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FREE VIBRATIONS OF RIGID MASSIVE RECTANGULAR FOUNDATIONS EMBEDDED IN A VISCOELASTIC HALF-SPACE

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A method is presented to analyse the free vibration of a rigid massive rectangular foundation buried to some extent beneath the surface of the ground. The response is evaluated using the complex eigenvalue analysis which is mathematically accurate and employs complex, nonclassical vibration modes. Taking the frequency dependence of dynamic impedance functions for the embedded foundation into account, the dimensionless damped natural frequency for each mode is obtained as a single real root of transcendental real frequency equation. The numerical results including the dynamic interaction effects are compared with those obtained at neglected radiation damping in the supporting medium and based on static data. It is shown that the embedment of the foundation and the state of backfill considerably affect its complex eigenvalues.

1. Introduction

Free vibrations of real structures undergo a gradual decrease of amplitude with time. This characteristic of vibration is referred to as damping. For a rigid block bonded to a flexible semi-infinite soil medium, the source of damping is the dissipation of energy by waves propagating away from the foundation. This damping is termed 'radiation damping'. To the radiation damping must be added the material damping which represent the energy dissipated through friction between the soil grains (Richart, Woods and Hall (1970)). The damping due to a dynamic soil-structure interaction does not satisfy the Caughey-O'Kelly condition of classically damped systems (Caughey and O'Kelly (1965)). Such systems are said to be non-classically damped.

Impedances of a supporting soil medium which relate the forces to the displacements depend upon the frequency of the excitation (Wolf (1985)). This dependency distinguishes the considered problem from the standard non-classically damped systems of structural dynamics (Hurty and Rubinstein (1964)).

The objective herein is to present the corresponding data for free vibrations of rigid massive rectangular foundations embedded in a viscoelastic half-space including the frequency-dependent stiffness and damping coefficients of the unbounded supporting medium. A rigid foundation has six degrees of freedom that involve three rotations and three displacements. Therefore, in general, six coupled differential equations are required to describe completely the foundation motion. However, in practice, the problem is reduced to that of solving a smaller number of coupled differential equations as the knowledgeable designer capitalizes on symmetry in order to simplify the dynamic design. Assuming as in a typical problem a symmetry of the vibrating system about two horizontal axes, the vertical translation mode and the torsional rotational mode occur as uncoupled motions while the remaining modes resolve into two independent coupled motions, i.e. rocking-lateral and pitching-longitudinal. The damped natural frequency for each mode of vibration can be obtained as a real root of a transcendental real characteristic equation. If the damped natural frequency is estimated, the modal damping factor as well as the corresponding eigenvector for a coupled mode can be directly obtained from the closed-form formulas.

2. Statement of the problem

The block is assumed to be rigid and massive being perfectly bonded to the semi-infinite supporting medium along the interface Γ. Furthermore, the rigid block displays two orthogonal vertical planes of symmetry and has a rectangular plan $2B \times 2L$, $(L \geq B)$, Fig.1. The origin of the coordinate system is defined at its centre of mass and the coordinate axes are oriented with the principal axes of the rigid body, for which the mass products of inertia are equal to zero.

The response of an inert rigid block can be represented by the dimensionless vector

$$\bar{\mathbf{u}}(t) = \Big[\frac{u_x(t)}{B}, \frac{u_y(t)}{B}, \frac{u_z(t)}{B}, \Theta_x(t), \Theta_y(t), \Theta_z(t)\Big]^{\top} \tag{2.1}$$

which involves the normalized translation $(u_x/B, u_y/B, u_z/B)$ at the point of

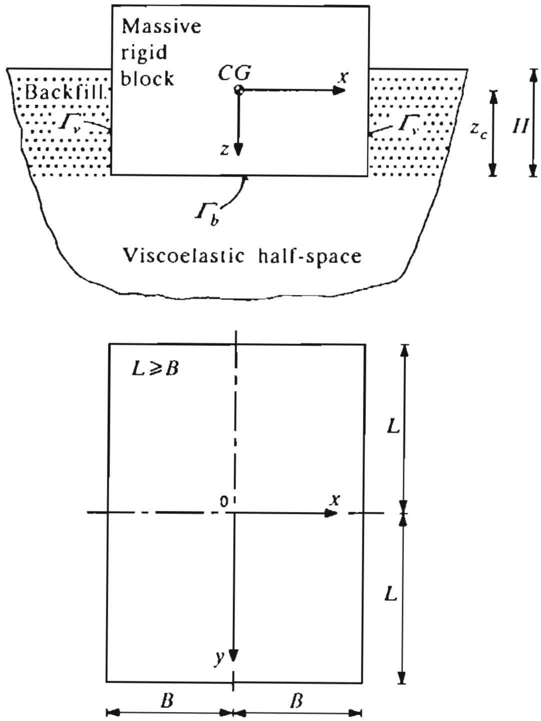

Fig. 1. System considered

reference and the rotation $(\Theta_x, \Theta_y, \Theta_z)$ about the coordinate axes; B denotes the half-width of the rectangular base.

Making use of dimensionless parameters, the equation of coupled free motion of the block can be written as

$$\bar{\mathbf{m}}\ddot{\bar{\mathbf{u}}} = -\bar{\mathbf{r}}(\bar{t}) \tag{2.2}$$

where $\bar{\mathbf{m}}$ is a normalized inertia matrix and $\bar{\mathbf{r}}(\bar{t})$ is a dimensionless reaction vector of a supporting medium. They are given by

$$\bar{\mathbf{m}} = \operatorname{diag}\{b_0, b_0, b_0, B_{rx}, B_{ry}, B_{rz}\} \tag{2.3}$$

$$\bar{\mathbf{r}}(\bar{t}) = \Big[\frac{P_x^r(\bar{t})}{GB^2}, \frac{P_y^r(\bar{t})}{GB^2}, \frac{P_z^r(\bar{t})}{GB^2}, \frac{M_x^r(\bar{t})}{GB^3}, \frac{M_y^r(\bar{t})}{GB^3}, \frac{M_z^r(\bar{t})}{GB^3}\Big]^\top \tag{2.4}$$

where

b_0	–	mass ratio, $b_0 = m/\rho B^3$
B_{rp}	–	inertia ratio, $B_{rp} = I_{mp}/\rho B^5$, $p = x, y, z$
m	–	mass of a rigid block
I_{mp}	–	principal mass moment of inertia of the block, $p = x, y, z$
P_p^r, M_p^r	–	reaction forces and moments of the supporting medium at the point of reference, $p = x, y, z$
G	–	dynamic shear modulus of the half-space
$\bar{t}$	–	dimensionless time, $\bar{t} = (V_s/B)t$
t	–	time
V_s	–	shear-wave velocity in the half-space, $V_s = \sqrt{G/\rho}$
ρ	–	mass density of the half-space material

and $(\dot{\ }) \equiv \frac{d}{dt}$.

To solve the equation of motion (2.2) it is necessary to know the reaction forces and moments of the supporting medium, $\bar{\boldsymbol{r}}(\bar{t})$, due to the general displacement mode $\bar{\boldsymbol{u}}(\bar{t})$ of the block. This relation depends upon the mathematical model of the supporting medium.

3. Model of the supporting medium

The dynamic reactions of supporting medium are developed both below the base Γ_b and on the sides Γ_v (Fig.1). The embedment conditions of the block are very complex as excavating and backfilling disturb the soil. Furthermore, the soil stiffness decrease in direction to the surface and the block may be separated from the soil.

It is difficult to take into account these effects accurately, then approximate approaches are justified. In this paper, it is assumed that the medium under the base of block is modelled as a viscoelastic half-space while the medium above the base is represented by an independent viscoelastic layer (cf Baranov (1967), Novak (1974)). The assumption allows to distinguish between the backfill and the underlying soil.

Distribution of contact stresses results from a solution to mixed boundary-value problem for a half-space and a sidelayer. Integration of the contact stresses over the block-medium interface $\Gamma = \Gamma_b \cup \Gamma_v$ leads to the total dynamic reaction vector at the point of reference

$$\bar{\boldsymbol{r}}(\bar{t}) = \bar{\boldsymbol{r}}^0(\bar{t}) + \bar{\boldsymbol{r}}^s(\bar{t}) \tag{3.1}$$

where $\bar{\boldsymbol{r}}^0(\bar{t}), \bar{\boldsymbol{r}}^s(\bar{t})$ are the reaction vectors of the half-space and of the sidelayer, respectively.

3.1. Base reaction

Distribution of contact stresses at the base of the rectangular foundation is estimated by discretization of the contact area into smaller subregions. The contact stresses within each subregion were considered to be constant and interactions between the regions were included by the flexibility concept of Wong and Luco (1976). Imposing the displacement boundary conditions of rigid body leads to a linear algebraic equation system for the contact stresses. More details on the numerical solution to this problem can be found in Sienkiewicz (1992). Finally, the reaction vector of the half-space is given in the frequency domain by the stiffness relation at the centre of the base of the block, e.g. at point $C(0,0,z_c)$ (Fig.1)

$$\bar{\boldsymbol{r}}_c^0(\bar{t}) = \bar{\mathbf{K}}_c^0(a_0)\bar{\boldsymbol{u}}_c \exp(ia_0\bar{t}) \tag{3.2}$$

where

$\bar{\boldsymbol{r}}_c^0(\bar{t})$	–	vector of reaction forces and moments at point C
$\bar{\boldsymbol{u}}_c$	–	vector of complex-valued amplitudes of displacements at point C
$\bar{\mathbf{K}}_c^0(a_0)$	–	normalized complex impedance matrix
a_0	–	dimensionless frequency factor, $a_0 = \omega B/V_s$
ω	–	angular frequency.

The normalized complex-valued impedance matrix can be written in the form

$$\bar{\mathbf{K}}_c^0(a_0) = \begin{bmatrix} \bar{K}^0_{HxHx} & 0 & 0 & 0 & \bar{K}^0_{HxRy} & 0 \\ 0 & \bar{K}^0_{HyHy} & 0 & \bar{K}^0_{HyRx} & 0 & 0 \\ 0 & 0 & \bar{K}^0_{VV} & 0 & 0 & 0 \\ 0 & \bar{K}^0_{RxHy} & 0 & \bar{K}^0_{RxRx} & 0 & 0 \\ \bar{K}^0_{RyHx} & 0 & 0 & 0 & \bar{K}^0_{RyRy} & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{K}^0_{TT} \end{bmatrix} \tag{3.3}$$

The terms $\bar{K}^0_{HpHp}$, $\bar{K}^0_{RpRp}$, $\bar{K}^0_{HpRq} = \bar{K}^0_{RqHp}$, $\bar{K}^0_{VV}$ and $\bar{K}^0_{TT}$ $(p,q = x,y)$ are the horizontal, rocking, coupling, vertical and torsional impedance functions, respectively.

Eq (3.2) must be transformed to a reference point at the center of mass (Weaver and Johnston (1987))

$$\bar{\boldsymbol{r}}^0(\bar{t}) = \bar{\mathbf{K}}^0(a_0)\bar{\boldsymbol{u}}\exp(\mathrm{i}a_0\bar{t}) \tag{3.4}$$

where

$$\bar{\mathbf{K}}^0(a_0) = \bar{\mathbf{T}}_{0c}\bar{\mathbf{K}}_c^0(a_0)\bar{\mathbf{T}}_{0c}^{\top} \tag{3.5}$$

The dimensionless transformation matrix $\bar{\mathbf{T}}_{0c}$ has the form

$$\bar{\mathbf{T}}_{0c} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & z_c/B & -y_c/B & 1 & 0 & 0 \\ -z_c/B & 0 & -x_c/B & 0 & 1 & 0 \\ y_c/B & x_c/B & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.6}$$

where x_c, y_c and z_c are the scalar components of a position vector $\boldsymbol{r}_{0c}$ directed from the center of mass to point $C(0,0,z_c)$.

It is of advantage to separate the real and imaginary parts of impedance matrix $\bar{\mathbf{K}}^0(a_0)$ in Eq (3.4) as follows

$$\bar{\boldsymbol{r}}^0(\bar{t}) = \Big[\bar{\mathbf{k}}^0(a_0) + \mathrm{i}a_0\bar{\mathbf{c}}^0(a_0)\Big]\bar{\boldsymbol{u}}\exp(\mathrm{i}a_0\bar{t}) = \bar{\mathbf{k}}^0(a_0)\bar{\boldsymbol{u}}(\bar{t}) + \bar{\mathbf{c}}^0(a_0)\dot{\bar{\boldsymbol{u}}}(\bar{t}) \tag{3.7}$$

where

$\bar{\mathbf{k}}^0(a_0)$ – normalized stiffness matrix, $\bar{\mathbf{k}}^0(a_0) = \mathrm{Re}\bar{\mathbf{K}}^0(a_0)$

$\bar{\mathbf{c}}^0(a_0)$ – normalized damping matrix, $\bar{\mathbf{c}}^0(a_0) = (1/a_0)\mathrm{Im}\bar{\mathbf{K}}^0(a_0)$

Re, Im – stand for the real and imaginary parts of the quantity that follows, respectively.

3.2. Side reaction

The disturbed soil adjacent to block sides (backfill) is modelled as an independent viscoelastic layer. The dynamic behaviour of the side-layer is governed by the three dimensional wave equation under the cylindrical plane strain condition. The corresponding boundary value problem for the rigid cylinder coupled with this medium has been solved by Novak, Nogami and Aboul-Ella (1977). The solution is mathematically accurate and the reactions of the medium to the motion of the cylinder are given by means of the closed-form expressions. The results may be used to estimate the reaction vector of the side-layer for the rectangular foundation by converting its rectangular base into an equivalent circular base with appropriate radius for translational and rotational modes of vibrations (Sienkiewicz (1992)).

The force-displacement relationship for the rigid block coupled with the side-layer is conveniently to write at point $E(0,0,z_c - H/2)$ (Fig.1)

$$\bar{\boldsymbol{r}}_e^s(\bar{t}) = \bar{\mathbf{K}}_e^s(a_0^s)\boldsymbol{u}_e \exp(\mathrm{i}a_0\bar{t}) \tag{3.8}$$

where

$\bar{\boldsymbol{r}}_e^s(\bar{t})$	–	vector of reaction forces and moments at point E
$\bar{\boldsymbol{u}}_e$	–	vector of displacement amplitudes at point E
$\bar{\mathbf{K}}_e^s(a_0^s)$	–	normalized complex-valued impedance matrix
a_0^s	–	dimensionless frequency factor for the side-layer, $a_0^s = \omega R/V_s^s$
V_s^s	–	shear-wave velocity in the side-layer, $V_s^s = \sqrt{G_s/\rho_s}$
R	–	radius of an equivalent circular base due to the mode of vibration of the rectangular block
G_s	–	dynamic shear modulus of the side-layer
ρ_s	–	mass density of the side-layer material.

The complex non-dimensional impedance matrix is as follows

$$\bar{\mathbf{K}}_e^s(a_0^s) = \tag{3.9}$$

$$= \begin{bmatrix} \bar{K}_{HxHx}^s(a_{01}^s) & 0 & 0 & 0 & 0 & 0 \\ 0 & \bar{K}_{HyHy}^s(a_{01}^s) & 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{K}_{VV}^s(a_{01}^s) & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{K}_{RxRx}^s(a_{02}^s) & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{K}_{RyRy}^s(a_{03}^s) & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{K}_{TT}^s(a_{04}^s) \end{bmatrix}$$

where the rocking impedance functions $\bar{K}_{RxRx}^s$ and $\bar{K}_{RyRy}^s$ are specified by a superposition of antisymmetric vertical displacements and non-uniform horizontal displacements

$$\bar{K}_{RxRx}^s(a_{02}^s) = \bar{K}_{AxAx}^s(a_{02}^s) + \frac{1}{12}\bar{h}^2\bar{K}_{HxHx}^s(a_{01}^s) \tag{3.10}$$

$$\bar{K}_{RyRy}^s(a_{03}^s) = \bar{K}_{AyAy}^s(a_{03}^s) + \frac{1}{12}\bar{h}^2\bar{K}_{HyHy}^s(a_{01}^s) \tag{3.11}$$

Functions $\bar{K}_{AxAx}^s$ and $\bar{K}_{AyAy}^s$ are the so called antisymmetric impedance functions (cf Novak et al. (1977), Sienkiewicz (1992)) and $\bar{h} = H/B$ is the

embedment ratio.
The dimensionless frequency parameters for the side-layer, a^s_{0j} $(j = 1,2,3,4)$, and for the half-space, a_0, are related by

$$a^s_{0j} = \alpha_j a_0 \qquad j = 1,2,3,4 \tag{3.12}$$

where the coefficients α_j depend on the mode of vibration and can be written as

$$\begin{aligned} \alpha_1 &= \sqrt{\frac{4\lambda\bar{\rho}}{\pi\bar{g}}} & \alpha_2 &= \sqrt[4]{\frac{16\lambda^3}{3\pi}}\sqrt{\frac{\bar{\rho}}{\bar{g}}} \\ \alpha_3 &= \sqrt[4]{\frac{16\lambda}{3\pi}}\sqrt{\frac{\bar{\rho}}{\bar{g}}} & \alpha_4 &= \sqrt[4]{\frac{16\lambda(1+\lambda^2)}{6\pi}}\sqrt{\frac{\bar{\rho}}{\bar{g}}} \end{aligned} \tag{3.13}$$

in which

$$\lambda = \frac{L}{B} \qquad \bar{\rho} = \frac{\rho_s}{\rho} \qquad \bar{g} = \frac{G_s}{G}$$

Eq (3.8) one should transform to the working point at the center of mass

$$\bar{\boldsymbol{r}}^s(\bar{t}) = \bar{\mathbf{K}}^s(a^s_0)\bar{\boldsymbol{u}}\exp(ia_0\bar{t}) \tag{3.14}$$

where

$$\bar{\mathbf{K}}^s(a^s_0) = \bar{\mathsf{T}}_{0e}\bar{\mathbf{K}}^s_e(a^s_0)\bar{\mathsf{T}}^{\mathsf{T}}_{0e} \tag{3.15}$$

The separation of the real and imaginary part of the matrix $\bar{\mathbf{K}}^s(a^s_0)$ in Eq (3.14) leads to

$$\bar{\boldsymbol{r}}^s(\bar{t}) = \Big[\bar{\mathbf{k}}^s(a^s_0) + ia_0\bar{\mathbf{c}}^s(a^s_0)\Big]\bar{\boldsymbol{u}}\exp(ia_0\bar{t}) = \bar{\mathbf{k}}^s(a^s_0)\bar{u}(\bar{t}) + \bar{\mathbf{c}}^s(a^s_0)\dot{\bar{u}}(\bar{t}) \tag{3.16}$$

where $\bar{\mathbf{k}}^s(a^s_0) = \mathrm{Re}\bar{\mathbf{K}}^s(a^s_0)$ is a normalized stiffness matrix of the side-layer and $\bar{\mathbf{c}}^s(a^s_0) = (1/a_0)\mathrm{Im}\bar{\mathbf{K}}^s(a^s_0)$ is the normalized damping matrix.

3.3. Model of material damping

The soil medium is a dissipative material. When strained cyclically, the medium exhibits a hysteretic stress-strain-time relation and energy is dissipated in each cycle, being represented by the area within the hysteresis loop (cf Richart, Woods and Hall (1970); Werno et al. (1985)).

The internal damping of homogeneous isotropic materials in the frequency domain is taken into account by expressing the elastic shear G and bulk K

moduli of the materials as complex-valued quantities $G^*(\mathrm{i}\omega)$ and $K^*(\mathrm{i}\omega)$ (*Nowacki (1963)). The mathematical formulas for the complex moduli can* be based on physical study of a mechanism which removes energy from the oscillating system (cf Solecki and Szymkiewicz (1964), Sorokin (1972)). The form can also result from a priori assumed viscoelastic models.

In soil dynamics, the two parameter Kelvin-Voigt model is often used (cf Sawinow (1972), Lipiński (1985)). But the model displays linear frequency dependent increase of damping what is not supported by measurements. A mechanism of energy dissipation in the soil generates the damping which is almost independent of the frequency of vibration over a finite range of frequencies (Richart, Woods and Hall (1970)). The physical behaviour of soils can be described using frequency-independent complex moduli. However, the constant hysteretic damping model leads to a noncausal behaviour, because the assumption that the dissipation of energy is frequency independent over all frequencies is not compatible with the principle of linearity and causality (cf Bracwell (1978), Ben-Menahem and Singh (1981)).

A more satisfactory model for behaviour of soil which comes under causality principle is a three parameter viscoelastic model consisting of a Kelvin-Voigt model and a linear spring element connected in series (cf Nowacki (1963); Ben-Menahem and Singh (1981)). The model gives the most general linear equation in stress, strain and their first time derivatives. It should be noted that the three parameter model describes the standard test of relaxation and creep with instantaneous elasticity and displays an asymptotic elastic behaviour. This rheological model has been adopted in the following, using the results of Gaul et al. (1988). The complex-valued shear G^* and bulk K^* moduli are given by

$$G^* = G\frac{1 + \mathrm{i}a_0\xi^G}{1 + \mathrm{i}a_0\frac{\xi^G}{2}} \tag{3.17}$$

$$K^* = K\frac{1 + \mathrm{i}a_0\xi^K}{1 + \mathrm{i}a_0\frac{\xi^K}{2}} \tag{3.18}$$

where

a_0 – dimensionless frequency factor

G – elastic shear modulus

K – elastic bulk modulus

ξ^G, ξ^K – non-dimensional damping ratios.

The number of viscous constants can be reduced from two to one putting forward the assumption $\xi^G = \xi^K = \xi$. It implies that the material is similarly viscoelastic in bulk and shear deformations and the Poisson ratio is real-valued, independent of frequency and equal to the Poisson ratio value for the corresponding elastic material. The simplified assumption $\xi^G = \xi^K = \xi$ is frequently used in soil dynamics (Gaul,Klein and Plenge (1988)).

4. Solution to the eigenfrequency problem

The stiffness and damping matrices describing the dynamic properties of the supporting medium in the frequency domain should be taken at a dimensionless frequency a_0^d with which the rigid block vibrates under the influence of initial conditions.

Substituting the formulae for interaction forces (3.7) and (3.16) into Eq (3.1) and (2.2) yields

$$\bar{\mathbf{m}}\ddot{\bar{\mathbf{u}}}(\bar{t}) + \bar{\mathbf{c}}(a_0^d)\dot{\bar{\mathbf{u}}}(\bar{t}) + \bar{\mathbf{k}}(a_0^d)\bar{\mathbf{u}}(\bar{t}) = \mathbf{0} \tag{4.1}$$

where $\bar{\mathbf{c}}$ is the total damping matrix and $\bar{\mathbf{k}}$ is the total stiffness matrix. The total dynamic stiffness coefficients have been re-scaled to give the static stiffnesses of an embedded rectangular foundation at the point of reference when the dimensionless frequency tends to zero.

The homogeneous equation (4.1) governs the free vibration of a rigid massive block supported by a semi-infinite flexible medium. The free motion depends upon the properties of the block and of the supporting medium that appear in the form of the following dimensionless parameters

- mass ratio $b_0 = m/\rho B^3$
- inertia ratios $B_{rp} = I_{mp}/\rho B^5$, $p = x, y, z$
- aspect ratio $\lambda = L/B$
- shear modulus ratio $\bar{g} = G_s/G$
- density ratio $\bar{\rho} = \rho_s/\rho$
- Poisson ratios ν and ν_s of the half-space and of the side-layer, respectively
- damping ratios ξ and ξ_s of the medium beneath the block base and of the side-layer, respectively
- embedment ratio $\bar{h} = H/B$
- block ratio $\bar{z}_c = z_c/B$.

A particular solution to Eq (4.1) is assumed to be of the form

$$\bar{\boldsymbol{u}}(\bar{t}) = \bar{\boldsymbol{e}}\exp(\mathrm{i}\bar{s}_0\bar{t}) \tag{4.2}$$

where $\bar{\boldsymbol{e}}$ denotes a dimensionless complex-valued eigenvector and $\bar{s}_0 = a_0^d + \mathrm{i}\delta_0^d$ is a normalized complex frequency. The solution contains an harmonic component of frequency a_0^d (dimensionless damped natural frequency) and a vibration decay component with dimensionless damping factor δ_0^d.

Substituting the solution (4.2) into the equation of motion (4.1) gives a set of equations

$$\Big[-\bar{s}_0^2\bar{\mathbf{m}} + \mathrm{i}\bar{s}_0\bar{\mathbf{c}}(\mathrm{Re}\bar{s}_0) + \bar{\mathbf{k}}(\mathrm{Re}\bar{s}_0)\Big]\bar{e} = \mathbf{0} \tag{4.3}$$

The non-trivial solution to Eq (4.3) requires that

$$\det\Big[-\bar{s}_0^2\bar{\mathbf{m}} + \mathrm{i}\bar{s}_0\bar{\mathbf{c}}(\mathrm{Re}\bar{s}_0) + \bar{\mathbf{k}}(\mathrm{Re}\bar{s}_0)\Big] = 0 \tag{4.4}$$

This requirement leads to a complex characteristic equation which has coefficients dependent upon a damped natural frequency $a_0^d = \mathrm{Re}\bar{s}_0$ that is to be searched. Then, the characteristic complex value $\bar{s}_0$ may be found only by an iteration process. Starting with an arbitrary small value of a_0^d, one can form an increasing sequence

$$a_0^{(1)} < a_0^{(2)} < \dots < a_0^{(n)} \tag{4.5}$$

For each value of $a_0^{(j)}$, the coefficients of frequency equation are fixed and the complex roots of the characteristic equation may be found using suitable algorithms. The criterion of selection is

$$|\mathrm{Re}\bar{s}_0 - a_0^{(j)}| < \varepsilon \qquad\qquad \mathrm{Im}\bar{s}_0 > 0 \tag{4.6}$$

where ε is an error tolerance (accuracy tolerance).

However, implementation of this general iterative scheme may leads to some numerical problems and even though the problems might be overcome, it is desirable to take advantage of the symmetry of the considered block-medium system. Namely, explicit formulae for the complex characteristic values $\bar{s}_{0j}$ may be established directly from the characteristic equation. Then, the damped natural frequency of the jth mode, a_{0j}^d, is obtained as the real root of the following jth real transcendental equation

$$f_j(a_0^d) \equiv \mathrm{Re}\bar{s}_{0j}(a_0^d) - a_0^d = 0 \tag{4.7}$$

Therefore, the problem is to find zero of the real frequency function $f_j(a_0^d)$, what can be done by well established and reliable procedures (cf Dahlquist and Björck (1974), Traub (1982), Stoer and Burlirsch (1983)).

If the natural frequency of the jth mode is estimated, the modal damping factor δ_{0j}^d follows from equation

$$\delta_{0j}^d = \operatorname{Im} \bar{s}_{0j}(a_{0j}^d) \tag{4.8}$$

For underdamped vibration modes, the characteristic values occur in complex dual pairs

$$\begin{aligned} \bar{s}_{0j} &= a_{0j}^d + \mathrm{i}\delta_{0j}^d \\ \bar{\bar{s}}_{0j} &= -a_{0j}^d + \mathrm{i}\delta_{0j}^d \end{aligned} \tag{4.9}$$

To the pair of the characteristic values (4.9) corresponds a complex conjugate pair of characteristic vectors $\bar{\boldsymbol{e}}$, $\bar{\bar{\boldsymbol{e}}}$.

For the sake of brevity, the explicit form of corresponding characteristic equations is presented only for the vertical mode of vibration.

4.1. Vertical mode of vibration

— frequency equation

$$f_v(a_0^d) \equiv \sqrt{\frac{\bar{k}_{33}(a_0^d)}{b_0} - \Big(\frac{\bar{c}_{33}(a_0^d)}{2b_0}\Big)^2} - a_0^d = 0 \tag{4.10}$$

— root a_{0v}^d

— modal damping factor δ_{0v}^d

$$\delta_{0v}^d = \frac{\bar{c}_{33}(a_{0v}^d)}{2b_0} \tag{4.11}$$

Note that the characteristic frequency equations contain stiffness $\bar{k}_{ij}$ and damping $\bar{c}_{ij}$ coefficients known only approximately what results from the approximate numerical solution to the corresponding mixed-boundary value problem in elastodynamics. Despite the fact that the very aim of finding the exact roots of the equations is meaningless, the approximate real roots can be found to the specified degree of accuracy. However, if the characteristic equation no roots within its domain of existence, the considered mode of vibration is overdamped.

5. Free vibration

Solution for the jth oscillating vibration mode is given by

$$\bar{\boldsymbol{u}}_j(\bar{t}) = \bar{A}_j \bar{\boldsymbol{e}}_j \exp(\mathrm{i}\bar{s}_{0j}\bar{t}) + \bar{\bar{A}}_j \bar{\bar{\boldsymbol{e}}}_j \exp(\mathrm{i}\hat{\bar{s}}_{0j}\bar{t}) \tag{5.1}$$

in which $\bar{A}_j$ is a complex-valued constant and $\bar{\bar{A}}_j$ is its complex conjugate.

The response of the block-medium system in coupled lateral and rocking motion to an arbitrary initial excitation is given by the superposition of two modal solutions

$$\bar{\boldsymbol{u}}(\bar{t}) = 2\sum_{j=1}^{2} \mathrm{Re}[\bar{A}_j \bar{\boldsymbol{e}}_j \exp(\mathrm{i}\bar{s}_{0j}\bar{t})] \tag{5.2}$$

Using the following decomposition

$$2\bar{A}_j \bar{\boldsymbol{e}}_j = \boldsymbol{\beta}_j + \mathrm{i}\boldsymbol{\gamma}_j \tag{5.3}$$

and the well-known identity between exponential and trigonometric functions, Eq (5.2) may be written as

$$\bar{\boldsymbol{u}}(\bar{t}) = \sum_{j=1}^{2} \exp(-\delta_{0j}^{d}\bar{t})\Big[\boldsymbol{\beta}_j \cos(a_{0j}^{d}\bar{t}) - \boldsymbol{\gamma}_j \sin(a_{0j}^{d}\bar{t})\Big] \tag{5.4}$$

where

$$\boldsymbol{\beta}_j = \mathrm{Re}(2\bar{A}_j \bar{\boldsymbol{e}}_j) \tag{5.5}$$

$$\boldsymbol{\gamma}_j = \mathrm{Im}(2\bar{A}_j \bar{\boldsymbol{e}}_j) \tag{5.6}$$

The real-valued dimensionless vectors $\boldsymbol{\beta}_j$ and $\boldsymbol{\gamma}_j$ are expressed in terms of the real and imaginary parts of the complex-valued characteristic vector $\bar{\boldsymbol{e}}_j$ and the complex-valued participation factor $\bar{A}_j$.

The characteristic vector $\bar{\boldsymbol{e}}_j$ can be normalized such that its first element has a unit real part and the corresponding imaginary part is zero

$$\bar{\boldsymbol{e}}_j = \begin{bmatrix} 1 \\ \bar{e}_{j2} \end{bmatrix} \qquad j = 1,2 \tag{5.7}$$

where

$$\bar{e}_{j2} = -\frac{\varepsilon_{j1} + \mathrm{i}\varepsilon_{j2}}{\varepsilon_{j3} + \mathrm{i}\varepsilon_{j4}} \qquad \text{or} \qquad \bar{e}_{j2} = -\frac{\varepsilon_{j3} + \mathrm{i}\varepsilon_{j4}}{\varepsilon_{j5} + \mathrm{i}\varepsilon_{j6}} \tag{5.8}$$

and

$$
\begin{aligned}
\varepsilon_{j1} &= \bar{k}_{11}(a_{0j}^d) - b_0[(a_{0j}^d)^2 - (\delta_{0j}^d)^2] - \delta_{0j}^d \bar{c}_{11}(a_{0j}^d) \\
\varepsilon_{j2} &= a_{0j}^d \bar{c}_{11}(a_{0j}^d) - 2b_0 a_{0j}^d \delta_{0j}^d \\
\varepsilon_{j3} &= \bar{k}_{15}(a_{0j}^d) - \delta_{0j}^d \bar{c}_{15}(a_{0j}^d) \\
\varepsilon_{j4} &= a_{0j}^d \bar{c}_{15}(a_{0j}^d) \\
\varepsilon_{j5} &= \bar{k}_{55}(a_{0j}^d) - B_{ry}[(a_{0j}^d)^2 - (\delta_{0j}^d)^2] - \delta_{0j}^d \bar{c}_{55}(a_{0j}^d) \\
\varepsilon_{j6} &= a_{0j}^d \bar{c}_{55}(a_{0j}^d) - 2B_{ry} a_{0j}^d \delta_{0j}^d
\end{aligned}
\tag{5.9}
$$

It can be shown that the complex-valued dimensionless participation factor $\bar{A}_j$ is given by

$$
\bar{A}_j = \frac{1}{\alpha_{j3}} \Big[\alpha_{j1}\bar{u}_1(0) + b_0\dot{\bar{u}}_1(0) + \alpha_{j2}\bar{u}_2(0) + B_{ry}\bar{e}_{j2}\dot{\bar{u}}_2(0)\Big] \tag{5.10}
$$

where

$$
\begin{aligned}
\alpha_{j1} &= (\mathrm{i}a_{0j}^d - \delta_{0j}^d)b_0 + \bar{c}_{11}(a_{0j}^d) + \bar{e}_{j2}\bar{c}_{15}(a_{0j}^d) \\
\alpha_{j2} &= (\mathrm{i}a_{0j}^d - \delta_{0j}^d)\bar{e}_{j2}B_{ry} + \bar{c}_{15}(a_{0j}^d) + \bar{e}_{j2}\bar{c}_{55}(a_{0j}^d) \\
\alpha_{j3} &= (\mathrm{i}a_{0j}^d - \delta_{0j}^d)(b_0 + B_{ry}\bar{e}_{j2}^2) + \alpha_{j1} + \alpha_{j2}\bar{e}_{j2}
\end{aligned}
\tag{5.11}
$$

and $\bar{u}_1(0)$, $\bar{u}_2(0)$ are the presummed initial dimensionless displacements and $\dot{\bar{u}}_1(0)$, $\dot{\bar{u}}_2(0)$ are the corresponding initial dimensionless velocities in coupled lateral and rocking free vibration (two degrees of freedom).

For uncoupled modes of vibration (one degree of freedom), the response of the system to presummed initial excitation is given by

$$
\bar{u}(\bar{t}) = \exp(-\delta_0^d\bar{t})[A_1\cos(a_0^d\bar{t}) - A_2\sin(a_0^d\bar{t})] \tag{5.12}
$$

where

$$
A_1 = \bar{u}(0) \qquad\qquad A_2 = -\frac{\dot{\bar{u}}(0) + \delta_0^d\bar{u}(0)}{a_0^d} \tag{5.13}
$$

6. Numerical results

Vertical and coupled lateral and rocking modes of vibration of a rigid rectangular massive foundation embedded in a viscoelastic half-space have been selected to make a parametric study.

Natural frequency of an underdamped mode a_{0i}^d is searched for as a single real root of the corresponding real transcendental equation. The process of finding the root consists of two stages:

- Establishing the possible smallest interval containing the root
- Reducing the interval to the specified degree of accuracy.

In the developed computer procedure, the selection of interval in which the function changes sign is done by the user and then the refining of the root is carried out by the bisection method combined with the secant method and the inverse quadratic interpolation (Forsythe, Malcolm and Moler (1977)) what gives the efficient and reliable algorithm. The length of the interval of uncertainty of the zero of the transcendental function was assumed as 10^{-6}.

It is claimed that the modelling of the embedment of the foundation by means of a local boundary gives results which are remarkably close to the results obtained by rigorous approaches. To verify this claim, the vertical vibration mode of a rigid massive square foundation embedded in a uniform elastic half-space was analysed by use the impedance functions resulting from the approach adopted in this study as well as obtained by a hybrid approach of Mita and Luco (1987). The hybrid approach was based on the use of Green's functions for the half-space combined with the finite element discretization of the finite portion of soil excavated for the foundation. This hybrid approach can attain excellent accuracy compared with solutions obtained by other methods as the finite element method and the indirect boundary integral equation approach (Apsel and Luco (1987)). The numerical values for the impedance functions presented by Mita and Luco (1989) in tabular form were completed by a spline interpolation (Ferziger (1981)).

The dimensionless damped natural frequency a_{0v}^d and the modal damping factor δ_{0v}^d calculated by use the two approaches to determine the impedance functions of embedded foundations are presented in Table 1 for a number of embedment ratios H/B. Inspection of Table 1 reveals that the agreement between the two sets of natural frequencies and modal damping factors is quite satisfactory.

Using the approach based on the local modelling of the embedment, the effect of soil-structure interaction on the complex eigenvalues of the rectangular foundation has been studied. The results illustrating the effect of embedment ratio $\bar{h}$ are shown in Fig.2 for the vertical mode and in Fig.3 for the coupled lateral and rocking mode of vibration, respectively. Full lines represent the values of damped natural frequency a_{0i}^d and modal damping factor δ_{0i}^d affected by the radiation and material damping and crossed lines show the values of

a^d_{0i} and δ^d_{0i} under control of the radiation damping only. Dotted lines in these figures show the undamped natural frequency a^n_{0i} computed as the root of the corresponding transcendental frequency equation at suppressed radiation damping and rhombed lines represent the values of natural frequency a^{ws}_{0i} computed from static data for the stiffness parameters.

Table 1. Vertical mode of vibration of a rigid massive square foundation embedded in a uniform half-space ($b_0 = 10$, $\nu = 0.25$). Comparison of the dimensionless damped natural frequency a^d_{0v} and modal damping factor δ^d_{0v} calculated by use two approaches to determine the impedance functions of embedded foundations; 1 – a hybrid approach of Mita and Luco (1987) and (1989), 2 – modelling of embedment by local boundary.

H/B	1		2	
	a^d_{0v}	δ^d_{0v}	a^d_{0v}	δ^d_{0v}
0.00	0.723	0.278	0.709	0.292
0.05	0.732	0.302	0.710	0.311
0.10	0.737	0.322	0.711	0.330
0.15	0.740	0.342	0.711	0.350
0.20	0.743	0.361	0.711	0.369
0.25	0.744	0.379	0.710	0.389
0.30	0.745	0.398	0.709	0.408
0.35	0.745	0.416	0.708	0.428
0.40	0.744	0.434	0.705	0.447
0.45	0.742	0.452	0.703	0.467
0.50	0.740	0.470	0.699	0.486
0.55	0.736	0.488	0.695	0.506
0.60	0.732	0.506	0.691	0.525
0.65	0.727	0.524	0.686	0.545
0.70	0.721	0.542	0.680	0.565
0.75	0.715	0.560	0.674	0.585
0.80	0.708	0.579	0.667	0.604
0.85	0.700	0.597	0.659	0.624
0.90	0.692	0.616	0.650	0.645
0.95	0.682	0.635	0.640	0.665
1.00	0.672	0.653	0.629	0.685

Examining Fig.2 and Fig.3, it can be seen, that the natural frequency for the vertical oscillation a^d_{0v} and the second natural frequency for the coupled mode a^d_{02} are highly damped and the damping significantly increases with the embedding of the foundation. It is demonstrated by curves representing the

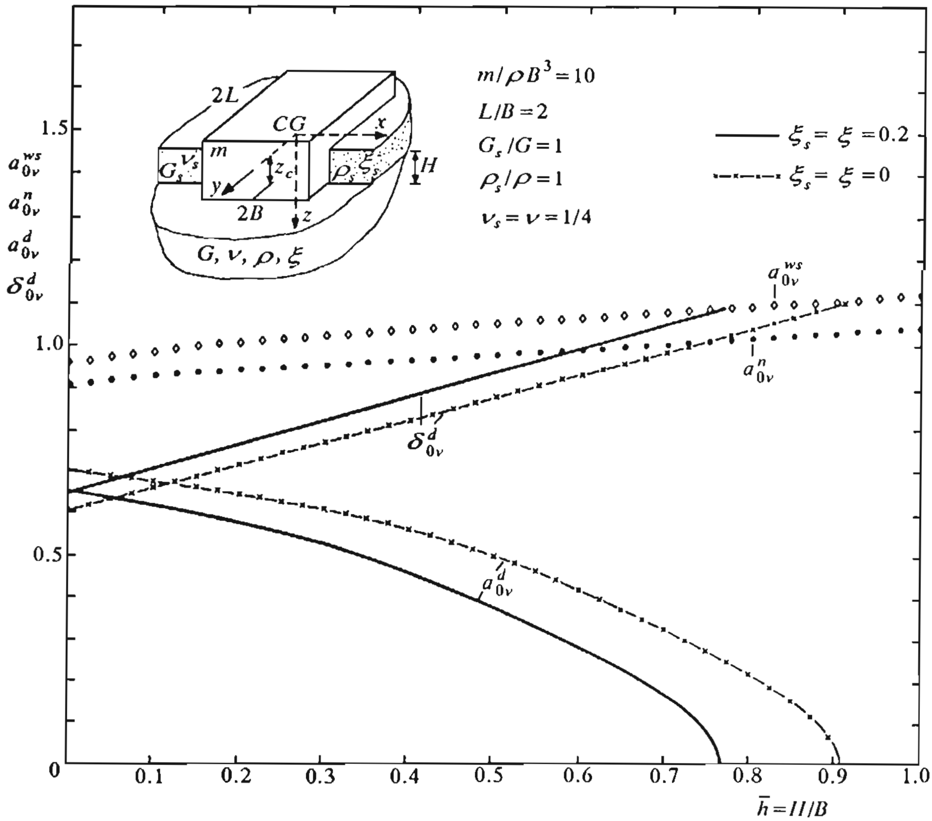

Fig. 2. Vertical mode of vibration. Variation of dimensionless natural frequencies a^d_{0v}, a^n_{0v}, a^{ws}_{0v} and modal damping factor δ^d_{0v} with the embedment ratio $\bar{h}$

modal damping factors δ^d_{0v} and δ^d_{02}. However, the modal damping factor of the first mode δ^d_{01} for the coupled rocking-lateral oscillation is low and the values of the first damped natural frequency a^d_{01} virtually coincide with the values of the first undamped natural frequency a^n_{01}.

Differencies between values of a^d_{0i} and a^n_{0i} indicate the effect of damping arising from the soil-structure interaction on the natural frequencies while the differences between a^n_{0i} and a^{ws}_{0i} illustrate the effect of inertia of the supporting medium.

It is known that the excavating and the backfilling disturb the soil surrounding the foundation. The backfill can reduce the embedment effectiveness quite substantially. In the considered model it is taken into account distinguishing between the shear modulus G_s and the mass density ρ_s of the sidelayer

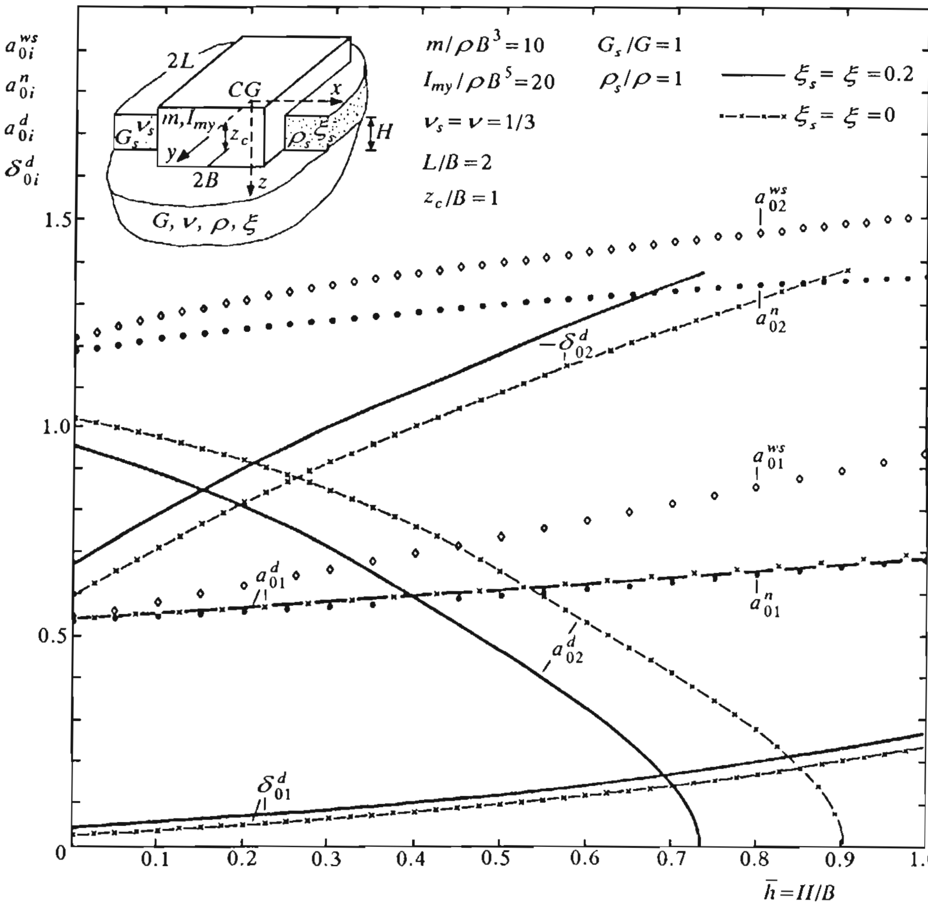

Fig. 3. Coupled rocking and lateral mode of vibration. Variation of dimensionless natural frequencies a_{0i}^d, a_{0i}^n, a_{0i}^{ws}, $i=1,2$, and modal damping factors δ_{01}^d, δ_{02}^d with the embedment ratio $\bar{h}$

from the shear modulus G and the mass density ρ of the underlying half-space assuming $G_s/G<1$ and $\rho_s/\rho<1$. The results for the vertical mode are given in Table 2 and for the coupled rocking and lateral mode of vibration in Tables 3 and 4.

Table 2. Vertical mode of vibration. Effect of state of backfill on dimensionless damped natural frequency a_{0v}^d and modal damping factor δ_{0v}^d of rigid massive rectangular foundation embedded in elastic or viscoelastic medium ($b_0 = 10$, $L/B = 2$, $\nu = 0.25$).

H/B	$\frac{G_s}{G} = 1$; $\frac{\rho_s}{\rho} = 1$		$\frac{G_s}{G} = 0.7$; $\frac{\rho_s}{\rho} = 0.85$		$\frac{G_s}{G} = 0.5$; $\frac{\rho_s}{\rho} = 0.75$	
	a_{0v}^d	δ_{0v}^d	a_{0v}^d	δ_{0v}^d	a_{0v}^d	δ_{0v}^d
elastic supporting medium $\xi_s = \xi = 0$						
0.00	0.698	0.604	0.698	0.604	0.698	0.604
0.10	0.674	0.656	0.679	0.644	0.682	0.635
0.20	0.644	0.709	0.656	0.684	0.664	0.667
0.30	0.607	0.762	0.630	0.725	0.644	0.699
0.40	0.562	0.817	0.600	0.765	0.622	0.731
0.50	0.504	0.873	0.564	0.807	0.597	0.763
0.60	0.419	0.934	0.522	0.849	0.569	0.795
0.70	0.325	0.990	0.468	0.893	0.537	0.828
0.80	0.223	1.045	0.392	0.942	0.499	0.862
0.90	0.063	1.101	0.312	0.987	0.453	0.897
1.00	–	–	0.249	1.030	0.392	0.934
viscoelastic supporting medium $\xi_s = \xi = 0.2$						
0.00	0.647	0.649	0.647	0.649	0.647	0.649
0.10	0.613	0.704	0.620	0.691	0.625	0.682
0.20	0.572	0.759	0.590	0.733	0.601	0.715
0.30	0.522	0.816	0.545	0.776	0.574	0.748
0.40	0.457	0.874	0.513	0.819	0.544	0.782
0.50	0.372	0.934	0.463	0.864	0.510	0.816
0.60	0.281	0.991	0.397	0.911	0.471	0.851
0.70	0.170	1.049	0.322	0.957	0.424	0.887
0.80	–	–	0.258	1.001	0.362	0.925
0.90	–	–	0.194	1.045	0.294	0.963
1.00	–	–	0.123	1.090	0.248	0.998

Table 3. Coupled rocking and lateral mode of vibration. Effect of state of backfill on dimensionless damped natural frequencies a_{01}^d, a_{02}^d and modal damping factors δ_{01}^d, δ_{02}^d of rigid massive rectangular foundation embedded in elastic medium ($b_0 = 10$, $B_{ry} = 20$, $L/B = 2$, $z_c/B = 1$, $\nu_s = \nu = 1/3$, $\xi_s = \xi = 0$).

H/B	First mode		Second mode	
	a_{01}^d	δ_{01}^d	a_{02}^d	δ_{02}^d
$G_s/G = 1$; $\rho_s/\rho = 1$				
0.00	0.541	0.029	1.020	0.599
0.10	0.554	0.040	0.975	0.712
0.20	0.569	0.053	0.917	0.818
0.30	0.583	0.068	0.847	0.915
0.40	0.599	0.084	0.761	1.005
0.50	0.615	0.103	0.652	1.090
0.60	0.631	0.124	0.530	1.167
0.70	0.648	0.147	0.409	1.239
0.80	0.665	0.173	0.275	1.309
0.90	0.683	0.202	0.055	1.376
1.00	0.700	0.235	–	–
$G_s/G = 0.5$; $\rho_s/\rho = 0.75$				
0.00	0.541	0.029	1.020	0.599
0.10	0.547	0.036	0.991	0.668
0.20	0.554	0.044	0.957	0.732
0.30	0.562	0.053	0.920	0.792
0.40	0.570	0.063	0.879	0.847
0.50	0.578	0.075	0.835	0.897
0.60	0.588	0.089	0.787	0.943
0.70	0.597	0.105	0.735	0.984
0.80	0.608	0.123	0.678	1.023
0.90	0.619	0.143	0.612	1.058
1.00	0.631	0.167	0.533	1.091

Table 4. Coupled rocking and lateral mode of vibration. Effect of state of backfill on dimensionless damped natural frequencies a_{01}^d, a_{02}^d and modal damping factors δ_{01}^d, δ_{02}^d of rigid massive rectangular foundation embedded in viscoelastic medium ($b_0 = 10$, $B_{ry} = 20$, $L/B = 2$, $z_c/B = 1$, $\nu_s = \nu = 1/3$, $\xi_s = \xi = 0.2$)

H/B	First mode		Second mode	
	a_{01}^d	δ_{01}^d	a_{02}^d	δ_{02}^d
$G_s/G = 1$; $\rho_s/\rho = 1$				
0.00	0.540	0.045	0.954	0.671
0.10	0.553	0.057	0.887	0.790
0.20	0.567	0.071	0.806	0.900
0.30	0.581	0.087	0.708	1.001
0.40	0.596	0.106	0.586	1.096
0.50	0.611	0.126	0.457	1.183
0.60	0.626	0.149	0.322	1.266
0.70	0.642	0.174	0.148	1.346
0.80	0.658	0.203	–	–
0.90	0.673	0.234	–	–
1.00	0.688	0.270	–	–
$G_s/G = 0.5$; $\rho_s/\rho = 0.75$				
0.00	0.540	0.045	0.954	0.671
0.10	0.546	0.052	0.911	0.744
0.20	0.553	0.061	0.863	0.811
0.30	0.560	0.071	0.811	0.873
0.40	0.567	0.082	0.756	0.930
0.50	0.575	0.096	0.696	0.982
0.60	0.584	0.111	0.631	1.029
0.70	0.593	0.128	0.556	1.073
0.80	0.602	0.147	0.477	1.112
0.90	0.612	0.170	0.418	1.149
1.00	0.622	0.195	0.370	1.185

7. Conclusions

It is well known from the structural dynamics that knowledge of the natural frequencies, the modal damping factors and the corresponding mode shapes of the linear system provides the insight into its dynamic action. The physical insight into the dynamic behaviour of a rigid massive rectangular block on soil medium has been realized by a continuum approach to modelling the soil taking the frequency dependence of foundation impedance functions into account. The impedance functions for the three-dimensional rigid embedded foundation have been determined by a procedure that takes due cognizance of the mixed boundary conditions at the surface of the half-space and the local boundary conditions at the vertical sides of the embedded block.

The natural frequencies obtained by including the dynamic interaction between the rigid massive foundation and the semi-infinite flexible medium have been compared with those obtained at neglected radiation damping in the elastic supporting medium and based on the static data. It indicates the significance of including interaction effects and adequate modelling of mechanisms governing dynamic soil-structure interaction in the presence of uncertainties which are an inherent part of engineering.

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Drgania swobodne sztywnych masywnych fundamentów o podstawie prostokątnej zagłębionych w półprzestrzeni lepkosprężystej

Streszczenie

Przedstawiono metodę analizy drgań swobodnych sztywnego masywnego fundamentu o podstawie prostokątnej, zagłębionego w podłożu. Zastosowano zespoloną analizę modalną wykorzystującą zespolone wartości własne oraz zespolone postacie drgań. Uwzględniając zależność zespolonych dynamicznych funkcji sztywności fundamentu od częstości drgań, bezwymiarową częstość drgań własnych tłumionych rozważanej formy głównej otrzymano jako pojedyńczy pierwiastek rzeczywisty odpowiedniego przestępnego równania charakterystycznego.

Wyniki obliczeń z uwzględnieniem dynamicznego oddziaływania pomiędzy fundamentem i podłożem porównano z wynikami otrzymanymi przy pominięciu tłumienia oraz z wynikami opartymi na danych statycznych. Pokazano, że zagłębienie fundamentu oraz stan gruntu otaczającego fundament wywierają istotny wpływ na zespolone wartości własne fundamentu.

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