MECHANIKA TEORETYCZNA
I STOSOWANA
Journal of Theoretical
and Applied Mechanics
1, 31, 1993

DYNAMICAL STABILITY OF A ROPE UNDERGOING SLOW CHANGES OF THE PARAMETERS

ANNA KUMANIECKA

JÓZEF NIZIOŁ

Cracow University of Technology

The longitudinal-transversal vibrations of a rope with varying length are considered. The analysis of parametrical resonances is a primary purpose of this paper. The dynamic state of investigated system is described by a nonlinear set of partial differential equations with boundary conditions varying in time. The physical nonlinearity and damping properties of the rope material as well as the dry friction between flakes are taken into account. Determination of the unstable regions by means of the balance harmonic method for the main, secondary and combination resonances respectively is done. Diagrams of the regions of instability are presented.
Influence of the physical nonlinearity and the character of the kinematic excitation are considered. The starting and the braking of winding machine is taken into consideration.

1. Introduction

The problem of stability of a rope has been studied for more then 20 years, cf the early research on the subject by Goroshko and Savin (1971). Considerable progress in this field was made due to Marczyk and Nizioł (1978), Ulshin (1975). All works mentioned consider longitudinal-transversal vibrations of a lifting rope only for a linear physical model and an uniform motion of the drum. Studies on the subject connected with stability are concentrated only on determination of the unstable region for the main resonance and for the first mode of the transversal oscillations. One can see that the wider analysis is necessary. The primary purpose of the work reported in the presented paper is, thus, the analysis of parametric resonance in the case of a nonlinear physical model.

2. Formulation of the problem

The investigated system consists of a drum rotating with a circumferential velocity $v(t)$, on which a steel rope loaded by a load Q is reeled (Fig.1).

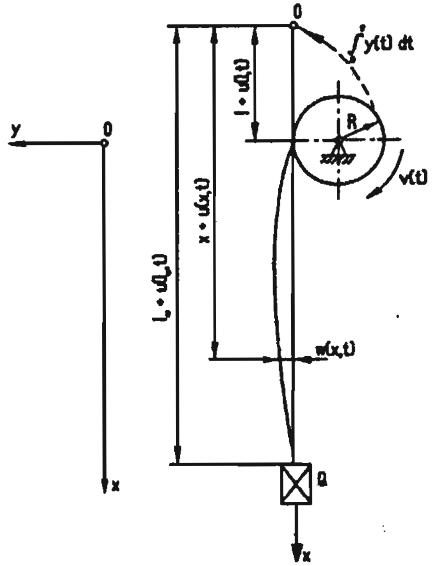

Fig. 1. General system model

The analysis of small longitudinal vibrations of such a system was presented by Kumaniecka and Nizioł (1992).

In the present paper the following simplifications and assumptions are adopted

- the longitudinal-transversal vibrations are small,
- the rope material is homogeneous,
- the rope material is physically nonlinear,
- the internal viscous damping of the material is according to the Voigth-Kelvin model,
- the dry friction between particular rope flakes exists and the stress resulting from dry friction forces is proportional to the absolute value of nonlinear elastic stress in the rope,
- the flexural stiffness is disregarded,
- the drum is perfectly rigid,
- the slip of the rope on the drum is neglected,

- a load Q is treated as a point load with one degree of freedom which can move along the $0x$ axis only,
- the kinematic excitation consists of the starting, uniform motion and braking of the winding machine.

All considerations are done in a movable reference system $0xy$, which starts at the point where the rope is fixed on the drum. The total length of the undeformed rope is l_0 and its part reeled on the drum is $l(t)$. The functions $u(x,t)$ and $w(x,t)$ describe longitudinal and transversal displacements, respectively.

The assumed physical model is presented in Fig.2.

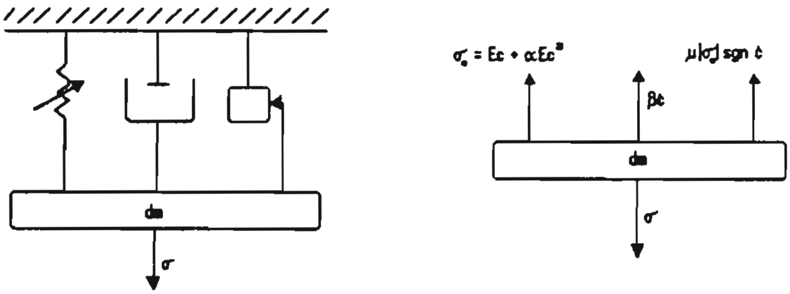

Fig. 2. Physical model

The stress in the rope is described by the following equation

$$\sigma(x,t) = E\varepsilon + \alpha E\varepsilon^3 + \beta\dot{\varepsilon} + \mu|\sigma_e|\mathrm{sgn}\dot{\varepsilon} \tag{2.1}$$

where

$$\sigma_e = E\varepsilon + \alpha E\varepsilon^3 \tag{2.2}$$

The internal force has the form

$$P(x,t) = AE\frac{\partial u}{\partial x} + \alpha AE\Big(\frac{\partial u}{\partial x}\Big)^3 + \beta A\frac{\partial^2 u}{\partial x\partial t} + \mu\Big|AE\frac{\partial u}{\partial x} + \alpha AE\Big(\frac{\partial u}{\partial x}\Big)^3\Big|\mathrm{sgn}\frac{\partial^2 u}{\partial x\partial t} \tag{2.3}$$

where the following notation is used

E – Young modulus,
A – the cross sectional-area,
$\dot{\varepsilon}$ – the strain rate,
AE – the longitudinal rigidity of the rope,
α – the coefficient of physical nonlinearity,
β – the coefficient of viscous damping,
μ – the overall coefficient of dry friction due to geometry and structure of the rope,
σ_e – the nonlinear elastic stress.

Since damping in the rope is complex and nonlinear we examine the equivalent viscous damping, which was widely discused by Kumaniecka (1992).

We introduce dimensionless variables and constants

$$
\begin{aligned}
&x = \xi l_0 && \kappa = \frac{v}{l_0 p_0} && P^* = \frac{P}{q l_0} && \lambda = \frac{3\alpha g^2 l_0^2}{c^4} \\
&\tau = \frac{ct}{l_0} && l^* = \frac{l}{l_0} && Q^* = \frac{Q}{q l_0} && T^* = \frac{cT}{l_0} \\
&b_1 = \frac{EAgl_0}{Qc^2} && b^2 = \frac{EAg}{qc^2} && b_3 = \frac{gl_0}{c^2} && p_0^* = \frac{p_0 l_0}{c} \\
&u^* = \frac{uc^2}{gl_0^2} && w^* = \frac{wc^2}{gl_0^2} && \beta^* = \frac{c\beta}{El_0} && \beta_z^* = \frac{c\beta_z}{El_0}
\end{aligned}
\tag{2.4}
$$

where

$u^*(\xi,\tau)$	–	the dimensionless longitudinal displacements,
$\omega^*(\xi,\tau)$	–	the dimensionless transversal displacements,
T^*	–	the period of the basic mode of longitudinal vibrations,
p_0^*	–	the frequency of the basic mode of longitudinal vibrations,
q	–	the specific weight,
c	–	the velocity of longitudinal wave propagation,
g	–	the acceleration of gravity,
λ	–	the dimensionless coefficient of physical nonlinearity,
β_z^*	–	the equivalent viscous damping coefficient,
κ	–	the coefficient of slow variability of the length of the rope.

3. Analysis of the equation of motion

Applying d'Alembert's law gives the following equations of motion

$$
\begin{aligned}
&\frac{\partial^2 u^*}{\partial \tau^2} - b^2\Big\{\Big[1 + \lambda\Big(\frac{\partial u^*}{\partial \xi}\Big)^2\Big]\frac{\partial^2 u^*}{\partial \xi^2} + c_z^*\frac{\partial^3 u^*}{\partial \xi^2 \partial \tau}\Big\} = 1 + \frac{dv^*}{d\tau} \\
&\frac{\partial^2 \omega^*}{\partial \tau^2} - b_3\frac{\partial}{\partial \xi}\Big(P^*\frac{\partial \omega^*}{\partial \xi}\Big) = 0
\end{aligned}
\tag{3.1}
$$

where

$$
\begin{aligned}
&c_z^* = \beta^* + \beta_z^* \\
&P^* = b^2\Big\{\Big[1 + \frac{\lambda}{3}\Big(\frac{\partial u^*}{\partial \xi}\Big)^2\Big]\frac{\partial u^*}{\partial \xi} + c_z^*\frac{\partial^2 u}{\partial \xi \partial \tau}\Big\}
\end{aligned}
\tag{3.2}
$$

The boundary conditions are

— for the lower end $\xi = 1$

$$
\begin{aligned}
&\frac{\partial^2 u^*}{\partial \tau^2}\Big|_{\xi=1} + b_1\Big\{\Big[1 + \frac{\lambda}{3}\Big(\frac{\partial u^*}{\partial \xi}\Big)^2\Big]\frac{\partial u^*}{\partial \xi} + c_z^*\frac{\partial^2 u^*}{\partial \xi \partial \tau}\Big\}\Big|_{\xi=1} = 1 + \frac{dv^*}{d\tau} \\
&w^*(1,\tau) = 0
\end{aligned}
\tag{3.3}
$$

— for the upper end $\xi = l^*(\tau)$

$$
\begin{aligned}
u^*(l^*,\tau) &= \int\limits_0^\tau \frac{\partial u^*(l^*,\tau)}{\partial \xi}\Big|_{\xi=l^*(\tau)} \frac{dl^*}{d\tau} d\tau \\
w^*(l^*,\tau) &= 0
\end{aligned}
\tag{3.4}
$$

The initial conditions are written in a general form

$$
\begin{aligned}
u^*(\xi,0) &= \varphi_1^*(\xi) & \qquad \omega^*(\xi,0) &= \varphi_3^*(\xi) \\
\frac{\partial u^*(\xi,\tau)}{\partial \tau}\Big|_{\tau=0} &= \varphi_2^*(\xi) & \qquad \frac{\partial w^*(\xi,\tau)}{\partial \tau}\Big|_{\tau=0} &= \varphi_4^*(\xi)
\end{aligned}
\tag{3.5}
$$

where $\varphi_i^*(\xi)$ for $i = 1,2,3,4$ are given functions.

The formula for the change of the length of the ropes has the form

$$
l^*(\tau) = b_3 \int\limits_0^\tau v^*(\tau)d\tau
\tag{3.6}
$$

The fact that during one period of oscillations the length changes insignificantly is of prime importance.

The solution of the first equation from Eq (3.1), that is the one for the longitudinal vibrations, was discussed by Kumaniecka (1992).

Because of the slowly-varying character of the function $l^*(\tau)$ that solution was obtained by means of the Galerkin and Bogolubow–Krylov–Mitropolski methods.

For the basic mode of oscillations the longitudinal displacement has the following form

$$
\begin{aligned}
u_0^*(\xi,\tau) &= a_0^*(\tau)\sin\beta_1\frac{\xi - l^*}{1 - l^*}\cos\varphi_0^* + (\xi - l^*)\Big(1 + \frac{dv^*}{d\tau}\Big)\Big(\frac{1}{b_1} + \frac{2 - \xi - l^*}{2b^2}\Big) + \\
&+ \int\limits_0^\tau \Big[\beta_1\frac{a_0^*(\tau)}{1 - l^*}\cos\varphi_0^* + \Big(1 + \frac{dv^*}{d\tau}\Big)\Big(\frac{1}{b_1} + \frac{1 - l^*}{b^2}\Big)\Big]\frac{dl^*}{d\tau}d\tau
\end{aligned}
\tag{3.7}
$$

where

a_0^*, φ_0^* – slowly-varying amplitudes and phases for the basic mode of longitudinal oscillations,

β_1 – the first eigenvalue.

When considering the equation of transversal vibrations the solution sought for is in the form

$$w^*(\xi,\tau) = \sum_{m=1}^{N} Z_m(\xi,\tau)\Omega_m(\tau) \tag{3.8}$$

where $Z_m(\xi, l^*)$ are slowly-varying eigenmodes of the transversal oscillations.

They are selected in the from

$$Z_m(\xi, l^*) = \sin\frac{m\pi(\xi - l^*)}{1 - l^*} \tag{3.9}$$

These functions satisfy the boundary conditions (3.3) and (3.4).

By means of the Galerkin method one obtains the following set of N equations

$$\sum_{m=1}^{N} A_{mn}\ddot{\Omega}_m + \sum_{m=1}^{N} B_{mn}\dot{\Omega}_m + \sum_{m=1}^{N} C_{mn}\Omega_m = 0 \tag{3.10}$$

where

$$\begin{aligned}
A_{mn} &= \int\limits_{l^*(\tau)}^{1} Z_m(\xi,l^*)Z_n(\xi,l^*)d\xi \\
B_{mn} &= 2\int\limits_{l^*(\tau)}^{1} \frac{dl^*}{d\tau}\frac{\partial Z_m(\xi,l^*)}{\partial l^*}Z_n(\xi,l^*)d\xi \\
C_{mn} &= \int\limits_{l^*(\tau)}^{1} \Big\{\frac{d^2l^*}{d\tau^2}\frac{\partial Z_m(\xi,\tau)}{\partial l^*}Z_n(\xi,l^*) + \Big(\frac{dl^*}{d\tau}\Big)^2\frac{\partial^2 Z_m(\xi,\tau)}{\partial l^{*2}}Z_n(\xi,l^*) - \\
&\quad -b_3\frac{\partial}{\partial\xi}\Big[P^*(\xi,\tau)\frac{\partial Z_m(\xi,\tau)}{\partial\xi}\Big]Z_n(\xi,l^*)\Big\}d\xi
\end{aligned} \tag{3.11}$$

Taking into account Eq (3.9) we obtain

$$\begin{aligned}
A_{mn} &= \begin{cases} 0 & \text{for} \quad n \neq m \\ \dfrac{1-l^*}{2} & \text{for} \quad n = m \end{cases} \\
B_{mn} &= \begin{cases} \dfrac{-2nm}{n^2-m^2}\dfrac{dl^*}{d\tau} & \text{for} \quad n \neq m \\ -\dfrac{1}{2}\dfrac{dl^*}{d\tau} & \text{for} \quad n = m \end{cases}
\end{aligned} \tag{3.12}$$

Due to complexity of C_{mn} their values are not explicitly given here. It was already done by Kumaniecka (1992).

Each equation of the set (3.10) can be written in the form of Hill equation

$$\begin{aligned}
&\ddot{\Omega}_n(\tau) + H_n(\tau)\Big[1 + H_{on}\cos(\varphi_k^* + \Theta_n)\Big]\Omega_n(\tau) = \\
&= \sum_{m\neq n}^{N} G_{mn}(\tau)\Big[1 + G_{onm}\cos(\varphi_k^* + \delta_{mn})\Big]\Omega_m(\tau) - \\
&-\Big[\frac{1}{2}h_{2n}\cos 2\varphi_k^* + \frac{1}{4}h_{3n}\cos 3\varphi_k^*\Big]\Omega_n(\tau) + \\
&+\sum_{m\neq n}^{N}\Big[\frac{1}{2}g_{2mn}\cos 2\varphi_k^* + \frac{1}{4}g_{3mn}\cos 3\varphi_k^*\Big]\Omega_m(\tau) + \\
&+\frac{l^*}{1-l^*}\dot{\Omega}_n(\tau) + \sum_{m\neq n}^{N}\frac{4nm}{n^2-m^2}\frac{l^*}{1-l^*}\dot{\Omega}_m(\tau)
\end{aligned} \tag{3.13}$$

The set of equations (3.10) can be written in the matrix from

$$\ddot{\mathbf{\Omega}} + \mathbf{B}\dot{\mathbf{\Omega}} + \mathbf{C}\mathbf{\Omega} = \mathbf{0} \tag{3.14}$$

where $\ddot{\mathbf{\Omega}}$, $\dot{\mathbf{\Omega}}$, $\mathbf{\Omega}$ are one column matrices.

The analysis of the properties of the $\mathbf{B}$ matrix elements was done by Kumaniecka (1992) and it turned out that Eqs (3.10) are coupled due to the "giroscopic" character of the $\mathbf{B}$ matrix elements for $i \neq j$.

When one analyzes the properties of the matrix $\mathbf{C}$ elements one comes to the conclusion that Eqs (3.10) are coupled not only due to the elements of $\mathbf{B}$ matrix but of the $\mathbf{C}$ matrix as well.

The most important fact is that the solution to the set (3.10) can not be limited only to the analysis of the nth equation as has been done till now by Ulshin (1975).

In the analyzed problem the solution to Eqs (3.10) is not of such importance as the determination of the regions of dynamic instability.

Because of the set (3.10) complexity we have chosen the set of only two equations for the analysis that follows.

It can be written in the form

$$\begin{aligned}
&\ddot{\Omega}_1 + \omega_1^2\Big(1 + c_{11}\cos\varphi_k^* + s_{11}\sin\varphi_k^* + c_{12}\cos 2\varphi_k^* + c_{13}\cos 3\varphi_k^*\Big)\Omega_1 - \\
&-\omega_{12}^2\Big(1 + \widehat{c}_{12}\cos\varphi_k^* + \widehat{s}_{21}\sin\varphi_k^* + c_{22}\cos 2\varphi_k^* + c_{23}\cos 3\varphi_k^*\Big)\Omega_2 + \\
&+b_{11}\dot{\Omega}_1 + b_{12}\dot{\Omega}_2 = 0 \\
&\ddot{\Omega}_2 + \omega_2^2\Big(1 + c_{21}\cos\varphi_k^* + s_{21}\sin\varphi_k^* + c_{22}\cos 2\varphi_k^* + c_{23}\cos 3\varphi_k^*\Big)\Omega_2 -
\end{aligned} \tag{3.15}$$

$$-\omega_{21}^2\left(1+\hat{c}_{11}\cos\varphi_k^* + \hat{s}_{11}\sin\varphi_k^* + c_{12}\cos 2\varphi_k^* + c_{13}\cos 3\varphi_k^*\right)\Omega_1 +$$

$$+b_{11}\dot{\Omega}_2 - b_{12}\dot{\Omega}_1 = 0$$

4. Analysis of the dynamic instability

The value of the internal force P^*, resulting from the longitudinal vibrations couples "parametrically" the transversal and longitudinal vibrations. Physically this means that for some values of parameters of the rope the longitudinal vibrations can excite the transversal vibrations with an increasing amplitude. From this it follows that there is a probability and a danger of the occurrence of parametric resonance.

In this chapter we are looking for the answer to the question

- what are the values of frequencies and amplitudes of the longitudinal vibrations for given parameters of the rope which cause the dynamic instability of the system.?

Due to the complexity of the considered set (3.15) the harmonic balance method has been chosen for determination of the boundaries of the parametric resonances.

For the analysis of this problem we have adopted the following method — the amplitudes a_0^* of the longitudinal vibrations are treated as changeable and for each of their values the interval of the frequency p_0^* is sought for.

Considering the main parametric resonance the solution on the boundaries of unstable regions is soughtfor in the form

$$\Omega_n = A_{n1}\sin\frac{1}{2}\varphi_k^* + B_{n1}\cos\frac{1}{2}\varphi_k^* + A_{n3}\sin\frac{3}{2}\varphi_k^* + B_{n3}\cos\frac{3}{2}\varphi_k^* \qquad n=1,2 \tag{4.1}$$

where k is the number of the longitudinal vibrations mode.

We assumed that

$$\frac{d\varphi_k^*}{d\tau} = p^*(\tau) \qquad\qquad \frac{d^2\varphi_k^*}{d\tau^2} = 0 \tag{4.2}$$

For the case when each of Ω_n consists of only one harmonic mode the solution is as follows

$$\Omega_n = A_{n1}\sin\frac{1}{2}\varphi_k^* + B_{n1}\cos\frac{1}{2}\varphi_k^* \qquad n=1,2 \tag{4.3}$$

In both cases we obtain a set of homogeneous equations. The condition for the existence of a nonzero solution leads to the determination of the soughtfor regions for the first and the second modes of the transversal vibrations. We also examine the possibility of the occurrence of the third mode.

If we consider the secondary parametric resonance the solution sought for has the form

$$\Omega_n = \frac{1}{2}b_{0n} + A_{n1}\sin\varphi_k^* + B_{n1}\cos\varphi_k^* + A_{n2}\sin 2\varphi_k^* + B_{n2}\cos 2\varphi_k^* \qquad n = 1,2 \tag{4.4}$$

When limiting the analysis to the first approximation

$$\Omega_n = \frac{1}{2}b_{0n} + A_{n1}\sin\varphi_k^* + B_{n1}\cos\varphi_k^* \qquad n = 1,2 \tag{4.5}$$

the boundaries of instability zone are described by the following condition

$$p^{*2} = \omega_i^2 - \frac{1}{2}\Big[\frac{1}{2}\omega_i^2(c_{i1}^2 + s_{i1}^2) + b_{11}^2\Big] \pm \frac{1}{2}\sqrt{\Delta_i} \tag{4.6}$$

where

$$\Delta_i = \Big[\frac{1}{2}\omega_i^2(c_{i1}^2 + s_{i1}^2) + b_{11}^2\Big] - 4\Big[\frac{1}{4}\omega_i^4(c_{i1}^2 - s_{i1}^2 - c_{i2})c_{i2} + b_{11}^2\omega_i^2\Big] > 0 \tag{4.7}$$

for $i = 1,2$ as we only consider the first and the second modes.

A vibrating nonlinear system with many degrees of freedom is rich in many kinds of resonances. Because of the particular type of coupling taken into account in the present constribution, periodic combination resonance is particulary interesting.

In our considerations the following assumption is adopted

$$p^* = p_1^* + p_2^* \tag{4.8}$$

where p_1^*, p_2^* are the frequencies of two components of the solution sought for

$$\Omega_n = A_{n1}\sin\psi_1^* + B_{n1}\cos\psi_1^* + A_{n3}\sin\psi_2^* + B_{n3}\cos\psi_2^* \qquad n = 1,2 \tag{4.9}$$

The following notation is used

$$\frac{d\psi_1^*}{d\tau} = p_1^* \qquad \frac{d\psi_2^*}{d\tau} = p_2^* \tag{4.10}$$

and the following assumptions are adopted

$$\frac{d^2\psi_1^*}{d\tau^2} = 0 \qquad \frac{d^2\psi_2^*}{d\tau^2} = 0 \tag{4.11}$$

The analysis was done for both, the case when two components are excited and for the case as follows

$$\Omega_n = A_{n1}\sin\psi_n^* + B_{n1}\cos\psi_n^* \qquad n = 1,2 \tag{4.12}$$

Using the balance harmonic method the set of equations was obtained

$$\begin{aligned}&4\Big[(-p_1^{*2}+\omega_1^2)(-p_2^{*2}+\omega_2^2)+p_1^*p_2^*b_{11}^2\Big] = \omega_{12}^2\omega_{21}^2\sqrt{(\widetilde{c}_{11}^2+\widetilde{s}_{11}^2)(\widetilde{c}_{21}^2+\widetilde{s}_{21}^2)}\\ &(-p_1^{*2}+\omega_1^2)p_2^* = (-p_2^{*2}+\omega_2^2)p_1^*\end{aligned} \tag{4.13}$$

The condition (4.8) and Eqs (4.13) determine the regions of the combination resonance.

5. Computations

For numerical calculations the following values of parameters have been chosen:

$$Q = 2\cdot 10^5[\mathrm{N}] \qquad \frac{\alpha}{E} = 0.043\cdot 10^{-16}[\mathrm{m^4/N^2}] \qquad EA = 2\cdot 10^8[\mathrm{N}]$$

$$q = 10^2[\mathrm{N/m}] \qquad \frac{\beta}{E} = 0.005[\mathrm{s}] \qquad c = 5\cdot 10^3[\mathrm{m/s}]$$

$$l_0 = 10^3[\mathrm{m}] \qquad \mu = 0.11$$

The starting, the uniform motion and the braking of the winding machine have been taken into account based on the diagrams given in Fig.3.

The diagrams of frequency $p^*(\tau)$ and amplitude $a^*(\tau)$ of the longitudinal vibrations were presented by Kumaniecka and Nizioł (1992).

Diagrams of the regions of the main, secondary and combination parametric resonances for the first and second modes of the transversal vibrations and for the arbitrarily chosen times $\tau = 0$ (the moment of the starting of the machine), $\tau = 22.5$ (from the starting interval), $\tau = 125$ (from the uniform motion interval) are presented in Fig.4, 5, 6.

The influence of decrease in stiffness of the system on the character of instability regions is considered. We also examine the effect of decrease in the value of weight Q^*. The result obtained is presented in Fig.7 only for the first mode of transversal vibrations and for $\tau = 22.5$. The change of physical nonlinearity has the influence on the location of instability regions. The graphs in Fig.8 shows this effect for the first mode of transversal vibrations and for $\tau = 22.5$.

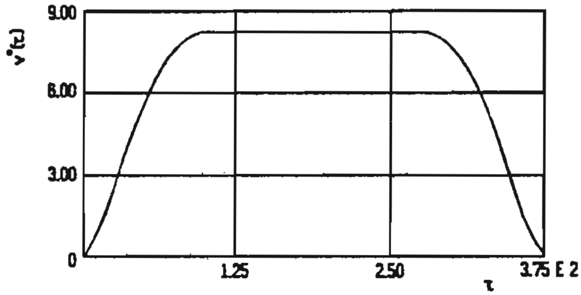

Fig. 3. Diagram of the velocity $v^*(\tau)$

6. Conclusions

The results presented in the present paper are of qualitative and quantitative character. The system considered serves as a model of a realistic system undergoing the variation of parameters. The nonlinear model which was adopted includes many features occurring during the work of ropes as a load carrying and winding elements. The set of differential equations of motion based on a physical model describes almost precisely the occurring processes.

Analysis of the dynamic instability leads us to the following conclusions

- for some values of the frequencies and the amplitudes of longitudinal vibrations, the motion of considered system can be unstable. The longitudinal vibrations bring about the increase amplitudes of the transversal vibrations;
- for the chosen parameters of rope only the first or the second modes can be parametrically extitated, but it requires a high amplitude of the longitudinal vibrations;
- the lowest value of frequency of the longitudinal vibrations, for which the third parametrically excited transversal mode occurs, is higher than the real value of frequency for the longitudinal vibrations. Having chosen the parameters of rope as in the present paper we can't excite the transversal oscillations even for high amplitudes of longitudinal vibrations;
- the higher the mode of transversal vibrations is, the wider the regions of instability occur;
- the value of peak coordinate of an unstable region along the amplitude axis depends on the coupling and it increases with the increase of velocity $v^*(\tau)$;

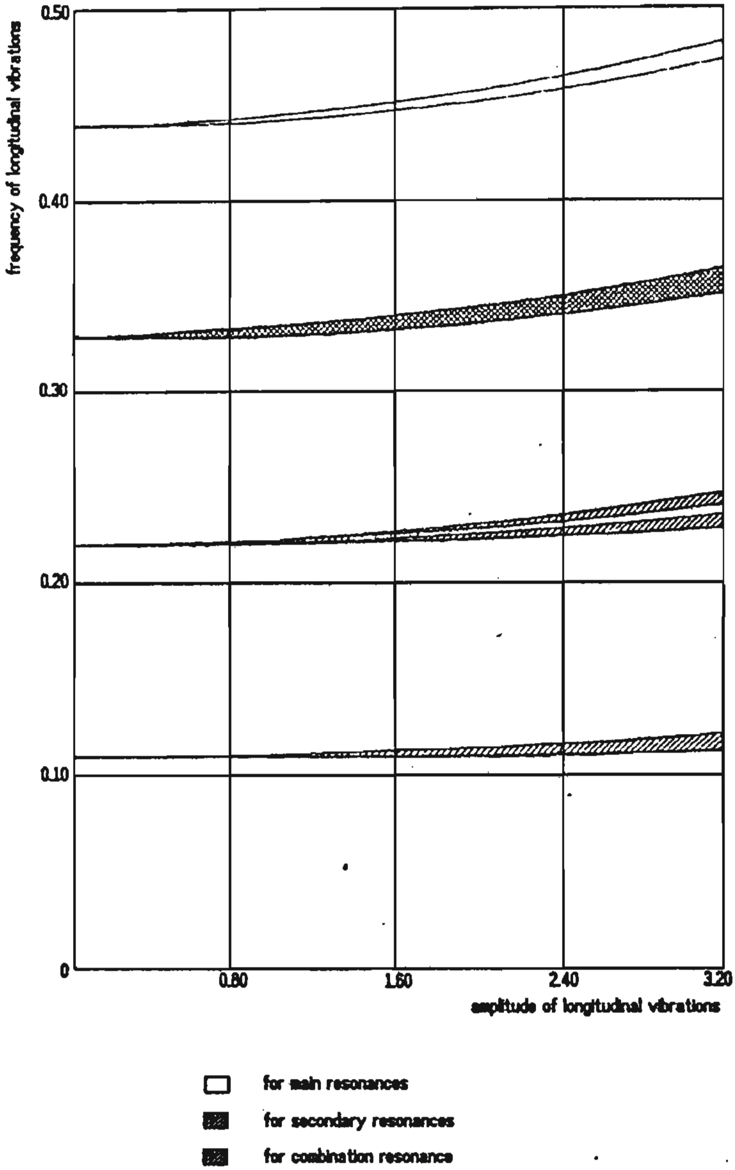

Fig. 4. Regions of: main, secondary and combination parametric resonances for first and second modes of the transversal vibrations for $\tau = 0$

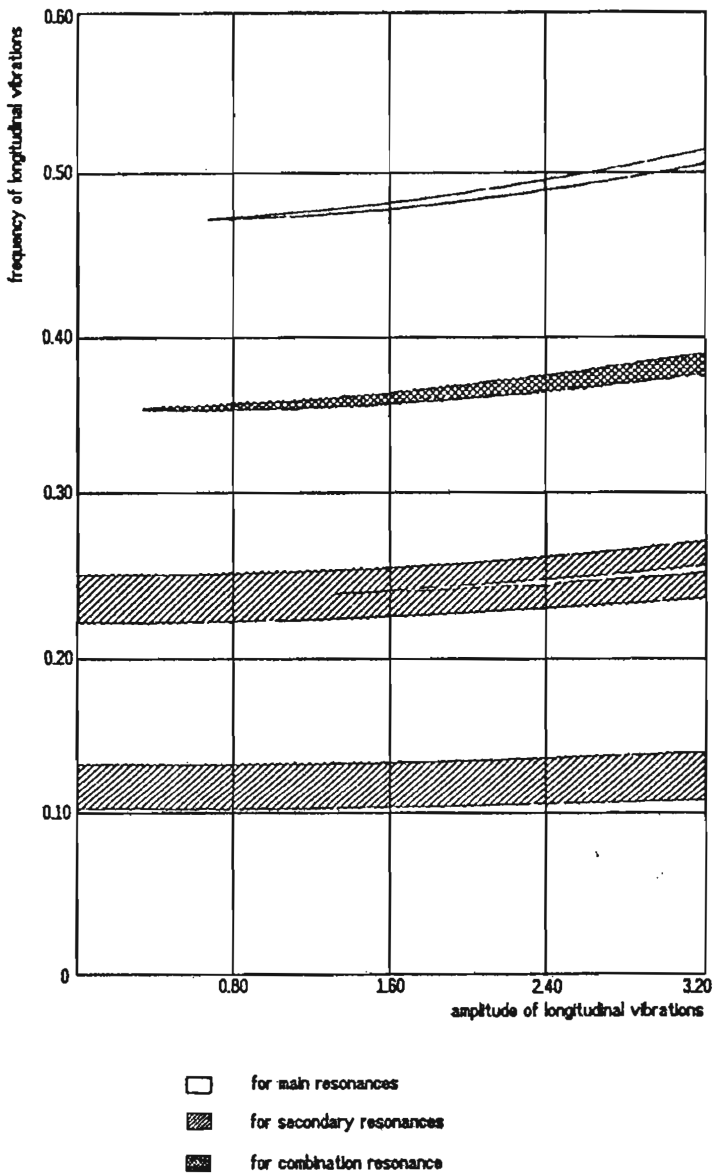

Fig. 5. Regions of: main, secondary and combination parametric resonances for first and second modes of the transversal vibrations for $\tau = 22.5$

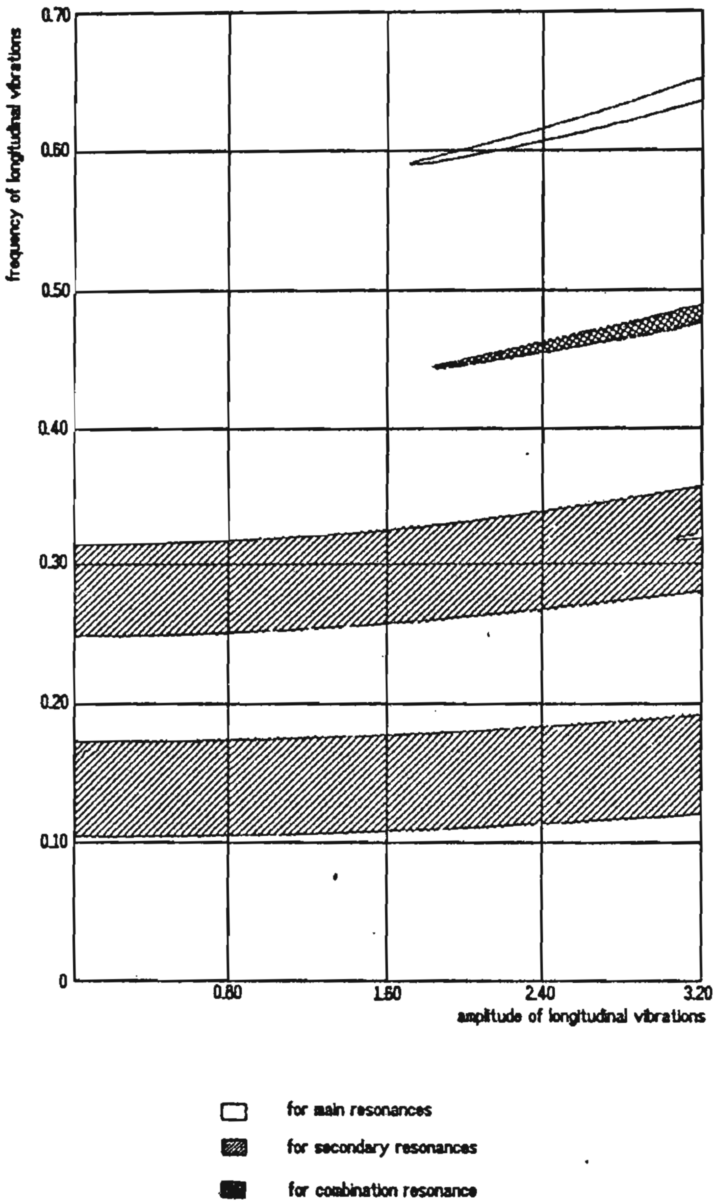

Fig. 6. Regions of: main, secondary and combination parametric resonances for first and second modes of the transversal vibrations for $\tau = 125$

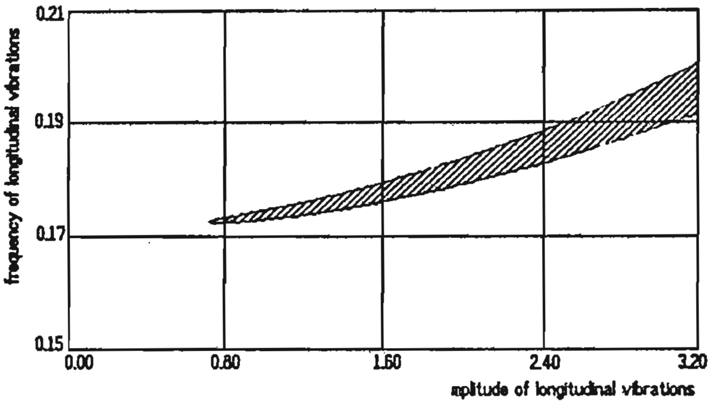

Fig. 7. Region of main parametric resonance for first mode of the transversal vibrations for $\tau = 22.5$, for load $0.5Q^*$

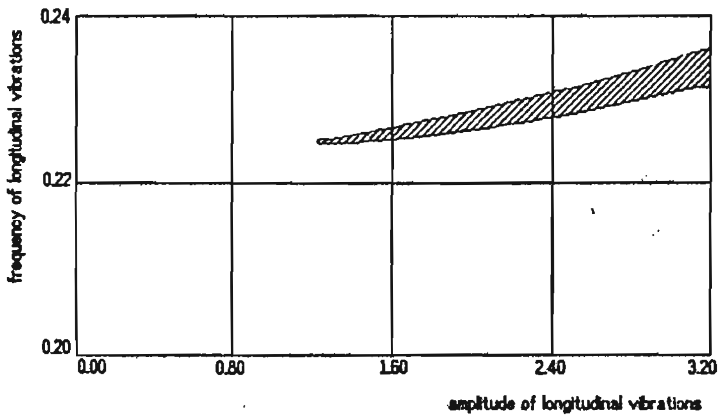

Fig. 8. Region of main parametric resonance for first mode of the transversal vibrations for $\tau = 22.5$, for load Q^* and for coefficient of physical nonlinearity 0.5λ

- for the velocity $v^* = 0$ the regions for the main, secondary and combination resonances have a similar width and they occur even for the amplitudes tending to zero;
- for small changes in the velocity v^* the unstable regions for main resonance are located in the neighborhood of doubled frequencies of the transversal vibrations for the considered first and second modes. The regions of secondary resonances are placed in the neighborhood of frequencies which are twice as low as the main resonances. The combination resonance occurs in the region of the arithmetical mean frequencies corresponding to the main resonances;
- as a result of the increasing $v^*(\tau)$ the broadening of the unstable regions for the secondary resonances is noticeable. It is due to the existence of the "giroscopic" in character elements of $\mathbf{B}$ matrix. The regions for the main and the combination resonances become narrower and the instability regions shift towards increasing values of the amplitudes a^*;
- as a result of the changes in the frequencies the instability regions shift upwards the p^* axis;
- the decrease in the load Q^* shifts the unstable regions downwards the frequency axis;
- the decrease of the physical nonlinearity has the effect of negligible narrowing of the zones of instability and it moves them down the frequency axis.

We end up with the following most important conclusion.

As a result of the coupling between the longitudinal and the transversal vibrations the parametric resonance is theoretically possible, but its occurrence requires high amplitudes of the longitudinal vibrations. It can occur only in emergencies (i.e. impulses resulting in high increase of the longitudinal amplitude a^*).

References

1. KUMANIECKA A., 1992, *Dynamika lin o zmiennej długości*, Ph.D.thesis, Cracow University of Technology
2. KUMANIECKA A., NIZIOŁ J., 1992, *The vibration of physically nonlinear rope of slowly variable length*, Nonlinear Vibration, 25
3. NIZIOŁ J., MARCZYK S., 1978, *Drgania poprzeczno-wzdłużne lin o zmiennej długości*, Engineering Transactions, 27, 3, 403-415
4. SAVIN G.N., GOROSHKO O.A., 1971, *Vvedenie v mekhaniku deformiruemykh odnomernykh tel peremennoĭ dliny*, Naukova Dumka, Kiev
5. ULSHIN P.I., 1975, *Ob ustoĭchivosti dvizheniya kanatov obladayushchikh fizicheskoĭ nelineĭnostyu*, Prikladnaya Mekhanika, 11, 6, 127-131

Stateczność dynamiczna liny z wolnozmiennymi parametrami

Streszczenie

W pracy przedstawiono analizę drgań poprzeczno-wzdłużnych lin stalowych o zmiennej długości. Analizę teoretyczną i obliczenia numeryczne wykonano na przykladzie lin wyciągowych. W analizie uwzględniono nieliniową sztywność wzdlużną, wiskotyczne tłumienie oraz tlumienie tarciem suchym pomiędzy splotkami liny proporcjonalnym do sily wzdlużnej. W budowie modelu matematycznego tarcie suche zastąpiono tarciem lepkim, przyjmując energetyczne kryterium równoważności.
Matematyczny model sprowadzono do nieliniowych równań różniczkowych cząstkowych typu hiperbolicznego z zależnymi od czasu warunkami brzegowymi. Badano drgania przy nawijaniu liny, biorąc pod uwagę wymuszenie kinematyczne, uwzględniające fazę rozruchu, ruchu ustalonego i hamowania.
Przeanalizowano problemy występowania rezonansów parametrycznych: glównych, pobocznych i kombinowanego. Zbadano wpływ parametrów liny i charakteru wymuszenia kinematycznego na ukształtowanie obszarów dynamicznej niestateczności.

Manuscript received July 13, 1992; accepted for print September 29, 1992