

ON THE MICROLOCAL PARAMETER METHOD IN MODELLING OF PERIODICALLY LAYERED THERMOELASTIC COMPOSITES

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This note treats of papers connected with the boundary value problems of periodically layered elastic composites. The problems were solved within the framework of the homogenized models with microlocal parameters.

1. Introduction

The problems of periodically layered thermoelastic materials have received wide attention (see, for example, monographs Christensen (1980), Jones (1975), Bensoussan et al. (1978), Achenbach (1975), Sanchez-Palencia (1980), Bakhvalov and Panasenko (1980), Tsai and Hahn (1980), Guz (1982) and references therein). A variety of exact, approximate and purely numerical methods are available for the solution of boundary value problems of the laminated bodies. However, the applications of homogenized models seem to be more suitable. One of them is the homogenized model with microlocal parameters given by Woźniak (1986), Woźniak (1987a,b,c), Matysiak and Woźniak (1987), (1988). These models, in the case of thermoelastic periodically layered composites, are governed by a system of linear partial differential equations for unknown macrodeformations, macrotemperatures and by a system of linear algebraic equations for extra unknowns called microlocal parameters. The microlocal parameters make to evaluate not only mean but also local values of stresses and heat fluxes in every material component of the composite.

The homogenized model with microlocal parameters have been applied to the following stationary two-dimensional and three-dimensional problems of periodic-layered composites.

2. Two-dimensional problems

2.1. Problems connected with methods of solution

In the papers of Kaczyński, Matysiak (1987a) and (1988a) the complex potential method was adopted for the problems of two-layered periodic elastic and thermoelastic composites, respectively.

2.2. Problems of cracks in periodically layered composites

The closed-form solutions of the interface crack problems in microperiodic two-layered elastic space were given by Kaczyński and Matysiak (1988b) – case of single crack, Kaczyński and Matysiak (1989a) – case of a system of interface cracks. The papers of Matysiak (1989), Kaczyński and Matysiak (1989b), (1991a) contains the solutions of the interface crack problems in thermoelastic laminated space. The influence of isolated body forces and concentrated heat sources on the stress distributions in microperiodic layered space was analyzed by Kaczyński and Matysiak (1993a) (1994) and Kaczyński (1993a). The crack normal to the layering was discussed by Pusz (1992). Paper of Kaczyński et al. (1994) contains an analysis of stress intensity factors in a laminated elastic layer weakened by a Griffith crack.

The solutions of the crack problems in the periodic two-layered elastic composites obtained within the framework of the homogenized models with microlocal parameters are characterized by the square-root singularities and it makes to overcome the physical contradiction arising from the peculiar, oscillatory behaviour of crack border stresses in the conventional formulation of interface cracks (see, for example, Erdogan (1972)).

2.3. Contact problems of periodically layered composites

The two-dimensional static punch problems in the presence of friction for a periodically layered half-space were considered by Kaczyński and Matysiak (1988c), (1993b). The stationary problem of a rigid thermoinsulated punch sliding over the boundary surface of a periodic two-layered thermoelastic half-space is solved in paper of Matysiak and Pauk (1995). The heat generation in the contact area is assumed to be caused by friction forces. The obtained

results show that by the choice of the components of composite it is possible to obtain the necessary level of the contact temperature and the stress fields.

2.4. Other problems

The paper of Matysiak and Pauk (1993) contains the analysis of stress distribution caused by gravitation forces in a periodic stratified elastic layer resting on a rigid foundation. The surface of substrate is assumed to be a plane with an infinitely long slender overlay or a narrow excavation having rectangular cross-sections. In the paper of Jevtushenko et al. (1994) the problem of elastic thin inclusion on an interface in the microperiodic two-layered composite is considered. The influence of the microperiodic layered structure on singular stress distribution caused by concentrated loads in the laminated half-space is discussed by Kaczyński and Matysiak (1987b).

3. Three-dimensional problems

3.1. Problems of cracks in periodically layered composites

Within the framework of the thermoelasticity with microlocal parameters the solutions of three-dimensional problems of interface cracks in periodically layered composites are presented by Kaczyński (1993b), (1994). The paper of Kaczyński et al. (1995) contains an analysis of stress distribution in periodically stratified elastic layer weakened by an interface central penny-shaped crack. The stress intensity factors are obtained in terms of the solution of Fredholm integral equation, which is solved numerically.

3.2. Contact problems of periodically layered composites

The axisymmetric Bousinesq problem for a periodic two-layered half-space is solved by Pusz (1989).

4. Conclusion

Within the framework of the homogenized models with microlocal parameters the following problems of wave propagation in the periodic layered elastic composites are solved:

- Plane harmonic waves in periodic multilayered space (cf Jakubowska and Matysiak (1987))
- Surface waves in a periodic two-layered half-space (cf Bielski and Matysiak (1992)) .

By using the homogenization procedure presented by Woźniak (1986), (1987a,b,c) the homogenized model of multilayered elastic plates are derived by Matysiak and Nagórko (1989a,b). The homogenized model of periodic stratified fluid-saturated porous solids are obtained by Matysiak (1992), (1994) and discussed by Kaczyński R. and Matysiak (1993). The problem of the modelling of heat conduction problem in laminated bodies were considered by Matysiak and Woźniak (1986).

The presented above review of papers shows a wide possibilities of applications of the homogenized models with microlocal parameters of the periodic layered bodies. The analytical structure of equations of the homogenized models is not more complicated that of the governing relations of the homogeneous bodies. These models describe not only mean but also local stresses heat fluxes in every lamina. They can be also applied to linear as well as nonlinear problems and to elastic and unelastic materials.

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O metodzie parametrów mikrolokalnych w modelowaniu periodycznych warstwowych kompozytów termosprężystych

Streszczenie

W notce omówiono prace związane z zagadnieniami brzegowymi periodycznie warstwowych sprężystych kompozytów.

Zagadnienia te zostały rozwiązane w ramach homogenizowanych modeli z parametrami mikrolokalnymi.

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