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AVERAGED FORMULATIONS OF NON-STATIONARY PROBLEMS FOR STRATIFIED POROUS MEDIA

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The aim of this contribution is to investigate the length scale effect on the behaviour of periodically stratified saturated porous media, representing subsoils made of thin silt and clay layers or periodically reinforced soil structures. To this end two engineering models of a layered subsoil are formulated: the structural model, involving the thickness of the soil layering and the effective modulus model, where the layering thickness is scaled down. It is shown that in a description of non-stationary processes the length-scale effects play the crucial role and cannot be neglected.

1. Introduction

The subject of this paper is a periodically layered two constituent fully saturated porous medium, representing a stratified subsoil (made of warved clays or representing periodically reinforced soil structures) in a consolidation process. As it is known, due to the heterogeneity of many stratified media their direct description is very complicated and cannot be used for calculational purposes. Hence, different averaged models of these media have to be formulated and applied to analysis of engineering problems. The aim of this contribution is to propose and compare two averaged models:

(i) The structural model, describing the layering length-scale effect on the behaviour of the medium

(ii) The effective modulus (homogenized) model, where the layering thickness is scaled down and hence, the length-scale effects related to the periodic structure of the medium are neglected.

The final result of this contribution consists of the fact that the proposed structural model of a stratified medium under consideration yields a deeper insight into the non-stationary phenomena then that resulting from the asymptotic homogenization procedure.

The considerations will be carried out taking into account the simplified linear consolidation theory for fully saturated linear-elastic porous media (de Josselin de Jong (1963), Verruijt (1969)) and introducing certain inertial effects related to the skeleton motion. More general starting point, taking into account the concept of partial stresses (Biot (1941)) and the dynamic coupling between motions of a skeleton and a liquid, will be treated separately. The obtained results can be applied to soil consolidation processes in which macro-inertia forces are neglected.

The proposed macro-modelling method for a periodically stratified medium constitutes a generalization of the refined approach to the macro-elastodynamics of periodic composites (Woźniak (1993), Woźniak at al. (1993)), being based on a similar procedure.

Denotations. Throughout the paper subscripts i, j, k, l run over $1, 2, 3$ being related to the orthogonal cartesian coordinate system $0x_1x_2x_3$ in the physical space. The region in this space occupied by the undeformed periodically layered saturated medium is denoted by Ω while Ω', Ω'' stand for collections of thin layers of Ω made of two separate constituents. We assume that the x_3-axis is normal to the interfaces between layers, by l', l'' we denote the thicknesses of the adjacent layers occupied by two constituents of the stratified medium and hence $l \equiv l' + l''$ is the thickness of representative (repetitive) layer of the subsoil. The points of the physical space are denoted by $\boldsymbol{x} \equiv (x_1, x_2, x_3)$ and t stands for the time coordinate. The remaining denotations, related to the direct description of the two-component micro-heterogeneous porous subsoil are:

u_i – displacements

s_{ij} – effective stresses

s_i – boundary tractions

b_i – body forces

ρ – mass density

C_{ijkl} – elastic modulae

n_i – unit normal outward to the boundary $\partial\Omega$ of Ω

p – pore fluid pressure

q_i – pore fluid discharge

q – fluid outflow across boundary

β – fluid compressibility modulus
η – porosity
k_{ij} – permeability modulae
e – pore fluid dilatational strain.

The constituents of the medium are assumed to be homogeneous and isotropic; hence

$$C_{ijkl} = \Big[K(x_3) - \frac{2}{3}G(x_3)\Big]\delta_{ij}\delta_{kl} + G(x_3)(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

$$k_{ij} = k(x_3)\delta_{ij} \qquad \rho = \rho(x_3) \qquad \eta = \eta(x_3)$$

where $K(x_3)$, $G(x_3)$, $k(x_3)$, $\rho(x_3)$, $\eta(x_3)$ are l-periodic functions, attaining the constant values K', G', k', ρ', η' in Ω' and K'', G'', k'', ρ'', η'' in Ω''; here $K(x_3)$, $G(x_3)$, $k(x_3)$ represent compression, shear and permeability modulae, respectively.

2. Direct description of a stratified porous medium

Problems of stratified porous media, where thicknesses of layers are small compared to the characteristic length dimensions of the occupied region, can be investigated on three levels. On the micro-level we take into account the heterogeneity of the medium made of a pore liquid and a solid skeleton. On the mezzo-level we deal with the phenomenological properties of a porous body within every layer and the heterogeneity is exclusively due to the periodic stratification of the body. At last, on the macro-level the averaged properties of all constituents are introduced and a certain homogenized model of a porous medium is formulated. In this paper we start from equations describing the porous medium on the mezzo-level (the direct description) and propose a procedure leading to a certain new homogenized model representing the macro-behaviour of the body considered.

To formulate the direct description of the heterogeneous periodically layered porous medium under consideration, we shall use the simplified version of the consolidation theory (cf de Josselin de Jong (1963), Verruijt (1969)); for the time being we also introduce inertial terms related to the motion of the medium skeleton. The governing equations are:

(i) The balance equations (equations of motion and storage equation)

$$s_{ij,j} + \rho b_i - \rho\ddot{u}_i = 0 \qquad \dot{u}_{i,i} = -q_{i,i} + \eta\dot{e}$$

together with the boundary conditions: $s_{ij}n_j = s_i$, $q_i n_i = q$ and the continuity conditions on the interfaces between layers: $[s_{i3}] = 0$, $[q_3] = 0$, where $[\cdot]$ stands for a jump of a field across the interfaces. The aforementioned relations can be also written down in the weak form of conditions

$$
\begin{aligned}
\int\limits_{\Omega} s_{ij}\delta u_{i,j}\, dv &= \oint\limits_{\partial\Omega} s_i\delta u_i\, da + \int\limits_{\Omega} \rho(b_i - \ddot{u}_i)\delta u_i\, dv \\
\int\limits_{\Omega} q_i\delta p_{,i}\, dv &= \oint\limits_{\partial\Omega} q\delta p\, da + \int\limits_{\Omega} (\dot{u}_{i,i} - \eta\dot{e})\delta p\, dv
\end{aligned}
\tag{2.1}
$$

which have to hold for every system of test functions $\delta u_i(\boldsymbol{x})$, $\delta p(\boldsymbol{x})$, $\boldsymbol{x} \in \overline{\Omega}$.

(ii) The constitutive equations, given by the stress relation (effective stress principle), the fluid discharge relation (Darcy's law) and the fluid compressibility relation

$$
\begin{aligned}
s_{ij} &= C_{ijkl}e_{kl} + \delta_{ij}p & \qquad q_i &= k_{ij}p_{,j} \\
e_{kl} &= u_{(k,l)} \equiv \frac{1}{2}(u_{k,l} + u_{l,k}) & \qquad e &= \beta p
\end{aligned}
\tag{2.2}
$$

The simplified consolidation theory described by Eqs (2.1) and (2.2) can be applied to fully saturated loose soils in which deformations are caused by the rearrangement of single incompressible grains constituting the skeleton of the porous medium. The skeleton in the continuum approximation is subjected to small strains $e_{kl} = u_{(k,l)}$ and its response is given by the effective stresses $C_{ijkl}e_{kl}$. The porosity and mass density in the theory under consideration are assumed to be time independent. The above version of the consolidation theory is usually applied to the description of quasi-stationary processes, in which inertia terms in Eq (2.1) are neglected. However, for the time being, we retain in Eq (2.1) the inertia forces $\rho\ddot{u}_i$ related to the motion of the skeleton of the porous medium (in the sequel we shall neglect what will be called macro-inertia forces, cf Section 3 below) and hence, we assume that the outflow of pore fluid in the consolidation process is rather slow and is not included into the inertia terms in Eq (2.1). Thus, using Eqs (2.1) and (2.2) we deal with a certain first approximation in description of processes occurring in fully saturated porous media. This approximation was chosen for a starting point of analysis mainly for the sake of its simplicity.

For stratified porous media modulae C_{ijkl}, k_{ij} and porosity η are l-periodic functions of x_3, where the thickness l of two adjacent layers made of two different constituents is assumed to be very small compared to the minimum characteristic length dimension L of the region Ω, $l \ll L$. Hence, a direct applications of Eqs (2.1) and (2.2) to the analysis of special engineering problems is not convenient. That is why Eqs (2.1) and (2.2) will be used only as the starting point for the modelling procedure proposed in this contribution.

3. Modelling procedure

In order to formulate modelling hypotheses leading to the proposed averaged models of the medium under consideration, we recall, following Woźniak (1993), two auxiliary concepts. The first is the concept of the macro function $F(\cdot)$, continuous and defined on Ω, related to a certain macro-accuracy parameter λ_F, which for every $\boldsymbol{x}', \boldsymbol{x}'' \in \Omega$, such that $\|\boldsymbol{x}' - \boldsymbol{x}''\| < l$, satisfies condition $|F(\boldsymbol{x}') - F(\boldsymbol{x}'')| < \lambda_F$. If similar conditions also hold for all derivatives of F (including time derivatives provided that $F(\cdot)$ depends on t) then $F(\cdot)$ will be called the regular macro function. It is evident that in the averaged descriptions of phenomena occurring in the periodically stratified media we should deal exclusively with macro functions, i.e. functions that are not highly oscillating within separate layers. The second auxiliary concept is that of the shape function $h(\boldsymbol{x})$, $\boldsymbol{x} \in \boldsymbol{R}$, which is continuous, l-periodic and in $[0,l]$ takes the values: $h(0) = h(l) = l/2$, $h(l') = -l/2$, being linear in $[0,l']$ and $[l',l]$. This function describes from the quantitative viewpoint the character of possible disturbances in displacements and a pore liquid pressure caused by the inhomogeneous stratified structure of the medium.

Using the aforementioned concepts we shall formulate three modelling hypotheses:

Macro-constraint hypothesis. In the stratified porous medium under consideration the displacements and pore pressure will be assumed in the form

$$
\begin{aligned}
u_i(\boldsymbol{x},t) &= U_i(\boldsymbol{x},t) + h(x_3)V_i(\boldsymbol{x},t) \\
p(\boldsymbol{x},t) &= P(\boldsymbol{x},t) + h(x_3)R(\boldsymbol{x},t)
\end{aligned}
\tag{3.1}
$$

where $U_i(\cdot,t)$, $V_i(\cdot,t)$, $P(\cdot,t)$, $R(\cdot,t)$ are regular macro functions, constituting new basic unknowns. Fields U_i, P are called macro-displacements and macro-pore pressure, respectively and fields V_i, R are called correctors. Terms hV_i, hR in Eqs (3.1) describe the disturbances in displacements and pore fluid pressure, respectively, caused by the inhomogeneity of the medium (this statement will be proved in the subsequent section).

Macro-balance hypothesis. The balance equations (2.1) are assumed to hold for

$$\begin{aligned} \delta u_i(\boldsymbol{x}) &= \delta U_i(\boldsymbol{x}) + h(x_3)\delta V_i(\boldsymbol{x}) \\ \delta p(\boldsymbol{x}) &= \delta P(\boldsymbol{x}) + h(x_3)\delta R(\boldsymbol{x}) \end{aligned} \tag{3.2}$$

where $\delta U_i(\cdot)$, $\delta V_i(\cdot)$, $\delta P(\cdot)$, $\delta R(\cdot)$ are arbitrary linear independent regular macro functions.

This assumption is strictly related to the previous one.

Macro-approximation of the balance equations. In calculation of integrals over Ω in Eqs (2.1) by means of Eqs (2.2) $\div$ (3.2), terms $\mathcal{O}(\lambda)$ will be neglected where λ is related to macro-functions in Eqs (3.1) and (3.2). In calculations of integrals over the boundary $\partial\Omega$ in these equations, terms $\mathcal{O}(l)$ will be neglected.

In order to apply this hypothesis we have to keep in mind that $h(x_3) \in \mathcal{O}(l)$ and that for any integrable l-periodic function $f(x_3)$ and macro function $F(\boldsymbol{x})$ the following formula holds

$$\int\limits_{\Omega} f(x_3)F(\boldsymbol{x})\,dv = <f> \int\limits_{\Omega} F(\boldsymbol{x})\,dv + \mathcal{O}(\lambda_F)$$

where

$$<f> \equiv \frac{1}{l}\int\limits_0^l f(x_3)\,dx_3$$

is the averaging operator.

Substituting the right-hand sides of Eqs (3.1) and (3.2) into Eqs (2.1) and (2.2), respectively, and combining together the resulting equations, applying the divergence theorem, using *the macro-approximation hypotheses* and taking into account the du Bois-Reymond lemma, we shall arrive at the system of eight governing equations in eight basic unknowns U_i, V_i, P and R. These equations will be represented below in the form involving the macro-balance equations and the macro-constitutive equations. Moreover, in the resulting equations we shall neglect terms depending on the second time derivatives of macro-displacements U_i, restricting the considerations to, what will be called macro-quasistationary processes. The results of the macro-modelling procedure outlined above will be summarized in the subsequent section and referred to as the structural model of the stratified porous medium under consideration.

4. Structural model (SM)

The structural model of the fully saturated stratified linear-elastic porous medium, obtained from Eqs (2.1), (2.2), (3.1) and (3.2) by the procedure similar to that given by (Woźniak (1993)), is governed by:

(i) The macro-balance equations:

$$
\begin{aligned}
&S_{ij,j} + <\rho> b_i = 0 \\
&\delta^2 <\rho> \ddot{V}_i + S_i = 0 \\
&<\beta\eta> \dot{P} - Q_{i,i} - \dot{U}_{i,i} = 0 \\
&\delta^2 <\beta\eta> \dot{R} + Q = 0
\end{aligned}
\tag{4.1}
$$

which hold for every $\boldsymbol{x} \in \Omega$ and $t > 0$ and where we have denoted $\delta^2 \equiv l^2/12$. Fields S_{ij} and Q_i are referred to as the macro-stresses and the pore fluid macro-discharge, respectively. We restrict ourselves to the macro-quasistationary processes neglecting the macro-inertial terms $<\rho> \ddot{U}_i$ in Eq $(4.1)_1$.

(ii) The natural boundary conditions (on $\partial\Omega$ and for $t > 0$)

$$
S_{ij}n_j = s_i \qquad\qquad Q_i n_i = q \tag{4.2}
$$

(iii) The macro-constitutive equations

$$
\begin{aligned}
&S_{ij} = <C_{ijkl}> U_{k,l} + <C_{ijk3}h_{,3}> V_k + \delta_{ij}P \\
&S_i = <C_{i3kl}h_{,3}> U_{k,l} + <C_{i3k3}(h_{,3})^2> V_k \\
&Q_i = <k_{ij}> P_{,j} + <k_{i3}h_{,3}> R \\
&Q = <k_{3j}h_{,3}> P_{,j} + <k_{33}(h_{,3})^2> R
\end{aligned}
\tag{4.3}
$$

Substituting the right-hand sides of Eqs (4.3) into Eqs (4.1) we arrive at the system of eight equations with constant coefficients for macro-displacements U_i, macro-pore pressure P and correctors V_i, R. It is easy to conclude that:

(i) Formulae $(4.1)_2$ and $(4.1)_4$ for correctors are ordinary differential equations and hence, V_i, R are independent of boundary conditions, constituting certain internal balance variables depending on the initial conditions only.

(ii) For homogeneous media: $< C_{ijk3}h_{,3} > = C_{ijk3} < h_{,3}> = 0$, $< k_{i3}h_{,3}> = k_{i3} < h_{,3}> = 0$, and for trivial initial conditions $V_i(\boldsymbol{x},0) = 0$, $\dot{V}_i(\boldsymbol{x},0) = 0$, $R(\boldsymbol{x},0) = 0$, $\boldsymbol{x} \in \Omega$, we obtain that the correctors are equal identically to zero: $V_i = 0$, $R = 0$. It means that the correctors describe the effect of the heterogeneity on the body behaviour.

(iii) The proposed model depends on the length parameter l (since $\delta^2 = l^2/12$) and hence, describes the length-scale effect on the behaviour of the medium.

(iv) The effect of the length parameter on the body behaviour takes place only in non-stationary problems.

(v) For stationary problems correctors are governed by linear algebraic equations and can be eliminated from Eqs (4.1) ÷ (4.3) by means of

$$
\begin{aligned}
V_i &= -D_{ij} < C_{j3kl}h_{,3}> U_{k,l} \\
R &= - < k_{33}(h_{,3})^2 >^{-1} < k_{3j}h_{,3}> P_{,j}
\end{aligned}
$$

where D_{ij} are elements of the 3×3 matrix inverse to the non-singular matrix of elements $< C_{i3j3}(h_{,3})^2 >$.

(vi) The proposed model can be applied only to the macro-quasistationary processes in which the inertia terms related to the macro-motion of the medium can be neglected.

5. Effective modulus model (EMM)

In engineering problems composite bodies with a micro-periodic structure are analyzed, as a rule, in the framework of different effective modulus theories. These theories are usually derived from the direct description of periodic non-homogeneous media either by scaling the microstructure down, cf Bensoussan at al. (1980), Bakhvalov and Panasenko (1984), or by certain averaging procedures which take into account stress and strain continuity conditions across the interfaces between constituents, cf Jones (1975). In order to derive from Eqs (2.1) and (2.2) the effective modulus model (EMM) for a stratified porous medium we shall apply *the constraint* and *balance hypotheses* formulated in Section 3, and then setting $l \to 0$. Hence, the EMM is not able to describe length-scale effects on the body bahaviour. After some simple calculations, following the procedure outlined at the end of Section 3, from Eqs (2.1) and (2.2) we obtain:

(i) The macro-balance equations

$$S_{ij},_j + <\rho> b_i = 0 \qquad <\beta\eta> \dot{P} - Q_i,_i - \dot{U}_i,_i = 0 \tag{5.1}$$

which hold for every $\boldsymbol{x} \in \Omega$, $t > 0$

(ii) The natural boundary conditions

$$S_{ij}n_j = s_i \qquad Q_i n_i = q \tag{5.2}$$

for $\boldsymbol{x} \in \partial\Omega$ and $t > 0$

(iii) The macro-constitutive equations

$$S_{ij} = C^{\text{eff}}_{ijkl} U_k,_l + \delta_{ij} P \qquad Q_i = k^{\text{eff}}_{ij} P,_j \tag{5.3}$$

where C^{eff}_{ijkl}, k^{eff}_{ij} are called the effective modulae, being defined by

$$\begin{aligned} C^{\text{eff}}_{ijkl} &\equiv <C_{ijkl}> - <C_{ijm3}h,_3> D_{mn} <C_{kln3}h,_3> \\ k^{\text{eff}}_{ij} &\equiv <k_{ij}> - \frac{<k_{i3}h,_3><k_{j3}h,_3>}{<k_{33}(h,_3)^2>} \end{aligned} \tag{5.4}$$

and D_{ij} are determined by conditions $D_{ij} <C_{j3k3}(h,_3)^2> = \delta_{ik}$.

The basic unknowns in the governing equations of EMM are macro-displacements U_i and a liquid macro-pressure P. It can be seen that for stationary problems the SM and the EMM coincide.

6. Example: SM versus EMM

In order to compare the SM and the EMM let us consider a thick stratified layer bounded by the coordinate planes $x_3 = \pm H$, subjected to compressive boundary normal tractions given by $s_3 = 0$ for $t \leq 0$ and $s_3 = s =$ const for $t > 0$, and having impervious boundaries: $Q_3 = 0$ for $x_3 = \pm H$ and every t. Moreover, let be known the initial pore liquid pressure $p = P_0 + h(x_3)R_0$, P_0, R_0 – constants, and assume that initial values of correctors V_i are equal the zero, $V_i = \dot{V}_i = 0$ for $t = 0$. The problem under consideration is independent of coordinates x_1, x_2 and under denotations $U \equiv U_3(x_3,t)$, $V \equiv V_3(x_3,t)$, $C \equiv C_{3333}(x_3)$, $k \equiv k_{33}(x_3)$, and neglecting body forces, we obtain from Eqs (4.1) and (4.3) the system of equations

$$
\begin{aligned}
&<C>U_{,33}+<Ch_{,3}>V_{,3}+P_{,3}=0\\
&\underline{\delta^2<\rho>\ddot{V}}+<Ch_{,3}>U_{,3}+<C(h_{,3})^2>V=0\\
&<k>P_{,33}+<kh_{,3}>R_{,3}+\dot{U}_{,3}-<\beta\eta>\dot{P}=0\\
&\underline{\delta^2<\beta\eta>\dot{R}}+<kh_{,3}>P_{,3}+<k(h_{,3})^2>R=0
\end{aligned}
\tag{6.1}
$$

which have to hold for $x_3 \in (-H,H)$, $t>0$. At the same time natural boundary conditions (4.2) yield

$$
\begin{aligned}
&<C>U_{,3}+<Ch_{,3}>V+P=-s\\
&<k>P_{,3}+<kh_{,3}>R=0
\end{aligned}
\tag{6.2}
$$

for $x_3=\pm H$, $t>0$ and initial conditions for $x_3 \in (-H,H)$, $t=0$ are

$$
\begin{aligned}
&P=P_0 \qquad\qquad R=R_0\\
&V=0 \qquad\qquad \dot{V}=0
\end{aligned}
\tag{6.3}
$$

where P_0, R_0 were assumed constant. Under denotations

$$
\begin{aligned}
&\alpha \equiv \frac{<C(h_{,3})^2>C^{\rm eff}}{<\rho><C>} &&C^{\rm eff} \equiv <C> - \frac{<Ch_{,3}>^2}{<C(h_{,3})^2>}\\
&\mu = \frac{<Ch_{,3}>}{<\rho><C>} &&\nu \equiv \frac{<Ch_{,3}>}{1+\beta<\eta><C>}\\
&\kappa^2 \equiv \alpha+\mu\nu &&\gamma \equiv \frac{<Ch_{,3}>^2\beta<\eta>(1+\beta<\eta><C>)}{<C><C(h_{,3})^2>(1+\beta<\eta>C^{\rm eff})}
\end{aligned}
$$

the solution to Eqs (6.1)÷(6.3) is given by

$$
\begin{aligned}
&U=-\frac{P_0+s}{<C>}x_3-\gamma(P_0+s)\Big(1-\cos\frac{\kappa t}{\delta}\Big)x_3\\
&P=P_0-\frac{\mu\nu}{\kappa^2}(P_0+s)\Big(1-\cos\frac{\kappa t}{\delta}\Big)\\
&V=\frac{\mu}{\kappa^2}(P_0+s)\Big(1-\cos\frac{\kappa t}{\delta}\Big)\\
&R=R_0\exp\Big(-\frac{\alpha t}{\delta}\Big)\\
&x_3\in[-H,H] \qquad\qquad t\geq 0
\end{aligned}
\tag{6.4}
$$

where we have assumed that $U = 0$ for $x_3 = 0$. Let us observe that the resultants of macro-inertia forces $<\rho> \ddot{U}(x_3, t)$ (neglected in Eq $(6.1)_1$) are equal to zero.

The equations of EMM can be obtained from Eqs (6.1) by neglecting the underlined terms; at the same time initial conditions for V and R drop out from Eqs (6.3). The solution to the problem under consideration within a framework of the EMM has the form

$$
\begin{aligned}
U &= -\frac{P_0 + s}{C^{\rm eff}} x_3 & \qquad P &= P_0 \\
V &= \frac{\mu}{\alpha}(P_0 + s) & \qquad R &= 0 \\
x_3 &\in [-H, H] & \qquad t &\geq 0
\end{aligned}
\tag{6.5}
$$

Let us observe that if $R_0 = 0$ then the length-scale on the behaviour of the saturated medium takes place only if $s \neq -P_0$; for $s = -P_0$ both the SM and EMM yield the trivial solution: $U = 0$, $P = P_0$, $V = 0$, $R = 0$. Solutions (6.4) and (6.5) also coincide for homogeneous media (in this case $\mu = \gamma = 0$ since $<Ch_{,3}> = C <h_{,3}> = 0$) and for stationary problems.

7. Conclusions

Taking into account the solutions to the problem formulated in Section 6, given by Eqs (6.4) within a framework of the SM and by Eqs (6.5) within a framework of the EMM, it is easy to compare results related to both models. First of all let us observe that applying the boundary tractions $s_3 = s$ to a saturated stratified medium, on condition that $s \neq -P_0$, we obtain the strain-oscilations $u_{3,3} = U_{,3} + h_{,3}(x_3)V$ (terms $O(\lambda)$ being neglected) and the pore pressure $p = P + h(x_3)R$. This physical effect is not described by the effective modulus theory. These oscilations are caused by the stratified material structure of the body, i.e., for homogeneous media they disappear. It can be also seen that the disturbances $h(x_3)R_0$ in the initial liquid pressure are strongly attenuated. This phenomenon is also not described by the EMM, since for the problem under consideration the EMM yields solutions (6.5) which are constant in time.

The comparison of solutions (6.4) and (6.5), which is related to the special problem formulated in Section 6, leads to the conclusion that only the SM yields the proper description of this problem. Also in a general case the initial disturbances in strains and fluid pressure (given by the initial values of correctors V_i and R, respectively) cannot be detected using the EMM, since within a framework of the effective modulus theories there are no initial conditions for V_i and R. The general conclusion is that in the non-stationary problems the microstructural model of periodically stratified porous media, proposed in this contribution, constitutes the better tool of analysis then the known effective modulus theories, which are not able to describe properly many time-dependent phenomena.

At the end of this paper it has to be emphasized that the above considerations were based on the approximated version of the consolidation theory mainly in order to obtain a simple form of resulting equations. More general approach to mechanics of inhomogeneous porous media, which takes into account the dynamic coupling between different fluxes, is now under investigation and will be published in a forthcaming paper.

References

1. BAKHVALOV N.S., PANASENKO G.P., 1984, *Osrednenie Processov v Peryodicheskikh Sredakh,* (in Russian), Moskva, Nauka
2. BENSOUSSAN A., LIONS J. L., PAPANICOLAOU G., 1980, *Asymptotic Analysis of Periodic Structures,* Amsterdam, North-Holland
3. BIOT M.A., 1941, General Theory of Three-Dimensional Consolidation, *J. Appl. Phys.,* **12**, 155-164
4. JONES R., 1975, *Mechanics of Composite Materials,* New York, St. Louis, London, McGrow-Hill
5. DE JOSSELIN DE JONG, 1963, Tree-Dimensional Consolidation, (in Dutch), *L.G.M. - Mededelingen,* **7**, 57-73
6. VERRUIJT A., 1977, Generation and Dissipation of Pore Water Pressure, *Finite Elements in Geomechanics,* New York, John Wiley & Sons, 293-317
7. WOŹNIAK C., 1993, Refined Microdynamics of Periodic Structures, *Arch. Mech.,* **45**, 3, 295-304
8. WOŹNIAK C., WOŹNIAK M., KONIECZNY S., 1993, A Note on Dynamic Modelling of Periodic Composites, *Arch. Mech.,* **45**, 6, 779-783

Uśredniony model niestacjonarnych procesów w porowatych ośrodkach warstwowych

Streszczenie

Celem opracowania jest zbadanie wpływu wielkości mikrostruktury na zachowanie się periodycznie uwarstwionych ośrodków porowatych. W tym celu wprowadzono dwa uśrednione modele takich ośrodków: model mikrostrukturalny uwzględniający grubość uwarstwienia, oraz model asymptotyczny, wynikający z przejścia z grubością uwarstwienia do zera. Wykazano, że w opisie procesów niestacjonarnych efekt skali mikrostruktury nie może być pomijany.

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