters are unknown. Among the primary parameters that are related to A and P are the depth of flow, y, and the top width of the channel section at the free water surface, T. These unknowns would be handled either by trial and error or by implementing an implicit solution procedure such as the Newton-Raphson method (NRM) or Regula-Falsi method (RFM). To implement these implicit solution procedures, Equation (3) is written as

$$F(\xi) = nQP^{\frac{2}{3}} - cA^{\frac{5}{3}}S^{\frac{1}{2}} \tag{4}$$

where ξ is the unknown and $F(\xi)$ is the function to be reduced to zero once the implicit solution procedure converges to the solution.

Implicit Solution Procedures

There are several procedures that can be utilized in solving implicit equations. The two procedures used in this paper include the NRM and RFM.

Newton-Raphson Method: This method (NRM) requires successive evaluation of both the function and its derivative to achieve the solution. Successive iterations are expressed as (Chapra and Canale, 1988)

$$\xi_{(j+1)} = \xi_{(j)} - \frac{F\ (\xi_{(j)})}{\left(\frac{dF\ (\xi_{(j)})}{d\xi}\right)} \tag{5}$$

where ξ is the unknown, $F\ (\xi)$ is the evaluated function, and j is the iteration counter. The NRM requires one initial estimate to commence the solution process.

In order to implement Equation (5), the derivative of Equation (4) must be established. This derivative is defined as

$$\frac{dF(\xi)}{d\xi} = \frac{2}{3}nQP^{-\frac{1}{3}}\frac{\partial P}{\partial \xi} - \frac{5}{3}\ cA^{\frac{2}{3}}S^{\frac{1}{2}}\frac{\partial A}{\partial \xi} \tag{6}$$

where $\partial P/\partial \xi$ and $\partial A/\partial \xi$ are the partial derivatives of P and A, respectively, with respect to the unknown ξ. These partial derivatives should be evaluated for the selected channel section before Equation (6) is utilized.

Regula-Falsi Method: This method (RFM) requires two initial estimates to commence the solution process while the derivative of the evaluated function is not needed. One of these estimates is held constant while the other is updated on succeeding iterations. The RFM is very useful in instances when the derivative of the evaluated function may not be determined. The consecutive iterations of the Regula-Falsi procedure can be expressed by (Chapra and Canale, 1988)

$$\xi_{(j+1)} = \frac{\xi_c F(\xi_{(j)}) - \xi_{(j)}F(\xi_c)}{F(\xi_{(j)}) - F(\xi_c)} \tag{7}$$

where ξ is the initial constant estimate. The number of iterations is dependent on the speed of convergence to the solution.

Modeled Cross Sections

The Manning equation was applied in the developed computer model for the design of four general cross sections of open channels. These include trapezoidal, parabolic, round-cornered rectangular, and circular sections (refer to Figure 1). The modeled general cross-sections represent the most commonly used geometric sections of open channels. Each of these sections is commonly used under slightly different field conditions or is the resultant of natural conditions. For instance, the trapezoidal cross section is most widely used with unlined earth canals since it provides side slopes for stability.

Trapezoidal Section: The trapezoidal section is one of the geometric sections that could be handled by the developed model. Rectangular and triangular channel sections can also be handled by the developed model since each represents a special case of the trapezoidal section. The rectangular or triangular cross sections result from setting the inverse of side slope, m, or the bottom width, b, of the trapezoidal channel section to zero, respectively. For the trapezoidal channel cross section, the wetted perimeter, P, and the cross-sectional area of flow, A, are expressed as

$$P = b + 2y\sqrt{m^2 + 1} \tag{8}$$

$$A = by + my^2 \tag{9}$$

where b is the bottom width of the channel, y is the depth of flow, and m is the inverse of side slope (refer to Figure 1a). For the three implicit cases where either y, b, or m is unknown, the partial derivatives of P and A in Equation (4) become

a. y is the unknown:

$$\frac{\partial P}{\partial \xi} = \frac{dP}{dy} = 2\sqrt{m^2 + 1} \tag{10}$$

$$\frac{\partial A}{\partial \xi} = \frac{dA}{dy} = b + 2my \tag{11}$$

b. b is the unkown:

$$\frac{\partial P}{\partial \xi} = \frac{dP}{db} = 1 \tag{12}$$

$$\frac{\partial A}{\partial \xi} = \frac{dA}{db} = y \tag{13}$$

c. m is the unknown:

$$\frac{\partial P}{\partial \xi} = \frac{dP}{dm} = \frac{2my}{\sqrt{m^2 + 1}} \tag{14}$$

$$\frac{\partial A}{\partial \xi} = \frac{dA}{dm} = y^2 \tag{15}$$

Parabolic Section: For the parabolic section, parameters P and A are expressed as

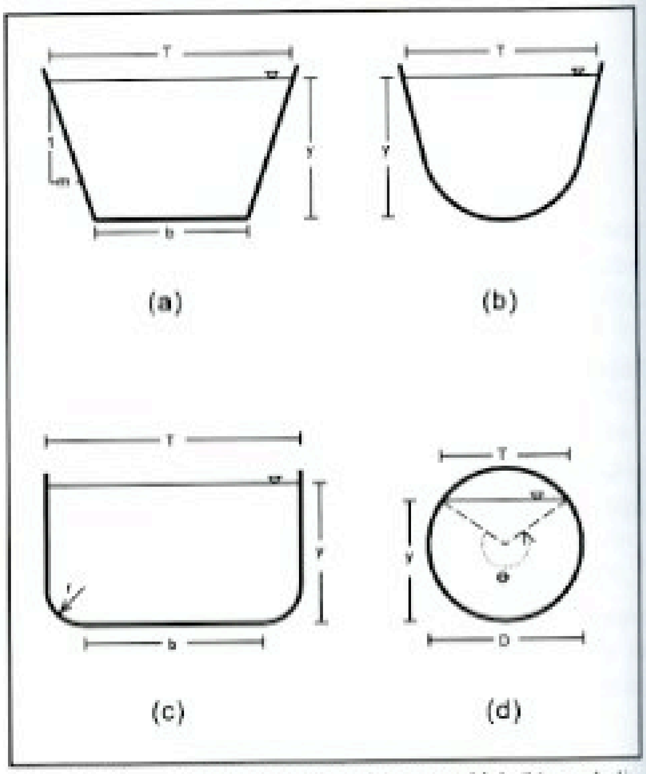

Figure 1. General channel sections: (a) trapezoidal, (b) parabolic, (c) round-cornered rectangular, and (d) circular.

$$P = T + \frac{8y^2}{3T} \tag{16}$$

$$A = \frac{2}{3} Ty \tag{17}$$

where T is the top width of the channel section at the free water surface (refer to Figure 1b). For the two implicit cases where either y or T is unknown, the partial derivatives of P and A in Equation (4) become

a. y is the unknown:

$$\frac{\partial P}{\partial \xi} = \frac{dP}{dy} = \frac{16}{3}\frac{y}{T} \tag{18}$$

$$\frac{\partial A}{\partial \xi} = \frac{dA}{dy} = \frac{2T}{3} \tag{19}$$

b. T is the unknown:

$$\frac{\partial P}{\partial \xi} = \frac{dP}{dT} = 1 - \frac{8}{3}\frac{y^2}{T^2} \tag{20}$$

$$\frac{\partial A}{\partial \xi} = \frac{dA}{dT} = \frac{2y}{3} \tag{21}$$

Round-Cornered Rectangular Section: For the round-cornered rectangular section (refer to Figure 1c), parameters P and A are expressed as

$$P = (\pi - 2)\ r + b + 2y \tag{22}$$

$$A = \left(\frac{\pi}{2} - 2\right) r^2 + by + 2ry \tag{23}$$

where r (with $y > r$) is the radius of the corner. For the three implicit cases where either y, b, or r is unknown, the partial derivatives of P and A in Equation (4) become

a. y is the unknown:

$$\frac{\partial P}{\partial \xi} = \frac{dP}{dy} = 2 \tag{24}$$

$$\frac{\partial A}{\partial \xi} = \frac{dA}{dy} = b + 2r \tag{25}$$

b. b is the unknown:

$$\frac{\partial P}{\partial \xi} = \frac{dP}{db} = 1 \tag{26}$$

$$\frac{\partial A}{\partial \xi} = \frac{dA}{db} = y \tag{27}$$

c. r is the unknown:

$$\frac{\partial P}{\partial \xi} = \frac{dP}{dr} = (\pi - 2) \tag{28}$$

$$\frac{\partial A}{\partial \xi} = \frac{dA}{dr} = 2\left(\frac{\pi}{2} - 2\right) r \tag{29}$$

Circular Section: For the circular section (refer to Figure 1d), parameters P and A are expressed as

$$P = \frac{\theta D}{2} \tag{30}$$

$$A = \frac{(\theta - \sin\theta)\ D^2}{8} \tag{31}$$

where D is the internal diameter of the circular cross section and θ is the angle. The depth of flow, y, and top width, T, can be expressed as a function of θ and D as follows:

$$P = \left(\frac{\theta - \sin\theta}{\sin\frac{\theta}{2}}\right) \frac{D}{8} \tag{32}$$

$$T = D \sin\frac{\theta}{2} \tag{33}$$

For the implicit case when θ is unknown, the partial derivatives of P and A in Equation (4) become

$$\frac{\partial P}{\partial \xi} = \frac{dP}{d\theta} = \frac{D}{2} \tag{34}$$

$$\frac{\partial A}{\partial \xi} = \frac{dA}{d\theta} = \frac{D^2}{8}(1 - \cos\theta) \qquad (35)$$

Model Description

The developed computer model is called *STYLIST*. *STYLIST* has five major options that are accessible from the main menu. The first option allows the user to select the menu of the circular channel sections (refer to Figure 2). This menu has 4 options that will enable the user to determine Q, n, S, or y. The first three unknowns are calculated explicitly while the last unknown is determined implicitly using either the NRM or the RFM. The second option of the main menu diverts the program to the menu of the trapezoidal, rectangular, and triangular cross sections. This menu has six options which include Q, n, S, and y in addition to m and b which are determined implicitly. The third option of the main menu displays the menu for parabolic sections which has similar options to the menu for circular channel sections. Also, this menu has an option for determining T implicitly. The fourth option of the main menu displays the menu for round-cornered rectangular sections. The options of this menu are similar to the options of the menu for trapezoidal sections except for m which is replaced by r. Finally, the fifth option of the main menu diverts the program to the set-up menu. The set-up menu has several options which are fundamental for running *STYLIST*. These options include the following: (a) selecting the system of units (English or SI units), (b) selecting the implicit solution procedure (Newton-Raphson or Regula-Falsi), (c) selecting the maximum allowable number of iterations, (d) setting the allowable error, (e) selecting the initialization mode for the implicit solution procedure (manual or automatic), (f) selecting the graphics mode for running STYLIST (text mode or high resolution graphics), and (g) saving the setup options.

The developed computer model requires an IBM compatible microcomputer with MS-DOS version 3.00 or higher. The program requires a machine with a 3.5" or 5.25" disk drive and at least 256 Kbytes of random access memory (RAM). The model has several graphical displays which are unavailable without a graphics adapter.

The developed model was verified and validated. The verification of *STYLIST* was carried out by checking the values of the computed variables against hand calculated values. This process was repeated until all the various options of the model were tested. Next, numerous runs were conducted to see whether the model was producing consistent results. A set of variables were defined and the resultant solution of the selected unknown was determined. A different unknown was then selected and the model was executed using the results of the previous run together with the initial set of variables. The results from this run must be consistent with the previous run for the selected channel cross section for the model to be performing satisfactorily. This process was repeated until all the components of the developed model were covered. After completing this step, it was clear that the model was functioning correctly.

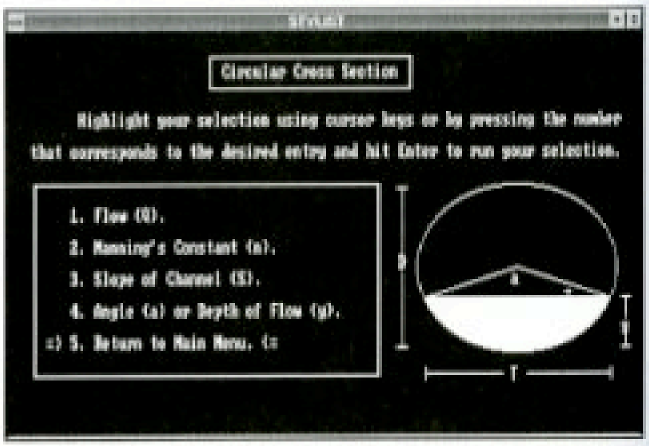

Figure 2. Sample menu of STYLIST (a = θ).

Results and Discussion

The *STYLIST* model was used to study the rate of convergence, the convergence error, and the effect of initial guess on convergence for the two implicit solution procedures. Many comparison runs were conducted to assess the more desirable solution procedure. These comparisons were reiterated for the different geometric sections of open channels which could be handled by the model.

In order to compare the two implicit solution procedures, the results from one of the analyses that were conducted will be reported. Since these results were consistent for other sections as well, the discussion here will be limited to a trapezoidal section with a flow rate of *7.72 m^3/sec*, flow depth of *1.25 m*, channel bed slope of *$1.00X10^{-4}$*, side slope of *1.00*, and Manning's roughness coefficient of 0.0150.

The base width of the trapezoidal channel described above was selected as the unknown in this example. Figure 3 illustrates the effect of fixing the number of iterations on the convergence process for the two implicit solution procedures. A fixed maximum allowable number of iterations was used for the runs of the model which produced the data in Figure 3. The graphs in this figure show that when the allowable number of iterations was increased, the resultant value of the unknown moved closer to the actual solution (*8 m*). The NRM showed no oscillations and the convergence occurred exponentially to the actual solution. On the other hand, the RFM showed damped sinusoidal oscillation as the maximum allowable number of iterations was increased from *0* to *20*. The NRM converged to the actual solution after 6 iterations within an allowable error of $1X10^{5}$ *m* while the RFM needed more than *20* iterations to reach the solution within the same allowable error. These results may vary based on the initial guess.

Figure 4 depicts a plot of the required number of iterations versus the desirable level of accuracy. This figure shows that the RFM needed more iterations to converge to the desired level of accuracy compared to the NRM. In general, the average number of iterations for the RFM was two to four times that of the NRM. The graph in Figure 4 also suggests that the rate of convergence of the RFM is slower than that of the NRM.

The speed of convergence is affected by the value of the initial guess. Figure 5 shows a plot of initial guesses versus the required number of iterations to converge to the actual solution within the preselected allowable error. This figure shows that the NRM needed fewer iterations to converge to the actual solution compared to the RFM. The average number of iterations for the RFM was 1.5 to 5 times that of the NRM. Figure 5 also shows that the RFM is more sensitive to the initial estimate compared to the NRM.

The results of the two implicit solution procedures were consistent regardless of the geometric shape of the channel section. It was observed that both methods diverge away from the actual solution under some circumstances. The NRM would diverge from solution whenever the initial guess was very close or equal to zero. On the other hand, the RFM appeared to have difficulties when the initial guess was equal to the constant guess. The outcome diverged away from the actual solution when this situation occurred. Both methods appeared to have no convergence problems when other conditions prevailed. In general, the RFM needed more iterations to converge to the solution compared to the NRM.

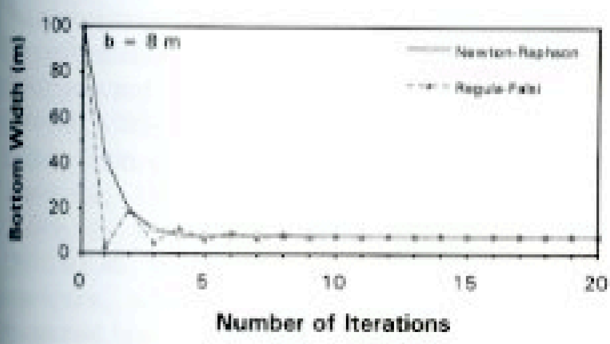

Figure 3. Effect of number of iterations on convergence.

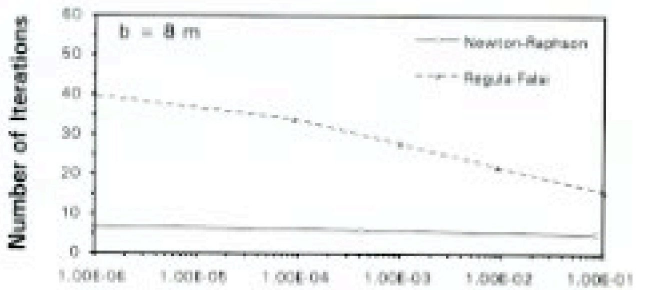

Figure 4. Effect of convergence error on the number of iterations.

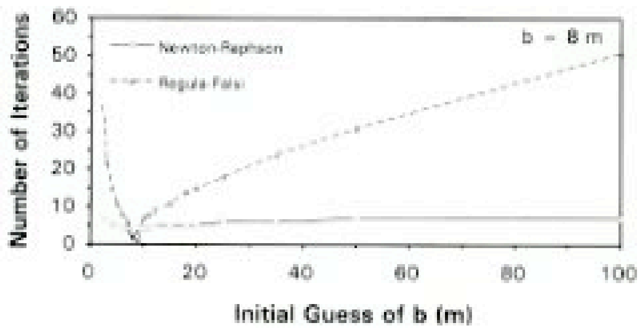

Figure 5. Effect of initial estimate on speed of convergence.

Conclusions

The developed computer model may be used as an effective educational tool for the hydraulic analysis of open channel flow. It allows for demonstrating the effect of varying the geometric parameters of an open channel on the hydraulics of flow. The model can be applied to the design of trapezoidal, rectangular, triangular, round-cornered rectangular, parabolic and circular cross sections. Also, it is also useful when designing or analyzing open channel distribution systems.

The considered two implicit solution procedures converged to the actual solution in almost all cases. The results revealed that the Regula-Falsi method demanded more iterations to converge to the final solution compared to the Newton-Raphson method, irrespective of the closeness of the initial guess to the actual solution. The Newton-Raphson method was less sensitive to the initial guess; therefore, it would be more applicable to the solution of implicit equations that result from the Manning equation. One disadvantage of the Newton-Raphson method would be its requirement of the derivative of the evaluated function before developing the solution. This precondition might not be possible for some odd channel cross sections. Under these circumstances, the Regula-Falsi method would be more desirable.

References

Chapra, S.C. and R.P. Canale. 1988. *Numerical Methods for Engineers* (2nd Edition). McGraw-Hill, New York.

Chow, V. T. 1959. *Open-Channel Hydraulics.* McGraw Hill, New York.

DeLong, L. L. 1989. Mass conservation: 1-D open channel flow equations. *Journal for Hydraulic Engineering*, ASCE, 115(2):263-269.

Hager, W. H. 1985. Modified venturi channel. *Journal of the Irrigation and Drainage Engineering*, ASCE, 3(1):19-35.

Hager, W. H. and P. U. Volkart. 1986. Distribution channels. *Journal of Hydraulic Engineering*, ASCE, 112(10):935-952.

Henderson, F. M. 1966. *Open Channel Flow.* Macmillan Publishing Company, New York.

Press, H. W., B. P. Flannery, S. A. Tuckolsky, and W. T. Vetterling. 1986. *Numerical Recipes: The Art of Scientific Computing.* Cambridge University Press, Cambridge, New York.

U. S. Soil Conservation Service. 1954. *Handbook of Channel Design for Soil and Water Conservation.* United States Department of Agriculture, Washington, D.C.

Published with the approval of the College of Agriculture, S.Q.U., as paper number 190895.