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UNSTEADY MHD MIXED CONVECTION FLOW PAST AN OSCILLATING PLATE WITH HEAT SOURCE/SINK

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Abstract:

This paper study unsteady MHD mixed convection flow past an infinite vertical oscillating plate through porous medium, taking account of the presence of free/forced convection and mass transfer. Using similarity transformation, the coupled non – linear governing equations are solved numerically by applying the combination of the base scheme sub-methods – midpoint, and a method enhancement scheme Richardson extrapolation technique together with Fehlberg fourth-fifth order Runge-Kutta shooting iteration method with degree four interpolant. The results are obtained for velocity, temperature, concentration. The effects of various material parameters are discussed on flow variables and presented by graphs.

Keywords: Mixed convection, magnetohydrodynamic flows, porous medium, mass transfer, oscillating plate generative reactions,Convergence, Runge – Kutta, Dufour effect, Soret effect, isotope separation

NOMENCLATURE

α	thermal conductivity
u, v	velocity components along x- and y- axes, respectively
C	concentration of the fluid
Dm	diffusion coefficient
T	fluid temperature
u_0	free steam velocity
C_∞	free stream concentration
T_∞	free stream temperature
Ha	Hartmann number
Q	heat generation coefficient
c_p	specific heat at constant pressure
C_w	surface concentration
T_w	surface temperature
B_0	magnetic induction
t	Time
Q	coefficient of heat transfer
g	acceleration due to gravity
k'	is the Darcy permeability
b	is the empirical constant
$\pm v_w$	is the suction/blowing parameter

Dimensionless Group

Grc	mass Grashof number
Grt	thermal Grashof number
$\Pr$	Prandtl number
Sc	Schmidt number
M	Hartmann Number
Du	Dufour number
Sr	Soret number

Greek symbols

θ	non - dimensional fluid temperature
λ	ratio of free stream velocity parameter to stretching sheet parameter
β	heat source/sink coefficient
β_c	coefficient of concentration expansion
β_τ	coefficient of thermal expansion
υ	kinematic viscousity
σ	electrical conductivity
ρ	density

Subscripts

w	condition on the wall
∞	ambient condition

1. Introduction

The study of Magneto-hydrodynamic (MHD) flows have stimulated considerable interest due to its important applications in cosmic fluid dynamics, meteorology, solar physics and in the motion of Earth's core (Cramer and Pai, 1973)]. In a broader sense, MHD has applications in three different subject areas, such as astrophysical, geophysical and engineering problems. Convection flow driven by temperature and concentration differences

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has been the objective of extensive research because such processes exists in nature and has engineering applications. The process occurring in nature includes photo-synthetic mechanism, calm-day evaporation and vaporization of mist and fog, while the engineering application includes the chemical reaction in a reactor chamber consisting of rectangular ducts, chemical vapour deposition on surfaces and cooling of electronic equipment. Sharma and Singh (2009) report the effects of Variable Thermal Conductivity and Heat Source/Sink on MHD flow near a stagnation point on a linearly stretching sheet. The analysis of propagation of thermal energy through mercury and electrolytic solution in the presence of external magnetic field and heat absorbing sinks has wide range of applications in chemical and aeronautical engineering, atomic propulsion, space science etc.

Recently, Ahmed and Ahmed (2004) analyzed two-dimensional MHD oscillatory flow along a uniformly Moving infinite vertical porous plate bounded by porous medium. Further, Ogulu and Prakash (2006) investigated the effects magnetic field on heat transfer unsteady flow past an infinite moving vertical plate with variable suction. Later on, the effects of unsteady free convective MHD flow through a porous medium bounded by an infinite vertical porous plate were investigated by Ahmed (2007). Ahmed (2010) studied mixed convection hydro-magnetic oscillatory flow and periodic heat transfer of a viscous incompressible and electrically conducting fluid past an infinite vertical porous plate. It was reported that the mean and transient velocity decreases with the increase in Prandtl number. Physically this is true because the increase in the Prandtl number is due to increase in the viscosity of the fluid, which makes the fluid thick and hence a decrease in the velocity of the fluid. Zueco and Ahmed (2010) studied combined heat and mass transfer by mixed convection MHD flow along a porous plate with chemical reaction in presence of heat source.

It was reported that as the chemical reaction parameter Kincreases, the temperature profiles decrease. Sharma et al. (2011) investigated the influence of chemical reaction and radiation on an unsteady MHD free convective flow and mass transfer through a viscous incompressible, electrically conducting fluid past an infinite vertical heated porous plate with suction, embedded in porous medium in the presence of a uniform transverse magnetic field, oscillating free stream and heat source by taking into account the viscous dissipation. They discovered that the magnitude of fluid temperature decreases with the increase of chemical reaction parameter, viscous dissipation effect and molecular diffusivity; while it increases with an increase of intensity of magnetic field and heat source. Also an increase in the Grashof number, leads to a rise in the magnitude of fluid velocity due to enhancement in buoyancy force. The peak value of the velocity increases rapidly near the porous plate as buoyancy force for heat transfer increases and then decays the free stream velocity. Sharma, Chand and Chaudhary (2011) report an approximate analysis of unsteady mixed convection flow of an electrically conducting fluid past an infinite vertical porous plate embedded in porous medium under constant transversely applied magnetic field, where The transient velocity increases with increase in distance from plate until it attains its maximum value (nearly $y = 1$), after which it decreases. Osman et al. (2011) studied thermal radiation and chemical reaction effects on the unsteady MHD convection through a porous medium bounded by an infinite vertical plate with heat source/sink. They observed that the presence of porous media increases the resistance flow resulting in a decrease in the flow velocity. Several other researcher have worked on MHD convection flow, Okedoye et al (2008) has good review of it.

Due to the importance of Soret (thermal-diffusion) and Dufour (diffusionthermo) effects for the fluids with very light molecular weight as well as medium molecular weight many investigators have studied and reported results for these flows. For the problem of coupled heat and mass transfer in MHD mixed convective flow of a conducting fluid through a porous medium in the presence of chemical reaction, the effect of both Dufour and Soret effects were neglected, on the basis that they are of a smaller order of magnitude than the effects described by Fourier's and Fick's laws. There are, however, exceptions the Soret effect, for instance, has been utilized for isotope separation and in mixture between gases and with very light molecular weight (H_2, He), and for medium molecular weight (H_2, air) the Dufour effect was found to be of considerable magnitude such that it cannot be neglected.

Hence, as a complementary study to that of Postelnicu (2004) and Alam and Rahman (2005), we propose to study the above-mentioned unsteady free - forced convection flow past an oscillating plate in a porous medium under the influence of transversely applied magnetic field.The aim of this paper is to present the numerical analysis of unsteady MHD mixed convective flow of a conducting fluid with variable properties through a porous medium in the presence of chemical reaction and heat source or sink when the plate is made to oscillate in time about a non-zero constant mean with a specified velocity.

2. Mathematical Formulation

We consider the mixed convection flow of an incompressible and electrically conducting viscous fluid along an infinite non conducting vertical flat plate through a porous medium. The *x* axis is taken along the plate in the vertically upward direction and *y* axis is taken normal to the plate. A magnetic field of uniform strength B_0 is applied in the direction of flow and the induced magnetic field is neglected. Initially, the plate and the fluid are at same temperature T_∞in a stationary condition with concentration level C_∞ at all points. At time $t > 0$ the plate starts oscillating in its own plane with a velocity $u_0 \cos \omega t$. Its temperature is raised to T_w and the concentration level at the plate is raised to C_w. The coordinates system and the configuration are shown in Fig. 1

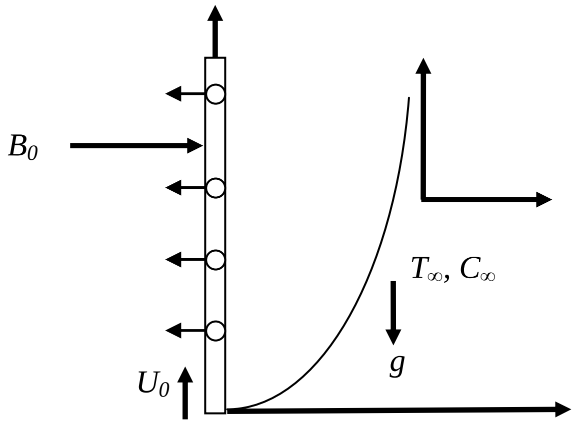

Fig. 1: Flow Configuration

In view of these, we consider that:

(i) All the fluid properties except density in the buoyancy force term are constant;
(ii) The influence of the density variations in other terms of the momentum and energy equations and the variation of the expansion coefficient with temperature is negligible;
(iii) The Eckert number and the magnetic Reynolds number are small so that the induced magnetic field can be neglected.
(iv) All the physical variables are independent of x, except possibly the pressure.
(v) The plate is subjected to a constant suction velocity.
(vi) There exists a first-order homogeneous chemical reaction with a constant rate K between the diffusing species and the fluid.

With foregoing assumptions using the Boussinesq approximation, the governing equations for the flow are given by:

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta_c(C - C_\infty) - \frac{\sigma B_0^{\ 2}}{\rho}u - \frac{\upsilon}{k'}u - \frac{b}{k'}u^2 \tag{2}$$

$$\rho c_p\left(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y}\right) = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{Dmk_T}{c_S c_P}\frac{\partial^2 c}{\partial y^2} + Q(T - T_\infty) \tag{3}$$

$$\rho\left(\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y}\right) = Dm\frac{\partial^2 C}{\partial y^2} + \frac{Dmk_T}{T_m}\frac{\partial^2 T}{\partial y^2} + A_0(C - C_\infty) \tag{4}$$

The boundary conditions are given by

$$u(y,0) = 0,\ T(y,0) = T_\infty,\ C(y,0) = C_\infty \ \forall\, y,\ t \le 0 \tag{5}$$

$$\left.\begin{aligned} &u(0,t) = U_0 \cos\omega t,\ T(0,t) = T_w,\ C(0,t) = C_w \ \ at\ y = 0,\ t > 0 \\ &u \to 0,\ T \to T_\infty,\ C \to C_\infty \qquad\qquad as\ y \to \infty, t > 0 \end{aligned}\right\} \tag{6}$$

Now integrating (1) we have, $v(y,t)=const$.

Then an appropriate value of *constant* for the problem under consideration is taking to be,

$$v=-v_w\frac{\upsilon}{h}, \tag{7}$$

where $v_w>0$ is the suction parameter and $v_w<0$ is the injection parameter and *h* is the scale parameter.

Let us introduce the non-dimensional variables

$$\eta=\frac{y}{\mathrm{h}},\; u=u_w f(\eta),\; \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\; \theta(\eta)=\frac{c-c_\infty}{c_w-c_\infty} \tag{8}$$

With the help of (7) and (8), the governing equations (2) – (4) reduce to

$$-\frac{h}{\upsilon}\frac{dh}{dt}\eta f'(\eta)-v_w f'(\eta)=f''(\eta)-\left(H_a^2+\frac{1}{k}\right)f(\eta)-\lambda f^2(\eta)+Grt\theta(\eta)+Grc\phi(\eta) \tag{9}$$

$$-\frac{h}{\upsilon}\frac{dh}{dt}\eta\theta'(\eta)-v_w\theta'(\eta)=\frac{1}{\mathrm{Pr}}\theta''(\eta)+\frac{Du}{\mathrm{Pr}}\phi''(\eta)+\alpha_0\theta(\eta) \tag{10}$$

$$-\frac{h}{\upsilon}\frac{dh}{dt}\eta\phi'(\eta)-v_w\phi'(\eta)=\frac{1}{Sc}\phi''(\eta)+\frac{Sr}{Sc}\theta''(\eta)+\alpha_1\phi(\eta) \tag{11}$$

Equations (9) to (11) are similar except for the term $\frac{h}{\upsilon}\frac{dh}{dt}$ where *t* appears explicitly. Thus the similarity condition required that $\frac{h}{\upsilon}\frac{dh}{dt}$ must be a constant.

Thus

$$\frac{h}{\upsilon}\frac{dh}{dt}=c$$

That is $h=\sqrt{c\upsilon t}$

Without loss of generality, we take $c=2$, and so $h=2\sqrt{\upsilon t}$, which define the well – established scaling parameter for unsteady boundary layer problem.

Hence equations (9) – (11) together with the boundary and initial conditions (5) and (6) becomes

$$f''(\eta)+(2\eta+c)f'(\eta)-\left(H_a^2+\frac{1}{k}\right)f(\eta)-\lambda f^2(\eta)+Grt\theta(\eta)+Grc\phi(\eta)=0 \tag{12}$$

$$\theta''(\eta)+\mathrm{Pr}(2\eta+c)\theta'(\eta)+Du\phi''(\eta)+\alpha_0\,\mathrm{Pr}\,\theta(\eta) \tag{13}$$

$$\phi''(\eta)+Sc(2\eta+c)\phi'(\eta)+Sr\theta''(\eta)+\alpha_1 Sc\phi(\eta) \tag{14}$$

$$\left.\begin{array}{ll} f(0)=\cos\tau,\; \theta(0)=1,\; \phi(0)=1 & \eta=0 \\ f(\eta)\to 0,\; \theta(\eta)\to 0,\; \phi(\eta)\to 0 & \eta\to\infty \end{array}\right\} \tag{15}$$

Where

$$Grt=\frac{g\beta_\tau h^2(T_w-T_\infty)}{\upsilon u_w},\; Grc=\frac{g\beta_c h^2(c_w-T_\infty)}{\upsilon u_w},\; k=\frac{k'}{h^2}$$

$$P_r=\frac{\mu c_P}{\alpha},\; \alpha_0=\frac{Qh^2}{\mu c_p}, Sr=\frac{k_T}{Tm}\left(\frac{T_w-T_\infty}{c_w-c_\infty}\right),\alpha_1=\frac{A_0h^2}{\mu}, Sc=\frac{\mu}{Dm}$$

$$Du=\frac{Dmk_T}{\alpha c_S c_P}\left(\frac{c_w-c_\infty}{T_w-T_\infty}\right),\quad \lambda=\frac{bu_w h^2}{\upsilon k'},\; H_a^2=\frac{\sigma B_0^2 h^2}{\rho\upsilon},\; \tau=\frac{\omega h^2}{4\upsilon}$$

The physical variables have their usual meanings as defined in nomenclature.

3. Numerical Computation

The set of equations (12) – (14) under the boundary conditions (15) have been solved numerically by applying the combination of the base scheme sub methods – midpoint (Hairer and Wanner (1996)), and a method enhancement scheme Richardson extrapolation (for detail discussion of the method see Ascher, Mattheij, and Russell (1995) and Ascher, and Petzold (1998)) technique together with *Fehlberg fourth-fifth order Runge-Kutta* shooting iteration method with degree four interpolant. The results are presented in Figs. (2) – (17)

3.1. Convergence.

In this paper, we considered RK methods with $s \geq 2$ and coefficients satisfying the hypotheses

$H1: a_{i\,j} = 0$ for $j = 1, \cdots, s;$

$H1:$ thesubmatrix $\widetilde{A} := (a_{i\,j})_{i,\,j\geq 2}$ is invertible;

$H2: b_i = a_{s\,i}$ for $j = 1, \cdots, s,$ i.e., the method is *stiffly accurate.*

In order to show the convergence criteria however, for convenience, we present here the theorems the detail proof is analogous to the one in Ascher and Petzold (1991)

THEOREM 1.0.*Let* Y_i, Z_i *be the solution* of (12, 13, 14) subject to (15) *and consider perturbed values* $\widetilde{Y}_i$, $\widetilde{Z}_i$ *satisfying*

(1.1) $$\left.\begin{aligned} \widetilde{Y}_i &= \widetilde{\eta} + h\sum_{j=1}^{s} a_{i\,j}\, f(\widetilde{Y}_j, \widetilde{Z}_j) + h\delta_i \\ 0 &= g(\widetilde{Y}_i) + \theta_i \end{aligned}\right\} \qquad i = 1, \ldots, s$$

with $\widetilde{Z}_1 := \zeta$*. In addition to the assumptions of Theorem* 4.1, *suppose that*

(1.2) $\widetilde{\eta} - \eta = O(h), \;\; \widetilde{Z}_i - \zeta = O(h), \delta_i = O(h), \qquad \theta_i = O(h^2).$

Then we have for $h \leq h_0$ *the estimates*

(1.3) $\left\|\widetilde{Y}_i - Y_i\right\| \leq C\left(\left\|\widetilde{\eta} - \eta\right\| + h^2\left\|\xi - \zeta\right\| + h\left\|\delta\right\| + \left\|\theta\right\|\right),$

(1.4) $\left\|\widetilde{Z}_i - Z_i\right\| \leq \dfrac{C}{h}\left(h\left\|\widetilde{\eta} - \eta\right\| + h\left\|\xi - \zeta\right\| + h\left\|\delta\right\| + \left\|\theta\right\|\right)$

where $\delta = (\delta_1, \ldots, \delta_s)^T$, $\left\|\delta\right\| = \max_i \left\|\delta_i\right\|$ *and similarly for* θ.

Remarks.

1) The conditions (1.2) ensure that all terms $O(\cdot)$ in the proof below are small.

2) We introduce the notation $\Delta\eta = \widetilde{\eta} - \eta,\; \Delta\zeta = \xi - \zeta,\; Y = (Y_1, \ldots, Y_s)^T,\; \Delta Y = \widetilde{Y} - Y,$ $\left\|\Delta Y\right\| = \max_i \left\|\Delta Y_i\right\|$ and similarly for the z-component. Over a multiple-vector a tilde '~' indicates the removal of its first subvector, e.g. $\widetilde{Y} = (Y_2, \ldots, Y_s)^T$.

THEOREM 2.0*.In addition to the assumptions of Theorem 1.0, suppose that the conditions C(q), D(r) and the hypothesis H3 hold, and that* $(g_y f)(\eta, \zeta) = O(h^k)$ *with* $k \geq 1$. *Then we have*

(2.1) $\widetilde{Y}_s - Y_s = P(\eta, \zeta)(\widetilde{\eta} - \eta) + O\left(h\left\|\widetilde{\eta} - \eta\right\| + h^{m+2}\left\|\xi - \zeta\right\| + h\left\|\xi - \zeta\right\|^2 + h\left\|\delta\right\| + \left\|\theta\right\|\right),$

(2.2) $\widetilde{Z}_i - Z_i = R(\infty)(\xi - \zeta) + O\left(\left\|\widetilde{\eta} - \eta\right\| + h\left\|\xi - \zeta\right\| + \left\|\delta\right\| + \left\|\theta\right\|/h\right)$

where $m = \min(k-1,\; q-1,\; r) \geq 0,$ *R is the stability function, and P is the projectordefined under the condition* (1.3) *by*

(2.3) $P := I_n - Q,\; Q := f_z (g_y f_z)^{-1} g_y.$

Remarks.

The important result consists in the factor h^{m+2} in front of $\left\|\xi - \zeta\right\|$ in (2.1) - (2.4).

Theorem 2.0 yields the main component for the convergence proof of RK methods with singular RK matrix A.

4. Results and Discussion

Here we have investigated numerically MHD mixed convection flow past an oscillating plate with heat source or sink. In order to point out the effects of various parameters on flow characteristic, the following discussion is set out. In simulation, the values of the Prandtl number are considered to be 0.70, 1.70, 2.97 and 4.34 that corresponds to helium, sulfur dioxide, methyl chloride and water, respectively. The values of the Schmidt number are chosen to represent the presence of species water vapour (0.60). Numerical results have been obtained for different values of flow condition and are presented graphically.

Special cases

(i) In the absence of magnetic field i.eM= 0, the results of the present paper are reduced to those obtained by Pop, Grosan, Pop (2004) and Mahapatra and Gupta (2002).
(ii)In the absence of magnetic field, heat source/sink and variable thermal conductivity, the results of the present paper are reduced to those obtained by Pop, Grosan, Pop (2004) inthe absence of radiation effect with constant thermal conductivity and Mahapatra and Gupta (2002) in absence of viscous dissipation and constant thermal conductivity.
(iii) In the absence of chemical reaction, the result of the present paper are reduced to those obtained by Sharma and Singh (2008)
(iv) In the absence of Dufour and Soret effect, the results of the present paper are reduced to those obtained by Osman, Abo-Dahab, and R. A. Mohamed1 (2011)

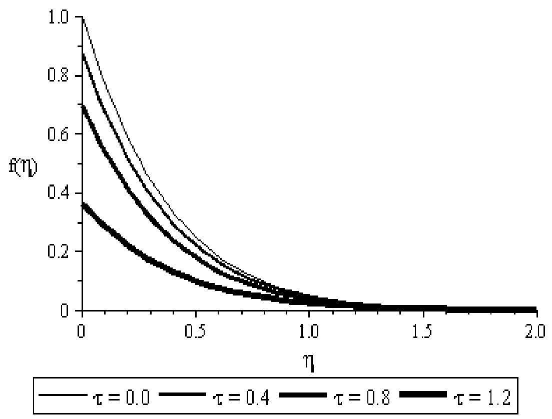

Fig. 2: Velocity distribution for various τ

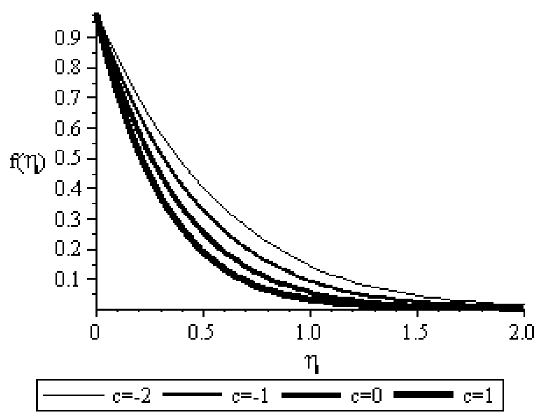

Fig. 3: Velocity distribution for various c

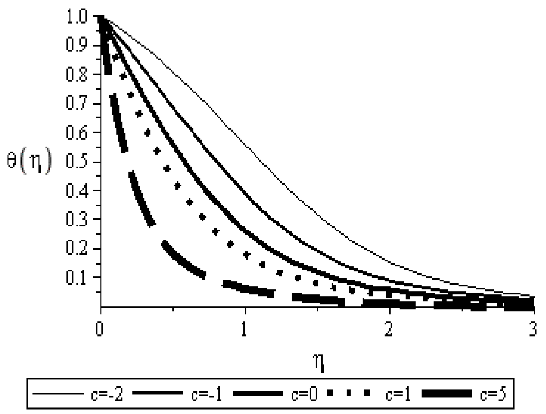

Fig. 4: Temperature distribution for various c

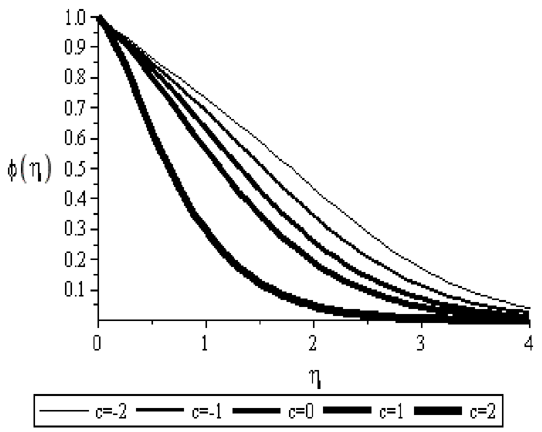

Fig. 5: Concentration distribution for various c

In Fig. 2 represents the velocity profiles due to the variations in ωt. It is evident from the figure, under the chosen condition that the maximum velocity is at the plate. Furthermore, the magnitude of the velocity decreases with increasing phase angle (ωt). Figs. 3, 4 and 5 reveal the velocity, temperature and species concentration variations with suction/injection parameter. It is observed that in case of cooling of surface (an increase in Gr), decrease in injection rate decreases velocity, temperature and species concentration distribution, while increase in suction rate increases the fields. It is found that the velocity decreases with increase in magnetic parameter. It is because that the application of transverse magnetic field will result a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. The presence of a porous medium increases the resistance to flow resulting in decrease in the flow velocity, since *1/K*is an addition to Ha^2.

This behaviour is depicted by the decrease in the velocity as K decreases and when $K = \infty$ (*i.e.* the porous medium effect is vanished) the velocity is greater in the flow field as shown in Fig. 6. In Figs. 7 and 8, it is observed that greater cooling of surface (an increase in Grt) and increase in Grc results in an increase in the velocity, respectively. It is due to the fact increase in the values of thermal Grashof number and mass Grashof number has the tendency to increase the thermal and mass buoyancy effect. This gives rise to an increase in the induced flow. Furthermore, the velocity near the plate is greater than at the plate. The maximum velocity attains near the plate and is in the neighbourhood of point $\eta = 0.4$. After $\eta > 0.4$, the velocity decreases and tends to zero as $\eta \rightarrow \infty$. Figs. 9 and 10 display the effects of Du (Dufour number) on the velocity and temperature fields for the case Grt> 0 and Grc> 0. It is seen that increase in Dufour number brings about increase in both the velocity and temperature distribution in the flow field, while in Fig.11, it is sown tat concentration distribution increases with an increase in Sr (Soret number) with maximum concentration near the plate for higher value of Sr.

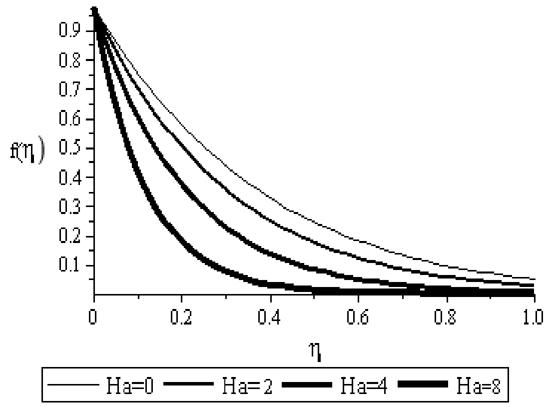

Fig. 6: Velocity distribution for various Hartmann number

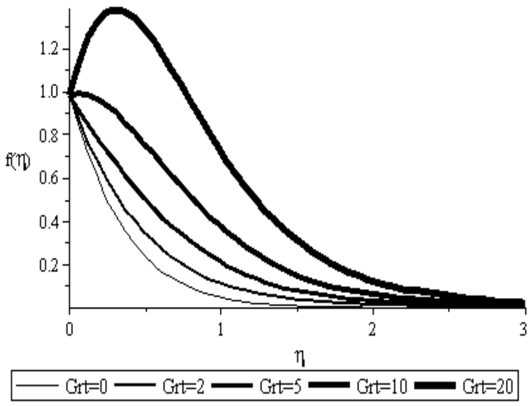

Fig. 7: Velocity distribution for various Thermal Grashof number

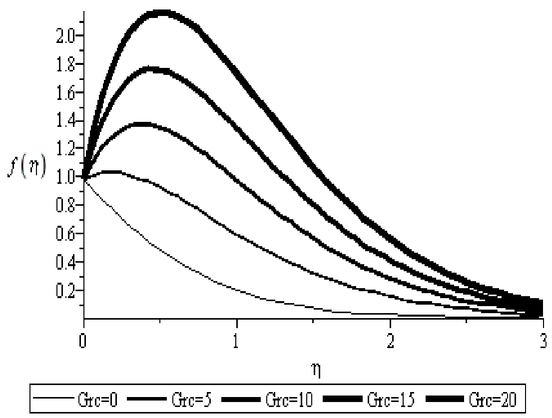

Fig. 8: Velocity distribution for various mass Grashof number

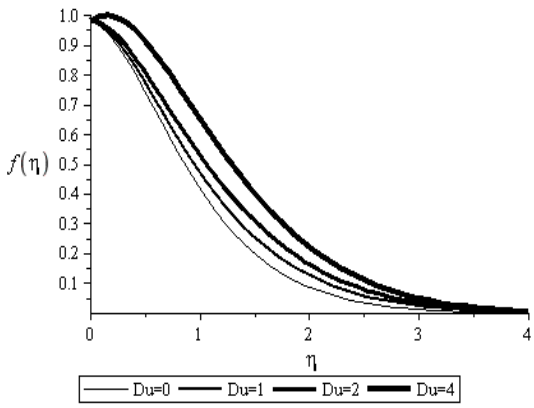

Fig. 9: Velocity distribution for various values of Dufor number

Figs. 12, 13 and 14 depict the effect of internal heat generation/absorption on velocity, temperature and concentration fields respectively. It is observed that internal heat generation increases the velocity, temperature and concentration distribution wile international eat absorption reduces the velocity and temperature distribution. Furthermore the magnitude of temperature is maximum at the plate whereas for internal eat generation the maximum velocity and concentration is near the wall and then decays to zero asymptotically. Effect of chemical reactivity on velocity, temperature and concentration fields are sown in Figs. 15, 16 and 17 respectively. It should be noted here that, $\alpha_1 < 0$ correspond to destructive chemical reaction and $\alpha_1 > 0$

correspond to generative chemical reaction. It is observed that increase in generative chemical reaction increases the velocity and concentration distribution but it reduces the flow temperature distribution. While the reverse is the case for destructive chemical reaction. Furthermore the magnitude of temperature is maximum at the plate whereas for generative chemical reaction, the maximum velocity and concentration is near the wall and then decays to zero asymptotically.

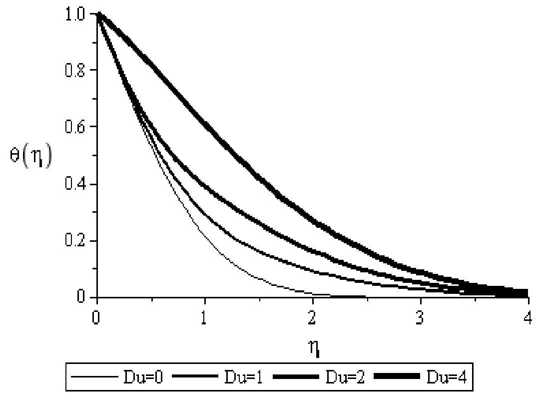

Fig. 10: Temperature distribution for various values of Dufor number

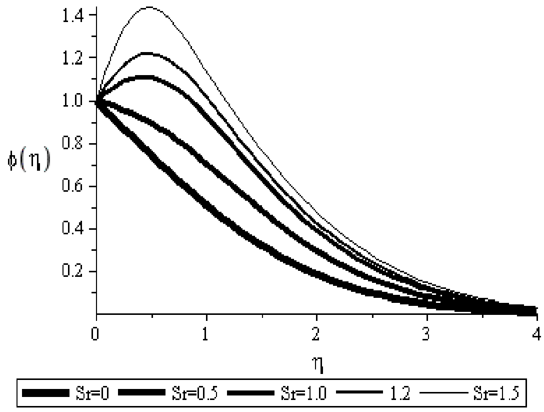

Fig. 11: Concentration distribution for various values of Soret number

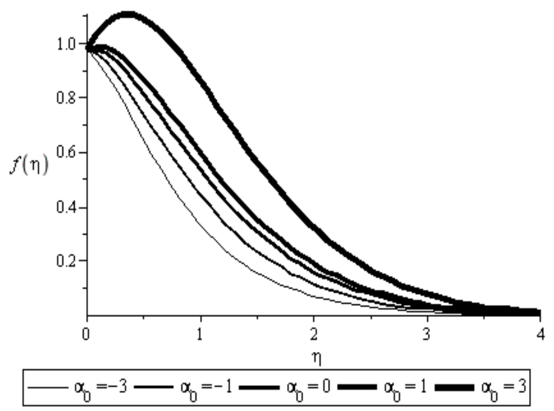

Fig. 12: Velocity distribution for various values of reaction parameter

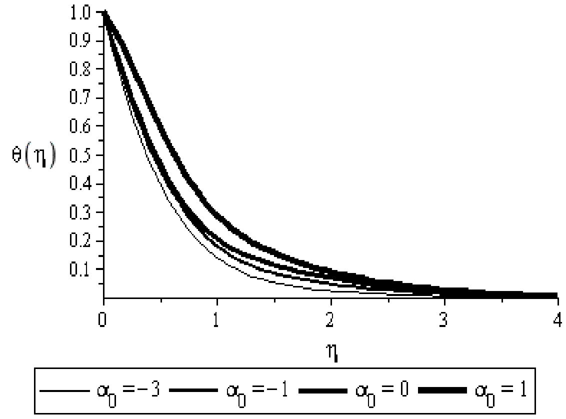

Fig. 13: Temperature distribution for various values of reaction parameter

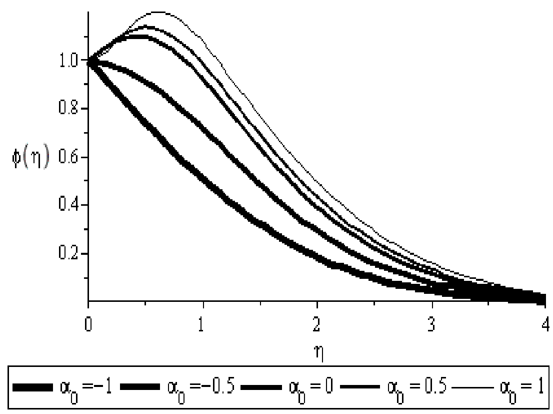

Fig. 14: Concentration distribution for various values of reaction parameter

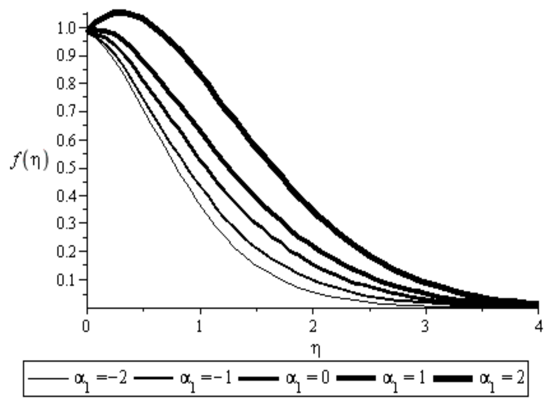

Fig. 15: Velocity distribution for various values of heat parameter

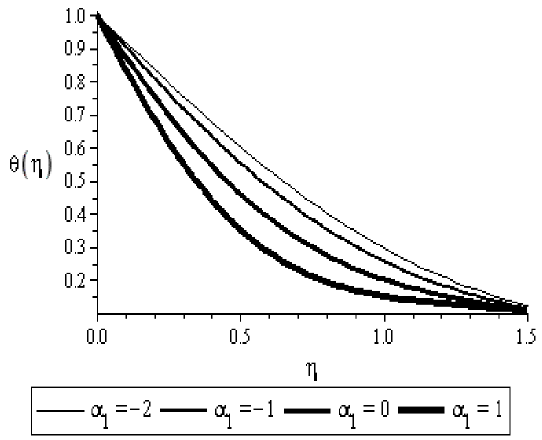

Fig. 16: Temperature distribution for various values of heat parameter

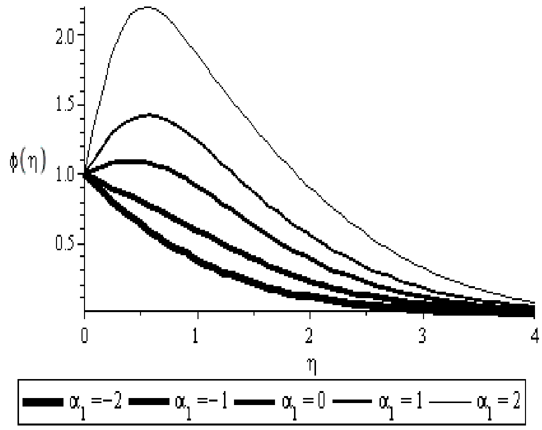

Fig. 17: Concentration distribution for various values of heat parameter

5. Conclusion

In this paper effect of temperature dependent thermal conductivity on MHD free convection flow along a vertical flat plate have been studied numerically. Implicit finite difference method together with Keller box scheme is employed to integrate the equations governing the flow. Comparison with previously published work is performed and excellent argument has been observed. From the present numerical investigation, following conclusions may be drawn:

- For increased value of magnetic parameter, the velocity profile decreases but the temperature profile increases slightly.
- In case of cooling of the plate (Grt> 0), the velocity decreases with an increase in phase angle, injection and magnetic parameter. On the other hand, it increases with an increase in the value of thermal Grashof number and mass Grashof number, suction parameter, Dufour number, internal heat generation and generative chemical reaction.
- The local skin friction coefficient decreases as well as the surface temperature distribution increase with the increase in values of the magnetic parameter.
- The concentration decreases with an increase in injection and increases with increase in Soret number, internal heat generation and generative chemical reaction.
- The temperature decreases with an increase in injection and increases with increase in Dufour number, suction, internal heat generation and generative chemical reaction.

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