Fuzzy optimal control of a poisoning-pest model by using α-cuts

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Abstract— In this article, a dynamical system represents the poisoning-pest model is considered. At First a mathematical model for the poisoning-pest model is simulated. Since there is no exact number of pests it is natural to consider the variables as fuzzy variables. Thus we need to consider a fuzzy dynamical system to the poisoning-pest model. To solve such a fuzzy optimal control system, using α-cut, and Zadeh's extension principle, one can convert this system to a non-fuzzy optimal control system. The final optimal control problem is solved by discrelization method.

Index Terms—Fuzzy optimal control, Poisoning-pest model, Zadeh's extension principle, Fuzzy solution, Generalized differentiability

I INTRODUCTION

Hukuhara differentiability (H-differentiability) for fuzzy functions was originally introduced by Puri and Raelescu in [1]. After that Kelva [2] discussed the properties of differentiable fuzzy function using Hukuhara derivative. Fuzzy differential equations are studied in several papers [3, 4]. But Hukuhara derivative has a disadvantage : the fuzziness of solution increases when time goes on.

Bede and Gal in [5] introduced the generalized differentiability. The presented differentiability has not this disadvantage. Apparently the disadvantage of generalized differentiability of a function compared to H-differentiability is that a fuzzy differential equation has no unique solution.

Whenever differential equation and control functions are fuzzy in an optimal control problem, we are facing with a fuzzy optimal control problem. The so-called problem considered by many authors, for example: Diamond and Kandel in [4] showed the existence of the fuzzy optimal control for the system $\dot{\tilde{x}}(t) = a(t)\odot\tilde{x}(t) \oplus \tilde{u}(t), \tilde{x}(0) = \tilde{x}_0$. Najariyan and Farahi in [6, 7] found new techniques respectively for solving linear fuzzy controlled systems with fuzzy initial conditions and fuzzy optimal linear control systems with fuzzy coefficients by using α-cuts.

One of the application of optimal control problems is the problem of controlling pests. Many attempts have been made in this area (see [8, 9]). Often times we are not dealing with an exact number of pests when we want to control pests. In such cases, researchers have used of the theory of fuzzy (see [10]).
This article is based on minimizing the number of pests. Because the exact number of pests is not known for us so we associate with a optimal fuzzy control.

This paper is organized as follows: In Section 2 we present basic definitions and theorems of fuzzy numbers and operations of fuzzy numbers. Also in this section we have discussed the definition of Zadeh's extension principle and generalized differentiability. In Section 3 we define optimal fuzzy control of a poisoning-pest model problem. In Section 4, we applied the technique to a real poisoning-pest model. Finally, Section 5 will give a conclusion briefly.

2 Basic concepts

Let Ω be a set in $\mathbb{R}$, then a fuzzy subset $\tilde{\mu}$ of Ω is defined by its membership function, $\tilde{\mu}(t)$, which produces values in $[0,1]$ for all t in Ω. So, $\tilde{\mu}(t): \Omega \to [0,1]$.
A fuzzy number is a convex, normalized fuzzy set of the real line $\mathbb{R}$ whose membership function is piecewise continuous and we show it as $\mathcal{F}(\Omega)$. A triangular fuzzy number $\tilde{\mu}$ is defined by three numbers $a < b < c$ where the base of the triangle is the interval $[a, c]$ and its vertex is at

$t=b$. Triangular fuzzy numbers will be written as $\tilde{\mu}=(a,b,c)$ (see[11]).

If $\tilde{\mu}$ is a fuzzy number then an α-cut of $\tilde{\mu}$, written $\tilde{\mu}_\alpha$ is defined as:

$$\tilde{\mu}_\alpha=[\tilde{\mu}]_\alpha=\begin{cases}\{x\in\Omega|\tilde{\mu}(x)\geq\alpha\}, & 0<\alpha\leq 1\\ \overline{\{x\in\Omega|\tilde{\mu}(x)>0\}}, & \alpha=0,\end{cases}$$

where $\bar{A}$ denotes the closure of $A\subset\Omega$ and $\tilde{\mu}_0$ is the support of $\tilde{\mu}$, (see[12]).

In this paper, we show the lower bound of $\tilde{\mu}_\alpha$, as $\underline{\tilde{\mu}}_\alpha$ and the upper bound of it as $\bar{\tilde{\mu}}_\alpha$.

Definition 1 (Zadeh's extension principle). Let Z be a cartesian product of universes, that is $Z=Z_1\times Z_2\times\ldots\times Z_r$ and $\tilde{\mu}_1,\tilde{\mu}_2,\ldots,\tilde{\mu}_r$ be r fuzzy sets in $Z_1,Z_2,\ldots,Z_r$ respectively and Y is a given space. Each function $f\colon Z\to Y$ induces corresponding function $\tilde{f}=\mathcal{F}(Z_1)\times\mathcal{F}(Z_2)\times\ldots\times\mathcal{F}(Z_r)\to\mathcal{F}(Y)$ (i.e., $\tilde{f}$ is a function mapping fuzzy sets in Z to fuzzy sets in Y) defined for each fuzzy set $\tilde{\mu}\in Z$ by

$$\tilde{f}(\tilde{\mu}_1,\tilde{\mu}_2,\ldots,\tilde{\mu}_r)(y)=\begin{cases}\sup\limits_{(Z_1,Z_2,\ldots,Z_r)=f^{-1}(y)}\min\{\tilde{\mu}_1(z_1),\tilde{\mu}_2(z_2),\ldots,\tilde{\mu}_r(z_r)\}, & f^{-1}(y)\neq\phi,\\ 0 & f^{-1}(y)=\phi,\end{cases}$$

where f^{-1} is the inverse of f. The function $\tilde{f}$ is said to be obtained from f by the extension principle.

An important result of Zadeh's extension principle is the characterization of the image levels of a fuzzy set through $\tilde{f}$, where f is a continuous function. Therefore if $f\colon\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ is a continuous function then according to Zadeh's extension principle one can extend f to $\tilde{f}\colon\mathcal{F}(\mathbb{R})\times\mathcal{F}(\mathbb{R})\to\mathcal{F}(\mathbb{R})$ by the equation

$$\tilde{f}(\tilde{\mu},\tilde{v})(z)=sup_{z=f(s,t)}min(\tilde{\mu}(s),\tilde{v}(t)). \quad (1)$$

It is well known that

$$\tilde{f}_\alpha(\tilde{\mu},\tilde{v})=f(\tilde{\mu}_\alpha,\tilde{v}_\alpha),\alpha\in[0,1],\tilde{\mu}\in\mathcal{F}(\mathbb{R}),\tilde{v}\in\mathcal{F}(\mathbb{R}). \quad (2)$$

Using Zadeh's extension principle the operations of addition, $\oplus$, multiplication, $\otimes$, and scalar multiplication, $\odot$, on the $\mathcal{F}(\mathbb{R})$ are defined respectively by

$$(\tilde{\mu}\oplus\tilde{v})(s)=sup_{t\in\mathbb{R}}min\{\tilde{\mu}(t),\tilde{v}(s-t)\},$$
$$(\tilde{\mu}\otimes\tilde{v})(s)=sup_{t\in\mathbb{R}}min\{\tilde{\mu}(t),\tilde{v}(s/t)\},$$

and

$$(\lambda\odot\tilde{\mu})(s)=\begin{cases}\tilde{\mu}(\frac{s}{\lambda}), & \lambda\neq 0,\\ \chi_{\{0\}}, & \lambda=0\end{cases}$$

where $\chi_{\{0\}}$ is the characteristic function of 0. It is clear that the following properties are true for all α-cuts

$$[\tilde{\mu}\oplus\tilde{v}]_\alpha=\tilde{\mu}_\alpha+\tilde{v}_\alpha \quad ,[\lambda\odot\tilde{\mu}]_\alpha=\lambda\tilde{\mu}_\alpha \quad ,\alpha\in[0,1],$$

and

$$[\tilde{\mu}\otimes\tilde{v}]_\alpha=[min\{\underline{\mu}_\alpha\underline{v}_\alpha,\underline{\mu}_\alpha\bar{v}_\alpha,\bar{\mu}_\alpha\underline{v}_\alpha,\bar{\mu}_\alpha\bar{v}_\alpha\},max\{\underline{\mu}_\alpha\underline{v}_\alpha,\underline{\mu}_\alpha\bar{v}_\alpha,\bar{\mu}_\alpha\underline{v}_\alpha,\bar{\mu}_\alpha\bar{v}_\alpha\}].$$

According to the definition of operations of addition, scaler multiplication, the operation subtraction, $\ominus$, is similarly defined .

Definition 2 [5] Let $\tilde{u},\tilde{v}\in\mathcal{F}(\mathbb{R})$. If there exists $\tilde{w}\in\mathcal{F}(\mathbb{R})$ such that $\tilde{u}=\tilde{v}\oplus\tilde{w}$, then $\tilde{w}$ is called the Hukuhara-difference of $\tilde{u}$ and $\tilde{v}$ and it is denoted by $\tilde{u}\ominus_H\tilde{v}$.

Definition 3 [5]. Let $\tilde{x}\colon T\in\mathbb{R}\to\mathcal{F}(\mathbb{R})$ and $t_0\in T$. We say that $\tilde{x}$ is differentiable at t_0 if :

(I) there exists an element $\dot{\tilde{x}}(t_0)\in\mathcal{F}(\mathbb{R})$ such that, for all $h>0$ sufficiently near to 0, there are $\tilde{x}(t_0+h)\ominus_H\tilde{x}(t_0),x(t_0)\ominus_H\tilde{x}(t_0-h)$ and the limits

$$lim_{h\to 0^+}\frac{\tilde{x}(t_0+h)\ominus_H\tilde{x}(t_0)}{h}=lim_{h\to 0^+}\frac{\tilde{x}(t_0)\ominus_H\tilde{x}(t_0-h)}{h}=\dot{\tilde{x}}(t_0),$$

or

(II) there is an element $\dot{\tilde{x}}(t_0)\in\mathcal{F}(\mathbb{R})$ such that, for all $h<0$ sufficiently near to 0, there are $\tilde{x}(t_0+h)\ominus_H\tilde{x}(t_0),\tilde{x}(t_0)\ominus_H\tilde{x}(t_0-h)$ and the limits

$$lim_{h\to 0^-}\frac{\tilde{x}(t_0+h)\ominus_H\tilde{x}(t_0)}{h}=lim_{h\to 0^-}\frac{\tilde{x}(t_0)\ominus_H\tilde{x}(t_0-h)}{h}=\dot{\tilde{x}}(t_0).$$

Theorem 1 *[12]. Let $\tilde{x}\colon T\to\mathcal{F}(\mathbb{R})$ be a function and denote $\tilde{x}_\alpha(t)=[\underline{x}_\alpha(t),\bar{x}_\alpha(t)]$ for each $\alpha\in[0,1]$. Then:*

(i) If $\tilde{x}$ is differentiable in the first form (I), then $\underline{x}_\alpha$ and $\bar{x}_\alpha$ are differentiable functions and $\dot{\tilde{x}}_\alpha(t)=[\dot{\underline{x}}_\alpha(t),\dot{\bar{x}}_\alpha(t)]$.

(ii) If $\tilde{x}$ is differentiable in the second form (II), then $\underline{x}_\alpha$ and $\bar{x}_\alpha$ are differentiable functions and $\dot{\tilde{x}}_\alpha(t)=[\dot{\bar{x}}_\alpha(t),\dot{\underline{x}}_\alpha(t)]$.

Now we consider the fuzzy initial value problem

$$\dot{\tilde{x}}(t)=\tilde{f}(t,\tilde{x}(t)),\tilde{x}(0)=\tilde{x}_0 \quad (3)$$

where $\tilde{f}\colon[0,T]\times\mathcal{F}(\mathbb{R})\to\mathcal{F}(\mathbb{R})$ is obtained by Zadeh's extension principle from a continuous function $f\colon[0,T]\times\mathbb{R}\to\mathbb{R}$, Note that $\tilde{f}$ is continuous because f is continuous (see [13]), and by (2) we have $\tilde{f}_\alpha(t,\tilde{x})=f(t,\tilde{x}_\alpha)$ where $f(t,A)=\{f(t,a)|a\in A\}$. Associated with (3) we can consider the following crisp differential equation

$$\dot{x}=f(t,x(t)),x(0)=x_0 \quad (4)$$

where $\dot{x}(t)$ is the derivative of a crisp function $x\colon[0,T]\to\mathbb{R}$. For more details see [14].

Theorem 2 *Let $\tilde{x}\in\mathcal{F}(\mathbb{R})$. Suppose that f is a continuous function and for each $x_0\in\mathbb{R}$ there exists a unique solution $x(.,x_0)$ for (4) and that $x(t,.)$ is continuous in $\mathbb{R}$ for each $t\in[0,T]$. Then:*

(i) If $\tilde{f}$ is nondecreasing with respect to the second argument, then the fuzzy solution of (3) and the solution of (4) via the derivative in the first from (I) are identical.

(ii) If $\tilde{f}$ is nonincreasing with respect to the second argument, then the fuzzy solution of (3) and the solution of (4) via the derivative in the second from (II), if it exists, are identical.

Proof 1 See [12]

3 Optimal control of the poisoning-pest model

In this section we present a fuzzy optimal control for the poisoning-pest model. we are going to determine the sufficient amount of poison to kill the approximate number of pests. Suppose that $\tilde{x}(t)$ is the pest density, $\tilde{u}(t)$ is the speed of poison insufflation at time $t\geq 0$, where $\tilde{x}$ and $\tilde{u}$ are fuzzy numbers. So we hope that in interval time $[0,t_f]$, the pest density is reduced desirable. Consider the pest density at $t=0$ is $\tilde{x}_1$. We want to minimize the cost of the poison and the harm done to the crop. The fuzzy control model of the poisoning-pest is as follows:

$$\dot{\tilde{x}}(t) = \tilde{F}(\tilde{x}(t)) \ominus \tilde{u}(t) \otimes \tilde{x}(t), \quad (5)$$

where the initial condition $\tilde{x}(0) = \tilde{x}_1$, and the final condition is $\tilde{x}(t_f) = \tilde{x}_f$. The function $\tilde{F}(\tilde{x}(t))$ can be written as $r \odot (\tilde{x} \oplus (\tilde{x} \otimes \tilde{x}) \odot \frac{-1}{k})$,where r is the growth rate of the pest density and k is the maximum pest density of the environment (see [15] for more details). The objective function is as:

$$\min \ J(\tilde{x}(t), \tilde{u}(t)) = \int_0^T [\tilde{g}(\tilde{x}(t)) \oplus \tilde{h}(\tilde{u}(t))] dt. \quad (6)$$

Suppose that the function $\tilde{g}(\tilde{x}(t))$ denote the cost of pest harm and the function $\tilde{h}(\tilde{u}(t))$ denotes the expense of the poison at the time $t \geq 0$.

4 Application

We consider the following fuzzy optimal control problem (see[15]):

$$\dot{\tilde{x}}(t) = r \odot \left(\tilde{x} \oplus (\tilde{x} \otimes \tilde{x}) \odot \frac{-1}{k}\right) \ominus \tilde{u}(t) \otimes \tilde{x}(t) \quad (7)$$

with cost function:

$$J(\tilde{x}(t), \tilde{u}(t)) = \int_0^{t_f} [\tilde{g}(\tilde{x}(t)) + \tilde{h}(\tilde{u}(t))] dt \ , \quad (8)$$

where $r = \frac{10}{9}$, $k = 20$, $t_f = 10$,$\tilde{x}(0) = \tilde{5} = (4,5,6)$, $\tilde{x}(t_f = 10) = \tilde{1} = (0,1,2)$, $\tilde{g}(\tilde{x}(t)) = 10 \odot \tilde{x}(t)$, and $\tilde{h}(\tilde{u}(t)) = 2 \odot \tilde{u}(t)$.

So, the fuzzy optimal control problem can be written as:

$$\min \quad J(\tilde{x}(t), \tilde{u}(t)) = \int_0^{10} [10 \odot \tilde{x}(t) \oplus \tilde{u}(t)] dt$$

$$\dot{\tilde{x}}(t) = \frac{10}{9} \odot (\tilde{x} \oplus (\tilde{x} \otimes \tilde{x}) \odot \frac{1}{-20}) \ominus \tilde{u}(t) \otimes \tilde{x}(t)$$

$$\tilde{x}(0) = (4,5,6), \tilde{x}(T = 10) = (0,1,2)$$

We solve this problem using α-cuts technique. We consider $\tilde{x}_\alpha = [\underline{x}_\alpha, \bar{x}_\alpha]$ and $\tilde{u}_\alpha = [\underline{u}_\alpha, \bar{u}_\alpha]$. In the problem $\tilde{f}(\tilde{x}, \tilde{u}) = \frac{10}{9} \odot (\tilde{x} \ominus \frac{1}{20} \odot (\tilde{x} \otimes \tilde{x})) \ominus \tilde{u}(t) \otimes \tilde{x}(t)$, is obtained by Zadeh's extension principle from a continuous function $f(x,u) = \frac{10}{9}(x(t) - \frac{1}{20}x^2) - u(t)x(t)$, Since $u(.)$ is a bounded function, it is not difficult to show that $f(x,u)$ is an increasing function with respect to $x(t)$ for all $|u(t)| \leq 1$ in [0,10], so we must use the first form (I) derivative. Now $\dot{\underline{x}}_\alpha = f(\underline{x}_\alpha, \underline{u}_\alpha, \bar{u}_\alpha)$ and $\dot{\bar{x}}_\alpha = f(\bar{x}_\alpha, \underline{u}_\alpha, \bar{u}_\alpha)$. the objective function is the average of $10\underline{x}_\alpha + 2\underline{u}_\alpha$ and $10\bar{x}_\alpha + 2\bar{u}_\alpha$.

So one interfaces the following non-fuzzy optimal control problem:

$$minJ = \frac{1}{2}\int_0^{10} \left(10\underline{x}_\alpha(t) + 10\bar{x}_\alpha(t) + 2\underline{u}_\alpha(t) + 2\bar{u}_\alpha(t)\right) dt \quad (9)$$

$$st: \quad \dot{\underline{x}}_\alpha(t) = \frac{10}{9}\left(\underline{x}_\alpha(t) - \frac{(\underline{x}_\alpha^2)(t)}{20}\right) - \bar{u}_\alpha(t)\underline{x}_\alpha(t) \quad (10)$$

$$\dot{\bar{x}}_\alpha(t) = \frac{10}{9}\left(\bar{x}_\alpha(t) - \frac{(\bar{x}_\alpha)^2(t)}{20}\right) - \underline{u}_\alpha(t)\bar{x}_\alpha(t) \quad (11)$$

where the initial conditions are $\underline{x}_\alpha(0) = 5\alpha + 4(1-\alpha), \bar{x}_\alpha(0) = 5\alpha + 6(1-\alpha)$ and final conditions are $\underline{x}_\alpha(10) = \alpha, \bar{x}_\alpha(10) = \alpha + 2(1-\alpha)$ for all $\alpha \in [0,1]$. Because $\underline{u}_\alpha \in [-1,1]$ and $\bar{u}_\alpha \in [-1,1]$,so one can define function $\underline{u}_\alpha = A_1 \sin(t \times \frac{\pi}{4})$, and $\bar{u}_\alpha = A_2 \sin(t \times \frac{\pi}{4})$ where $A_1 \in [-1,1], A_2 \in [-1,1]$. We solve this problem by discretization method (see for more details [16]), the solutions have obtained for $\alpha = 0, 0.25, 0.5, 0.75, 1$. The solutions of $\underline{x}_\alpha, \bar{x}_\alpha$ and $\underline{u}_\alpha, \bar{u}_\alpha$ are shown in Figure 1 and Figure 2 respectively.

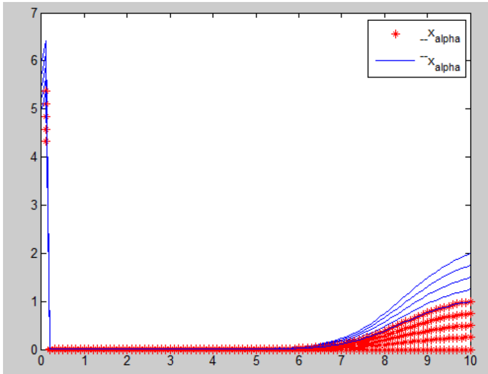

Figure 1: The number of pests , $\underline{x}_\alpha, \bar{x}_\alpha$

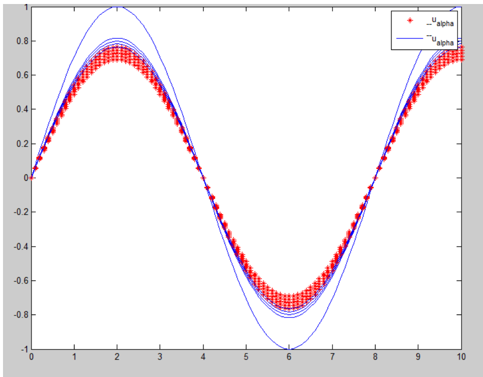

Figure 2: The speed of poison, $\underline{u}_\alpha, \bar{u}_\alpha$

5 CONCLUSION

Optimal fuzzy control theory is applied to a poisoning-pest problem. By applying α-cuts, and using Zadeh's extension principle the fuzzy optimal control of a poisoning-pest system, extended to a new form involve in lower and upper state and control.

Based on diseretization method, the above metioned non-fuzzy optimal control problem is solved.

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