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PLANE BENDING LOW-CYCLE FATIGUE INVESTIGATIONS OF 45-STEEL[1]

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The results of low-cycle fatigue investigations of the 45-steel, which were conducted under plane bending conditions are presented in this paper. Fatigue tests were carried out for five levels of the total strain ε_{ac} = 0.04, 0.02, 0.01, 0.008, 0.005. Three different heights of specimens (H = 8 mm, H = 12 mm, H = 16 mm) were used for evaluating the influence of strain gradients in specimens. Strains in specimens were measured in a zone of maximal strains (extensometer). Test results are shown on fatigue curves and cyclic strain curves. The analysis of fatigue curves shows the influence of the specimen height on fatigue life. The maximal values of fatigue life were found for the specimen of H = 16 mm height, whereas the minimal values appeared for the specimen of H = 8 mm height. The comparative analysis of fatigue life under conditions of axial and plane bending shows the essential differences between the results obtained. These differences come up to 300% for large values of strains. Taking into consideration the shapes and flows of cyclic strain curves and static bending curves the dependence of cyclic properties of the tested steel on the total strain amplitude on the lateral surface of the specimen was noticeable. Basing on the stress curves in bended specimens analysis, the influence of their parameters on the fatigue life was observed. Three parameters: the tangent gradient, the plastic gradient and the thickness of plastic zone for the stress curves analysis were proposed.

Notations

ε_{ac} – amplitude of total strain
ε_{ap} – amplitude of plastic strain

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ε_{ae} – amplitude of the elastic strain
E – Young modulus
ε'_f – cyclic plastic strain coefficient
σ'_f – fatigue strength coefficient
c – exponent of cyclic plastic strain
b – exponent of the fatigue strength
$2N_f$ – reversals to failure
σ_{Ec} – cyclic strain limit
ε_{Ec} – amplitude of cyclic elastic strain for σ_{Ec}
σ_a – amplitude of cyclic elastic strain for σ_{Ec}
K' – coefficient of cyclic strain curve
n' – exponent of cyclic strain curve.

1. Introduction

In modern methods of fatigue calculations (cf Neuber (1961), Sharpe (1991), Blatt et al. (1993), Hoffmann and Seeger (1989), Harkegard and Stubstad (1992)) the behaviour of material in the notch area until crack initiation is modelled by a smooth specimen axially loaded on the equivalent level of strain and stress existing at notch roots. This assumption does not take into account the occurrence of strong stress and strain gradients in the notch zone. However, replacing a small notch area by a normalized specimen does not include the size effect.

It is difficult to analyse the influence of strain gradient on fatigue life basing on the results of investigations of notched specimens, because the basic problem consists in accurate determination of strains and stresses in the notch area. This problem was overcome in the present investigations since the specimens of rectangular cross-section were tested under the plane bending test conditions. A diagrammatic approach to this problem is presented in Fig.1.

The graphical representation of the strain distribution in an smooth specimen was shown in Fig.1a. The uniform strain in a section has the value ε_{ac} and the uniform stress has the value σ_a for this specimen. In notched specimen, substantial differences between the values of the strains and stresses in the section were noted. In the upper part of Fig.1b, the plots of variation of stress σ_a and strains, which at the notch root reach ε_{ac} level and in the lower part of the figure the modelled smooth specimen – inscribed in the notch specimen – is presented.

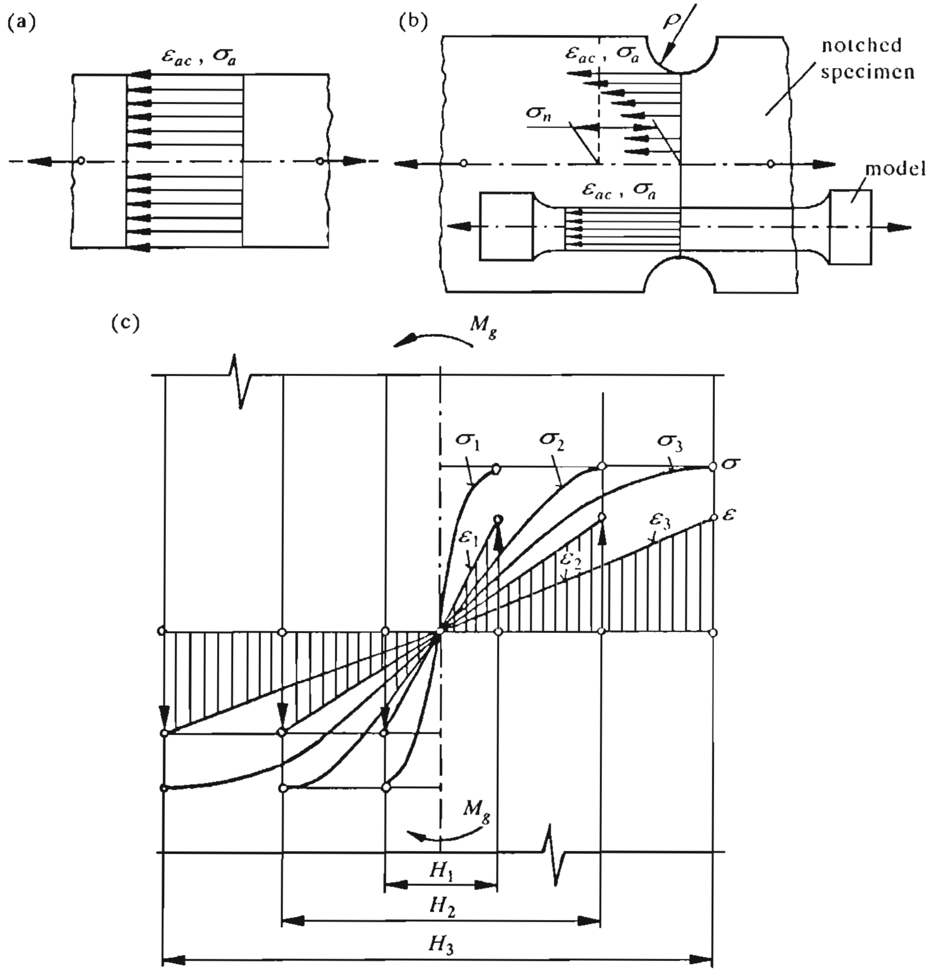


Fig. 1. Graphical representation of strain and stress distributions in a specimen: (a) – smooth, in the case of axial loading; (b) – notched, in the case of axial loading; (c) – smooth, in the case of bending, for three different heights H

In this model variations of ε_{ac} and σ_a in the specimen are equivalent to those presented in Fig.1a. In the case of bended specimen (Fig.1c), distribution of strains and stresses depends on the height H of the specimen and for high specimens is close to the case presented in Fig.1a, whereas for short specimens distribution is close to the situation presented in Fig.1b.

There have been some papers investigating low-cycle fatigue under plane or rotary bending conditions (cf Abel (1987), Goss et al. (1986), Urashima and Nishida (1987)). The aforementioned Authors presented a comparative approach. The influence of loading on fatigue life was investigated. It was found that the basic agent, which affected the fatigue life was the amplitude of plastic strain on the specimen surface.

The fundamental aim of our work is the analysis of the specimen height influence (strain gradient) on fatigue life and on the course of characteristic phenomena, which are observed in a low-cycle bending (stabilization, hardening and softening). Moreover, we want to determine the relation between some parameters, which describe the strain distribution in the bending specimen within the elastic-plastic range. Another task is also the comparative analysis of 45-steel fatigue life obtained under axial loading conditions and under variable bending conditions. An additional aim of this work is to give an appraisal of the grounds of modelling phenomena in a notch by smooth specimen under axial loading conditions.

2. Experimental investigations

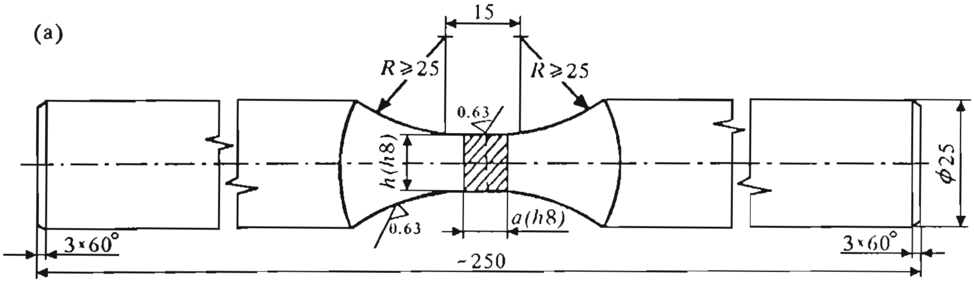
Fatigue investigations were conducted using smooth specimens, under four-point bending. Schemes of the specimen and the method of fastening with a own made device are presented in Fig.2, respectively.

The change of the gradient of strain was obtained by using specimens of different heights in investigations and changing the value strain ε_{ac}. Tests were carried out at five levels of the strain $\varepsilon_{ac} = 0.04, 0.02, 0.01, 0.008, 0.005$ for every height. Specimens were made of the normalized 45-steel (Table 1).

Table 1. Strength parameters of the normalized 45-steel

R_m [MPa]	R_e [MPa]	A_5 [%]	Z [%]	HB
730	435	25	44	190

The measurement part of the rectangular section was formed by mechanical working (milling and grinding) of circular bar. The constant section modulus



Specimen dimension

Dimension / Execution	a	h
1	5.63	16
2	10.00	12
3	22.50	8

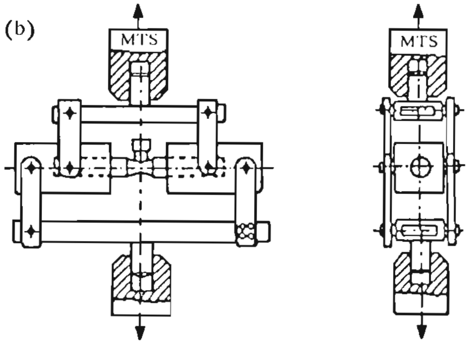


Fig. 2. The specimen for investigations (a) and the test equipment for realisation of plane bending (b)

for bending Wx was assumed for specimens of different heights.

Fatigue investigations were performed by means of an "MTS" strength test machine under constant total strain (i.e. ε_{ac} = const). Strains were measured using the extensometer of basis 10 mm, which was fastened onto the specimen zone, where the maximal total strains existed. The loading frequency, failure criterion and the method of results the recording were assumed in the way complying with the Polish Standard PN-84/H-04334.

3. Test results

3.1. Fatigue life of bending specimens

Results of fatigue lives for different loading levels under plane bending are presented in Table 2.

Table 2. Fatigue live results of tests carried out undes plane bending and axial loading conditions

	Loading										
	Bending									Axial	
	$H = 8$ mm			$H = 12$ mm			$H = 16$ mm			$\emptyset = 12$ mm	
ε_{ac} [%]	N_f	ε_{ap}	M_g [Nm]	N_f	ε_{ap}	M_g [Nm]	N_f	ε_{ap}	M_g [Nm]	N_f	ε_{ap}
1	2	3	4	5	6	7	8	9	10	11	12
4	45	0.0331	256	50	0.0337	250	60	0.0338	245	12	0.0345
	40	0.0341	250	55	0.0342	245	65	0.0341	235	17	0.0312
	37	0.0344	260	50	0.0339	255	65	0.0345	250	10	0.0356
2	190	0.0155	220	300	0.0157	202	320	0.0155	212	85	0.0161
	205	0.0154	210	270	0.0158	210	335	0.0156	210	80	0.0165
	215	0.0152	215	290	0.0157	205	330	0.0153	205	86	0.0168
1	770	0.0068	175	850	0.0071	176	1375	0.0067	170	650	0.0071
	800	0.0068	180	950	0.0068	184	1300	0.0064	165	620	0.0071
	770	0.0067	170	1000	0.0068	180	1250	0.0066	180	615	0.0067
0.8	1800	0.0051	160	2100	0.0052	161	2300	0.0050	158	1350	0.0054
	1850	0.0050	156	2150	0.0053	159	2350	0.0051	162	1300	0.0052
	1950	0.0049	150	2100	0.0052	150	2300	0.0052	163	1300	0.0054
0.5	5200	0.0027	138	5350	0.0028	129	6700	0.0026	140	4200	0.0030
	5700	0.0025	135	6100	0.0029	134	6500	0.0026	135	4150	0.0028
	5400	0.0026	140	6250	0.0028	137	6600	0.0026	131	3825	0.0031

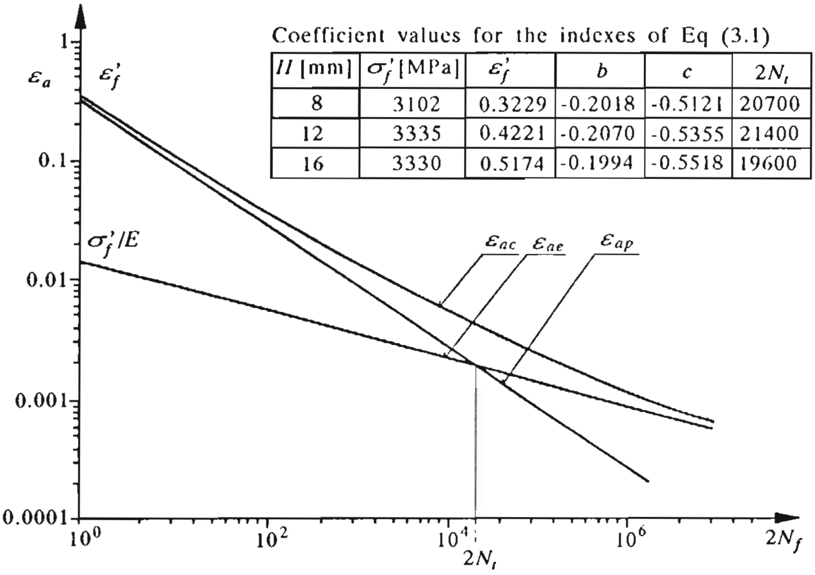
The test results concerning axial bending conditions (cf Szala and Mroziński (1993)) are enclosed in columns 11 and 12 with for the comparative analysis. Fatigue curves were determined in compliance with the Polish Standard PN-84/H-04334, applying the Morrow's formula (Morrow et al. (1965)) in the form

$$\frac{\Delta\varepsilon_{ac}}{2} = \frac{\Delta\varepsilon_{ae}}{2} + \frac{\Delta\varepsilon_{ap}}{2} = \frac{\sigma'_f}{E}(2N_f)^b + \varepsilon'_f(2N_f)^c \tag{3.1}$$

Fig.3 shows a graph of the fatigue life for one height of the specimen ($H = 8$ mm). For the estimation of the specimen height influence on the course of strains in bending specimens the values of the coefficients and exponents of Eq (3.1) for all examined specimens in the table are confronted.

3.2. Static and cyclic properties

Static tests of three specimens of each type were made under variable bending conditions for evaluation of the static properties of 45-steel. The same test machine, which had been used in fatigue investigations was used in the static experiments. Hardening and softening phenomena in 45-steel as a function of the total strain level and the specimen dimensions can be studied



Coefficient values for the indexes of Eq (3.1)

H [mm]	σ_f' [MPa]	ε_f'	b	c	$2N_t$
8	3102	0.3229	-0.2018	-0.5121	20700
12	3335	0.4221	-0.2070	-0.5355	21400
16	3330	0.5174	-0.1994	-0.5518	19600

Fig. 3. Fatigue curves ($\varepsilon_{ac} = 2\%$)

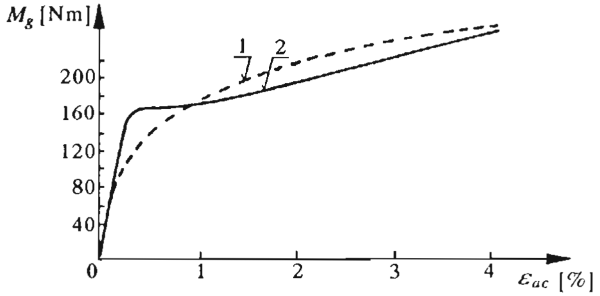


Fig. 4. Plots of static (1) and cyclic (2) bendings, respectively

using the plots of monotonic and cyclic bending, which are presented in Fig.4. Whereas the phenomena of stabilization for particular levels of the total strain and for the different dimensions of the specimen are illustrated on the plots showing the changes of the bending moments as a function of the number of variable loading cycles in Fig.5. The charts are plotted in a coordinate system "stress σ_a – strain ε_{ac}" or "stress σ_a – number of loading reverses $2N_f$" in the case of low-cycle fatigue investigations under axial loading conditions. In the case of bending, the procedure of determination of the stresses is considerably more complicated than in the case of axial loading and the proper model of the material has to be assumed. In this work we only observed changes in the bending moment M_g for estimation of phenomena of stabilization, hardening or softening. Considering the proved slight influence of the height of the specimen on the mutual site of the cyclic stress-strain curve and static tensil curve as well as on the changes in bending moment versus the cycles number these graphs are limited only to the specimen of height $H = 12$ mm.

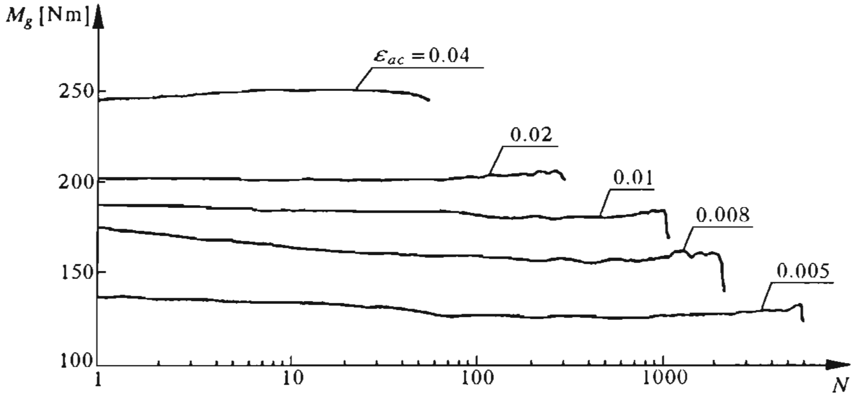


Fig. 5. Changes in bending moments as a function of the number of cycles

4. Test results analysis

4.1. Fatigue life analysis

The fatigue test results analysis was made from the point of view of the specimen height influence (influence of the gradient) and by comparing the test results in the cases of bending tension, respectively. The comparative analysis of coefficients b and c for three kinds of specimens made it possible to show specimen height influence on the plastic strain in a bended specimen. The height of specimen has not affected changes in the elastic strain. The fatigue curves $\varepsilon_{ac} = f(2N_f)$ for bending and axial loading are shown in *Fig.6*. *Taking into account* the course of the fatigue curves in the case of bending we can see that within the whole range of strains, the fatigue lives of particular specimens ratios can be approximately written as: 1 : 1.24 : 1.45 (respectively for specimens H = 8 mm, 12 mm and 16 mm).

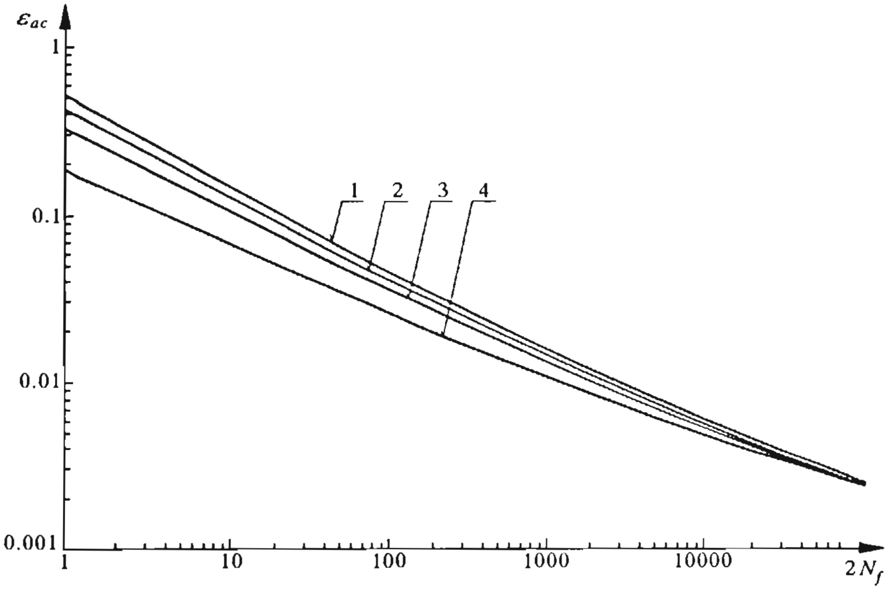


Fig. 6. Fatigue life in the case of bending (1,2,3) and tension (4); 1 – H = 16 mm; 2 – H = 12 mm; 3 – H = 8 mm; 4 – tension

It follows from the presented comparison of the obtained fatigue lives (Fig.6), that the values of fatigue lives of specimens made of 45-steel, determined under bending conditions differ significantly from the fatigue lives determined in the axial bending tests. On same levels of the total strains the differences have been on the average from 30% for low strain levels, up to 300% for large values of total strains. These differences have resulted from the different distributions of strains. These substantial differences in fatigue lives give way to doubts referring to the accuracy of modelling of notch elements by means of the axially loaded smooth specimen.

4.2. Fatigue properties analysis

Analysing Fig.4 it was noted that the cyclic properties of 45-steel under bending conditions depend on the amplitude of the total surface strain. For strains ε_{ac} of less than 0.8%, material has been characterized by cyclic softening. Material has been characterized by cyclic stability in the range of strains: $0.8\% < \varepsilon_{ac} < 1\%$. And throughout all inconsiderable hardening is characteristic of the material in the same case for strains $\varepsilon_{ac} > 1\%$.

The cyclic properties of tested steel agree with those obtained for the same material investigated under axial loading conditions, which have been presented by Szala and Mroziński (1993), Osiński et al. (1988). In evaluation of cyclic features in the case of bending as a function of the total strain on the surface, the course the strains in bending section (Fig.1) should be taken into account.

In the analysis of the changes in bending moment as a function of the number of cycles $M_g = f(N_f)$ (Fig.5), let us note that for realized strains $\varepsilon_{ac} < 2\%$ all this periods of softening, stabilization and inconsiderable hardening have occured during the fatigue tests. For strains $\varepsilon_{ac} = 2\%$ and $\varepsilon_{ac} = 4\%$ the phenomenon of softening is not observed but hardening and stabilization exist. A comparative analysis of changes in the amplitude stress during axial loading presented by Szala and Mroziński (1993) and changes in a bending moment proves great similarity of changes in both parameters.

4.3. Analysis of the stress diagram

Considering a linear strain distribution in the bended specimen it is possible to determine a stress distribution in bended specimens for the realized strain levels. In this paper the influence of strain value and the height of a specimen on changes of same parameters of a strain graph was analysed. Fig.7 shows strain graphs in the bended specimens for one of the values of total strain realized during the testing ($\varepsilon_{ac} = 2\%$). The parameters of the strain diagram can be: the tangent gradient G_{st}, the plastic gradient G_{pl} and a thickness of plastic zone S ($S = H - e/H \times 100\%$).

Strain graphs in bended specimens were plotted by using a cyclic strain diagram, determined on the grounds of author's investigations (cf Szala and Mroziński (1993)). The values of coefficient K' and exponent n', were determined for a tension and compression loading (tension: $K' = 1728$ MPa, $n' = 0.2513$; compression: $K' = 1885$ MPa, $n' = 0.2617$). An elastic strain

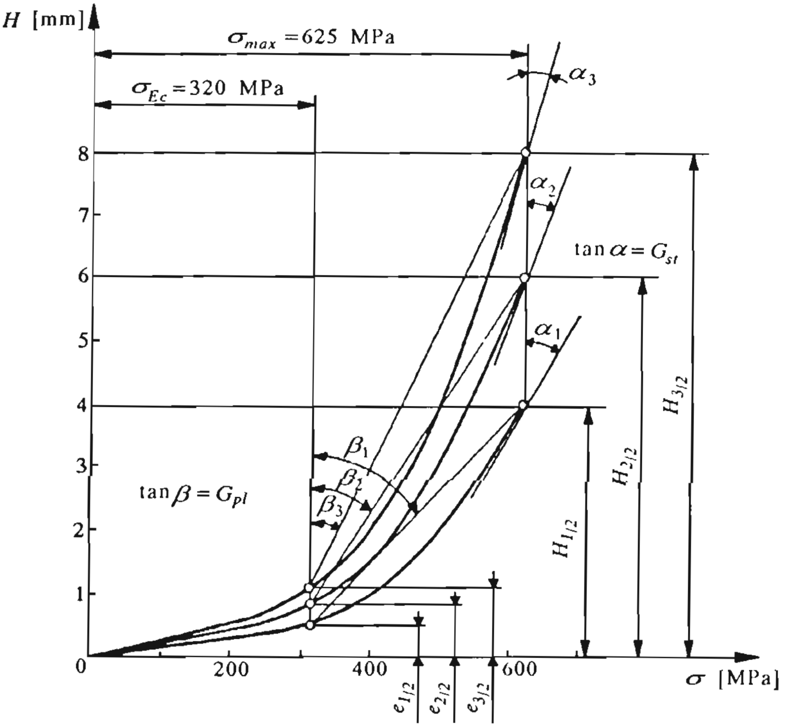


Fig. 7. Plots of stress for bending loading conditions ($\varepsilon_{ac} = 2\%$)

ε_{ac} agreed with a cyclic limit of elasticity σ_{Ec} as determined by a method suggested by Meisel et al. (1993). Due to insignificant differences σ_{Ec} for compression and tension half cycles, the following values: $\sigma_{Ec} = 320$ MPa and $\varepsilon_{ac} = 0.25\%$, were assumed for further analysis. The results of calculating G_{st}, G_{pl} and S, are shown in the Table 3. Fig.8 shows changes in calculated values due to strain ε_{ac} on the specimen surface. An analysis of graphs shown in Fig.8a allows the observation of increase in values of G_{st} and G_{pl} together with increase in the total strains. The largest values of all this parameters were obtained for the specimen of height $H = 8$ mm, and the smallest for the specimen of height $H = 16$ mm.

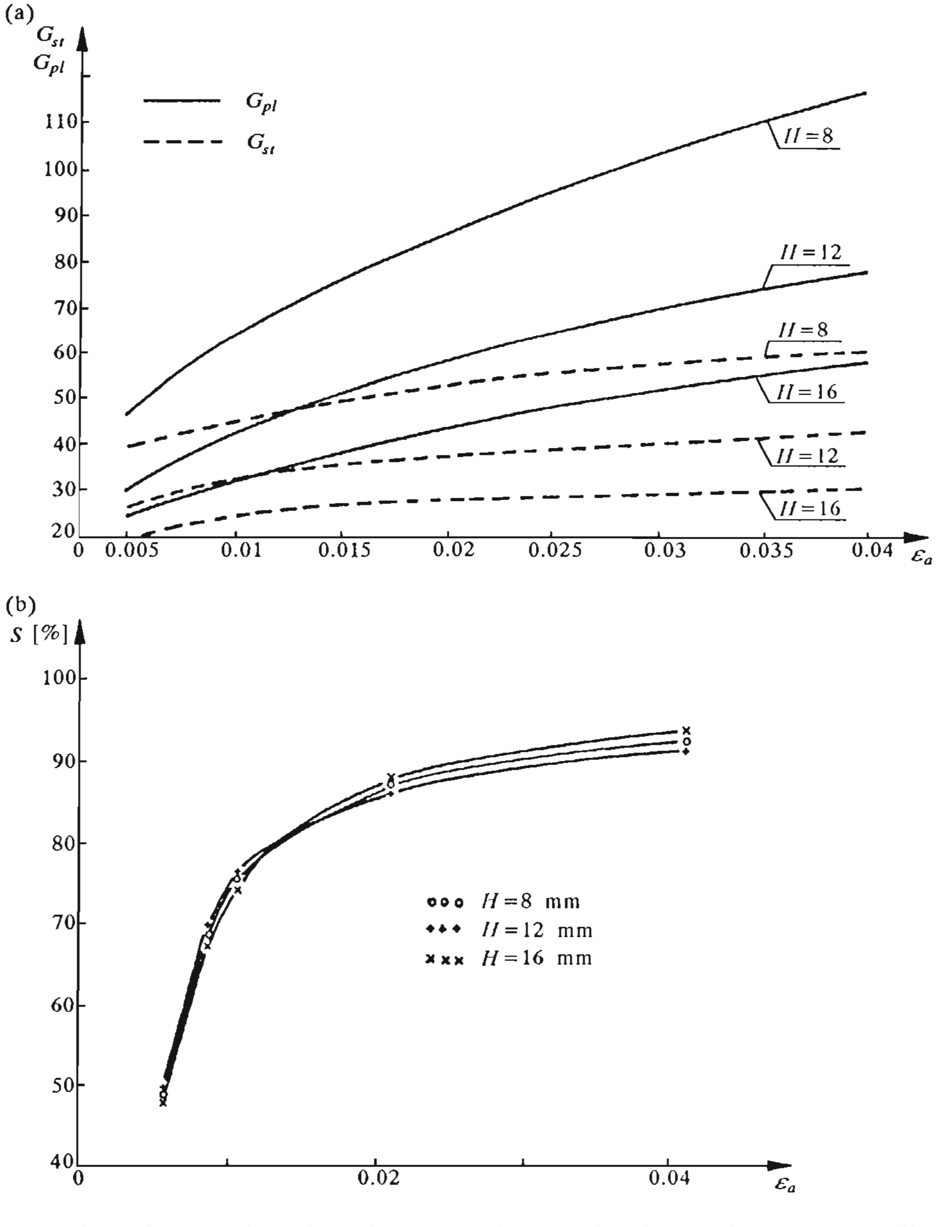


Fig. 8. The influence of total strain on a surface on the changes in tangent gradient G_{st} and plastic gradient G_{pl} (a), a thickness of plastic zone S (b)

Table 3. Comparison of values G_{st}, G_{pl} and S

ε_{ac} [%]	$H = 8$ mm			$H = 12$ mm			$H = 16$ mm		
	G_{st}	G_{pl}	S [%]	G_{st}	G_{pl}	S [%]	G_{st}	G_{pl}	S [%]
1	2	3	4	5	6	7	8	9	10
0.5	40.2	46.3	48	25.3	30.7	48	20.1	24.4	50
0.8	43.1	59.5	70	28.1	40.2	65	22.3	29.1	69
1.0	45.7	64.6	79	34.2	43.5	82	25.7	32.3	75
2.0	52.4	88.2	88	37.4	59.3	87	27.4	44.2	88
4.0	60.5	117.3	94	43.5	78.2	93	30.5	58.7	94

A comparative analysis of the tests results demonstrates the relationship between fatigue life and tangent or plastic stress gradients. The contrary to the gradients G_{st} and G_{pl} a participation of thickness of plastic zone does not depend on the specimen height (Fig.8b). Together with the increase of strain ε_{ac} on a surface the parameter S for all specimens, increases to the maximum values (95% for $\varepsilon_{ac} = 0.04$).

5. Conclusions

Baseing on the performed analysis of results of low-cycle fatigue tests of 45-steel under conditions of plane bending the following conclusions can be drawn:

- The fatigue lives in the normalized 45-steel under conditions of axial loading and plane bending differ significantly for the same total strain. These differences amount to 30% for low strain levels and up to 300% for large values of the total strains.

- Low-cycle fatigue life depends significantly on the height of the specimen. In the whole range of analysed strains specimens differ in fatigue life on the average like 1 : 1.24 : 1.45 respectively for specimens of heights $H = 8$ mm; 12 mm; and 16 mm and these differences increase as the total strains increase.

- Cyclic properties under conditions of plane bending depend on the total strain $\varepsilon_{ac} < 0.8\%$, when the material is subject to softening. In the range $0.8\% < \varepsilon_{ac} < 1.0\%$ the stabilisation is characteristic for the material, wheres for $\varepsilon_{ac} > 1.0\%$ the material characterizes by little hardening. The changes of $M_g = f(N_f)$, for strains $\varepsilon_{ac} = 4\%$ and

$\varepsilon_{ac} = 2\%$, show the stabilization of features in the whole range of life. Periods of softening and stabilization can be seen in the plots of strains $\varepsilon_{ac} = 1.0; 0.8; 0.5\%$.

- The parameters of the stress graph in a bended specimen G_{st} and G_{pl} depend on a strain on the surface and on the height of the specimen. In assumed conditions of testing we proved the influence of a plastic gradient and a tangent stress on the difference in obtained life. The thickness of plastic zone for the same total strains on the surface of a bended specimen doesn't depend on the height of a specimen.
- Modeling of the behaviour of a notched structural component by means of axially loaded smooth specimens causes a shift of the calculation results in a so called "safe assessment area". It arises from the fact that the fatigue life in the case of axial loading is lower than the fatigue life under bending loading conditions.

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Niskocyklowe badania zmęczeniowe stali 45 w warunkach płaskiego zginania

Streszczenie

W pracy przedstawiono wyniki badań zmęczeniowych zrealizowanych w warunkach płaskiego zginania. Próby zmęczeniowe przeprowadzono na pięciu poziomach odkształcenia całkowitego $\varepsilon_{ac} = 0.04$; 0.02; 0.01; 0.008; 0.005. W celu oceny wpływu gradientu odkształcenia na trwałość podczas badań stosowano trzy różne wysokości próbek ($H = 8$ mm; $H = 12$ mm; $H = 16$ mm). Odkształcenia próbki mierzono w strefie występowania maksymalnych odkształceń. Wyniki badań przedstawiono w formie wykresów trwałości, wykresów cyklicznego odkształcenia oraz wykresów naprężeń w próbce w funkcji jej wysokości. Analiza wykresów trwałości wykazała wpływ wysokości próbki na trwałość zmęczeniową. Najwyższe trwałości stwierdzono dla próbki o wysokości $H = 16$ mm, natomiast najniższe dla próbki o wysokości $H = 8$ mm. Analiza porównawcza trwałości w warunkach obciążenia osiowego i płaskiego zginania wykazała występowanie istotnych różnic uzyskanych rezultatatów badań dochodzących do 300% dla dużych odkształceń. Na podstawie wzajemnego położenia wykresów cyklicznego odkształcenia oraz statycznego zginania stwierdzono zależność własności cyklicznych badanej stali od amplitudy odkształcenia całkowitego na powierzchni próbki. Na podstawie analizy wykresów naprężeń w próbkach zginanych stwierdzono wpływ niektórych parametrów wykresów naprężeń na trwałość zmęczeniową.

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