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DEFORMATION OF AN ANISOTROPIC NON-HOMOGENEOUS CYLINDER

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The solution to the problem of stress-deformation state of hollow thick-walled laminated cylinders within the framework of 3D anisotropic elasticity theory is proposed. The author studies the influence of non-homogeneity and anisotropy of elastic properties of materials on the stress level and deformability of wound cylinders. Different layer conjugations, types of loading and cylinder boundary conditions are taken into account. A rational scheme of reinforcement can be found by changing properties of the material.

Key words: non-homogeneous, anisotropy, winding scheme

1. Formulation of the problem

Many structural elements in modern technology have the form of cylinder made of layers of composite materials, which are composed through winding, due to the necessity for receiving products with smaller weight and still the required strength and stiffness. Different applied shell theories are used to solve these problems (cf Bolotin and Novichkov (1980); Grigolyuk and Kulikov (1988); Kominar (1981); Malmeister et al. (1980); Pankratova (1992)). It is important to find a rational scheme of reinforcement for such a thick-walled cylinder. Changing this scheme we will be able to influence strength and stiffness of the product. When determining the stress level and deformability of composite cylinders their characteristics as, for example, anisotropy and non-homogeneity of mechanical parameters, types of layer conjugations, types of loading, etc. should be taken into account. As our experience indicates we can be provided with a deeper insight into the stress-deformation state of the layered cylinder basing on the equations of 3D anisotropic elasticity theory. Among the available methods of solution to problems in 3D formulation we should point out the method for calculating stresses in the body made of anisotropic materials which have one plane of elastic symmetry (cf Pankratova (1992); Vasilenko and Pankratova (1984), (1990); Grigorenko et al. (1991)). Its practical significance consists in a possibility of using the obtained solution for orthotropic materials when the principal directions of anisotropy do not coincide with those, that take place in the structural elements made by winding.

2. Stress-deformation state of the anisotropic non-homogeneous cylinder

In this paper the equations of the 3D elasticity theory are used to study the influence of non-homogeneity and anisotropy of elastic properties of material of each layer of a cylinder exerted the choice of a winding scheme and types of layers conjugation, types of loading and boundary conditions, on the stress-deformation state of laminated cylinder. The non-homogeneous cylinder is described in cylindrical coordinate system z, θ, r. There is one plane of elastic symmetry tangent to each point of cylindric surface r =const (cf Vasilneko and Pankratova (1984)) or perpendicular to the axis of rotation z =const (cf Vasilneko and Pankratova (1990)). The Hook generalized constitutive law for the ith layer $(r_i \leq r \leq r_{i+1}, i = 1, 2, ..., N)$ has the form

$$\boldsymbol{e}^i = B^i \sigma^i + \boldsymbol{f}^i \tag{2.1}$$

where

$$\begin{aligned}
&\boldsymbol{e}^i = [e^i_z, e^i_\theta, e^i_r, e^i_{z\theta}, e^i_{r\theta}, e^i_{rz}] \\
&\sigma^i = [\sigma^i_z, \sigma^i_\theta, \sigma^i_r, \tau^i_{z\theta}, \tau^i_{r\theta}, \tau^i_{rz}] \\
&B^i = \|b^i_{lp}(i)\| \qquad l, p = 1, 2, ..., 6 \\
&b_{m4} = b_{m5} = b_{4m} = b_{5m} = 0 \qquad (m = 1, 2, 3, 6) \quad \text{for} \quad r = \text{const} \\
&b_{m5} = b_{m6} = b_{5m} = b_{6m} = 0 \qquad (m = 1, 2, 3, 4) \quad \text{for} \quad z = \text{const} \\
&\boldsymbol{f}^i = [\alpha^i_z T, \alpha^i_\theta T, \alpha^i_r T, \alpha^i_{z\theta} T, \alpha^i_{r\theta} T, \alpha^i_{rz} T]
\end{aligned}$$

when e^i_z, e^i_θ, e^i_r, $e^i_{z\theta}$, $e^i_{r\theta}$, e^i_{rz} are strain tensor components, σ^i_z, σ^i_θ, σ^i_r, $\tau^i_{z\theta}$, $\tau^i_{r\theta}$, τ^i_{rz} are stress tensor components. Elastic constants b^i_{lp}, coefficients of linear thermal expansion α^i_z, α^i_θ, α_r in the corresponding directions z, θ, r, coefficients of temperature shift $\alpha^i_{z\theta}, \alpha^i_{r\theta}, \alpha^i_{rz}$ are functions of the coordinate r.

This makes it possible to take into account arbitrary variability of the material properties through the thickness of the elastic cylinder.

Eqs (2.1) are also valid for an orthotropic cylinder principal directions of elasticity of which are turned about the normal to the surface r =const or z =const through the angle φ. In this case the elastic constants appear from the known equations (cf Lehniskiĭ) through the corresponding characteristics of an anisotropic material.

Taking into account the equilibrium equations, the strain-displacement relations and the Hook law for a non-homogeneous anisotropic body, the system of partial differential equations describing the stress-deformation state of laminated hollow elastic bodies may be derived. The functions $\boldsymbol{\sigma}^i$ are taken as the basic ones with the help of which we can formulate in stresses σ_r, τ_{rz}, $\tau_{r\theta}$ and in displacements u_r, u_z, u_θ the conditions for limiting surfaces $r = r_0$, $r = r_N$ and for the surfaces of conjugation of layers r_i. After making a series of transformations a system of partial differential equations for the ith cylinder layer is obtained (cf Pankratova (1992))

$$\frac{\partial \boldsymbol{\sigma}^i}{\partial r} = \sum_{k=1}^{6} B_k^i \boldsymbol{\sigma}_k^i + \boldsymbol{f}^i \tag{2.2}$$

where

$$B_k^i = \|b_{pq}^i(r)\| \qquad p, q = 1, 2, ..., 6$$
$$\boldsymbol{\sigma}^i = [\sigma_r^i, \tau_{rz}^i, \tau_{r\theta}^i, u_r^i, u_z^i, u_\theta^i]$$
$$\boldsymbol{f}^i = [f_1^i, f_2^i, ..., f_6^i]$$
$$\boldsymbol{\sigma}_1^i = \boldsymbol{\sigma}^i \qquad \boldsymbol{\sigma}_2^i = \frac{\partial \boldsymbol{\sigma}^i}{\partial z} \qquad \boldsymbol{\sigma}_3^i = \frac{\partial \boldsymbol{\sigma}^i}{\partial \theta}$$
$$\boldsymbol{\sigma}_4^i = \frac{\partial^2 \boldsymbol{\sigma}^i}{\partial z^2} \qquad \boldsymbol{\sigma}_5^i = \frac{\partial^2 \boldsymbol{\sigma}^i}{\partial z \partial \theta} \qquad \boldsymbol{\sigma}_6^i = \frac{\partial^2 \boldsymbol{\sigma}^i}{\partial \theta^2}$$

The solution to the equations must satisfy the following conditions on the limiting surfaces $r = r_0$, $r = r_N$

$$\sigma_r^0(r_0, z) = f_1^0(z) \qquad \sigma_r^N(r_N, z) = f_1^N(z)$$
$$\tau_{rz}^0(r_0, z) = f_2^0(z) \qquad \tau_{rz}^N(r_N, z) = f_2^N(z)$$
$$\tau_{r\theta}^0(r_0, z) = f_3^0(z) \qquad \tau_{r\theta}^N(r_N, z) = f_3^N(z)$$

Additionally the conditions for the surfaces of conjugation of layers and boundary conditions at the cylinder ends must be satisfied. The author takes into consideration the fact that different conditions imposed by layered material can be formulated on the surfaces of conjugation of layers. In the case of rigid conjugation of layers, when layers work together without slipping and breaking off, the continuity conditions $\sigma_j^i = \sigma_j^{i+1}$, $j = 1, 2, ..., 6$, $i = 1, 2, ..., N-1$ are formulated for all $\boldsymbol{\sigma}$ vector components. Sometimes these conditions can be violated, for example, when at the layers conjugation some of the components of $\boldsymbol{\sigma}^i$ can break, i.e. $\sigma_j^i \neq \sigma_j^{i+1}$. In particular, in the case of layers slipping, tangential stresses τ_{rz}, $\tau_{r\theta}$ on the conjugation surface are known and can be arbitrarily specified. In that case the tangential displacement components are discontinuous $u_z^i \neq u_z^{i+1}$, $u_\theta^i \neq u_\theta^{i+1}$. For the normal displacement components the continuity condition $u_r^i = u_r^{i+1}$ is met. When this condition is not satisfied, separation of layers takes place.

Here we consider the case when at the cylinder ends $z = 0, z = l$ such axial stresses σ_z and tangential stresses $\tau_{z\theta}$ are applied that the ends of cylinder neither shift in the axial direction nor rotate

$$u_z^i = u_\theta^i = 0 \qquad \tau_{rz}^i = 0 \tag{2.3}$$

Expanding all the fields into series of orthogonal trigonometric functions allows one the strict separation of the variables in Eq (2.2). Then the boundary problem is exactly transformed into a set of one-dimensional problems. The requirement of taking into account an arbitrary type of the layers conjugation leads to the necessity of solution of a multi-point boundary problem. As a result, for each ith layer we use the method of reduction of the boundary problem to the Cauchy problems. According to this method the solution is assumed in the form

$$\sigma_j^i = \sum_{v=1}^{l_i} \mu_v^i \sigma_{jv}^i \tag{2.4}$$

Here $l_i = n - p^j + 1$ (for this problem $n = 6$) is the number of solutions to the Cauchy problems for the ith layer, σ_{jv}^i is the solution for the ith layer of the non-homogeneous body of the vth Cauchy problem for the components of $\boldsymbol{\sigma}^i$ with numbers $j = 1, 2, ..., p^j$ using the formulated initial conditions, and for the components of $\boldsymbol{\sigma}^i$ with numbers $j = p^j + 1, ..., n$ with the following initial conditions

$$\sigma_{jv}^i = \begin{cases} 1 & \text{if} \quad j = p^i + v \qquad v = 1, 2, ..., n - p^i \\ 0 & \text{if} \quad j = p^i + v \qquad v = 1, 2, ..., n - p^i + 1 \end{cases}$$

Here

$$\sum_{v=1}^{l_i} \mu_v^i = 1 \tag{2.5}$$

Using the stable numerical method of discrete orthogonalization for integration of the constructed Cauchy problems, we obtain solutions with the required accuracy level. As a result, the solutions to the problem lead to evaluation of coefficients μ^i_v. These coefficients are found from the solution to the system of linear algebraic equations. This system can be derived taking into account Eq (2.5), the continuity conditions for the m components of σ, when crossing the surface between the ith and $(i+1)$th layers, the formulated s^i conditions on the external surfaces for each ith layer of the elastic cylinder for σ^i_j $j = 1, 2, ..., s^i$ and the conditions on the cylinder limiting surface r_N. Having defined μ^i_v, values of the resolving functions σ^i_j at the required points of integration within the interval $r_0 \leq r \leq r_N$ are found in accordance with Eq (2.4). Then, using the obtained functions we calculate all factors of the stress-deformation state of the considered non-homogeneous hollow elastic cylinder. Since the general solution to a three-dimensional problem is written in terms of double trigonometric series automatic summation subroutines were used in the algorithm. These subroutines enabled calculation of the values of stress and displacement fields in three mutually orthogonal directions at any point of the considered elastic cylinder.

3. Analysis of the cylinder fabricated by winding

On the basis of the approach presented above, let us study the influence of the choice of a winding scheme, modelled here by non-homogeneity and anisotropy of elastic material properties, on the stress state of the laminated cylinder (Fig.1). This cylinder is subjected to the surface load which changes according to $\sigma_r = \sigma_0 \sin^p(n\pi z/l)$. The case of smoth layers conjugation is considered here. The hollow cylinder is made of an orthotropic material (cf Grigorenko et al. (1991)) with the following elastic constants for the case r =const

$$\begin{array}{lll} E_z = 20.1E_0 & E_\theta = 1.6E_0 & E_r = 1.63E_0 \\ v_{z\theta} = 0.024 & v_{rz} = 0.324 & v_{r\theta} = 0.543 \\ G_{r\theta} = 0.548E_0 & G_{rz} = G_{z\theta} = 0.878E_0 & \end{array}$$

and for the case z =const these elastic characteristics were chosen as

$$\begin{array}{lll} E_z = 1.63E_0 & E_\theta = 1.6E_0 & E_r = 20.1E_0 \\ v_{z\theta} = 0.543 & v_{rz} = 0.324 & v_{r\theta} = 0.024 \\ G_{r\theta} = 0.548E_0 & G_{rz} = G_{z\theta} = 0.878E_0 & \end{array}$$

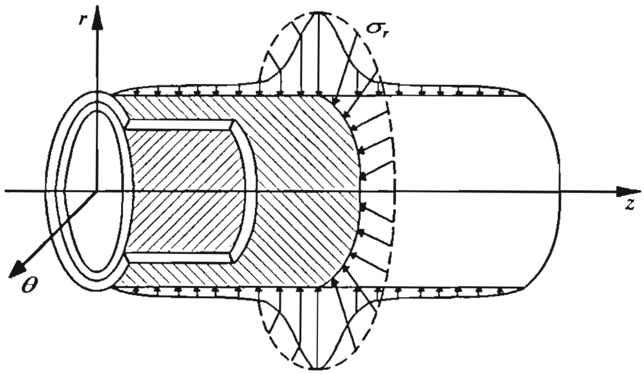

Fig. 1. Two-layer cylinder fabricated by winding

Principal directions of material elasticity for the case r =const are turned about the normal to the face surfaces (for the case z =const to the end surfaces) of the cylinder by the angle φ. Calculations are performed for

$$r_N = 1.2r_0 \qquad r_0 = 1 \qquad E_0 = 1$$
$$l = 2r_0 \qquad p = 9$$

and for different values φ: $0; \pi/12; \pi/6; \pi/4; \pi/3; 5\pi/12; \pi/2$.

Within the limits of unchangeable constant thickness the cylinder was divided into 3, 5 and 7 layers, respectively, with the following winding structure

Layers number	Number of the variant	Winding scheme layer number						
		1	2	3	4	5	6	7
3	1	φ	0	φ				
	2	0	φ	0				
5	1	φ	0	$-\varphi$	0	φ		
	2	0	φ	0	$-\varphi$	0		
7	1	φ	0	$-\varphi$	0	φ	0	$-\varphi$
	2	0	φ	0	$-\varphi$	0	φ	0

Some of the obtained results for the 3, 5 and 7 layers cylinder for two variants of the winding scheme are given in Table 1 and Table 2.

Table 1. Distribution of stresses σ_z, σ_θ, $\tau_{z\theta}$ and displacements u_r for the fixed section $z = 0.5l$ and the winding angle $\varphi = \pi/4$

Layers number	r	Number of the variant	σ_z/σ_0	σ_θ/σ_0	$\tau_{z\theta}/\sigma_0$	$\frac{u_r}{\sigma_0 E_0^{-1} r_0}$
			r =const			
3	1.0	1	0.8371	−3.1403	−0.0299	−1.7446
		2	0.1994	−1.8831	−0.0754	−1.9233
	1.2	1	−3.5346	−4.6467	−0.0560	−1.7439
		2	−4.1466	−4.8179	0.0619	−1.9233
5	1.0	1	0.9695	−3.1710	0.0340	−1.7766
		2	−0.0754	−1.8900	−0.0270	−1.8856
	1.2	1	−3.6019	−4.7323	0.0247	−1.7766
		2	−4.0604	−4.7206	−0.0260	−1.8856
7	1.0	1	0.0259	−3.1906	0.0026	−1.7931
		2	0.0282	−4.7771	−0.0145	−1.8674
	1.2	1	−3.6380	−1.8864	−0.0249	−1.7926
		2	−4.0191	−4.6738	0.0069	−1.8671
			z =const			
3	1.0	1	−2.9358	0.7551	−0.0371	−2.3212
		2	−2.2761	0.2572	−0.0650	−2.1886
	1.2	1	−4.3350	−3.2144	0.0352	−2.2189
		2	−5.2014	−4.3173	−0.0505	−2.1414
5	1.0	1	−2.8939	0.7585	−0.0454	−2.2950
		2	−2.3181	0.2422	−0.0399	−2.2098
	1.2	1	−4.2985	−3.1550	0.0248	−2.2042
		2	−5.2530	−4.3286	−0.0400	−2.1535
7	1.0	1	−2.8510	0.7462	0.0285	−2.2661
		2	−2.3604	0.2413	−0.0353	−2.2359
	1.2	1	−4.2531	−3.0776	0.0290	−2.1816
		2	−5.3271	4.4799	0.0153	−2.1694

All linear dimensions are referred to the unit of length r_0. In Table 1 there are results for stresses σ_z, σ_θ, $\tau_{z\theta}$ and displacements u_r on the limiting surfaces r_0, r_N in the middle section $z = 0.5l$.

Table 2. Distribution of stresses τ_{rz}, $\tau_{r\theta}$ and displacements ; u_z, u_θ for the fixed section $z = 0.4l$ and the winding angle $\varphi = \pi/4$

Layers number	r	Number of the variant	τ_{rz}/σ_0	$\tau_{r\theta}/\sigma_0$	$\frac{u_z}{\sigma_0 E_0^{-1} r_0}$	$\frac{u_\theta}{\sigma_0 E_0^{-1} r_0}$
			r =const			
3	1.07	1	−0.4756	0.0470	−0.1570	−0.0070
		2	−0.4197	0.0236	−0.1766	0.0160
	1.13	1	−0.4118	−0.0072	−0.0011	−0.0076
		2	−0.4618	0.1214	−0.0238	−0.0117
5	1.08	1	−0.4634	−0.0104	−0.1206	−0.0087
		2	−0.5002	0.0043	−0.1470	−0.0097
	1.12	1	−0.4771	−0.0091	−0.0423	0.0084
		2	−0.4748	−0.0064	−0.0436	−0.0102
7	1.09	1	−0.5004	−0.0033	−0.1087	0.0074
		2	−0.4869	−0.0044	−0.1130	−0.0070
	1.12	1	−0.4640	0.0032	−0.0362	0.0076
		2	−0.4884	−0.0037	−0.0497	0.0064
			z =const			
3	1.07	1	−0.4320	0.0171	−0.1984	−0.0330
		2	−0.4306	0.0226	−0.2008	0.0213
	1.13	1	−0.3869	0.0011	−0.0157	−0.0345
		2	−0.4577	0.0209	−0.0314	−0.0380
5	1.08	1	−0.4566	−0.0075	−0.1617	−0.0273
		2	−0.4722	0.0107	−0.1650	−0.0253
	1.12	1	−0.4654	−0.0089	−0.0566	0.0077
		2	−0.4558	0.0005	−0.0643	−0.0260
7	1.09	1	−0.4804	−0.0041	−0.1369	0.0890
		2	−0.4740	0.0013	−0.1384	0.0430
	1.12	1	−0.4503	−0.0001	−0.5940	0.0090
		2	−0.4696	−0.0012	−0.0618	−0.0144

In Table 2 there are results for stresses τ_{rz}, $\tau_{r\theta}$ and displacements u_z, u_θ on the surface of layers conjugation r_i in the section $z = 0.4l$ for the winding angle $\varphi = \pi/4$. As follows from the given results, choice of the first variant of the winding scheme lowers the level of the maximal absolute values of stresses σ_z, σ_θ more than by $10\% \div 40\%$. We should note that when the winding angle lies within the range $\pi/12 \leq \varphi \leq 5\pi/12$, circumferential displacements u_θ and tangential stresses $\tau_{r\theta}$, $\tau_{z\theta}$ appear, values of which are

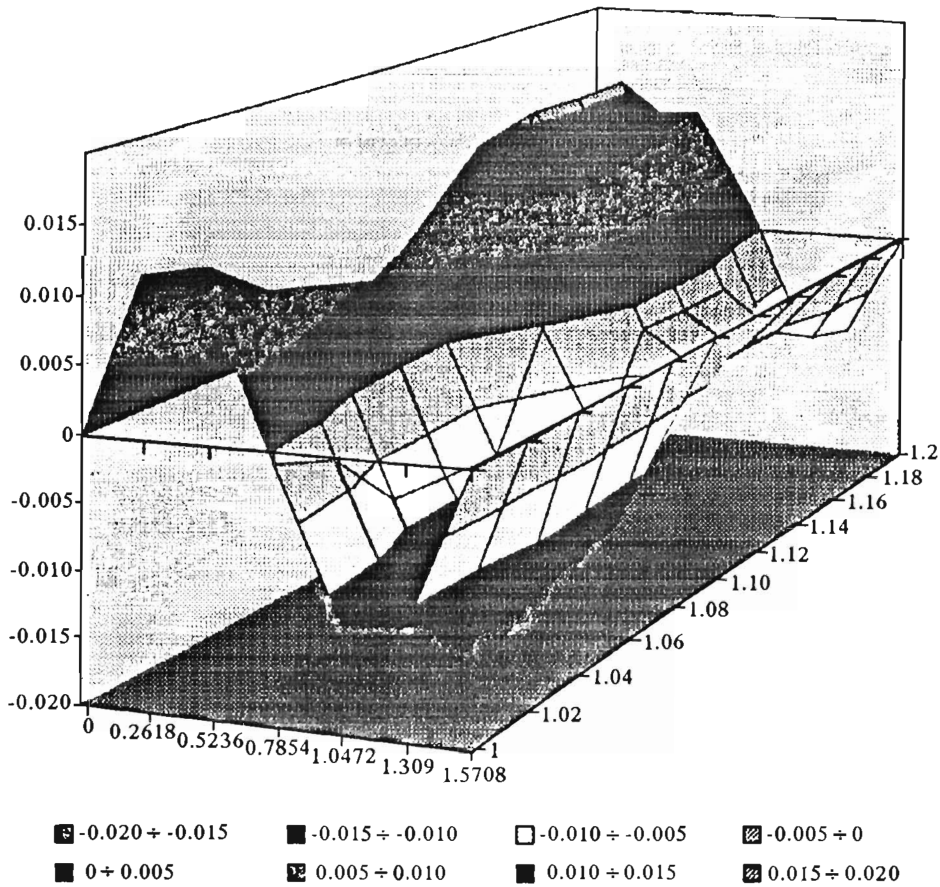

Fig. 2. Distribution of displacements u_θ through thickness versus winding angle φ

comparable to displacements u_z and stresses τ_{rz}. Distribution of u_θ and $\tau_{r\theta}$ through the thickness of the five-layer cylinder for the first variant of the winding according to the winding angle φ in section $z = 0.8l$ are shown in Fig.2 and Fig.3. Displacements u_θ reach their maximal values when the winding angle is $\varphi = \pi/4$.

Tangential stresses τ_{rz} change across the cylinder thickness obeying the square parabola law reaching their maximal values near the center of the section for $z = 0.4l$. Stresses $\tau_{r\theta}$, $\tau_{z\theta}$ and displacements u_θ change across the cylinder thickness in a non-linear, non-monotonous manner, revealing the character of distribution of the formulated winding scheme, reaching their maximal values at the cylinder length $z = 0.25l$. Radial displacements u_r

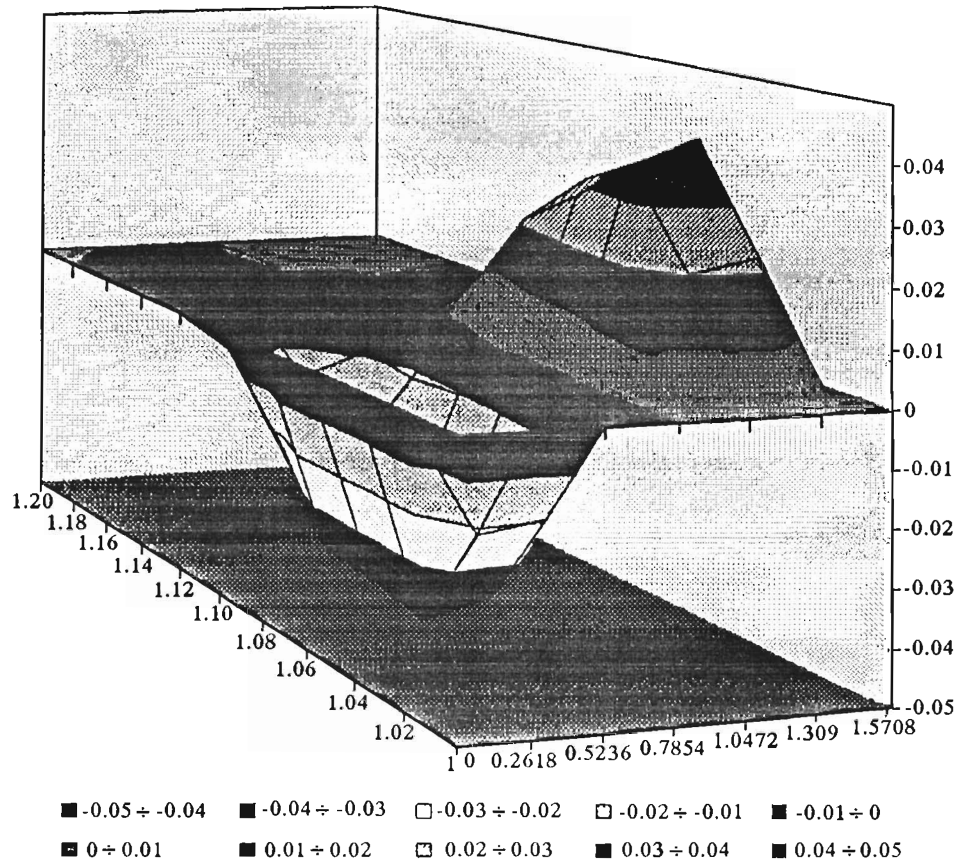

Fig. 3. Distribution of stresses $\tau_{r\theta}$ through thickness versus winding angle φ

across the cylinder thickness change insignificantly. Stresses σ_z change in amonotonic way along the cylinder length, and the stresses σ_θ near the section $z = 0.25l$ change their sign relative to the values at $z = 0$. Stresses $\tau_{z\theta}$ reach their maximal values at $z = 0.4l$.

4. Solution for the cylinder with free ends

Let us solve the problem of an anisotropic non-homogeneous cylinder with free ends, where

$$\sigma_z^i = \tau_{rz}^i = \tau_{z\theta}^i = 0 \qquad z = 0 \qquad z = l \tag{4.1}$$

We use the solution to the problem of tension of an anisotropic multi-layer cylinder subjected to the axial force P (cf Vasilenko and Pankratova (1984)), based on the solution with the boundary conditions (2.3) at the ends. This allows one to satisfy the boundary conditions for σ_z and $\tau_{z\theta}$ in the averaged form. Cylindrical surfaces for the stretched cylinder are free from stresses, and the boundary conditions at the ends have then the form

$$\begin{aligned} &\sum_{i=1}^{N} \int_{r_{i-1}}^{r_i} \sigma_z^i r\, dr = \frac{P}{2\pi} \\ &\sum_{i=1}^{N} \int_{r_{i-1}}^{r_i} \tau_{z\theta}^i r^2\, dr = \frac{M}{2\pi} \qquad \tau_{rz} = 0 \end{aligned} \tag{4.2}$$

As it was shown by Vasilenko and Pankratova (1984), the solution to the problem of tension of the anisotropic cylinder subjected to the axial force is described by the system of ordinary differential equations of the second order. The right-hand side of this system includes unknown constants C and D, which are calculated from the boundary conditions (4.2) using methods of the mean square approximation and changeable directions for minimization of the function of two variables. Following these methods the criterium of minimization is written as follows

$$\min C, D \sqrt{\Big[P - 2\pi \sum_{i=1}^{N} \int_{r_{i-1}}^{r_i} \sigma_z^i(C, D) r\, dr\Big]^2 + \Big[M - 2\pi \sum_{i=1}^{N} \int_{r_{i-1}}^{r_i} \tau_{z\theta}^i(C, D) r^2\, dr\Big]^2}$$

with the help of which the values of constants C and D are obtained (Fig.4). Influence of the winding effect in the cylinder with free ends is studied on the example of a two-layer hollow cylinder. Cylinder layers are wound by oriented fibres with equal and opposite angles relative the longitudinal axis. Principal directions of elasticity are turned towards coordinate axes z and θ through the angle φ in the inner layer and the angle $-\varphi$ in the outer layer. Material of the element is orthotropic and has the following characteristics (cf Grigorenko et al. (1991))

$$\begin{aligned} &a_{11} = E_0^{-1}/0.7 && a_{22} = a_{33} = E_0^{-1}/1.4 \\ &a_{12} = a_{13} = -0.068E_0^{-1}/1.4 && a_{23} = -0.4E_0^{-1}/1.4 \\ &a_{44} = E_0^{-1}/0.5 && a_{55} = a_{66} = E_0^{-1}/0.575 \end{aligned}$$

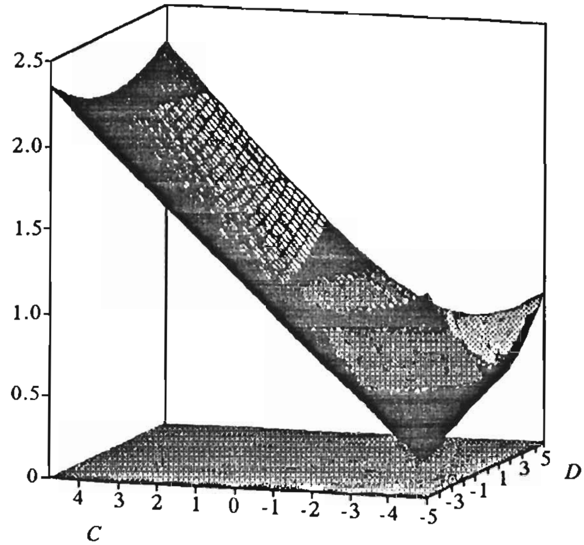

Fig. 4.

Pressure $\sigma_r = \sigma_0 \sin^9(n\pi z/l)$ is applied at the outer surface of the cylinder. The inner cylinder radius is r_0, the outer one is $r_N = 1.2 r_0$. Each layer has the same thickness. Calculations were made for

$$r_0 = 1 \qquad l = 2r_0 \qquad \varphi = 0,\ \frac{\pi}{12},\ \frac{\pi}{6},\ \frac{\pi}{4},\ \frac{\pi}{3},\ \frac{5\pi}{12},\ \frac{\pi}{2}$$

The results of calculations are given in Fig.5 (for the boundary conditions (2.3), (4.2) and (4.1)) in the form of distribution of stresses σ_z, σ_θ, $\tau_{z\theta}$ over the outer cylinder surface in the section $z = 0.5l$ according to the winding angle φ. As follows from the obtained results, stresses $\tau_{z\theta}$ arising in the wounded cylinder are comparable with the stresses σ_z, σ_θ and in the area $\pi/6 \le \varphi \le 5\pi/12$ contribute considerably to the stress-deformation state of the cylinder. Stresses σ_z that appear at the ends of the cylinder are smaller than their values arising far from the ends.

5. Solution for two-layer cylinder with non-rigid conjugation of layers

Let us consider the stress-deformation state in a two-layer anisotropic cylinder with non-rigid conjugation of layers. Local surface loading

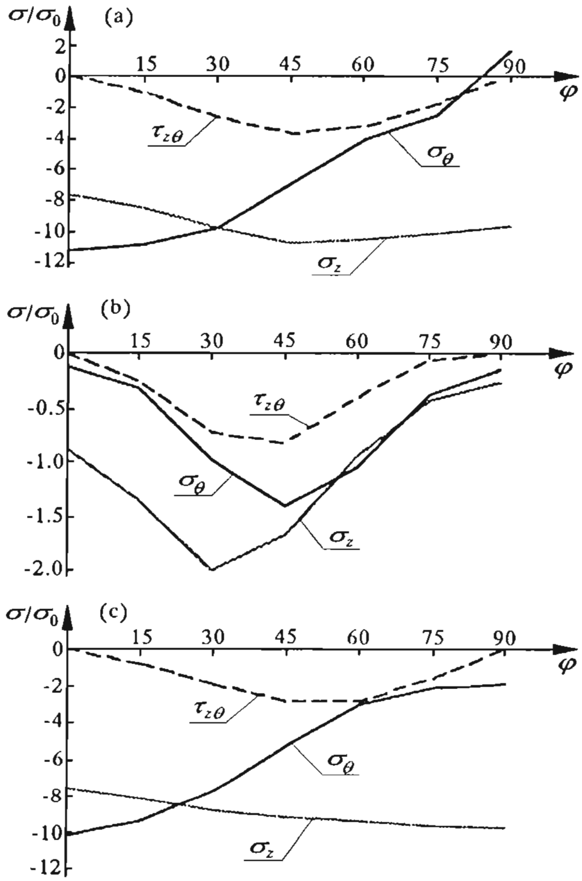

Fig. 5. Distribution of stresses σ_z, σ_θ, $\tau_{z\theta}$ versus winding angle φ; (a) – for a cylinder with boundary conditions (2.3), (b) – for a cylinder under axial force, (c) – for a cylinder with free ends

$\sigma_r = \sigma_0 \sin^9(n\pi z/l)$ is applied at the external cylinder surface and the loading $\sigma_r = \sigma_0 \sin(n\pi z/l)$ is applied at the internal cylinder surface. The internal cylinder layer is made of orthotropic material with the following elastic characteristics (cf Pankratova and Mukoed (1994))

$$
\begin{aligned}
&a_{11} = E_0^{-1}/138 && a_{22} = a_{33} = E_0^{-1}/9.7 \\
&a_{12} = a_{13} = -0.31E_0^{-1}/138 && a_{23} = -0.5E_0^{-1}/9.7 \\
&a_{44} = E_0^{-1}/3.2 && a_{55} = a_{66} = E_0^{-1}/6.9
\end{aligned}
$$

The external cylinder layer is made of the material turned through 90 degrees relative to the material of the internal layer and has the following elastic characteristics

$$a_{11} = E_0^{-1}/9.7 \qquad a_{22} = E_0^{-1}/138 \qquad a_{33} = E_0^{-1}/9.7$$
$$a_{12} = -0.31E_0^{-1}/9.7 \qquad a_{13} = -0.5E_0^{-1}/9.7 \qquad a_{23} = -0.31E_0^{-1}/138$$
$$a_{55} = E_0^{-1}/3.2 \qquad a_{44} = a_{66} = E_0^{-1}/6.9$$

Calculations have been made for $r_0 = 1$, $r_c = 1.1$, $r_N = 1.2$, $l = 2r_0$, $E_0 = 1$. The cases of rigid conjugation, slipping and separation of layers are considered. Some of the results are given in Fig.6 in the form of stress distribution σ_z across the thickness of the cylinder in the section $z = 0.125l$.

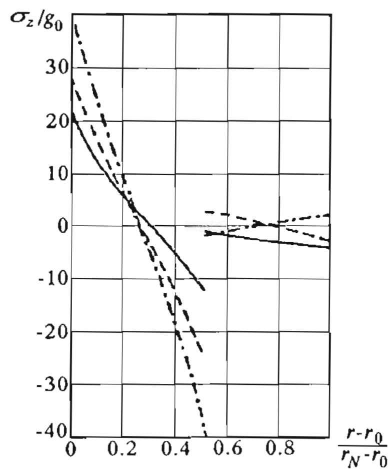

Fig. 6. Distribution of stresses σ_z through the thickness in section $z = 0.5l$

The results corresponding to the rigid conjugation, slipping and separation of layers are shown by solid, dashed and dash-doted lines, respectively. Tangential stresses σ_z change almost linearly across the thickness. Non-perfect bonding of layers increases the jump of σ_z to the order of one on the interfaces, when jumps in the values of circumferential stresses σ_θ do not exceed 30% in the case of slipping and 60% in the case of layers separation. This can be explained by the layers material structure. From the results it follows that radial stresses σ_r change in the nonlinear nonmonotonous manner along the cylinder length within the internal layer, reaching their maximal absolute values at the distance equal to the cylinder thickness from its end. This can be explained by specifics of the applied load. Here, tangential stresses τ_{rz} are equal to zero on the whole surface of layers conjugation for the cases of their non-rigid conjugation. In the case of rigid conjugation and slipping of layers, radial displacements u_r change insignificantly through the thickness of layers package, reaching 10% in their distribution.

In the case of layers separation displacements u_r, as we expected, have leap while going through the whole surface of layers conjugation, reaching a difference almost of the order of one in the section $z = 0.125l$, that is preconditioned by changing the character of σ_r. Displacements u_z change linearly across the thickness of cylinder layers. In the case of slipping and separation of layers these displacements change their sign, when crossing the interface between the adjoing layers leading to a jump of an order two relatively to their absolute values.

6. Conclusions

The analysis of stress-strain state of the non-homogeneous anisotropic wound cylinders points out the necessity for taking into account the effects caused by non-coincideness of the principal directions of elasticity with directions of the coordinate lines with due account of different manners of conjugation of cylinder layers, types of loading and boundary conditions. It has been shown that changing material properties we may select a rational scheme of winding of structural elements of the cylindrical form.

The solution to the problem, obtained here within 3D anisotropic elasticity with high degree of accuracy, may be applied as a standard in elaborating various more approximate models.

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Odkształcenie anizotropowego, niejednorodnego cylindra

Streszczenie

W pracy zaproponowano sposób wyznaczania stanu naprężenia i odkształcenia w wydrążonym laminatowym cylindrze grubościennym, w zakresie anizotropowej teorii sprężystości. Autorka, biorąc pod uwagę różne typy połączeń warstw, obciążeń oraz warunków brzegowych, bada wpływ własności sprężystych materiału, niejednorodności i anizotropowości, na poziom naprężeń oraz odkształcalność cylindrów powstałych przez nawijanie. Z przeprowadzonych badań wynika, że zmieniając własności materiału można otrzymać racjonalny schemat wzmocnienia.

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