

Research Article

A Game Theory Approach for Multi-agent System Resources Allocation against Outside Threats

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*Keywords*Multi-agent system
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This study proposes an integrated model for the deployment of multiagent resources for resisting outside threats. The proposed two-stage model applies the divide-and-conquer strategy to solve the resources allocation problem. First, the interactive actions between an external attack and a response agent are modeled as a non-cooperative game, after which the external threat value is derived from the Nash equilibrium. Second, the threat values of all response agents are utilized to compute each agent's Shapley value. Then an acceptable resource allocation of agents based on their expected marginal contribution creates a minimum set of resource deployment costs. The experimental results show that our approach is feasible as a means to mobilize search and rescue resources from a non-affected district and to improve relief efforts against earthquake damage. The Shapley value allocation approach proposed in this study; the percentage of resources allocation of districts is closer to death rate of each district than the proportional division of resources.

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This is an open access article distributed under the CC BY-NC 4.0 license (<http://creativecommons.org/licenses/by-nc/4.0/>).**1. INTRODUCTION**

Emergency Response Systems (ERS) and Homeland Security Advisory Systems (HSAS) are both centralized Multiagent Systems (MASs) which delegate multiple interacting agents to resist outside attacks. However, the effectiveness of these MASs as a means to defend entire large-scale geographic regions is constrained by the available resources. The system administrator faces a density of agent deployment dilemmas, where the disposition of more agents easily leads to higher costs. These systems also lack specific measures for rational decision-making, and do not apply mathematical models to capture the interactions between attacker and defender. The administrator of a MAS should have a tool to measure the strength of the attacks and the resistance capability of the response agents. By considering the utilities of moves available to the attacker and the defender, we can find a way to build a rating system for decision making [1].

Game theory tools provide analytical techniques that are already applied in many other research areas, where multiple agents compete and interact with each other within a specific system. In most multiple agent interactions, the overall outcome depends on the choices made by all self-interested agents. The goal is to make choices that optimize the outcome [2]. Game theory also provides general mathematical techniques for analyzing goal-conflict and cooperation between two or more individuals [3].

This study surveyed several game theoretic approaches to be applied when external attacks threaten the MAS. These MASs

include ERS, HSAS, and security force reallocation systems. However, there are common problems related to extension and scalability that are often encountered [4]. When these game theory models are played by more than four players (i.e., agents), it is not easy to compute the Nash Equilibrium (N.E.) of the payoff matrix in a multi-dimensional game. Because MASs consist of multiple agents, these computations are complex and therefore pose a challenge for classical game theory. The model uses two steps for solving the growing amount of MAS agent work. A framework is formed for deploying multiple agents using game theory, which analyzes the detected threats (or attacks) and allocates the resources of the MAS as much as efficiently as possible.

The game theory model is appropriate for analyzing the interactions of MAS agents and to deal with the Resource Allocation Problem (RAP) [5,6]. However, when many agents face external attacks, the whole MAS may suffer from limited resources to resist these attacks. The two-stage model applies the divide-and-conquer idea to divide the scalability problem into two parts. First, a non-cooperative game is applied to model the conflict of goals between an attacker and a response agent, after which the external threat value (i.e., the strength of the attacker) is derived from the Nash equilibrium. Second, a cooperative game is used to solve the RAP among MAS agents, in particular, by increasing the number of agents. The threat values of all response agents are utilized to compute each agent's Shapley value. Then an acceptable resource allocation of agents based on the expected marginal contribution creates a minimum set of resource deployment costs.

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The proposed model is applied to reallocate Search-and-Rescue (SAR) resources after a strong earthquake event, to maximize the capacity of the security response teams and minimize the number of emergency responders needed. The experimental results show that the model is feasible as a means of mobilizing SARs from a non-affected district and to improve relief efforts against earthquake damage.

2. MAS RESPONSE REGION

In general, the MAS agent is regarded as a software agent although they could be robots, humans or human teams, or even a combination of human-agent teams. MASs can represent self-organized and complex behaviors even when the individual skills of all their agents are simple [7]. In this study, various MASs including Intrusion Detection System (IDS), HSAS, and ERS are surveyed. The agents possess skills and offer emergency services to protect the MAS from external threats. A response agent can be a node in the IDSs software, a response crisis unit comprised of human teams in HSAS, or agencies which assist in dealing with any emergency, such as a detector, SAR-group, security force agency, etc. Figure 1 shows the interactions of an agent in the MAS under external threats (or attacks). As the external attacker chooses its moves to attack the MAS, the response agent uses their moves to resist the external attack.

This study assumes that the MAS architectures (e.g., ERS or HSAS) will face external threats, rather than internal attacks. Each agent has a possibility of an attack that is an external threat to the MAS. An attack is an event which happens in the MAS. In contrast, a threat is not an actual event. Therefore, we define an external threat as “a possibility of outside attack” created by an attacker who will attack the MAS, or by an unexpected disaster, such as a strong earthquake. In this situation, each agent possesses detection techniques, raises alerts, and improves emergency responses against the external attacks (e.g., large earthquake attacks) (see Figure 1).

Figure 2 shows the five interaction steps among SAR agents faced with external attacks.

- (i) Each agent faces a sudden attack in the external threat environment.
- (ii) The agent’s goal is to resist the external attack. He/she utilizes specific resources, such as fire fighters, rescue-groups, ambulances, equipment and statistical analysis to report and relieve damage. These specific techniques refer to the SAR agent’s capability which controls and handles its resources.
- (iii) Each agent reports to the coordinator (e.g., Emergency Operation Center: EOC) about its own capability and resources as it resists the attack.
- (iv) The only goal in this environment is the optimization of MAS resource allocation. After the coordinator measures the value of a threat to all agents and analyzes its decisions, he/she determines and commands all SAR agents to mobilize and redistribute resources.

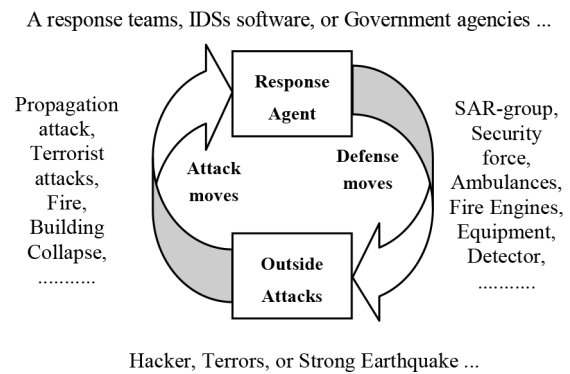


Figure 1 Behaviors and interactions between response agents and external attacks.

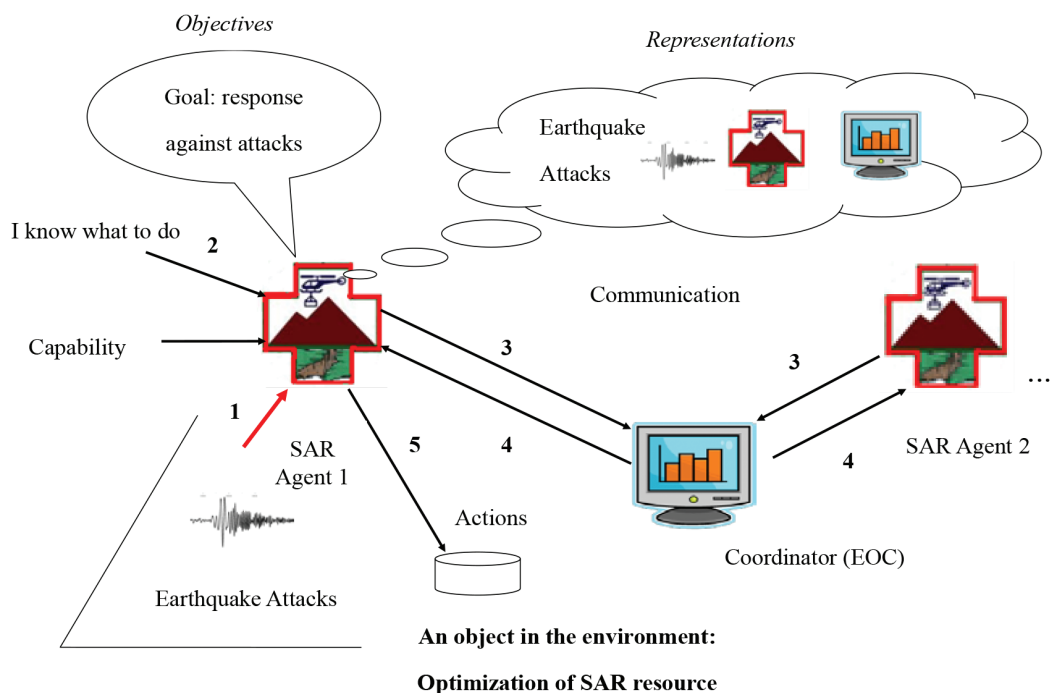


Figure 2 An illustration representing the interactions of an SAR agent with its external attacker and EOC.

- (v) According to the commands of the EOC, each agent begins to employ his/her own resources or make them available for someone else. In other words, some agents' resources are redistributed to the critical agents so as to achieve the goal of resisting serious attacks and discarding non-affected agents.

3. RELATED WORKS

Several game theory approaches have been applied in which external attacks threaten a MAS which could include ERS, HSAS, and security force reallocation systems. Various approaches have been applied to model the interactions between attackers and defenders for computer network intrusion detection. Burke [8] proposed a two-player game framework to model the information warfare between two players, an attacker and a system administrator. In his model, the mixed strategy equilibrium was used to construct a player's optimal solution. The strategies and scenarios of the model are simple extensions of a few simulations to analyze how it would behave in real situations.

Other approaches also have been applied to model the interactions between terrorists and defenders for counter-terrorism purposes. Paruchuri et al. [9] deployed a software assistant agent that aided police or other security agencies in randomizing their security schedules. They improved the security capability of anti-terrorism forces when dealing with multiple agents posing an intentional threat. The model maps the problem of check-point scheduling as a Bayesian Stackelberg game. The proposed model is faster than the multiple-LP method because it simply solves the mixed-strategy N.E. However, Bayesian concepts do not fully apply in real situations, and the model did not consider incomplete information regarding the type of adversary. The solution of the Bayesian N.E. is an NP computation. Their model also encountered the many-players problem, which involves the extension and scalability of the proposed model.

Alpcan and Basar [10] proposed the utilization of cooperative and non-cooperative game theory concepts to address some of the basic network security tradeoffs. They constructed a sensor warning system and a security game for making various decisions, analysis and, IDS control schemes. The sensor warning system generates a security risk value for the IDS agent (i.e., sensor). This value could have various levels and the calculation is based on simple detection output. However, the strategy between the IDS and the attacker was incomplete, because it did not consider the propagation attacks in a game, therefore their model could not satisfy the decision-making requirements for IDS control. Their proposed model consists of two independent schemes. Our study combines their proposed non-cooperative game with a cooperative game to solve the RAP in a MAS.

3.1. Why Game Theory?

Gupta and Ranganathan [11] surveyed several optimization approaches such as Genetic Algorithms (GA), the Bayesian Search (BS) method, as well as Random Search (RS) and Greedy Allocation (GS) methodologies. These algorithms cannot be applied to the

multi-crisis management problem because they do not provide individual rationality and some of the members of the population become dominant as the algorithm progresses. However, using game theory, scenarios that optimize multiple competing objectives can be modeled. Game theory presents interactions between self-interested agents and analysis as to which strategies can be designed that will maximize the benefits of an agent in a MAS. Many of the applications of game theory have been to analyze the negotiation and coordination of multiple agents. Game theory can be a useful tool for building future generations of mixed game theory and decision theory agents [3].

In a non-cooperative game, each player tries to utilize resources at minimum cost and the coordination is not enforced externally but is self-enforcing. All players optimize their decisions which maximize their payoffs in a non-cooperative game. N.E. is a solution concept for a non-cooperative game which identifies a prediction of the game outcome such that every player in the game is satisfied with respect to every other player [12]. N.E. provides a theoretical prediction of attack in a conflict situation, where individual MAS agents suffer from external attacks. The interactions between an external attacker and a response agent can be modeled as a non-cooperative game.

The Shapley value is a solution concept for cooperative games which computes the power index of an individual for cost allocation [13]. The cooperative game provides a suitable model for the design and analysis of response agent deployment, and it has been shown that the famous Shapley value rule satisfies many nice fairness properties [14]. The Shapley value also identifies a socially fair, good quality allocation for all agents. Here, the individual fairness for each player is optimal and the average fairness of the MAS is high. The social optimality property ensures that each player in the game receives the best utility for herself/himself and the complete MAS. A power index in the form of the Shapley value is applied to calculate the marginal contribution among agents and achieve a mutually agreeable division of cost for MAS deployment [2].

4. THE PROPOSED MODEL

There are two schemes applied in the proposed model. A simplified workflow chart describing the principles for optimal agent deployment is given in Figure 3. In the first scheme, the interaction processes between the attacker (or attacks) and the defensive agent are modeled as a two-person non-cooperative game (zero- or general-sum game), taking into account the interactions of security measures among agents and attackers. The proposed payoff functions utilize the external threat measures (such as defensive or offensive capability) for two players. The N.E. derived from these functions is calculated to assign a unique external threat value for the agent. Then, the second scheme constructs a resource allocation game. The power index of the Shapley value is applied to calculate the marginal contribution among MAS agents, to find a mutually agreeable division of cost for deployment. This study revises Owen's method [15] to propose a Shapley value formula. MAS agents are grouped into coalition groups based on the threat levels to provide an acceptable MAS deployment. The Shapley value can be used to assure the derivation of a unique solution for each agent in the MAS

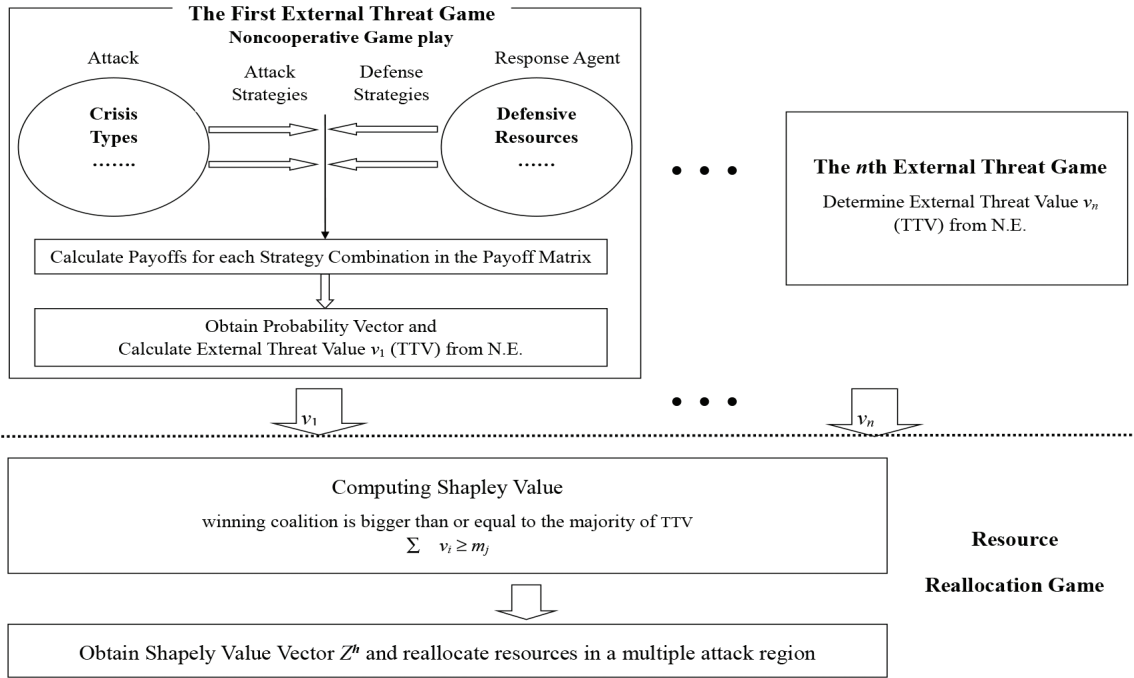


Figure 3 | Flowchart of the proposed model.

resource sharing interactions. Let $Z^h = \{\omega_1^h, \omega_2^h, \omega_3^h, \dots, \omega_i^h\}$ be the minimum set of response agents, $\forall_i \in N$, and N is the number of agents, which is subject to the threat level h . Finally, we obtain the appropriate Shapley value vector and compute the number of reallocation resources of each agent. These optimal reallocation resources of the agents in the emergency response enable us to mitigate the damage from external attacks.

4.1. The External Threat Game

The first game models the interactions of an attacker (or attack) and a response agent. When an agent confronts an external attacker, both face a competitive situation. The relationships between external attacker and response agents include pure and impure conflicts, and their behaviors are non-cooperative. Thus, in the first stage, a non-cooperative game is applied to model decisions and to analyze the best responses. The two-player non-cooperative game is defined as follows:

- (i) The set of players A : The first model only has two players. The attacker is player 1 and the response agent is player 2. $A = \{\text{Attacker, Response agent}\}$.
- (ii) Strategy space S_i : A set of all possible strategies available to two players in a game.
- (iii) Payoff π_i : The expected utility to a player as a function of the strategies chosen by an attacker or response agent.

Let $G = \langle A, (S_i), (\pi_i) \rangle$ be such a normal form game. Sometimes this game cannot determine a pure N.E. strategy, because there is probably no N.E. However, every finite normal form game has a mixed-strategy N.E [16]. Thus, this study can also derive another strategic game from G , called the “mixed extension” of G , in which

the set of actions A_i of each player i is the set of mixed strategies in G . More generally, suppose that player i has K pure strategies: $S_i = \{s_{i1}, \dots, s_{iK}\}$. Then, a mixed strategy for player i has a probability distribution $(p_{i1}, p_{i2}, \dots, p_{iK})$, where p_{iK} is the probability that player i will play s_{iK} , for $k = 1, \dots, K$. Since p_{iK} is a probability, this study requires $0 \leq p_{iK} \leq 1$ for $k = 1, \dots, K$ and $p_{i1} + \dots + p_{iK} = 1$, where p_i is used to denote an arbitrarily mixed strategy from the set of probability distributions over S_i , just as s_i is used to denote an arbitrary pure strategy from S_i .

Let J denote the number of pure strategies in S_1 and K indicate the number in S_2 . Now, $S_1 = \{u_1, u_2, \dots, u_j\}$ and $S_2 = \{d_1, d_2, \dots, d_k\}$ are rewritten using u_j and d_k to denote pure arbitrary strategies from S_1 and S_2 , respectively. A $J \times K$ payoff matrix is created for the external threat game based on the two players’ strategies and interactions (see Table 1).

If player 1 believes that player 2 will play the strategies $\{d_1, d_2, \dots, d_k\}$ with the probability $q = (q(d_1), q(d_2), \dots, q(d_k))$ then player 1’s expected payoff from playing the pure strategy u_j is [Equation (1)]

$$\sum_{k=1}^K q(d_k) \pi_1(u_j, d_k) \tag{1}$$

and player 1’s expected payoff from playing the mixed strategy $r = (r(u_1), r(u_2), \dots, r(u_j))$ is [Equation (2)]

$$\begin{aligned} M_1(r, q) &= \sum_{j=1}^J r(u_j) \left[\sum_{k=1}^K q(d_k) \pi_1(u_j, d_k) \right] \\ &= \sum_{j=1}^J \sum_{k=1}^K r(u_j) q(d_k) \pi_1(u_j, d_k) \end{aligned} \tag{2}$$

where $r(u_j), q(d_k)$ is the probability that player 1 plays u_j and player 2 plays d_k . Thus, $0 \leq r(u_j), q(d_k) \leq 1$ for $k = 1, \dots, K$ and $j = 1, \dots, J$; $r(u_1) + r(u_2) + \dots + r(u_j) = 1$ and $q(d_1) + q(d_2) + \dots + q(d_k) = 1$.

Table 1 | A payoff matrix for the external threat game

		Response agent				
		d_1	d_2	...	d_K	r -mix
Attacker	u_1	$\pi_1(u_1, d_1), \pi_2(u_1, d_1)$	$\pi_1(u_1, d_2), \pi_2(u_1, d_2)$...	$\pi_1(u_1, d_K), \pi_2(u_1, d_K)$	$r(u_1)$
	u_2	$\pi_1(u_2, d_1), \pi_2(u_2, d_1)$	$\pi_1(u_2, d_2), \pi_2(u_2, d_2)$...	$\pi_1(u_2, d_K), \pi_2(u_2, d_K)$	$r(u_2)$

	u_j	$\pi_1(u_j, d_1), \pi_2(u_j, d_1)$	$\pi_1(u_j, d_2), \pi_2(u_j, d_2)$...	$\pi_1(u_j, d_K), \pi_2(u_j, d_K)$	$r(u_j)$
	q -mix	$q(d_1)$	$q(d_2)$...	$q(d_K)$	

Player 1’s expected payoff from the mixed strategy r , given in Equation (2), is the weighted sum of the expected payoff for each of the pure strategies $\{u_1, u_2, \dots, u_j\}$, given in Equation (1), where the weights are the probabilities $r(u_1), r(u_2), \dots, r(u_j)$. Thus, for the mixed strategy $r(u_1), r(u_2), \dots, r(u_j)$ is the best response for player 1 and 2’s mixed strategy $q(d_k)$, and it must be that $r(u_i) > 0$ only if [Equation (3)]

$$\sum_{k=1}^K q(d_k)\pi_1(u_j, d_k) \geq \sum_{k=1}^K q(d_k)\pi_1(u_j, d_k) \quad (3)$$

for every u_j in S_1 . That is, for a mixed strategy to be the best response to q it must put a positive probability on a given pure strategy only if the pure strategy is itself the best response to q . Conversely, if player 1 has several pure strategies that are best responses to q , then any mixed strategy that puts all its probability on some or all of these pure-strategy best responses (and zero probability on all other pure strategies) is also the best response for player 2 to q [17].

This study computes player 2’s expected payoff when players 1 and 2 play the mixed strategies r and q , respectively. If player 2 believes that player 1 will play the strategies (u_1, u_2, \dots, u_j) with the probabilities $(r(u_1), r(u_2), \dots, r(u_j))$, then player 2’s expected payoff from playing the strategies (d_1, d_2, \dots, d_K) with the probabilities $(q(d_1), q(d_2), \dots, q(d_K))$ is [Equation (4)]

$$\begin{aligned} M_2(r, q) &= \sum_{j=1}^J r(u_j) \left[\sum_{k=1}^K q(d_k)\pi_2(u_j, d_k) \right] \\ &= \sum_{j=1}^J \sum_{k=1}^K r(u_j)q(d_k)\pi_2(u_j, d_k) \end{aligned} \quad (4)$$

Given $M_1(r, q)$ and $M_2(r, q)$ this study restates the N.E. requirement that each player’s mixed strategy is the best response to the other player’s mixed strategy: for the pair of mixed strategies (r^*, q^*) to be a N.E., r^* must satisfy [Equation (5)]

$$M_1(r^*, q^*) \geq M_2(r^*, q) \quad (5)$$

for every probability distribution r over S_1 , and q^* must satisfy [Equation (6)]

$$M_2(r^*, q^*) \geq M_2(r^*, q) \quad (6)$$

for every probability distribution q over S_2 , where r^* and q^* represent the optimal mixed strategy for the attacker and response agent respectively. A mixed strategy N.E. is similar to a stochastic state that predicts the outcome of a game and it can capture stochastic regularity. The attacker and agent have information about their payoffs from actions which were taken in the past; each player applies

these payoffs to form his belief about the future behaviour of the other players, and thereby formulate his optimal mixed strategy.

In Table 1, player 2’s expected payoff is computed when players 1 and 2 play mixed strategies r and q respectively. If the game G has no pure strategy N.E., a mixed N.E. pair (r^*, q^*) exists in this game, which is an optimal strategy [18]. The mixed N.E. for the probability vector is $r^* = \{r^*(u_1), r^*(u_2), \dots, r^*(u_j)\}$ with actions $\{u_1, u_2, \dots, u_j\}$ for the external attacker and the vector $q^* = \{q^*(d_1), q^*(d_2), \dots, q^*(d_K)\}$ with actions $\{d_1, d_2, \dots, d_K\}$ for the response agent. The external attacker’s expected payoff for a N.E. (pure or mixed strategy) is defined as its external threat value. Player 1 (i.e., the external attacker) obtains the positive payoff (+), because player 1 gains a profit from player 2’s resource responses. Player 2 (i.e., the response agent) obtains a negative payoff (-), which means that player 2 pays for player 1’s attack. When the response agent (i.e., player 2) gets a low expected payoff, the external attacker (i.e., player 1) gets a high expected payoff and the threat of external attack is high. Because player 1’s expected payoff always represents a gain (+), this study defines the v_i as the i th response agent’s threat value, given by [Equation (7)]

$$v_i = \pi_1(p^*, q^*) = \sum_{j=1}^J \sum_{k=1}^K p^*(u_j)q^*(d_k)\pi_1(u_j, d_k), u_j, d_k \in \text{N.E.} \quad (7)$$

Therefore, v_i is derived from the attacker optimal strategies’ expected payoff which represents the threat value in the first game model. The next proposed model uses the threat value v_i to compute the Shapley value of each agent within the cooperative game.

4.2. Resource Reallocation Game

In this study, it is assumed that a central planner of the Security of Center (SOC) or Emergency Operation Center (EOC) can command and control MAS resource allocation [19]. Each agent reports its degree of threat (or threat value) to the central planner. The central planner computes the Shapley value based on the threat values of all agents in order to command agents to dispatch resources. In this second game, the interactions of all response agents in the MAS response region are likened to the playing of a cooperative game. An efficient method is needed to decide the number and priority of the deployment of resources to various response events when multiple attacks occur simultaneously. The Shapley value is applied to create an optimal allocation of resources for the SOC commander during multiple attacks. We utilize the concept of the majority coalition in a party voting game to deal with MAS resource reallocation. A majority of voters can pass any bill in the majority game. The power of a voter will depend on how crucial that voter is to the formation of a winning coalition [15]. The Shapley value can

provide a measure of the power of each voter, which is represented by the power index. When the sum of external threat values of some attacks passes the threshold of the majority level, the formation of a winning coalition is enabled, and the power index of each attack can be computed.

We define $y: V \rightarrow R^+$ as a one-to-one function by assigning a positive real number to each element of v (i.e., external threat value) and $y(0) = 0, V = \{v_1, v_2, \dots, v_i\}, i \in n$. The majority of the attack level h_H is derived from a majority of all external threat values which represents the corresponding threshold value m_H . The allocation of the response agent resources is based on the concept of the majority of external threat values. Given the output vector of all external threat values of attacked events, the majority level is h_H if the sum of the external threat values of attacks is greater than or equal to m_H [Equation (8)]

$$h_H \text{ if } \sum_{k=1}^n v_k \geq m_H, m_H = v_{\text{Min}} + \left(\frac{v_{\text{Max}} - v_{\text{Min}}}{2} \right) \quad (8)$$

The external threat values of attacks can be grouped into the majority level h_H . This is divided by 2 from the maximum value v_{Max} to minimum value v_{Min} . All response agents can be modelled as an N -person game with $X = \{1, 2, \dots, N\}$, which includes a set of players (i.e., response agents) and each subset $V \subset X$, and where $v_j \neq 0, \forall j \in V$ is called a coalition [2]. The coalition of X response agent groups in the m th threshold of the threat level, and each subset of X (coalition), represents the observed threat pattern for different threat levels H . The aggregate value of the coalition is defined as the sum of the threat values of the response agent, $y(C) = \sum_{i \in C} v_i$ and is called a coalition function. Each response agent coincides with one or another given m thresholds of the threat level. Therefore, the different priorities for the response agent deployment are derived from the thresholds. Based on the external threat for each MAS response agent with respect to others, and the effect of the threshold values on various threat levels, the Shapley value represents the relative importance of each response agent. Now let $y(C) = \sum_{i \in C} v_i, v_i \in V, C \subset X$ be the value of coalition C with a cardinality of c . The Shapley value of the i th element of the response agent vector is defined by [Equations (9) and (10)]

$$\omega(i) = \sum_{C' \subset X} \frac{(c-1)!(n-c)!}{n!} [y(C) - y(C - \{i\})] \quad (9)$$

$$\Rightarrow \omega(i) = \sum_{C' \subset X} \frac{(c-1)!(n-c)!}{n!} \quad (10)$$

Equation (9) can be simplified to Equation (10), because the term $y(C) - y(C - \{i\})$ will always have a value of 0 or 1, taking the value 1 whenever C' is a winning coalition. If it is not a winning coalition, the terms $C - \{i\}$ and $y(C)$ are 0 [15]. Hence, the Shapley value is $\omega(i)$, where C' denotes the winning coalitions with $\sum_{i \in C'} v_i \geq m_i$. The Shapley value of the i th response agent output indicates the relative threat value for the thresholds m_H (i.e., threat levels). Therefore, a Shapley value vector shows the strength of all external attacks. We can use all Shapley values to compute the reallocation amount of emergency response resources of the agent. The numbers of resources of type k reallocated to the i th agent are defined by [Equation (11)]

$$e_k(i) = \omega(i) \times o_{\text{all},k}, i, k \in N \quad (11)$$

where $o_{\text{all},k}$ is the total number of resource available of type k in a MAS (such as ambulances when an earthquake occurs). The reallocated numbers of the i th agent's resources $e_k(i)$ are derived from the Shapley value of the i th response agent $\omega(i)$ multiplied by the total number of resource available of type $k, o_{\text{all},k}$. Finally, the commander can reallocate any kind of agents' resources so as to fairly deploy resources to resist attacks in multiple attack regions.

5. SAR RESPONSE CASE

The September 21, 2009 earthquake claimed 2297 fatalities, according to the National Fire Agency and Ministry of the Interior of Taiwan. Almost 90% of these victims died within 1 day of the event and 56% died within 6 hours (nearly 1284 victims) [20]. As we know, the key point to minimizing the total number of fatalities after an earthquake is the deployment of SAR efforts in the first few hours. In an emergency response region, there are usually a number of alarm districts-each equipped with resources that must be able to be deployed quickly and effectively after any kind of emergency. However, the SARs (e.g., fire fighters, rescue-groups, ambulances, and equipment) available to some affected districts may be limited.

A devastating earthquake brings with its multiple disasters such as fire, building collapse and a combination of fire and collapse. In cases of multiple emergencies, each affected district might possess different response capabilities. Some form of the central emergency management system must be present in order to allocate response resources. This study focuses on the problem faced by the emergency manager: which affected districts are the major ones and which ones are minor? Which affected districts require first aid and to need more SAR resources? This is a deployment efficiency problem. An emergency manager must find an optimal resource allocation for assigning response resources in space and time to the affected districts and improve the relief efforts against earthquake attacks.

We apply the proposed model to solve the SAR resources allocation problem. Two games are constructed, representing the two stages needed for the economical deployment of available resources. One is the earthquake damage game which calculates the value of the earthquake threat in each district. Another is the SAR resource reallocation game which efficiently mobilizes the available resources of the response agent to increase the survival rate of earthquake victims.

5.1. The Model Assumptions

Certain assumptions must be made when treating a multi-emergency and response management scenario as a game theory framework. The assumptions are listed below.

- (i) After the deadliest quakes, emergencies will occur in different districts in a time-overlapped manner, and resource requests will be simultaneously received by the central response agent managers (see Table 2). We assume the multi-emergency event is player 1 and the district response agent is player 2. Hence, two players can use a static single step. Each player plays a non-cooperative and a zero-sum game. A response agent manages one district. Three types of earthquake emergencies (i.e., fire, house collapse, and fire and house collapse) could

occur simultaneously when evaluating the value of the earthquake threat in a specific time period.

- (ii) The EOC has developed a training program to improve the emergency response agents' capabilities. As seen in Table 3, the annual emergency training gives us the time needed to perform search and rescue activities from each of the resource sites. These data are generated by the training program through annual earthquake drills or exercises.
- (iii) The first model is a normal form game for capturing the interactions between two players. This model assumes complete information; that is, every player knows the available strategies of every other player. Each district has a different number of resources available. In multiple emergencies, there is also complete information regarding the resource requirements, emergency priorities and the population density of the affected district (see Tables 2 and 4).

Table 2 | Emergency priorities, types, and the ratio of four resource requests

	Rescue groups	Ambulances	Fire engines	Machine and equipments	Priority (1–3)
Fire	0	1	2	0	1
Collapse	3	2	0	0.3	2
Fire and collapse	3	3	2	0.5	3

Table 3 | Average times needed to perform search and rescue activities from the resource sites in *i*th district

	Rescue groups	Ambulances	Fire engines	Machine and equipments
Fire	5	5	5	5
Collapse	5	5	5	5
Fire and collapse	10	10	10	10
Total	20	20	20	20

These average times (or min) are measured by the annual earthquake exercise.

5.2. Earthquake Damage Game

This game applied the first scheme of the proposed model, which is designed to obtain the threat value of the earthquake in each district. We assume that the multi-emergency event is player 1 and the district response agent is player 2. The interactions between multi-emergency event and response agent are modeled by a two-person game. It is assumed that both players are forming a set of non-cooperative players $I = \{I_1, I_2\}$, where I_1 generates multiple emergency events; and I_2 is the district response agent responding to allocate finite SAR resources, obtained by prioritizing the different districts. The parameters for determining the threat measures are defined in the following paragraphs.

5.2.1. Multi-emergency events

In the earthquake damage game, we assume the multi-emergency event to be player 1; three emergencies could happen simultaneously as a result of the player's actions. S_1 denotes the set of strategies available to player 1: $S_1 = \{u_1, u_2, u_3\} = \{\text{fire, house collapse, and fire and house collapse}\}$. The greater the seriousness of the emergency events, the more resources are needed. W denotes the set of resource requirements for multiple emergencies in one urban region. $W = \{w_1, w_2, \dots, w_n\}$. The variable $w_{j,k}$ denotes the number of resources required by u_j emergency event from resource d_k . Moreover, the priority for each emergency is related to the resource requirement. Typically, in a priority-based system, a high priority emergency needs more resources, and a lower priority emergency needs fewer resources. Each event is assigned a priority P on a scale of 1–3 indicating the severity of the emergency, which is used as a weight in the payoff function to facilitate the calculation of losses. p_j denotes the priority of the j th emergency: $p_j = 1, 2, 3$, where 1 is the lowest level and 3 is the highest. An emergency event on a densely populated target would increase the number of fatalities and thus would gain greater benefit for player 1, while the response agent must pay a higher rescue cost. β_i denotes the population density of the i th district in the whole region.

Table 4 | Population density and resource available in 15 disaster districts

Districts	The number of available resources				
	Population density	Rescue workers	Ambulances	Fire engines	Machine and equipments
A1 TAIPEI	94	350	250	600	100
A2 TAOYUAN	100	100	100	200	60
A3 HSINCHU	37	50	60	100	50
A4 MIAOLI	19	25	40	50	45
A5 TAICHUNG	73	125	100	250	65
A6 CHANGHUA	74	75	50	150	65
A7 NANTOU*	8	25	50	50	45
A8 YUNLIN	34	25	50	50	45
A9 CHIAYI	25	30	60	70	47
A10 TAINAN	52	80	80	180	60
A11 KAOHSIUNG	57	125	200	250	65
A12 PINGTUNG	19	50	40	100	50
A13 TAITUNG	4	20	20	30	44
A14 HUALIEN	4	20	20	30	44
A15 YILAN	13	25	20	40	45
Total	613	1125	1140	2150	830

*A strong earthquake occurred.

5.2.2. Response agent

Each alarm district has different response agents, which in turn have varied resources to prepare for multiple emergencies. In the earthquake damage game, the response agent is player 2, who holds four resources (i.e., four strategies). S_2 denotes the sets of strategies player 2 has access to: $S_2 = \{d_1, d_2, d_3, d_4\} = \{\text{rescue worker, ambulance, fire engine, and machine and equipment}\}$. O denotes the set of resources available to response agents in one district: $O = \{o_1, o_2, o_3, o_4\}$. o_k denotes the number of resources available at resource d_k , and $t_{j,k}$ denotes the time for resource d_k to reach the u_j emergency; $k \in 1, 2, 3, 4$. We assume these times are measured based on the annual emergency drill or exercise.

Two players will simultaneously make their strategic decisions. A 3×4 payoff matrix for the earthquake damage game is created based on two players' strategies and interactions as seen in Table 5. The payoff to player 1 for choosing a particular strategy when player 2 makes his selection can be represented as a gain by player 1 (+) or a loss for player 2 (-). In this model, a summation of the losses of player 2 is depicted, and player 1 tries to maximize this loss. Player 2 tries to minimize the losses. Player 1 gains a profit from player 2's effort of resource responses. Player 2 pays as a result of player 1's multi-emergency events. We assume that each player's aim in the game is to achieve as high a payoff for player as possible.

The payoff for the j th strategy for player 1 when player 2 chooses the k th strategy to the response can be formulated as [Equation (12)]

$$\pi_1 = \sum_{j=1}^3 \sum_{k=1}^4 \left(\frac{w_{i,k}}{o_{\text{all},k} - w_{j,k}} \right) t_{j,k} \times \beta_i \times p_j \quad (12)$$

where $o_{\text{all},k}$ is the total number of SAR resources of type k ; $w_{i,k}$ indicates the total number SAR resources needed of type k by the i th district. The proposed model assumes that the non-cooperative game is a zero-sum game; therefore, the payoff function of player 2 is given by [Equation (13)]

$$\pi_2 = - \sum_{j=1}^3 \sum_{k=1}^4 \left(\frac{w_{i,k}}{o_{\text{all},k} - w_{j,k}} \right) t_{j,k} \times \beta_i \times p_j \quad (13)$$

A mixed Nash equilibrium pair (r^*, q^*) exists in the normal form game if this game has no pure strategy Nash equilibrium, which is an optimal strategy. The mixed N.E. for the probability vector is $r^* = \{r^*(u_1), r^*(u_2), r^*(u_3)\}$ with actions $\{u_1, u_2, u_3\}$ for the Multi-emergency event and the vector $q^* = \{q^*(d_1), q^*(d_2), q^*(d_3), q^*(d_4)\}$ with actions

$\{d_1, d_2, d_3, d_4\}$ for the response agent. Now, the proposed model lets v_i be the i th threat value of a strong earthquake in a district which is the expected payoff of player 1. v_i is computed by Equation (7).

$$v_i = \pi_1(p^*, q^*) = \sum_{j=1}^3 \sum_{k=1}^4 p^*(u_j) q^*(d_k) \pi_1(u_j^*, d_k^*), u_j^*, d_k^* \in \text{N.E.} \quad (14)$$

5.3. Reallocation of SAR Resources Game

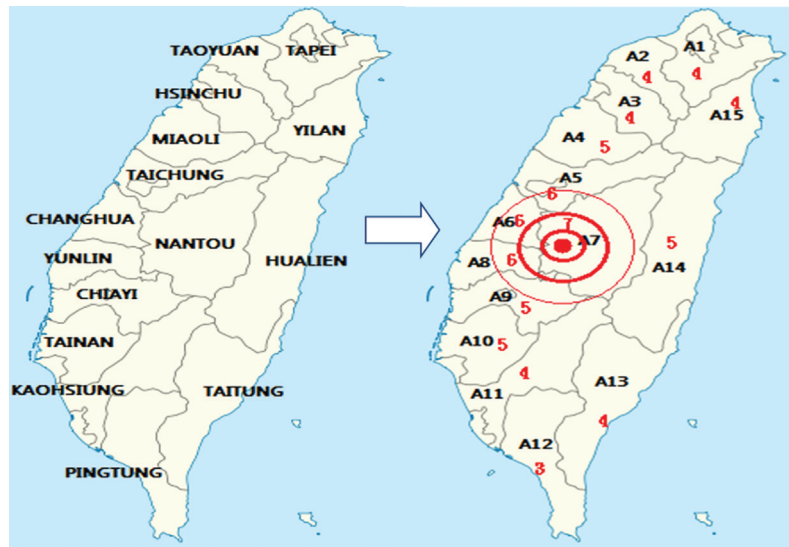
This game applies the second scheme of the proposed models as explained in Section 4.2. We assume that the 15 SAR districts of Taiwan are modeled as a 15-person cooperative game (see Figure 4). The threshold value m_H is generated by Equation (8). Given the m_H threshold of the majority value, we can compute the majority of coalitions which are pivotal to each district. The different priorities for response resource deployment are derived from the majority of coalitions. Based on the earthquake threat value for each district with respect to the others, and the impact of the threshold values on the majority of emergency level represents the relative importance of each district. Equations (9) and (10) are used to compute the Shapley value for each agent and the amount of reallocation of each agent's resources in Section 4.2. According to SAR resource reallocation, the EOC mobilizes resources from all districts to assure that even rescue of victims in multiple emergency events across the whole region is possible.

5.4. Simulation Experiments

The simulation takes advantage of the 921-earthquake data to verify the proposed model. We hypothesize that a strong earthquake occurs as it did in Jiji, Nantou County, Taiwan. The EOC deploys 15 response agents from the district (or county) in the whole region (from A1 to A15) and provides central management and monitoring for homeland security of Taiwan (see Figure 4). In the first stage, the information in Tables 2–4 is used, regarding the resource types and the availability, population density, emergency requests, emergency priorities, and time taken to reach the crisis area from each of the resource centers. Each district's emergency resource request is derived from the number of emergency calls (i.e., 119) multiplied by the ratio of the resource requests (see Tables 2 and 6). We model the non-cooperative game and generated simulation sets of threat measures for the strong earthquake and the response agent in each district. The payoff matrix of the zero-sum earthquake damage game for each district is modeled according to Equations (12)

Table 5 | A payoff matrix for the external threat game

		Response agent			
		d_1 (Rescue groups)	d_2 (Ambulances)	d_3 (Fire engines)	d_4 (Machine and equipments)
Multi-emergency events	u_1 (Fire)	$\left(\frac{w_{i,1}}{o_{\text{all},1} - w_{1,1}} \right) t_{1,1} \beta_i p_1$	$\left(\frac{w_{i,2}}{o_{\text{all},2} - w_{1,2}} \right) t_{1,2} \beta_i p_1$	$\left(\frac{w_{i,3}}{o_{\text{all},3} - w_{1,3}} \right) t_{1,3} \beta_i p_1$	$\left(\frac{w_{i,4}}{o_{\text{all},4} - w_{1,4}} \right) t_{1,4} \beta_i p_1$
	u_2 (Collapse)	$\left(\frac{w_{i,1}}{o_{\text{all},1} - w_{2,1}} \right) t_{2,1} \beta_i p_2$	$\left(\frac{w_{i,2}}{o_{\text{all},2} - w_{2,2}} \right) t_{2,2} \beta_i p_2$	$\left(\frac{w_{i,3}}{o_{\text{all},3} - w_{2,3}} \right) t_{2,3} \beta_i p_2$	$\left(\frac{w_{i,4}}{o_{\text{all},4} - w_{2,4}} \right) t_{2,4} \beta_i p_2$
	u_3 (Fire and collapse)	$\left(\frac{w_{i,1}}{o_{\text{all},1} - w_{3,1}} \right) t_{3,1} \beta_i p_3$	$\left(\frac{w_{i,2}}{o_{\text{all},2} - w_{3,2}} \right) t_{3,2} \beta_i p_2$	$\left(\frac{w_{i,3}}{o_{\text{all},3} - w_{3,3}} \right) t_{3,3} \beta_i p_3$	$\left(\frac{w_{i,4}}{o_{\text{all},4} - w_{3,4}} \right) t_{3,4} \beta_i p_3$



Epicenter is in the Nantou County and the Arabic numerals represent the surface wave magnitude.

Figure 4 | 15 Search and rescue (SAR) districts of Taiwan.

Table 6 | Fifteen examples for computing earthquake threat values and Shapley values

Districts	Number of emergency calls			Requirements				Earthquake threat values	Shapley values
	Fire	Collapse	Fire and collapse	Rescue groups	Ambulances	Fire engines	Machine and equipments		
A1 TAIPEI	5	50	40	275	225	180	46	160	0.086353
A2 TAOYUAN	5	10	10	65	55	50	11	80	0.036489
A3 HSINCHU	8	16	20	116	100	92	19.6	53	0.022389
A4 MIAOLI	8	20	30	158	138	126	26.6	37	0.015335
A5 TAICHUNG	30	200	200	1230	1030	860	206	618	0.334121
A6 CHANGHUA	5	150	150	605	505	410	101	89	0.040354
A7 NANTOU*	20	300	350	1990	1690	1430	333	610	0.334121
A8 YUNLIN	4	100	100	604	504	408	50.4	129	0.062895
A9 CHIAYI	10	10	20	200	170	160	17	31	0.013536
A10 TAINAN	8	8	16	212	196	188	19.6	75	0.034316
A11 KAOHSIUNG	4	4	8	40	36	36	3.6	15	0.005728
A12 PINGTUNG	2	2	20	68	66	66	6.6	23	0.011142
A13 TAITUNG	2	2	10	38	36	36	3.6	3	0.001931
A14 HUALIEN	2	2	10	38	36	36	3.6	3	0.001931
A15 YILAN	5	5	5	35	30	30	3	4	0.00129
Total	118	879	989					2229	1

*Epicenter.

and (13) and Table 5. Then, 15 earthquake threat values are calculated using Equations (12–14). Table 6 shows 15 earthquake threat values of the response agent derived from Nash equilibrium points by enumerating the strategies in the normal form game.

In the second stage, all earthquake threat values are utilized to compute the Shapley value for each district. Given the vector output for all the values, we use Equation (8) to compute the majority of the threat values; the threshold value (i.e., m_H) is 310. According to this threshold value 15 Shapley values of the district are calculated using Equation (10). The results obtained are shown in Table 6. Assume that all SAR resources available are unable to satisfy the requirements for all the earthquake-related emergencies occurring in Taiwan. Table 4 provides the experimental number of total available SAR resources. According to these experimental data, the number of reallocated SARs for each district can be computed using Equation (11). The results obtained are presented in Table 7.

5.5. Discussion

We analyze the application of the proposed model to determine whether the relief effort of SAR resources is improved after the 921-earthquake or not. Assume specific SAR resources for the entire Taiwan region. When the 921-earthquake occurred, the results calculated by our model guided the EOC to mobilize resources from all districts to assure even rescue of victims in multiple emergency events across the whole region (see Table 7). The number of people killed in all districts during the 921-Earthquake is revealed by the Ministry of the Interior of Taiwan statistics (see Table 7). Central Taiwan suffered major damage, particularly Nantou County and Taichung County. The majority of the casualties were concentrated in these two districts. Table 7 is interesting; because the proposed model dispatches most SAR resources to the two most severely affected districts. In addition, it shows the Average Distribution

Table 7 | Our proposed division of resources for 15 affected districts

Districts	Resource allocation after earthquake				Resource allocation rate after earthquake (%)				Average distribution rate (%)	Number of deaths	Death rate (%)	
	Rescue workers	Ambulances	Fire engines	Machine and equipments	Rescue Workers	Ambulances	Fire engines	Machine and equipments				
A1	TAIPEI	97	98	186	72	8.62	8.60	8.65	8.67	8.64	134	5.40
A2	TAOYUAN	41	42	78	30	3.64	3.68	3.63	3.61	3.64	1	0.04
A3	HSINCHU	25	26	48	19	2.22	2.28	2.23	2.29	2.26	0	0.00
A4	MIAOLI	17	17	33	13	1.51	1.49	1.53	1.57	1.53	6	0.24
A5	TAICHUNG	376	381	718	277	33.42	33.42	33.40	33.37	33.40	1305	52.58
A6	CHANGHUA	45	46	87	33	4.00	4.04	4.05	3.98	4.01	30	1.21
A7	NANTOU [*]	376	381	718	277	33.42	33.42	33.40	33.37	33.40	916	36.91
A8	YUNLIN	71	72	135	52	6.31	6.32	6.28	6.27	6.29	82	3.30
A9	CHIAYI	15	15	29	11	1.33	1.32	1.35	1.33	1.33	7	0.28
A10	TAINAN	39	39	74	28	3.47	3.42	3.44	3.37	3.43	1	0.04
A11	KAOHSIUNG	6	7	12	5	0.53	0.61	0.56	0.60	0.58	0	0
A12	PINGTUNG	13	13	24	9	1.16	1.14	1.12	1.08	1.12	0	0
A13	TAITUNG	2	2	4	2	0.18	0.18	0.19	0.24	0.20	0	0
A14	HUALIEN	2	2	4	2	0.18	0.18	0.19	0.24	0.20	0	0
A15	YILAN	1	1	3	1	0.09	0.09	0.14	0.12	0.11	0	0
Total		1125	1140	2150	830	100	100	100	100	100	2482	100

^{*}Epicenter.

Table 8 | Proportional division of resources for 15 affected districts

Districts	Magnitude	Resource allocation after earthquake				Resource allocation rate after earthquake (%)				Average distribution rate (%)	Number of deaths	Death rate (%)
		Rescue workers	Ambulances	Fire engines	Machine and equipments	Rescue workers	Ambulances	Fire engines	Machine and equipments			
A1	TAIPEI	4	63	64	121	47	5.63	5.63	5.63	5.63	134	5.40
A2	TAOYUAN	4	63	64	121	47	5.63	5.63	5.63	5.63	1	0.04
A3	HSINCHU	4	63	64	121	47	5.63	5.63	5.63	5.63	0	0.00
A4	MIAOLI	5	79	80	151	58	7.04	7.04	7.04	7.04	6	0.24
A5	TAICHUNG	6	95	96	182	70	8.45	8.45	8.45	8.45	1305	52.58
A6	CHANGHUA	6	95	96	182	70	8.45	8.45	8.45	8.45	30	1.21
A7	NANTOU [*]	7	111	112	212	82	9.86	9.86	9.86	9.86	916	36.91
A8	YUNLIN	6	95	96	182	70	8.45	8.45	8.45	8.45	82	3.30
A9	CHIAYI	5	79	80	151	58	7.04	7.04	7.04	7.04	7	0.28
A10	TAINAN	5	79	80	151	58	7.04	7.04	7.04	7.04	1	0.04
A11	KAOHSIUNG	4	63	64	121	47	5.63	5.63	5.63	5.63	0	0
A12	PINGTUNG	3	48	48	91	35	4.23	4.23	4.23	4.23	0	0
A13	TAITUNG	3	48	48	91	35	4.23	4.23	4.23	4.23	0	0
A14	HUALIEN	5	79	80	151	58	7.04	7.04	7.04	7.04	0	0
A15	YILAN	4	63	64	121	47	5.63	5.63	5.63	5.63	0	0
Total			1125	1140	2150	830	100	100	100	100	2482	100

^{*}Epicenter.

Rate (ADR) for Nantou County (i.e., 33.4%) which almost matches the death rate (i.e., 36.91%). Therefore, the proposed model is able to improve relief effort to counteract damage from external attacks.

We also evaluated the proposed division of resources by comparing the results obtained with those from the proportional division. The epicenter of the 921-earthquake was in Nantou County (i.e., agent A7) where the seismic intensity measured 7 on the Richter scale. Different magnitudes were measured at other counties in Taiwan when the 921-earthquake occurred. We assume that the higher the magnitude in a district, the more SARs resources needed. Therefore, we can use the proportional division to compute the amount of reallocation of each agent's resources. The numbers of resources of type *k* reallocated to the *i*th agent are defined by [Equation (15)]

$$p_k(i) = \left(\frac{M_i}{M_{all}} \right) \times O_{all,k}, i \in N \quad (15)$$

where $o_{all,k}$ is the total number of SAR resources of type *k*; M_i indicates the seismic intensity of the *i*th agent in the district when a serious quake occurred. The reallocation of the *i*th agent's resources in district $p_k(i)$ is derived from the proportion of M_i/M_{all} multiplied by the total number of resources available of type *k* $o_{all,k}$. Table 8 shows the experimental results for 15 districts (or agents) using the proportional division of resources.

In Figure 5, it can be seen that with the Shapley value allocation approach proposed in this study, the average distribution of resource rate is closer to the death rate of each district than

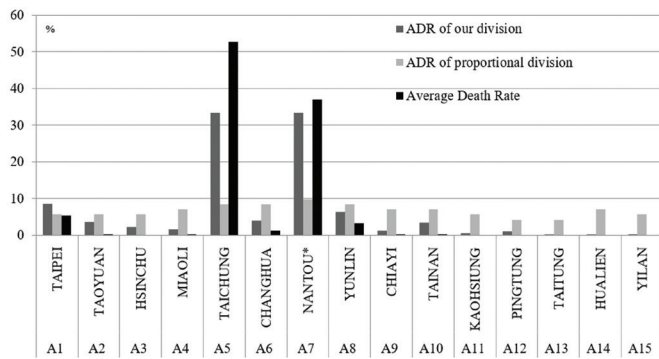


Figure 5 | Comparison of our division and proportional division.

the proportional division of resources. The results of this comparison support the idea of the proposed model, which creates a minimum set of resource allocation costs when the MAS confronts external attacks. The EOC rapidly commands all agents and efficiently mobilizes and redistributes SAR resources for seriously affected districts.

6. CONCLUSION

IDS, HSAS, and the emergency response system deploy multiple interacting agents to resist outside attackers. The resources of MAS are constrained for defending entire large-scale geographic regions. The emergency manager faces problems in agent resource deployment. The divide-and-conquer idea is utilized to improve the scalability of resource allocation. A two-stage model is proposed in this study to combine a non-cooperative game with a cooperative game to provide acceptable reallocation of resources for the MAS. The proposed model is applied to one strong earthquake attack case and is able to allocate resources efficiently in the overall system. Future work will design a computer-based decision-support system based on the proposed model. Real data will be applied and the feasibility of resource allocation from an organization's emergency operational center point of view verified.

CONFLICTS OF INTEREST

The authors declare they have no conflicts of interest.

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