Mediating early number learning: specialising across teacher talk and tools?

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Abstract

In this paper we locate our work in the context of claims of poor performance amongst South African learners in primary mathematics, and gaps in the knowledge base of primary mathematics teachers. Our focus is on the analysis of three Grade 2 Numeracy teachers' actions with artifacts in the context of increasing resource provision in the South African national policy landscape. Using ideas of mediation drawn from the Vygotskian tradition, and developed by Michael Cole and Jim Wertsch, we identify actions with the artifacts that suggest shortcomings in teachers' understandings of the mathematical structures that underlie the design of the resources. Drawing on the work on modeling and tool use in the Dutch Realistic Mathematics Education tradition, we note disruptions in openings for pedagogy to provide 'models of' increasingly sophisticated strategies, which might provide children with 'models for' working more efficiently (and ultimately 'tools to' think with), disruptions arising from teachers presenting only concrete unit counting based models of early number calculations. Within a policy context where improving the resource situation is a priority, we argue for more attention to longitudinal support to teachers to develop understandings of number and its progression that allow them to see the significance of the mathematical structures that are figured into the design of the artifacts that are increasingly available for use.

Introduction

Performance in mathematics at all levels of the schooling system in South Africa continues to be described in terms of a 'crisis' (Fleisch, 2008). The recently introduced Annual National Assessment (ANA) – standardised national tests taken by all primary grades in mathematics and language – continue to show very low levels of attainment in the Foundation (Grades 1–3) and Intermediate Phases (Grades 4–6), with, in 2011, national mean performances in the Grade 3 numeracy test standing at 28% and at 30% in the Grade 6 Mathematics test (Department for Basic Education, 2011b).

In this context, research and policy attention has shifted back to the primary years with a range of issues cited as factors contributing towards this crisis of performance including lack of curriculum clarity and inadequate resources for teaching. Policy responses to this situation have resulted in a more prescriptive curriculum together with the provision of resources (artifacts such as abaci and hundred squares) within the Foundations for Learning (FFL) campaign, introduced in 2009. The policy of prescribing the content, sequencing and pacing – backed up by the increasing provision of standardised termly tests and resources – continues into the recent 2012 introduction of the mandatory Curriculum and Assessment Policy Statements (CAPS) (Department for Basic Education, 2011a).

In 2011 we began the 5-year research and development focused Wits Maths Connect–Primary Project, working with ten government primary schools in one district. Our observations during the first year suggested problems with take up of the resources provided. Crucially our data suggested that even in the context of tightly prescribed content coverage with provision of supporting artifacts, a lack of in-depth attention to teachers' understandings of the mathematical structure of number embedded in their talk and in their use of resources to represent mathematical ideas often prevented coherent and progressively sophisticated opportunities to work with early number. In this paper we explore possible reasons for this particularly with respect to teachers' pedagogic content knowledge.

The policy context

In recent years the structure and policy presentation of mathematics curricula in the primary years has reverted to a format that incorporates prescription of content, sequencing, pacing and progression. In 2008, a Government Gazette signaled the introduction of the 'Foundations for Learning' (FFL) campaign (Department of Education, 2008a). This campaign incorporated the introduction of the ANA, alongside the prescription of a minimum of an hour of mathematics teaching each day – which was to include at least ten minutes focused on mental arithmetic skills. 'Daily teacher activities for numeracy in Grades 1– 3' were also detailed in this document, with the following activities specified for whole class work:

Count with whole class according to their level

– Count using a number square
– Count on the number line
– Count forwards and backwards
– Count forwards and backwards from a given number to a given number
– Count in multiples
– Odd and even numbers, etc. (p.17)

Coupled with these activities, a set of 'Recommended resources for Numeracy in Grades 1–3' was also specified (p.18). For display on walls and teacher whole class work, this list included resources like a number line, a large 100 square, and a large abacus. At learner level, the list included an individual 100 square, place value cards, counters, and an individual small abacus.

The FFL Assessment Framework (Department of Education, 2008b) also introduced the notion of termly curriculum 'milestones' – described thus:

> The milestones (knowledge and skills) derived from the Learning Outcomes and Assessment Standards from the National Curriculum Statement for Languages and Mathematics (Grades 1–3) have been packaged into four terms for each grade to facilitate planning for teaching. These milestones explain the content embedded in the Learning Outcomes and Assessment Standards.
>
> These milestones have been further written into manageable units to assist you to develop the required assessment tasks per term (p.iii).

Research findings from South Africa show however that provision of detailed curriculum statements and resources in and of themselves may not be a key policy lever for raising standards. The work of Ensor and colleagues (2009), for example, presents evidence of teaching that holds learners back through focusing on concrete counting based strategies rather than supporting the shift over time towards more abstract calculation based strategies. The analysis leading to this finding incorporated a focus on 'specialisation strategies' (following Dowling, 1998), which include specialisation of content and modes of representation. Specialisation of content in early number refers to tasks that shift over time from counting, to calculation by counting, to calculating without counting. Specialisation of modes of representation correspondingly relates to shifts from concrete representations towards more abstract symbolic representations of number. Ensor *et al.*'s findings indicated a high prevalence (89% of the total time across the Grades 1– 3) of counting/ calculation by counting-oriented tasks. The authors noted that 'concrete apparatus for counting, and for calculating by-counting, is visible in all three grades, with sustained use through Grades 1, 2 and 3' (p.21).

These findings relating to teaching of early number are consistent with broader findings at the learner level which suggest the ongoing predominance of unit counting and repeated addition/subtraction strategies well into the late Intermediate Phase (Schollar, 2008).

Policy that provides artifacts with minimal induction into the form and function of such artifacts must either assume that teachers have the pedagogic content knowledge to use the resources in ways that move learners from unit counting, or assume that the specification for skills in the curriculum bridges the gap. However, a central thread of evidence in South Africa related to the crisis in performance points to weaknesses in primary teachers' mathematics content knowledge and pedagogic content knowledge (Carnoy, Chisholm, Chilisa, Addy, Arends, Baloyi *et al*., 2011; Taylor, 2010), with this thread having significant international parallels (Ball, Hill, and Bass, 2005; Ma, 1999). Carnoy *et al.*'s (2011) study indicated low levels of content knowledge in relation to Grade 6 mathematics content, and significant positive associations between teachers' content knowledge, pedagogic content knowledge and the time they spent teaching mathematics. This study also noted specific weaknesses in what the authors referred to as 'mathematical knowledge in teaching', defined in the following terms:

> the degree to which teacher can appropriately integrate the use of the instructional techniques with the mathematical concept being taught and its effectiveness on learner learning. This also includes the use of correct language to clearly convey mathematical ideas (p.102).

Within the broader field of study into mathematical knowledge for teaching, our focus here is on aspects of what Hill, Ball and Schilling (2008) have identified as aspects of 'specialised content knowledge', which includes the need to represent and explain mathematical ideas, and 'knowledge of content and students', which includes awareness of typical student development sequences.

The research

In this paper we pick up on the issue of teachers' integration of 'instructional techniques' with learning outcomes. Our focus is on teachers' use of artifacts to mediate learners' developing number-sense. Through this focus we examine what this can tell us about teachers' content knowledge and pedagogic content knowledge (PCK).

Our data are drawn from three Grade 2 Numeracy lessons and, given the extensive focus on number in the Foundation Phase Numeracy curriculum, we emphasise meanings and strategies related to number sense. Analysis of teachers' practices indicated difficulties in using abaci and hundred squares – both resources advocated within the policy context – as effective mediating means. We argue that these difficulties are linked to limited pedagogic content knowledge regarding number structure and its presentation to learners. Our motivation for doing this work is twofold. First, we seek to understand misalignments between teacher talk, actions and artifacts to inform our work in the primary mathematics focused Wits Maths Connect–Primary project, and the work of others engaged in teacher development. Second, we aim to contribute to the policy process with evidence of some ways in which policy artifacts are being taken up in classrooms and what needs to be done to improve effectiveness within take-up.

We begin with a summary of the ways in which number sense is described in the international literature and some of the practices and artifacts advocated for teaching aimed at developing number sense. The analytical base that we draw from is based on the Vygotskian notion of artifact mediated action, and follow-up work that distinguishes between artifacts taken up in the ways that build on the meanings connected to the cultures from which they are drawn (ideality) and artifacts taken up in ways that relate to other aspects of their materiality. We do this by linking the work of post-Vygotskian theorists such as Wertsch and Cole with the theory linking models and tools arising from the Freudenthal institute.

Following an outline of the data sources, we present three episodes of teaching involving the use of policy-advocated artifacts and provide an analysis of these episodes based on concepts related to the theory of mediated action. In each case we argue that the teachers' work with the artifacts proceeds in ways that are unlikely to support the development of learners' number sense. We conclude with a discussion of the ways in which our evidence suggests the need to move beyond both mere provision of classroom artifacts and claims of lack of content/pedagogical content knowledge, and instead to move towards longitudinal teacher support based on the use of artifacts in ways that build on their form and relate to the meanings of number, operations and problem-solving.

Number sense and teaching for number sense

McIntosh, Reys and Reys' (1992) widely cited paper describes number sense in terms of three key areas: knowledge and facility with numbers, knowledge and facility with operations, and applying this knowledge and facility with number and operations to a range of computational settings (p.4). Application skills based in strong number sense would demonstrate use of operations in ways that are well connected both to the numbers being operated on, and to the problem context. A recurring feature of the application category is the requirement for 'flexibility' and 'efficiency' of strategy – a point echoed by Kilpatrick, Swafford and Findell (2001) in their description of strategies backed by mathematical proficiency as being both 'effective' and 'efficient'. Flexibility depends on being able to adapt selected strategies to a range of problem settings. Central to the building of efficiency in early number learning is the adoption of 'non unit counting' strategies, which in turn, involve the reification of at least some counting processes into numerical objects (Sfard, 2008), or in the words of Gray (2008): "a shift in attention from the objects of the real world to objects of the arithmetical world – numbers and their symbols" (p.82). Such reification allows initially for 'count on' based strategies, and subsequently for the creation and use of strategies based on calculating with grouped numbers – using '5' and '10' in particular within early number learning as benchmarks for calculating 'through'.

Writing focused on progression within number sense has noted the importance of representations in supporting this reification of counting processes into numerical objects. Askew (1998) draws a distinction between 'structured' and 'unstructured' materials – with the former viewed as resources that are 'usually designed to embody a particular mathematical idea' (p.13). In contrast, 'unstructured' resources tend to be everyday materials or objects used for counting or measuring. Structured resources include artifacts such as dice that support 'subitizing' (the immediate perception of small quantities without unit counting) and artifacts that provide experience of a range of representations showing 'pair-wise', 'five-wise' and groups of ten based number representations (Bobis, 1996; Gravemeijer, Cobb, Bowers, and Whitenack, 2000) that can support later understanding of operations within the decimal number system. In this view, artifacts such as abaci and 100 squares that provide visual representations of number in the 1–100 range as constitute-able through a quantity-based decomposition into tens and units – e.g. seeing 54 as constituted by 50 and 4, with the 50 further broken down into five 10s (Thompson, 1999) – would be structured resources, whereas cubes and counters would tend to fall into the unstructured resource category.

Given the widespread acceptance of the importance of actions on objects forming the basis for well-connected understandings of number ideas, artifacts that can be operated on, and that lead into and link with representations of these ideas, are viewed as critical to early mathematical learning. Thus there is wide agreement that resource provision is an important aspect of supporting the development of numeracy. But whilst representations in early number learning must include actions on objects, early number sense has been described as needing to go beyond this to develop well-connected networks between the language of a problem, the actions on objects that physically 'replay' or model the problem, and the diagrams and representations that provide a record of these actions or models, leading eventually to symbolic representations (Haylock and Cockburn, 2008). In turn, and related to progression in terms of representations, research points to limited number sense being related to representations that are closer to concrete actions and good number sense being related to more compressed symbolic representations (Askew and Brown, 2003). Closely related to this progression of representational forms, Steffe, Von Glasersfeld, Richards, and Cobb (1983) note that progression in counting strategies can be related to being able to work across a range of count-able units with perceived items at the lowest level through to abstract number units at the highest level of competency.

Across all these discussions, a common theme is the idea of teachers modelling coherently connected and increasingly sophisticated and efficient strategies for solving number problems, using suitable artifacts and meditating the key structures within these artifacts to represent their problem-solving processes. This teacher modelling provides the basis from which learners can appropriate and use similar actions for themselves, actions that latterly become internalised psychological constructs for learners to work with. In the language of the research coming out of the Freudenthal institute (Gravemeijer and Stephan, 2002), the teacher initially provides a *model of* how to act on and talk about artifacts. For example, a child adding eight and four might count out eight and then count on four. The teacher provides a model of how this can be done on the abacus by using the ten structure to slide across eight beads without counting them in ones and also modelling how the four could be added as two followed by two. There is no suggestion in this *model of* that what the teacher shows is an accurate mirroring of what the child does: the child's actions provide the impetus for the teacher's actions, but the teacher shows how the artifact can be acted on in ways that mirror the child's actions and, where possible, drawing attention to how the artifact may be acted upon more efficiently. Thus increasingly sophisticated strategies are related to gradually more compressed representations, which in turn, need to be based on more compressed actions on mediating artifacts.

As the children watch the teacher frequently act and talk in such ways, they begin to appropriate the actions and language for themselves, acting on the artifacts in similar ways to the teachers' modelling: the artifact becomes a *model for* the learner to operate on. Ultimately, if the artifact is well designed (for supporting reification and mental 'actions'), this experience of acting on material artifacts supports the development of *tools for* thinking whereby the learner has internalised the language and actions to the point of being able to operate in such ways without the physical presence of the artifacts. (The theory does not suggest that this interiorisation of *tools for* is a result of a simple internalisation of the physical action, but studies do suggest that models such as the number line do fit with how the brain may process quantities additively (Dehaene, 1999).)

Analytical concepts

The central focus of this paper is on three excerpts of Grade 2 Numeracy teaching in which artifacts were deployed to support problem-solving on a given task (the problems being numerical calculations the answers to which the learners do not have rapid recall). Wertsch's (1991) concept of mediated action, drawn from his studies of Vygotsky's notion of mediation (Vygotsky, 1981), is used to analyse teacher actions effected using selected tools to solve a problem they have set. For Wertsch (1995), mediated action rests on 'individual-operating-with-mediational-means' (p.64) as the central unit of analysis. In this linked agent-artifact formulation, the possibilities for transformation of action are present but Wertsch emphasises the need for agent take up of the artifact:

> this is not to say that the meditational means somehow act alone. An individual using the new mediational means had to change as well, since it obviously called for new techniques and skills (p.67).

In the first instance here, the teacher is the individual using the 'new mediational means' and, as argued above, the use has to provide an appropriate *model of* the use of the artifact. There is little point in teachers acting on, say an abacus, in ways that do not make use of the form and function of the artifact, or in ways that could have been done with an unstructured collection of discrete objects. This combined focus on agent-artifact is central to our focus on ruptures in the process of take up (in effective ways) of the resources being inserted via the policy landscape, ruptures that appear to relate to a lack of use of the mathematical structure that underlies the design of the selected resource, which in turn is linked to teachers' PCK.

Cole's (1996) distinction between 'artifacts' and 'tools' is useful here, and he draws on the distinction made by Bakhurst (1991) between the 'materiality' and 'ideality' of artifacts to make his point. Bakhurst notes that:

> as an embodiment of purpose and incorporated into life activity in a certain way – being manufactured for a reason and put into use – the natural object acquires a significance. This significance is the 'ideal form' of the object, a form that includes not a single atom of the tangible physical substance that possesses it (p.182).

The 'physical substance' constitutes the 'materiality' of the artifact. For Cole, an artifact is transformed into a tool when the nature of its use corresponds with the purposes recognised by the culture – i.e. when its use embodies ideality rather than materiality. Tool use, rather than simply artifact use, is associated with the transformation of mental functioning, following Vygotsky's (1981) formulation:

> The inclusion of a tool in the process of behavior (a) introduces several new functions connected with the use of the given tool and with its control; (b) abolishes and makes unnecessary several natural processes, whose work is accomplished by the tool; and alters the course and individual features of all the mental processes that enter into the composition of the instrumental act, replacing some functions with others (i.e. it re-creates and

> reorganizes the whole structure of behavior just as a technical tool re-creates the whole structure of labor operations) (p.139–140).

The Dutch research inserts a stage into this movement of artifact to tool, from materiality to ideality: the learner's use of an artifact as a model for carrying out an operation. At this stage the boundaries between artifact and tool are blurred: two learners could be acting in similar ways, but one might simply be mimicking what the teacher has done, while the other could have come to appreciate the ideality of the artifact and be closer to appropriating it as a tool for thinking. The analysis of this movement is beyond the scope of this paper, but we draw attention to it to highlight the importance of teachers appreciating the ideality of artifacts and providing appropriate models of how to act on them.

Our interest in doing the analysis, within the context of the teacher development work in this project, is to understand the kinds of tasks, artifacts and experiences over time that we might need to provide teachers with access to in order to build settings within which teachers' understandings are attuned to the mathematical structures that the artifacts are intended to model that are starting to become more available through the policy landscape.

Data sources

The data presented in this paper is drawn from our 'baseline' observations of Grade 2 Numeracy teaching at the start of 2011 as we launched the Wits Maths Connect–Primary (WMC-P) project. We observed, videotaped and transcribed single lessons taught by 33 of the 41 Grade 2 teachers across the ten schools in our sample – with a view to more broadly gaining insights about the nature of teaching and learning across our schools, and the contexts of teachers' work. In this paper three episodes from these observations are selected because they are typical of a number of lessons observed that showed evidence of artifacts being used in ways that seemed to bypass the mathematical structures built into their design. In all cases we have other instances in our data of the same kind of actions being enacted by other teachers in the Grade 2 sample, suggesting the wider presence of the phenomena of rupture between task, artifact and talk that we seek to describe and analyse here.

The excerpts

Excerpt 1

Ms. C works in a township/informal settlement school where mathematics is taught in Home Language. Ms. C uses a mix of Tsonga and English in her teaching, and often stresses vocabulary across the languages to support children's learning of mathematical vocabulary in both. In the focal lesson, Ms. C announced: 'Today we are going to do subtraction'. The vocabulary of subtraction actions in English and Tsonga is introduced in terms of 'minus'/'asoosa'/'minus, we take away something and throw it away'. A large abacus sat on the teacher's desk and learners each had a small abacus in front of them.

The series of examples listed below was worked through, some as whole class demonstration (WC) with the teacher modeling actions on her large abacus, and some set as seatwork tasks for individuals to complete (SW):

5 – 3 (WC) 4 – 0 (WC) 6 – 2 (SW) 8 – 5 (SW) 10 – 5 (SW)

Across each of the whole class demonstrations, unit counting of beads on the abacus was used to count out the first number, and then to 'take away' the second number, and then to count out, again in ones, what was left. In the seatwork tasks, some learners were seen also to be unit counting in this way to solve the problems, whilst others appeared to be adding the two numerals rather than subtracting. Ms. C went round checking learners' abacus representations of each calculation and the actions they enacted to produce answers. She noted the addition representations and emphasised: 'We are not adding'.

Following these examples, Ms. C announced in English, and then in Tsonga:

> *Now we are going to minus bigger numbers of two digits minus-ing only one digit. Nothing is changing when we minus the bigger numbers. You are still going to use your abacus and count what you will be given, right.*

She wrote 12–4 on the board, and then said:

> *Let me show you how we count two numbers that we must minus with only one number. Let's count 12 balls.*

Twelve beads were counted out in ones, as before, with the count now using two rows of the abacus; ten on the top row, two on the one below. She asked at this point:

> *How many lines did I use?*

The class responded: 'Two', to which Ms. C replied:

> *Two lines because it's twelve. It has to jump to another row right?*

The teacher then proceeded to take away four through unit counting, sliding back, the two beads (one at a time) on the second row and then two (again singly) on the top line. The remaining beads were then counted out one by one to get eight.

Excerpt 1 analysis

The abacus is the key mediating tool in use in this excerpt. The structure of the abacus – with ten rows each carrying ten beads – provides a physical form of the '10–based' structure of numbers to 100 in the decimal system. However, almost no recourse is made to this structure within the strategies presented by Ms. C. Further, as noted by Ensor *et al.* in their study (2009), all the calculations presented are solved through unit counting processes involving 'count all' strategies.

Two key questions emerge from this initial commentary. The first, related to the lack of take up of the '10-ness' structure of the artifact, is whether the abacus was an appropriate tool in relation to the tasks set? On the one hand the calculations set could be answered with an unstructured collection of discrete objects – which is essentially the way in which the abacus is being used here (the fact that they are beads on a rod simply making it easy to keep track of the unit counting). This would be classified as take up of the materiality, rather than the ideality, in Bakhurst's (1991) terms of the mediating artifact (and consequently, limiting the potential for this laying the basis of a psychological tool). Clearly, for some learners in the class who did not yet appear to have made sense of subtraction as an operation artifacts that allow for the building of a connection between the ‘taking away’ action being presented here and the subtraction problem in word and symbolic form, would seem to be the priority.

The second question relates to whether the structure of 10 was worth pursuing for this set of calculations, most of which involved subtracting single digit numbers from (at most) two digit numbers less than 20. The unit count strategy effectively ‘produces’ the answers to all of these questions. Ms. C’s consistent use of unit counting suggests that ‘getting the answer’ was the object of her mediated action, but this leaves aside the notions of flexibility and efficiency of strategy identified as important in the number sense literature. Greater efficiency is certainly possible with a grouped count strategy, and the structure of the abacus would support the modelling of 12 in the last calculation (12–4) as a composite number made up of 10 and 2, each of which could be moved across on the abacus as grouped objects with a single swift action rather than in a one-by-one fashion. Ms. C drew children’s attention to the fact that 12 required beads from 2 rows of the abacus, but made no explicit reference to the breakdown into a ten and units. Her expression of this idea therefore remained at an everyday, rather than at a more specialised level based on the decimal structure of number. Further, her actions created the ten and unit values through counting processes rather than as conceptual objects in Gray’s (2008) terms. The teacher’s actions were limited in terms of being a ‘model of’ that might provide the basis for learners’ later use of the abacus as a ‘model for’ calculating using the base-ten structure.

For efficiency to be pursued, it would appear that teacher and learners require prior experiences with moving interim (1–10) numbers of beads with a single shift, in ways that would support reifications of smaller numbers that could lead into discussion of the 10-ness structure of the abacus, and the ability to very quickly represent 12 as a quantity on an abacus using this structure. In this more nuanced artifact mediated action we see possibilities associated with working with learners at various stages of the counting, calculating by counting, and calculating hierarchy – and actions that learners could appropriate with different degrees of understanding thus encouraging the artifact/tool and material/ideal shifts noted by Cole (1996). Thus whilst Ensor *et al.* point out shifts in specialisation of representation from concrete to symbolic representations, we suggest that there are gradations *within* these representational categories as well that can relate to building calculating that is likely to involve some reified number facts and some counting – a feature noted as prevalent in children's work with number across a range of attainment (Thompson, 1995).

Excerpt 2

In Ms. K's classroom, a mix of Zulu interspersed with English was used within a lesson focused on addition. The following episode occurred early in the lesson. Ms. K had distributed a worksheet with the following table drawn at the top, and two further similar tables (with different addends) below it:

+	8	16	4	20	12	24
4						

Each learner had an individual abacus in front of him or her, and the teacher had a large abacus at the front. The following explanation unfolded for how to work out answers using the abacus.

T: Let us do Number 1. We read it together [pointing on the board].

T & class: 4 plus 8. Ha! [Teacher picked up the large abacus that had eight beads showing on the top row from a previous unit count out activity. Teacher held the abacus up and moved back the eight beads before counting out four beads through unit counting with the class.]

T & class: 1, 2, 3, 4. Is that correct?

Class: Yes.

T: Plus 8. [With the class chanting, eight beads were counted out one by one on the second row of the abacus.] 1, 2, 3, 4, 5, 6, 7, 8. Now before you do it, let's work it out. [Eight beads in the second row were counted again with whole class chant in unit count followed by '9, 10, 11, 12' as the first row beads were added in.] Twelve! Which means the answer is – ?

Class: Twelve.

T: The answer is twelve. [Teacher wrote 12 into the box beneath 8 on the worksheet copy stuck on the board.]

Subsequently, 4 + 16 was also worked out using a unit count of beads across three rows of the abacus – four on the top row, ten on the second and six on the third, as were the remaining sums in the table.

Analysis of Excerpt 2

This excerpt is similar in many ways to excerpt 1, and also based on the use of the abacus artifact. We see once again that mediation relied on unit count based strategies to represent both addends, and counting all to produce the answer. However, the number range was higher, and combined with addition as the operation in focus, resulted in some answers going above 20.

A difference of interest relating to artifact use in this episode is the way in which the addition was represented on the abacus. For example 4 + 8 was represented by four beads pulled across one by one on the top row, followed by eight beads pulled across on the second row. In Ms. C's excerpt, subtraction proceeded by taking (in 12 –4) the two on the second row away followed by two more on the first row, which is close to modeling 12–4 as 12–2–2. In contrast Ms. K's modeling, where the two numbers were pulled from two different rows, disrupts the possibility of bridging through 10 at all. Therefore, although the numbers being operated on are larger, in one sense, the mediation in this excerpt is more rooted in the realm of concrete counting than in Excerpt 1.

Excerpt 3

Ms. S worked in a township/informal settlement school and taught a mixed Grade 2/3 class. Within the focal lesson, opening activities were based on whole-class chanting of days of the week, and 10s to a 100. The class was then asked to use fingers and/or the individual 100 squares they had on their desks to count in 1s, 2s and 5s to 100 – something that not all learners appeared able to do.

At this stage, pointing to the large 100 square stuck on the blackboard, Ms. S announced:

> *I want to see if you know the number that you count, I'm going to hide a number and I want you to show or tell me [. . .] On our chart I will hide one number and you tell me which I hide.*

She stuck a small blank square over the number 15 on her 100 chart and asked the class which number was covered. She prefaced learner answers with: 'Did you see which number I hid?' 'Fifteen' was audibly called out, but the first two children the teacher asked said the answer was 'Fifty'. The third child offered 'Fifteen', and the class, in chorus, agreed that this was correct. The blank square was then stuck over the number 44, and the same question posed: 'Did you see which number I hid?' The following interaction then ensued:

Lr1:	34
T:	Is it 34?
Class:	No
T:	[…] Is she correct [pointing at another learner]?
Lr2:	24
T:	24? Did you see it well [pointing at another learner]?
Lr3:	45
Class:	No. [T asks another learner.]
Lr4:	44
T:	Is he correct?
Class:	Yes
T:	Let's count and check starting from 41, do you see it? [The teacher pointed to the numbers on the 100 square from 41, pointing to each number as the class counted from 41– 44.]

This was followed by an episode where the number 26 was hidden and asked for, with the right answer given by a learner. The next questions asked were: 'What number comes after 26?' and 'What number comes before 26?' – with both of these numbers visible before and after the small blank square on the 100 square on the board.

Analysis of Excerpt 3

A key feature of Ms. S' work within this excerpt was her recurring reference to 'seeing' the hidden number. Given this recurrence, we are unsure as to whether the object of the activity was simply to recall the hidden number by remembering what it looked like before it disappeared from view or to use the structure of the square to deduce the hidden number. Her mediated actions suggest recall more often than deduction using the 10-based structure of the 100 square, although the episode involving 44 suggests that the teacher did have some awareness of the sequence of number. Even within this one section, the explanation that is used to 'check' whether 44 is correct was based on reciting the number list from 41 – again, an everyday rather than a specialised form of expression that makes no explicit reference to the ways in which rows of a 100 square are structured and how looking down the columns is an effective strategy: there was no reference at all to the column structure of 100 squares, which, like the abacus, is a representation that embodies the 10-based structure. Modelling of how to find the number that would use the row and column structure – 44 is in the row that contains all the numbers in the 'forties' and the column that contains all the numbers with 4 in the units – was visibly absent. Learners' attempts to identify 44 – with 34 and 24 offered – suggest that for some children the vertical patterns in the square had begun to have some ideality but this was not picked up and worked with by the teacher. Within the highly partial take up of the base-ten numerical structure, we see materiality once again within mediation rather than ideality.

Conclusions

In our analysis we have brought together the theoretical concepts associated with mediated action: the notion of an agent working with an artifact to model the ideal in the expectation that learners might appropriate actions and come to achieve a desired object. Within each of our excerpts we argue that our analyses of the teacher-artifact mediation observed was likely to restrict any tool use to unit counting. While this may be located in an individualised view of limitations in individual teachers' PCK we also want to argue that there is a cultural dimension to these limitations and return to Wertsch's work to examine this.

Vygotsky's (1981) assumption about the fundamental role of mediating tools (psychological as well as physical tools) in mental function was that:

> by being included in the process of behavior, the psychological tool alters the entire flow and structure of mental functions. It does this by determining the structure of a new instrumental act (Vygotsky, 181, p.137).

Here, the presence and use of artifacts is viewed as naturally leading to their appropriation. Wertsch's position (1995) differs somewhat from Vygotsky's by expanding the focus on tools into the sociocultural setting in which they are located, and points out the take-up of artifacts in the dialectic between tools and thinking as reliant on a further condition:

> the meditational means that shape mental functioning and action more generally are inherent aspects of, and hence serve as indexes of, a sociocultural setting (p. 64).

In our empirical problem this second condition is problematic. The push to get resources into schools via the FFL policy suggests that (in poorer schools at least) these resources were not previously a part of the socio-cultural setting, and thus we cannot assume that the mathematical structure of a resource that might be apparent within certain sociocultural settings (including those of the policy makers) is also a feature of the sociocultural setting of these schools. We have shown that material, rather than ideal, use of the artifact was a common feature across all three of the empirical excerpts presented in this paper. The almost complete lack of reference or use of the 10-based structure of both the abacus and the 100 square raises questions relating to these teachers' awareness of this structure from both a content knowledge and pedagogical content knowledge perspective. But we suggest that this is a consequence of absence in the sociocultural setting, rather than deficit in the teachers' knowledge.

Our data, collected in early 2011 in the context of FFL with its explicit attention to resource provision suggests that a wider range of resources (including some that embody features related to more symbolic representations of number such as 100 squares and abacuses) are making their way into schools. Our analysis shows though that provision of artifacts that support more abstract ways of working with number is insufficient in improving teaching and learning. Understanding of the ways in which early counting progresses into mental procedures – expressed succinctly by Askew and Brown (2003) as: ‘count all, count on from the first number, count on from the larger number, use known facts and derive number facts’ (p.6), and how progressive counting leads into more abstract number concepts (Gelman and Gallistel, 1986) – would appear to be required in the socio-cultural setting, prior to being able to recognise the potential usefulness of these presences in artifacts that model these reified structures.

Research tells us that carefully structured experiences with children focused on developing more compressed operational understandings and the more abstract notions of number that these are founded on and further develop are possible (Askew, Bibby, and Brown, 2001). Our future work is to develop structured experiences for teachers that can work at this content knowledge level whilst working at the pedagogical action level as well. Our analysis suggests that this is needed in order to support the development of mediated actions that allow for the ‘specialised’ mediation required to capitalise on the improving resource situation in ways that have impact at the level of children’s learning.

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