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MATHEMATICAL MODELLING IN MEASUREMENTS OF UNSTEADY FLOWS WITH ULTRASONIC FLOWMETERS

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In the paper a mathematical model of unsteady flow through an ultrasonic flowmeter has been presented. This mathematical model is based on the Reynolds equation for the axial component of velocity vector and $k-\varepsilon$ turbulence model equations. Unsteady flow was given by suitable formulation of a function representing the gradient of pressure in equation of motion. The equations of the model were numerically solved with the finite-difference method. The model was used for analysis of the influence of velocity profile, variable time, on the flowmeter indications. Pulsating flows and flows with velocity leap were tested. From the numerical testes it results that variability of the velocity profiles time is connected with important changes in sensitivity coefficient of the ultrasonic flowmeter. Variability of this coefficient is mainly influenced by a pulsation amplitude of the mass flux. Small influence of pulsation on the flowmeter indications can be observed only in the case of slow-changing flows, for which the Strouhal number is $\mathrm{Sh} < 0.1$. In the case of flows with velocity leap the maximum changes of the sensitivity coefficient are of $4 \div 6\%$. For flows where the maximum value of Strouhal number in the range of velocity leap is less than 0.1, the range of sensitivity coefficient changes is not wide and it does not exceed the range resulting from increase of the Re number.

1. Introduction

Unsteady flows are often found in industrial systems and they are usually connected with transient processes. They can be also caused by the action of control systems. Also disturbances in systems may be periodic and caused by operation of pumps and piston compressors. Unsteady states in systems produce additional errors in mass stream measurements. These errors depend on dynamic properties of the measuring system used and parameters characterizing the disturbance of flow (cf Dobrowolski and Pospolita (1987); Dijstelbergen (1964); Launder and Spalding (1972)). Most of traditional flowmeters have got inertial elements and therefore the measurements of quick-changing streams are not possible. Thus, flowmeters with no inertial elements, for example, ultrasonic flowmeters should be applied to measurements but at first one should to make sure if the changeable with time velocity profiles influence the accuracy of flowmeter. Ultrasonic flowmeters are more and more often used in practice because they are precise and they do not cause any additional resistance to flow. Ultrasonic flowmeters in which the velocity of fluid is determined from the difference between times of acoustic signal passages in direction of flow and reversely, are applied most often. Such flowmeters are usually used for measurements of liquid flows; in gas flows it is difficult to obtain intense acoustic vibration because the acoustic resistance is weak. There are, however, some reasons for the limited application of ultrasonic flowmeters to measurements of unsteady flows. The measurement procedure is the first limitation for most ultrasonic flowmeters. The measuring system of flowmeter (Fig.1) usually consists of three basic blocks: primary element, receiver and digital system. The primary element generates pulses activating ultrasonic converters. The converters are usually activated by rectangular pulses, tens of nanoseconds wide. The receiver collects ultrasonic signals and generates a signal corresponding to the time of ultrasonic wave course between heads for a given number of activations. The digital system performs a function of control system, calculates and averages results of measurements and makes communication with external devices possible. Such a kind of measurements limits a possibility of application of ultrasonic flowmeters to measurements of quick-changing flows. For example, flowmeters made by DANFOSS have got time-constants not less than 1 s [15]. There are, however, ultrasonic flowmeters with very good dynamic properties and they can be applied to measurements of instantaneous value of the pulsating stream with frequency of tens Hz (Dordain (1979). The second limitation is – in the case of unsteady flows – the velocity profile, variable with time. In such a case it is necessary to analyse if and how the changing velocity profile influences accuracy of the flowmeter. Only few Author (cf Dobrowolski and Pospolita (1987); Dordain (1979); Hamidulin and Karamowicz (1989)), discuss some aspects of this problem, so a full metrological analysis is not possible. Suitable experiments are complicated and requirements for the measuring apparatus are very high, so the problem must be solved in the theoretical way.

Notation

c	–	velocity of the acoustic wave in the flowing medium
c_1, c_2	–	empirical constants of a turbulence model
D	–	pipeline diameter
$h_{\dot{M}}$	–	dimensionless amplitude of the pulsating mass flux
k	–	kinetic energy of turbulence
k_u	–	sensitivity coeficient
L	–	distance between converters
$\dot{M}$	–	mass flux
p	–	pressure
r	–	radial direction
Re	–	Reynolds number
R	–	pipe radius
Sh	–	Strouhal number
t	–	time
U	–	axial component of the velocity
U_A	–	mean velocity in the pipe section
U_R	–	velocity averaged along the radius
y_p	–	distance of the point P from the wall
z	–	axial direction
α	–	angle between the pipeline axis and the straight line connecting converters
$\sigma_k, \sigma_\varepsilon, \sigma_\mu$	–	empirical constants of a turbulence model
ϕ	–	variable
Φ	–	time averaged value of the variable ϕ
ε	–	rate of dissipation of turbulent kinetic energy
ν	–	kinematic viscosity
ν_{ef}	–	effective viscosity
ρ	–	mass density of fluid
ω	–	frequency of the mass flux pulsation
Ω	–	acoustic signal frequency
$\Delta\beta$	–	phase displacement between acoustic waves travelling between converters
$\overline{}$	–	time averaged value

2. Mathematical model of the ultrasonic flowmeter

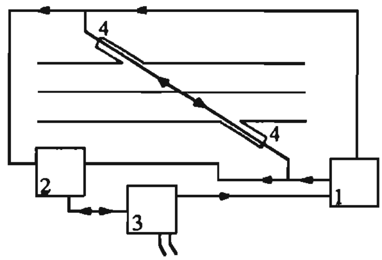

Fig. 1. The one-way ultrasonic flowmeter; 1 – primary element, 2 – receiver, 3 – digital system, 4 – ultrasonic converters

Let us consider a one-way ultrasonic flowmeter, in which the velocity is measured in the plane of the pipeline diameter D (Fig.1). The average velocity in the pipeline section U can be written as (cf Kremlevski (1989))

$$U_A = k_u U_R \tag{2.1}$$

where

k_u – sensitivity coefficient

U_R – velocity averaged along the radius

$$U_R = \frac{1}{2R} \int\limits_{-R}^{R} U\ dr \tag{2.2}$$

For example, in the case of ultrasonic phase flowmeter the measured signal is (cf Kremlevski (1989))

$$\Delta\beta = 2L\Omega \cos \frac{\alpha U_R}{c^2} \tag{2.3}$$

where

$\Delta\beta$ – phase displacement between acoustic waves

L – distance between converters

α – angle between the pipeline axis and the straight line connecting converters

c – velocity of the acoustic wave in the flowing medium

Ω – acoustic signal frequency.

Sensitivity coefficient k_u takes into account influence of the velocity profile on the flowmeter indications.

For determining the velocity profile, variable with time, equations representing the fluid motion in the pipeline were solved in a numerical way. The mathematical model of the flow was derived on the following assumptions:

- The flow is fully developed, so all the derivatives except for the pressure derivative are zero
- Density and laminar viscosity coeficients are constant
- Mean parameters characterizing the turbulent flow are determined by the group averaging (Watkins (1977)). Then an arbitrary parameter Φ is defined as

$$\Phi(z,r,t) = \lim_{m\to\infty} \frac{1}{2m+1} \sum_{n=-m}^{m} \phi(z,r,t+nT) \tag{2.4}$$

  - where T is a period of stream pulsation. Due to such a definition of mean values it is possible to analyse the flow, variable with time, for which the magnitude of changes corresponds to the period of pulsation
  - the pressure gradient in a pipe is a periodic function with the component constant or the pressure gradient makes a linear increase of the mass stream flux (leap of velocity).

A velocity profile of the fully-developed pulsating turbulent fluid flow in the pipeline is described by the Reynolds equation the axial component U of the velocity vector

$$\frac{\partial U}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\Big(r\nu_{ef}\frac{\partial U}{\partial r}\Big) - \frac{1}{\rho}\frac{\partial p}{\partial z} \tag{2.5}$$

where the effective viscosity is expressed by

$$\nu_{ef} = \nu + c_\mu \frac{k^2}{\varepsilon} \tag{2.6}$$

Kinetic energy of turbulence k and its dissipation rate ε are determined from equations of the model of turbulence (cf Launder and Spalding (1972))

$$\frac{\partial k}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\Big(r\frac{\nu_{ef}}{\sigma_k}\frac{\partial k}{\partial r}\Big) + \nu_{ef}\Big(\frac{\partial U}{\partial r}\Big)^2 - \varepsilon \tag{2.7}$$

$$\frac{\partial \varepsilon}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\Big(r\frac{\nu_{ef}}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial r}\Big) + \frac{\varepsilon}{k}\Big[c_1\nu_{ef}\Big(\frac{\partial U}{\partial r}\Big)^2 - c_2\varepsilon\Big] \tag{2.8}$$

Empirical constants in Eqs (2.5) ÷ (2.8) are

$$c_\mu = 0.09 \qquad c_1 = 1.43 \qquad c_2 = 1.92$$
$$\sigma_k = 1 \qquad \sigma_\varepsilon = 1.3$$

For Eqs (2.5) ÷ (2.8) the following boundary conditions were assumed. In the symmetry axis a condition $\partial\Phi/\partial r = 0$ was assumed for all the variables.

On a pipe wall the boundary conditions are formulated according to the model $k - \varepsilon$ for a large Reynolds number (cf Launder and Spalding (1972)). The velocity U on the wall is 0. If it is assumed, that the point p of difference mesh near the wall lies in the region of developed turbulence, then the velocity U_p, is described by the following logarithmic formula

$$\frac{U_p}{\sqrt{\frac{\tau_w}{\rho}}} = \frac{1}{\chi} \ln \frac{E y_p \sqrt{\frac{\tau_w}{\rho}}}{\nu} \tag{2.9}$$

where y_p is the distance of point P from the wall, $\chi = 0.42$ and $E = 9.7$ are the empirical constants, respectively. Assumption that the shear stresses τ are constant along the segment connecting points w on the wall and P in the boundary layer, respectively, implies

$$\tau_p = \tau_w = \rho k_p \sqrt{c_\mu} \tag{2.10}$$

Compiling the above formulae we obtain the identity

$$\tau_p = \frac{\rho \chi U_p \sqrt[4]{c_\mu}\sqrt{k_p}}{\ln \frac{E y_p \sqrt[4]{c_\mu}\sqrt{k_p}}{\nu}} \tag{2.11}$$

which relates the shear stress on the wall to the kinetic energy of turbulence and the component of velocity vector parallel to the wall.

The amount of energy dissipation ε_p at the boundary point can be found from the relation

$$\varepsilon_p = \frac{\sqrt[4]{c_\mu^3}\sqrt{k_p^3}}{\chi y_p} \tag{2.12}$$

The amount of kinetic energy of turbulence k_p at the point adjacent to the wall is found from the general equation of balance with the diffusion neglected. A component representing the dissipation $\rho\varepsilon$ in the equation of transport is estimated a follows

$$\rho\varepsilon = \frac{\sqrt[4]{c_\mu^3 k_p^6}}{\chi} \ln \frac{E y_p \sqrt[4]{c_\mu}\sqrt{k_p}}{\nu} \tag{2.13}$$

The system of equations (2.5) $\div$ (2.8) was solved with the finite-difference method. As a result of discretization of the system difference equations were obtained and their general form was (Dobrowolski and Pospolita (1987))

$$a_P^{\Phi}\Phi_P^{n+1} = a_N^{\Phi}\Phi_N^{n+1} + a_S^{\Phi}\Phi_S^{n+1} + R + PS_U + M_P\Phi_P^n \tag{2.14}$$

The formulae for coefficients a_P^{Φ}, a_N^{Φ}, a_S^{Φ} and M_P, S_U result from the assumed way of discretization of Eqs (2.5), (2.7) and (2.8). Some details of the difference network assumed and discretization of the mathematical model equations are given, among others, by Dobrowolski and Pospolita (1987), Dobrowolski et al. (1991). The solution was based on the SIMPLE method, Patankar (1980). Since the difference scheme for coordinate t is unexplicit, the system of difference equations was solved iteratively at each time level. The iterative process was continued up to occurrence of the residual test for convergence. Then calculations for the next time level were done.

3. Effect of unsteady stream on sensitivity coefficient of the flowmeter

In the problem considered, the velocity profile for a large mass stream results from a kind of stream variability and the initial conditions. There are many possible unsteady states and many possible velocity distributions, so it is not easy to establish the values of parameters characterizing the unsteady flow, which can be used as criteria of usability of the flowmeter at a permissible additional error of measurement. Thus, velocity profiles, variable with time, were considered for unsteady flows being very similar to those occurring in practice. Pulsating flows and flows with violent velocity leaps were analysed. Some sample velocity distributions for pulsating, fully-developed turbulent flow in a pipe are shown in Fig.2. Calculations were done for flows with the same amplitude of pulsation $h_{\dot{M}}$ but different values of the Strouhal number of hydrodynamic similarity $\mathrm{Sh} = \omega D/\bar{U}_A$, where ω is frequency of the stream pulsation, $\bar{U}_A$ is the flow velocity averaged in time. It has been found that the increase of Sh number is connected with the greater changes of velocity profiles in comparison with the steady flow. In the case of large amplitudes of pulsation of fluid flux a reverse flow may occur near the walls. The calculation results for the pulsating flow in a pipe, made according to the mathematical model and algorithm presented, were compared with the experimental data available and a good agreement was obtained (Pospolita (1992)). Changes of

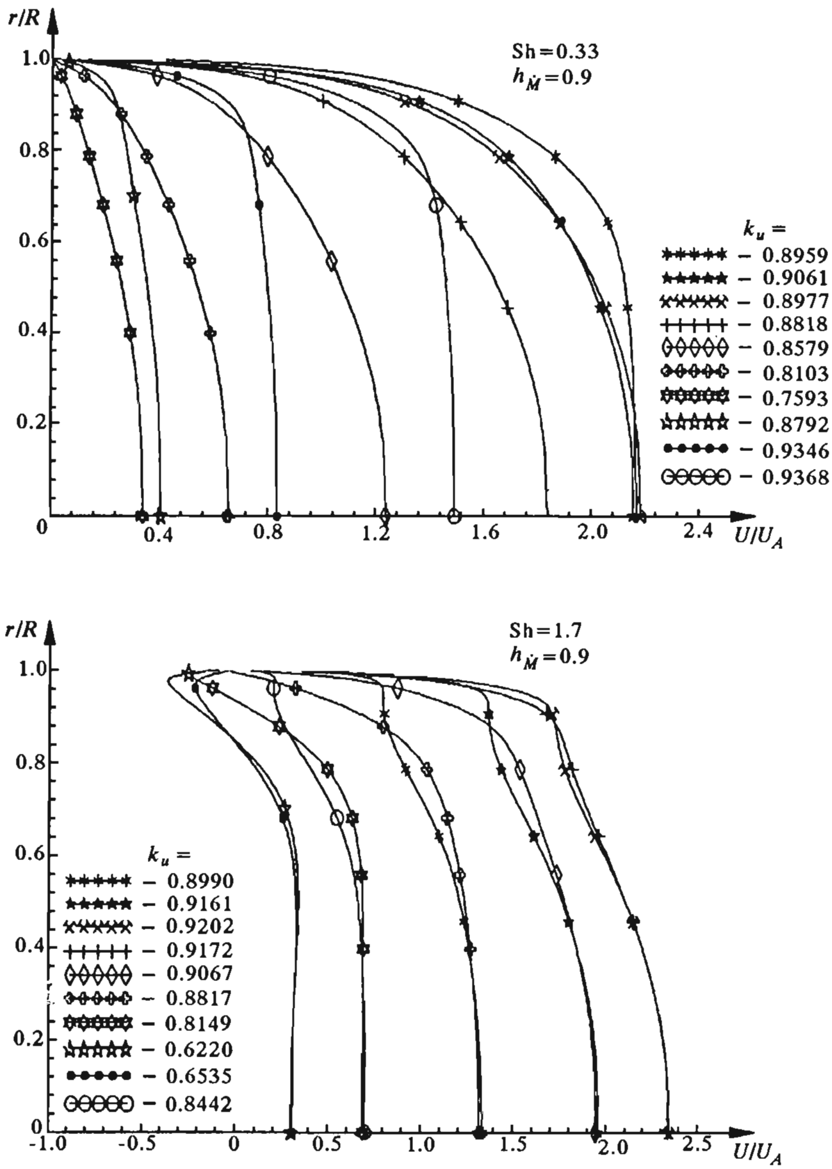

Fig. 2. Velocity profiles and coefficient k_u while stream pulsation; $\bar{U}_A, \bar{k}_u$ – quantities averaged at time, $h_{\dot{M}}$ – relative amplitude of mass stream pulsation

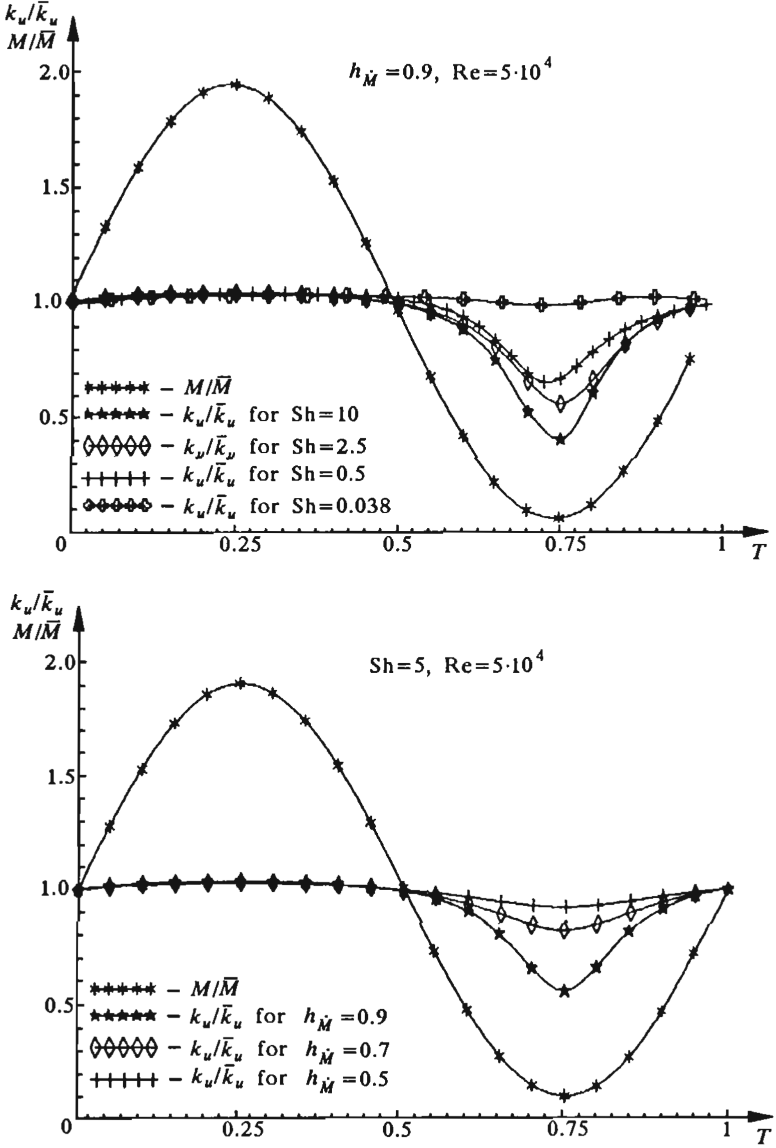

Fig. 3. Influence of the Strouhal number and amplitude of mass stream pulsation on the range of changes of k_u while pulsation

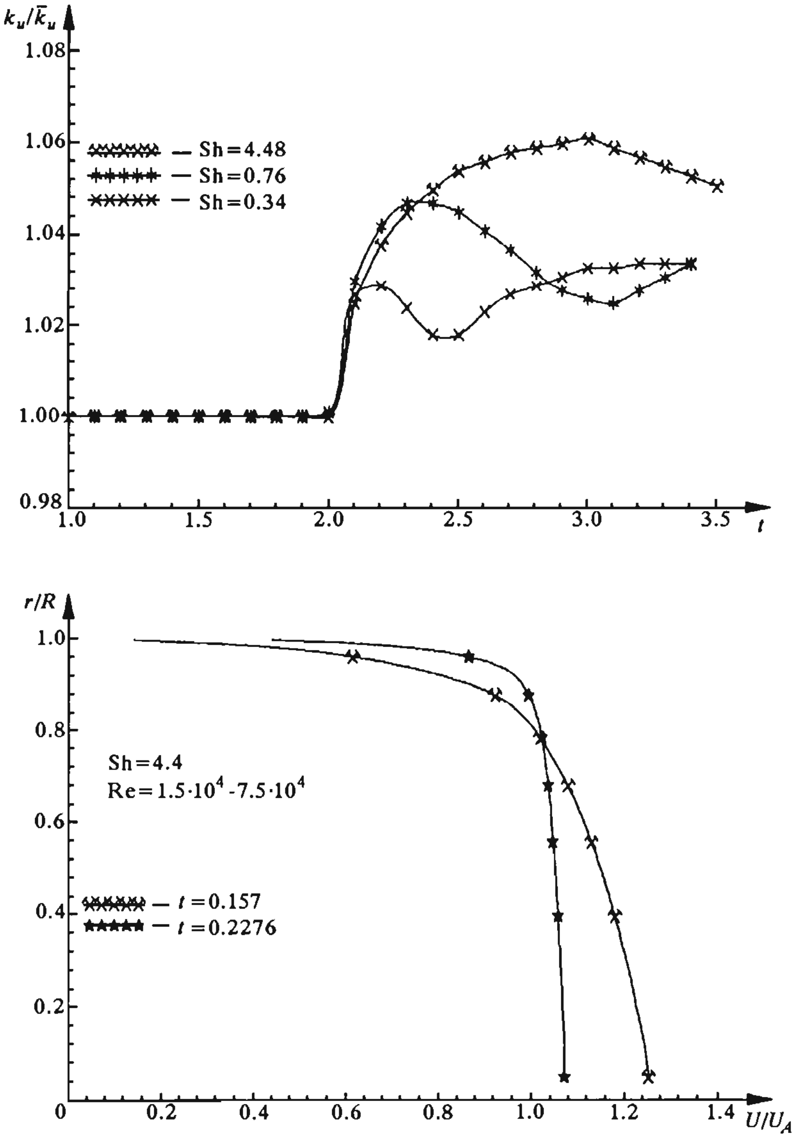

Fig. 4. Values of k_u for the velocity leap, velocity profiles before the leap and for the extremum k_u ($\bar{k}_u$ – directly before the leap)

coefficient k_u are result frm the profile changes. Fig.3 shows the influence of pulsation amplitudes and Strouhal number on changes in sensitivity coefficient k_u. From numerical investigations it results that the relative amplitude of stream pulsation is the parameter exerting the largest influence on variability of the coefficient. The range of changes k_u widens with the increase in stream pulsation amplitude. It is important that these changes are observed mainly in that part of pulsation period where the flow reaches its minimum values. It is connected with the maximum changes of the velocity profile in that part and even with occurence of reversion flow in a part of the flow section. For a given pulsation amplitude the range of changes of k_u widens together with increase in Strouhal number Sh.

The presented results as well as the results of further numerical computations show that in case of slowly – changeable flows, where $\mathrm{Sh} < 0.01$, the range of changes of k_u is not large even for high relative amplitudes of stream pulsation $(0.8 \div 0.9)$ and results from changes in instantaneous values of Re numbers.

For flows with $h_{\dot{M}} < 0.5$ the changes of k_u within a pulsating cycle reach $2 \div 3\%$, independently of the stream pulsation frequency. These comments concern the flows in which relamination of the stream does not occur (high Re). In the case of flows in which violent differences velocity are observed, the flows accelerated from the steady motion were tested for Reynolds numbers $1.3 \cdot 10^4 \div 7 \cdot 10^4$. Fig.4 shows the changes in k_u versus time for a velocity leap. Relative time on the X-axis from 1 to 2 corresponds to the steady flow, from 2 to 3 to the velocity leap. Quantity of $\bar{k}_u$ corresponds to the value of sensitivity coefficient for the steady flow before the velocity leap. From the analysis it results that for short times of a leap the relative value of $k_u/\bar{k}_u$ undergoes the largest changes, i.e., about 6%. From calculations it also results that the largest change in the coefficient value can be observed just after the leap, next k_u tends to its value for the steady flow, proper for a given Re after the velocity leap. In Fig.4 the Strouhal number $\mathrm{Sh} = \frac{dU_A}{dt}\frac{D}{U_A^2}$ characterizes turbulence in the form of velocity leap. A number Sh given in Fig.4 is the largest one within the leap considered. For flows in which the maximum value within the velocity leap is less than 0.1, the range of changes of k_u is small and it does not exceed the range resulting from the increase of Re.

References

1. BOROŃ R., SZEBESZCZYK J., PIETRASZAK S., 1990, Przeplywomierze ultra-

dźwiękowe z nakładanymi głowicami do pomiaru strumienia objętości cieczy w rurociągach, *Metrologia i Systemy Pomiarowe*, 7, 159-172

2. DOBROWOLSKI B., POSPOLITA J., 1987, Theoretical Estimation of the Applicability Range of Differential Pressure Type Flowmeters in Presence of Pulsation of the Mass Flux, *Arch. Mech.*, **39**, 6, 571-588

3. DOBROWOLSKI B., KABZA Z., POSPOLITA J., 1991, Theoretische und Experimentelle Untersuchungen des Einflusses der Pulsationsströmung auf die Charakteristiken der Drosselgeräte, *Fortschritt Berichte VDI*, Strömungstechnik, 193, Düsseldorf

4. DORDAIN J.J., 1979, Steady-State and Unsteady Liquid Flow-Rate Measurements. Characteristics and Performance of the Inera Ultrasonic Flowmeter, *ACTA IMEKO*

5. DIJSTELBERGEN H.H., 1964, Rotametr dynamics, *Chemical Engineering Science*, **19**, 856-865

6. HAMIDULIN W.K., KARAMOWICH W., 1989, *Ultrazvukovye kontrolno-izmeritelnye ustroǐstva i sistemy*, Izdatelstvo Leningradskogo Universiteta, Leningrad

7. IGUCHI M., OHMI M., 1982, Transition to Turbulence in a Pulsatile Pipe Flow, Part 2, Characteristics of Reversing Flow Accompanied by Relaminarization, *Bull. of the JSME*, **25**, 208, 1529-1536

8. KREMLEVSKI P.P., 1989, *Raschodomery i scotchiki kolichestva*, Mashinostroenie, Leningrad

9. LAUNDER B.E., SPALDING D.B., 1972, *Mathematical Models of Turbulence*, New York, Academic Press

10. LEE W.F., KIRIK M.S., BONNER J.A., 1975, Gas Turbine Flowmeter Measurement of Pulsating Flow, *Journal of Engineering for Power, Trans. of ASME*, 531-539

11. OHMI M., IGUCHI M., 1982, Occurence of Turbulentburts in an Oscillatory Pipe Flow, *Technology Reports of the Osaka University*, **32**, 1643, 113-120

12. OHMI M., IGUCHI M., 1980, Flow Pattern and Friction Losses in Pulsating Pipe Fow, Part 3: General Representation of Turbulent Flow Pattern, *Bull. of the JSME*, **23**, 186, 2029-2036

13. PATANKAR S.V., 1980, *Numerical Heat Transfer and Fluid Flow*, Hemisphere Publ. Corp., New York

14. POSPOLITA J., 1992, Teoretyczna ocena możliwości pomiaru pulsującego strumienia masy przepływomierzem ultradźwiękowym, *Pomiary, Automatyka, Kontrola*, 1, 8-11

15. SONOFLO: Ultrasonic Flowmeters, Materialy informacyjne firmy Danfoss

16. WATKINS A.P., 1977, Flow and Heat Transfer in Piston/Cylinder Assemblies, Ph.D. Thesis, University of London

Zastosowanie modelowania matematycznego do oceny możliwości pomiarów przepływów nieustalonych przepływomierzem ultradźwiękowym

Streszczenie

W pracy przedstawiono model matematyczny nieustalonego przepływu przez jednodrogowy przepływomierz ultradźwiękowy. Model matematyczny sformułowano w oparciu o równanie Reynoldsa dla składowej osiowej wektora prędkości i równania $k-\varepsilon$ modelu turbulencji. Nieustalony przepływ zadawano odpowiednio formułując postać funkcji wyrażającej gradient ciśnienia w równaniu ruchu. Równania modelu rozwiązano numerycznie metodą różnic skończonych. W oparciu o przedstawiony model matematyczny dokonano analizy wpływu zmiennego w czasie profilu prędkości na wskazania przepływomierza ultradźwiękowego. Badano przepływy pulsujące oraz przepływy charakteryzujące się skokiem prędkości. Przeprowadzone badania numeryczne wykazały, że ze zmianami w czasie profilu prędkości wiążą się istotne zmiany współczynnika czułości przepływomierza ultradźwiękowego. Zasadniczy wpływ na zmienność tego współczynnika ma amplituda pulsacji strumienia masy. Jedynie dla przepływów wolnozmiennych, charakteryzujących się liczbami Strouhala $\mathrm{Sh} < 0.01$ wpływ pulsacji strumienia na wskazanie przepływomierza jest znikomy. W przypadku przepływów charakteryzujących się skokiem prędkości, maksymalne zmiany współczynnika czułości są rzędu $4 \div 6\%$ i występują bezpośrednio po zadaniu zaburzenia. Dla przepływów, dla których maksymalna wartość liczby Sh w obrębie skoku prędkości jest mniejsza od 0.1 zakres zmian współczynnika czułości jest niewielki i praktycznie nie wykracza poza zakres zmian wynikający ze wzrostu liczby Re.

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