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ON CERTAIN METHODS OF DETERMINING THE ELLIPSES AND ELLIPSOIDS OF THE POSITIONING ACCURACY OF ROBOT MANIPULATORS[1]

WOJCIECH SZCZEPIŃSKI

ZBIGNIEW WESOŁOWSKI

Institute of Fundamental Technological Research
Polish Academy of Sciences, Warsaw

When analysing the problem of positioning accuracy of robot manipulators it is important to know how large may random deviations of the hand be from the desired position if the joint positioning errors possess a normal distribution. Two simple methods of determining the ellipses and ellipsoids of probability concentration are proposed. One of them consists in finding at first the polygon or polyhedron of the positioning accuracy, and then in finding the ellipse or ellipsoid of the principal axes and second order moments coinciding with those of the polygon or polyhedron, respectively. In the second of the proposed methods a computer generates random Gaussian deviations from the desired joint positions. The calculated numerous positioning errors are forming an elliptical or ellipsoidal pattern demonstrating good agreement with theortically obtained ellipses or ellipsoids of probability concentration.

1. Introduction

Small changes in position of the hand of a manipulator are caused among others by small random deviations Δq_i from the desired nominal joint coordinate q_i^0. In the case of a revolute joint the deviation Δq_i corresponds to a small rotation $\Delta\Theta_i$ with respect to the desired angle Θ_i^0. In the case of a prismatic joint Δq_i corresponds to a small linear deviation Δl_i from the desired distance l_i^0.

The problem of the positioning accuracy of robot manipulators has been analysed in several papers and books. Basic notions are discussed by Paul (1982). A mathematical model of random positioning errors has been developed by Kumar

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and Waldron (1981). The problem of calculating the ellipsoids of the positioning accuracy has been shortly mentioned by Antshev et al. (1988).

Three following sources of the positioning errors may be distinguished (Kumar and Waldron, 1981)

1. errors in positioning the joints accurately;
2. dynamic errors due to elastic deflections of individual members of the manipulator;
3. mechanical clearances in the system.

In the present paper only errors in positioning the hand accurately due to random Gaussian errors in positioning the joints will be analysed. A particular positioning error of the hand may be represented by a displacement vector, components of which represent deviations from the desired nominal coordinates of the hand. Since the joint positioning errors have random magnitudes in each of the repeated cycles the end point of such a vector will have random coordinates. Analysing a large number of repeated cycles of the movement of the manipulator we have to deal with the problem of probability concentration of the distribution of all the displacement vectors end points.

For manipulators operating in two dimensions the probability concentration may be represented by an ellipse of equal probability. For a general case when manipulator operates in three dimensions the probability concentration may be represented by a certain ellipsoid. We shall present below two approximate methods of determining such ellipses and ellipsoids for robot manipulators.

Let us assume that hand positioning errors result from the random errors in joint positions. A joint position error is treated as a small random rotation or displacement from the desired position of the joint. The errors Δq_i are assumed to be distributed according to a normal distribution. Thus the density of probability that the joint positioning error is of a magnitude Δq_i is given by the formula

$$\varphi(\Delta q_i) = \frac{1}{\sigma_i\sqrt{2\pi}} \exp\Big[-\frac{1}{2}\Big(\frac{\Delta q_i}{\sigma_i}\Big)^2\Big] \tag{1.1}$$

where σ_i is the standard deviation, value of which depends on the accuracy of the joint labeled by the index i.

In the present work an approximate theoretical procedure for determining the probability concentration ellipses and ellipsoids has been used (cf Cramer, 1946). At first we shall assume that instead of the normal distribution, Eq (1.1), of joint positioning errors, this distribution is uniform with the same standard deviation σ_i.

Then we shall find for such uniform distribution the regions within which all the possible error displacements of the hand will be located. For manipulators operating in two dimensions such regions take the form of certain polygons, while for manipulators operating in three dimansions they have the form of certain polyhedrons. As the final step of calculations we shall find the orientation and dimensions od the ellipses or ellipsoids of the same principal axes and second order moments as those of the corresponding polygons or polyhedrons, respectively.

2. Ellipses of positioning accuracy concentration

We shell now analyse the positioning accuracy of manipulators operating in two dimensions. Let us assume a Cartesian coordinate system XY. Any position of a chosen reference point of the hand is defined by its two coordinates. Each coordinate is a certain function of the joint position parameters $q_i = q_i^0 + \Delta q_i$

$$X = X(q_1, q_2, ..., q_n)$$
$$Y = Y(q_1, q_2, ..., q_n) \tag{2.1}$$

To analyse the hand positioning errors we shell use a local coordinate system xy with the axes parallel to the corresponding axes of the basic system XY and the origin at the desired position of the reference point on the hand. The positioning error will be represented by a displacement vector $\boldsymbol{v}$ with the components

$$x = X - X^0 \qquad\qquad y = Y - Y^0 \tag{2.2}$$

where X^0, Y^0 define the desired position of the hand and X, Y are the actual coordinates of the hand position.

We shall assume that the concentration of the two-dimensional distribution of the hand positioning error may be represented in the reference system xy by a certain ellipse

$$\left(\frac{x}{\sigma_x}\right)^2 + \left(\frac{y}{\sigma_y}\right)^2 - 2\rho_{xy}\frac{x}{\sigma_x}\frac{y}{\sigma_y} = \text{const} \tag{2.3}$$

where σ_x, σ_y are the standard deviations and ρ_{xy} is the correlation coefficient.

According to the approximate procedure used in this paper we shall at first assume that the distribution of joint positioning errors is uniform with the same standard deviation as that of the original Gaussian distribution. Note that the two standard deviations are equal if the errors in the uniform distribution are limited by two extreme values $\pm\sigma_i\sqrt{3}$.

It has been demonstrated by Szczepiński (1955) that when joint positioning errors vary within the two extreme values, then the end point of any error displacement vector of the hand will lie inside a certain polygon bounded by several pairs of parallel straight lines. The equations of these lines are

$$\frac{\partial Y}{\partial q_r}x - \frac{\partial X}{\partial q_r}y = \sum_{i=1}^{n} \begin{vmatrix} \frac{\partial X}{\partial q_i} & \frac{\partial X}{\partial q_r} \\ \frac{\partial Y}{\partial q_i} & \frac{\partial Y}{\partial q_r} \end{vmatrix} \Delta q_i \tag{2.4}$$

Writing consecutively such equations for all joint positioning parameters q_r we obtain the equations of several families of parallel straight lines. Their extreme positions constitute the edges of the polygon of the positioning accuracy.

Now having found the polygon we can calculate its second order (inertia) moments and then find the orientation of its principal axes 1, 2 and second order principal moments J_1, J_2. The principal radii a and b of the ellipse of probability concentration can be calculated by solving the following system of equations

$$\frac{1}{4}\pi a^3 b = J_1 \qquad \frac{1}{4}\pi a b^3 = J_2 \tag{2.5}$$

2.1. Examples of application

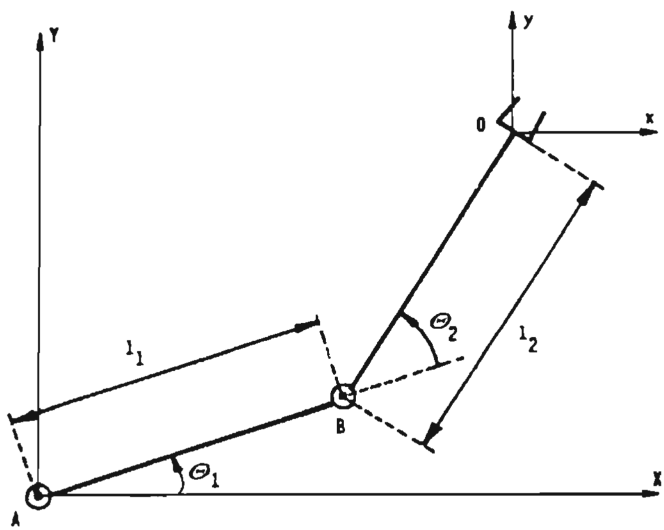


Fig. 1.

As the working examples we shall determine the ellipses of probability concentration of the positioning accuracy for a simple manipulator with two revolute joints shown schematically in Fig.1.

The position of the hand (point 0) is determined by the functions of two independent variables Θ_1 and Θ_2

$$X = l_1 \cos\Theta_1 + l_2 \cos(\Theta_1 + \Theta_2)$$
$$Y = l_1 \sin\Theta_1 + l_2 \sin(\Theta_1 + \Theta_2) \tag{2.6}$$

In the particular examples we assume

$$l_1 = l_2 = 1000 \text{ mm}$$

Standard deviations for the normal distribution of joint positioning errors are taken to be

$$\sigma_{\Theta_1} = \sigma_{\Theta_2} = 0.01 \text{ rad}$$

The polygon of the positioning accuracy will be bounded by extreme positions of straight lines determined by the equations (see Eq (2.4))

$$\frac{\partial Y}{\partial \Theta_1} x - \frac{\partial X}{\partial \Theta_1} y = \begin{vmatrix} \frac{\partial X}{\partial \Theta_2} & \frac{\partial X}{\partial \Theta_1} \\ \frac{\partial Y}{\partial \Theta_2} & \frac{\partial Y}{\partial \Theta_1} \end{vmatrix} \Delta\Theta_2 \tag{2.7a}$$

for the first family of lines and

$$\frac{\partial Y}{\partial \Theta_2} x - \frac{\partial X}{\partial \Theta_2} y = \begin{vmatrix} \frac{\partial X}{\partial \Theta_1} & \frac{\partial X}{\partial \Theta_2} \\ \frac{\partial Y}{\partial \Theta_1} & \frac{\partial Y}{\partial \Theta_2} \end{vmatrix} \Delta\Theta_1 . \tag{2.7b}$$

for the second family. Here $\Delta\Theta_1 = \Delta\Theta_2 = \pm 0.01\sqrt{3}$ rad.

2.1.1. *Example 1*

Desired position of the hand is determined by the following joint positioning angles

$$\Theta_1 = 0 \qquad \Theta_2 = \frac{1}{2}\pi \tag{2.8}$$

Making use of relations (2.6) and Eqs (2.7) we find that the polygon of the positioning accuracy is bounded by two pairs of straight lines

$$x + y = \pm 17.3 \text{ mm} \qquad y = \pm 17.3 \text{ mm}$$

The polygon is shown in Fig.2.

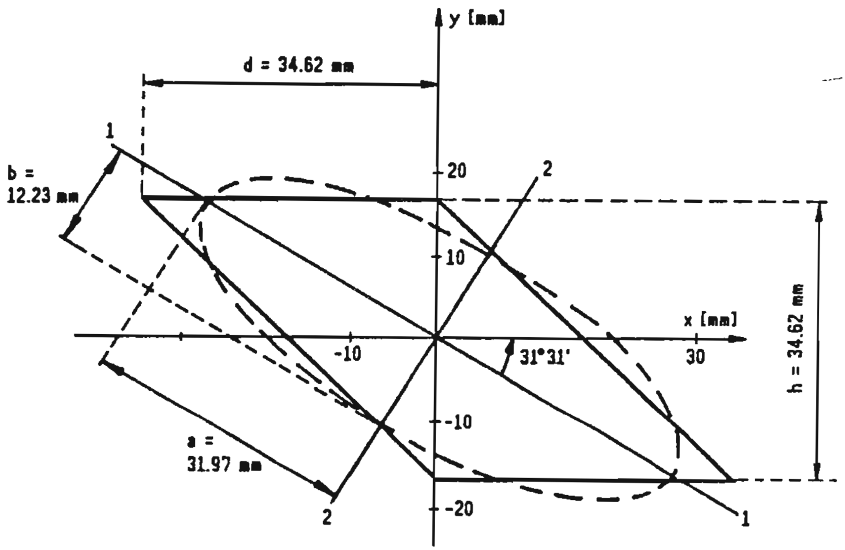


Fig. 2.

The second order (inertia) moments of the polygon with respect to the reference axes x and y are

$$J_x = \frac{1}{12} dh^3 = 12 \cdot 10^4 \text{ mm}^4$$

$$J_y = \frac{1}{6} d^3 h = 24 \cdot 10^4 \text{ mm}^4$$

and the mixed second order moment is

$$J_{xy} = \frac{1}{12} d^2 h^2 = 12 \cdot 10^4 \text{ mm}^4$$

Now the principal second order moments of the polygon can be found by constructing the Mohr circle. These principal moments are

$$J_1 = 31.4 \cdot 10^4 \text{ mm}^4 \qquad J_2 = 4.6 \cdot 10^4 \text{ mm}^4$$

The principal axis 1 makes the angle of $31°31'$ with the x-axis as shown in Fig.2.

The principal radii of the ellipse of probability concentration can now be calculated by solving the system of equations (2.5). Finally we obtain

$$a = 31.97 \text{ mm} \qquad b = 12.23 \text{ mm}$$

The ellipse is shown in Fig.2.

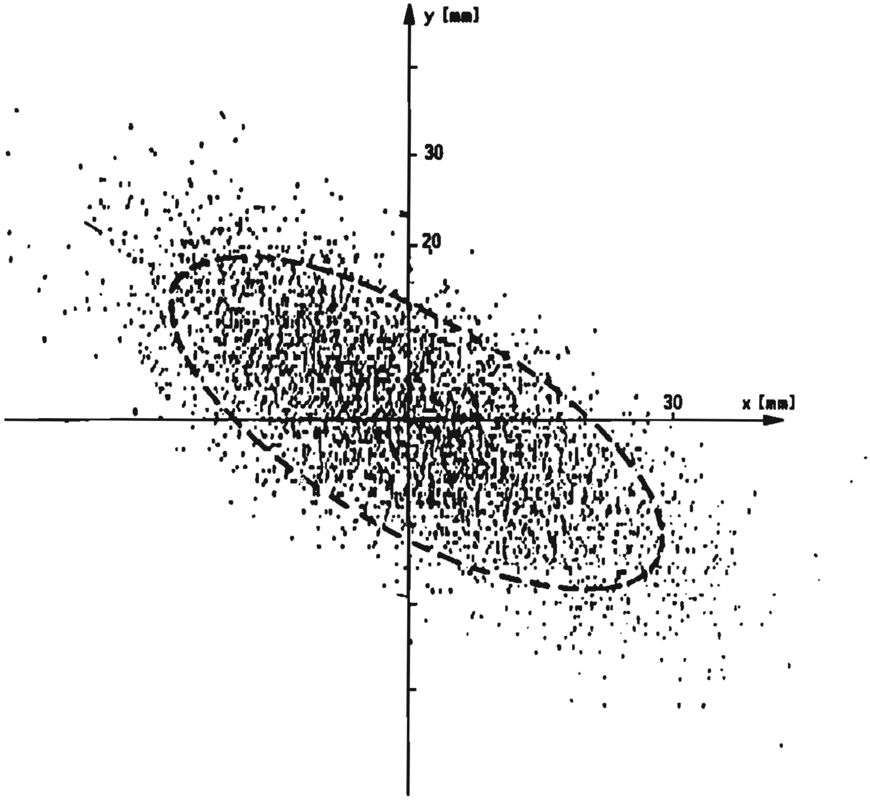


Fig. 3.

The theoretically calculated ellipse from Fig.2 has been compared with the results of a numerical experiment. Random small Gaussian deviations from the desired nominal joint positions Eq (2.8) have been numerically generated by a program for a personal computer calculating the displacement of the hand from its nominal position. Calculated displaced positions are shown in Fig.3 as the corresponding points. Altogether five thousand repeated cycles of the movement have been numerically simulated with randomly generated joint positioning errors. The theoretical ellipse from Fig.2 is also shown in Fig.3 for ready comparison. It can be seen that the theoretical ellipse coincide well with the assembly of points obtained in the numerical experiment.

2.1.2. Example 2

The desired position of the manipulator's hand shown in Fig.1 is determined by the following values of joint position angles

$$\theta_1 = 0 \qquad \theta_2 = \frac{3}{4}\pi$$

Repeating the procedure described in Example 1 we find the theoretical ellipse of probability concentration of the positioning accuracy.

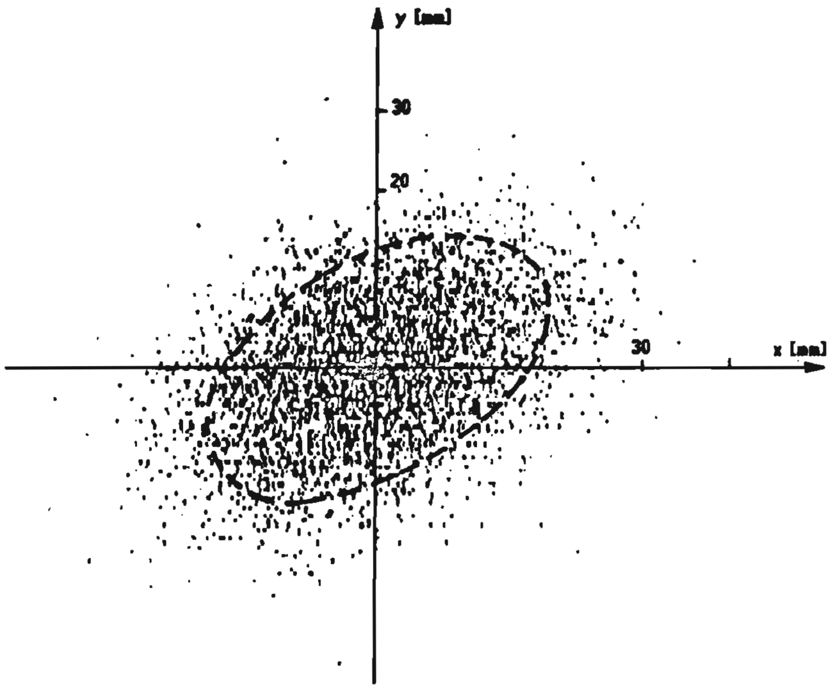


Fig. 4.

In Fig.4 the ellipse is compared with the results of a numerical experiment analogous to that described in the previous example.

2.1.3. *Example 3*

The desired position of the hand is now determined by the joint position angles

$$\theta_1 = 0 \qquad \theta_2 = \frac{1}{4}\pi$$

The theoretical ellipse is compared in Fig.5 with the result of a numerical experiment as in the previous examples.

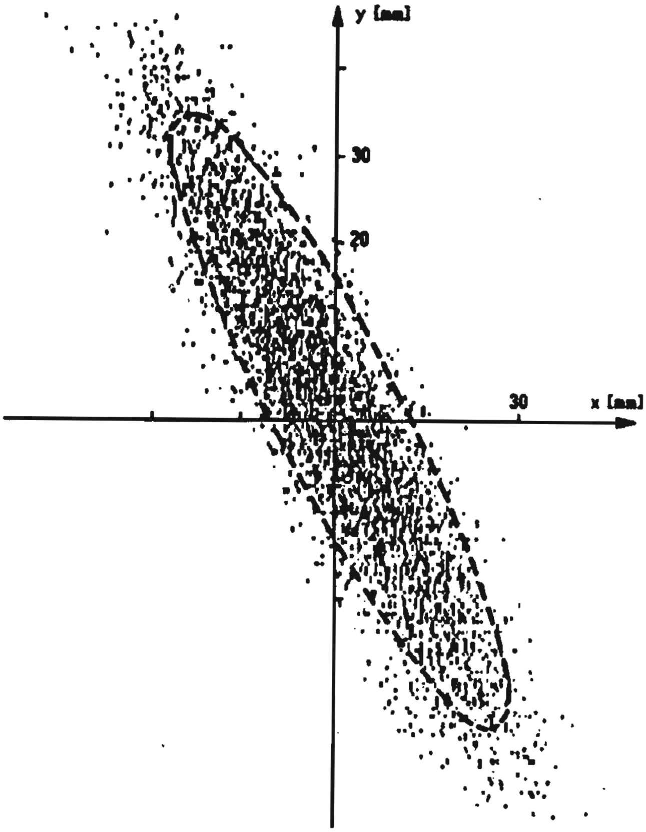


Fig. 5.

3. Ellipsoids of positioning accuracy concentration

Now we shall analyse the positioning accuracy for more general cases of manipulators operating in three dimensions. In a Cartesian coordinate system any position of a chosen reference point of the hand is defined by its three coordinates. Each coordinate is a certain function of joint positioning parameters $q_i = q_i^0 + \Delta q_i$

$$\begin{aligned} X &= X(q_1, q_2, ..., q_n) \\ Y &= Y(q_1, q_2, ..., q_n) \\ Z &= Z(q_1, q_2, ..., q_n) \end{aligned} \tag{3.1}$$

To analyse hand positioning errors we shall introduce a local coordinate sytem xyz with the axes parallel to the respective axes of the basic system XYZ and the origin at the desired position of the hand. Any positioning error will be represented by a certain vector v with the components

$$x = X - X^0 \qquad y = Y - Y^0 \qquad z = Z - Z^0 \tag{3.2}$$

where X^0, Y^0, Z^0 define the desired position of the hand and X, Y, Z are the actual coordinates of the hand position.

We shall asume that the concentation of three-dimensional distribution of the hand positioning errors may be reresented in the reference system xyz by a certain ellipsoid

$$\begin{aligned}&\Big(\frac{x}{\sigma_x}\Big)^2 + \Big(\frac{y}{\sigma_y}\Big)^2 + \Big(\frac{z}{\sigma_z}\Big)^2 - 2\rho_{xy}\frac{x}{\sigma_x}\frac{y}{\sigma_y} - 2\rho_{yz}\frac{y}{\sigma_y}\frac{z}{\sigma_z} - \\ &-2\rho_{zx}\frac{z}{\sigma_z}\frac{x}{\sigma_x} = \text{const}\end{aligned} \tag{3.3}$$

where σ_x, σ_y, σ_z are standard deviations and ρ_{xy}, ρ_{yz}, ρ_{zx} are correlation coefficients, respectively.

Similarly as in the two-dimensional problem we shall at first assume that the distribution of joint positioning errors is uniform and that it is limited by the extreme values $\mp\sigma_i\sqrt{3}$.

It has been demonstrated in the previous paper (Szczepiński, 1991) that when the joint positioning errors vary within certain limits (joint positioning tolerances), then the end points of all vectors of the error displacements from the desired position of the hand will lie inside a certain polyhedron bounded by a family of pairs of parallel planes. The equations of these planes have the following form

$$\begin{aligned}&\begin{vmatrix} \frac{\partial Y}{\partial q_r} & \frac{\partial Y}{\partial q_s} \\ \frac{\partial Z}{\partial q_r} & \frac{\partial Z}{\partial q_s} \end{vmatrix} x + \begin{vmatrix} \frac{\partial Z}{\partial q_r} & \frac{\partial Z}{\partial q_s} \\ \frac{\partial X}{\partial q_r} & \frac{\partial X}{\partial q_s} \end{vmatrix} y + \begin{vmatrix} \frac{\partial X}{\partial q_r} & \frac{\partial X}{\partial q_s} \\ \frac{\partial Y}{\partial q_r} & \frac{\partial Y}{\partial q_s} \end{vmatrix} z = \\ &= \sum_{i=1}^{n} \begin{vmatrix} \frac{\partial X}{\partial q_r} & \frac{\partial X}{\partial q_s} & \frac{\partial X}{\partial q_i} \\ \frac{\partial Y}{\partial q_r} & \frac{\partial Y}{\partial q_s} & \frac{\partial Y}{\partial q_i} \\ \frac{\partial Z}{\partial q_r} & \frac{\partial Z}{\partial q_s} & \frac{\partial Z}{\partial q_i} \end{vmatrix} \Delta q_i\end{aligned} \tag{3.4}$$

The end point of the vector of hand positioning errors moves along one of such planes when two joint positioning errors Δq_r and Δq_s change, while all remaining joint positioning errors are kept constant.

We shall obtain two extreme positions of these planes by taking appropriately the extreme values of the joint positioning errors $\Delta q_i^+ = +\sigma_i\sqrt{3}$ or $\Delta q_i^- = -\sigma_i\sqrt{3}$.

Taking consecutively all the possible combinations of pairs of joint positioning errors Δq_r and Δq_s as changing parameters we obtain equations of various families of parallel planes and then their extreme positions forming the faces of the polyhedron of positioning accuracy.

Now following the known procedure, Cramer (1946), we can find the orientation and dimensions of the ellipsoid (3.3), which should have the same principal axes and second order moments as the polyhedron.

Having found the polyhedron in the space of positioning errors we can calculate its second order moments (volume inertia moments) with respect to the reference axes x, y, z. As the next step we can find the orientation of the principal axes 1,2,3 of the polyhedron and also its second order principal moments J_1, J_2, J_3.

The principal radii a, b, c of the ellipsoid can be calculated by solving the system of equations

$$\begin{aligned} \frac{4}{15}\pi a^3 bc &= J_1 \\ \frac{4}{15}\pi a b^3 c &= J_2 \\ \frac{4}{15}\pi abc^3 &= J_3 \end{aligned} \tag{3.5}$$

where J_1, J_2, J_3 are the second order principal moments calculated for the polyhedron.

3.1. Examples of application

As the working examples we shall determine the positioning accuracy ellipsoids for the hand of a simple 4-R manipulator with four revolute kinematic pairs. The manipulator is shown schematically in Fig.6.

Joint positions are determined by three positioning angles $\Theta_1, \Theta_2, \Theta_3$. Position of the joint with the axis 4-4 has no influence on the position of the hand.

The position of the hand in the basic reference system XYZ is determined by the coordinates

$$\begin{aligned} X &= \left[l_1 \cos\Theta_2 + l_2 \cos(\Theta_2 + \Theta_3)\right] \cos\Theta_1 \\ Y &= l_1 \sin\Theta_2 + l_2 \sin(\Theta_2 + \Theta_3) \\ Z &= \left[l_1 \cos\Theta_2 + l_2 \cos(\Theta_2 + \Theta_3)\right] \sin\Theta_1 \end{aligned} \tag{3.6}$$

These coordinates are functions of three random independent variables $\Theta_1, \Theta_2, \Theta_3$. Linear dimensions l_1 and l_2 do not change their values. Thus for the problem in question general relations (3.1) are written in the particular form (3.6).

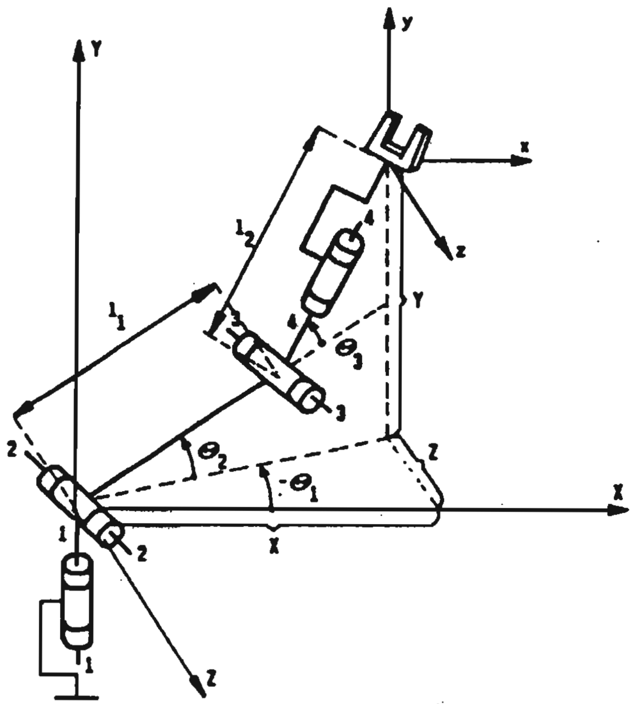


Fig. 6.

Numerical examples for the manipulator shown in Fig.6 will be calculated for the following data

$$l_1 = l_2 = 1000 \text{ mm} \tag{3.7}$$

Standard deviations for the normal distribution of joint positioning errors are taken to be

$$\sigma_{\Theta_1} = \sigma_{\Theta_2} = \sigma_{\Theta_3} = 0.01 \text{ rad}$$

These standard deviations are taken deliberately large in order to demonstrate that the linear theory used when Eq (3.4) were derived may be applied in a wide range of joint positioning errors similarly as in two-dimensional problems discussed in Section 2.

The polyhedron of positioning accuracy will be bounded by the extreme positions of planes determined by the equations (see Eq (3.4))

$$\begin{vmatrix} \frac{\partial Y}{\partial \Theta_r} & \frac{\partial Y}{\partial \Theta_s} \\ \frac{\partial Z}{\partial \Theta_r} & \frac{\partial Z}{\partial \Theta_s} \end{vmatrix} x + \begin{vmatrix} \frac{\partial Z}{\partial \Theta_r} & \frac{\partial Z}{\partial \Theta_s} \\ \frac{\partial X}{\partial \Theta_r} & \frac{\partial X}{\partial \Theta_s} \end{vmatrix} y + \begin{vmatrix} \frac{\partial X}{\partial \Theta_r} & \frac{\partial X}{\partial \Theta_s} \\ \frac{\partial Y}{\partial \Theta_r} & \frac{\partial Y}{\partial \Theta_s} \end{vmatrix} z = \sum_{i=1}^{3} \begin{vmatrix} \frac{\partial X}{\partial \Theta_r} & \frac{\partial X}{\partial \Theta_s} & \frac{\partial X}{\partial \Theta_i} \\ \frac{\partial Y}{\partial \Theta_r} & \frac{\partial Y}{\partial \Theta_s} & \frac{\partial Y}{\partial \Theta_i} \\ \frac{\partial Z}{\partial \Theta_r} & \frac{\partial Z}{\partial \Theta_s} & \frac{\partial Z}{\partial \Theta_i} \end{vmatrix} \Delta\Theta_i \tag{3.8}$$

where $\Delta\Theta_i = \pm 0.01\sqrt{3}$ rad.

3.1.1. Example 1

The desired position of the hand is determined by the following nominal values of joint positioning angles

$$\Theta_1 = \Theta_2 = 0 \qquad \Theta_3 = \frac{1}{2}\pi \tag{3.9}$$

Making use of Eqs (3.6) and (3.8) we find that the polyhedron of the positioning accuracy is bounded by three pairs of planes. The equations of these planes are

$$\begin{aligned} x + y &= \pm 17.3\,\text{mm} \\ y &= \pm 17.3\,\text{mm} \\ z &= \pm 17.3\,\text{mm} \end{aligned} \tag{3.10}$$

The polyhedron is shown in Fig.7.

The second order moments of the polyhedron with respect to the reference planes are

$$J_{xx} = \frac{1}{6} d^3 he = 8.26 \cdot 10^6\ \text{mm}^5$$

$$J_{yy} = \frac{1}{12} dh^3 e = 4.13 \cdot 10^6\ \text{mm}^5$$

$$J_{zz} = \frac{1}{12} dhe^3 = 4.13 \cdot 10^6\ \text{mm}^5$$

and the mixed second order moment with respect to the planes xz and yz is

$$J_{x-y} = -\frac{1}{12} d^2 h^2 e = -4.13 \cdot 10^6\ \text{mm}^5$$

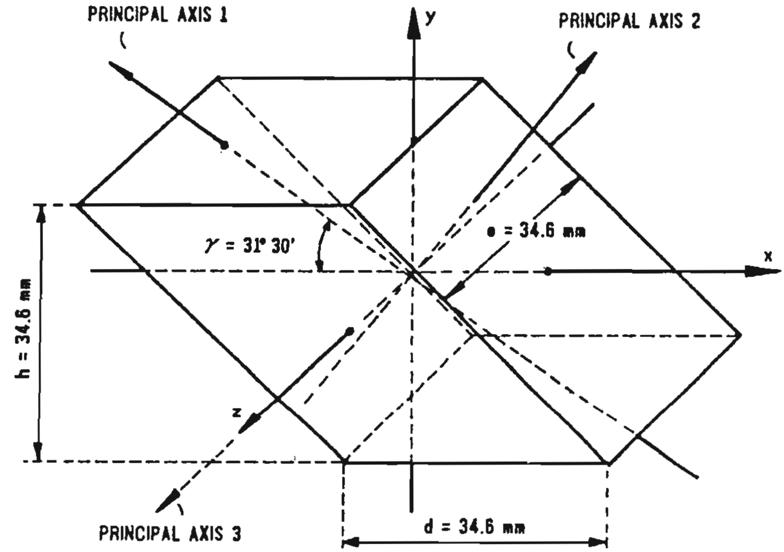


Fig. 7.

Now principal second order moments of the polyhedron can be found by constructing Mohr circles. These principal second order moments are

$$J_1 = 10.85 \cdot 10^6 \text{ mm}^5$$
$$J_2 = 1.58 \cdot 10^6 \text{ mm}^5$$
$$J_3 = 4.13 \cdot 10^6 \text{ mm}^5$$

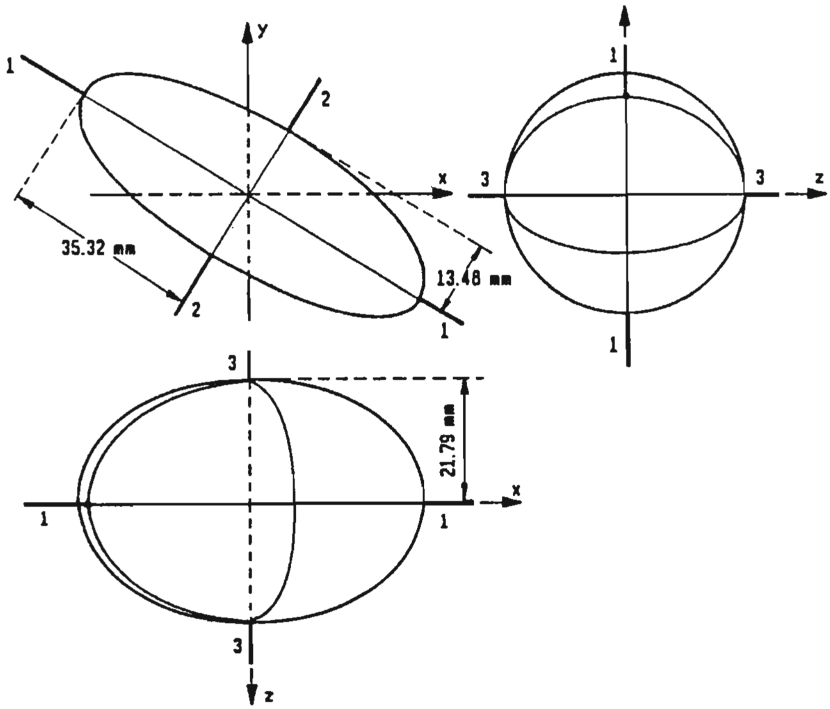
The principal axis 1 makes the angle $\gamma = 31°30'$ with the direction of the x-axis as shown in the figure.

The principal radii of the ellipsoid of probability concentration can now be calculated by solving equations (3.5). Finally we obtain

$$a = 35.32 \text{ mm} \qquad b = 13.48 \text{ mm} \qquad c = 21.79\text{mm}$$

Three projections of this ellipsoid are shown in Fig.8.

The theoretical ellipsoid has been compared with the results of a numerical experiment. Random small Gaussian deviations from the desired joint positions (3.9) have been numerically generated by a program for a personal computer calculating the displacement of the hand from its nominal position. Calculated displaced positions are shown in Fig.9 as projections of the corresponding points. Altogether five thousand repeated cycles of the movement have been numerically simulated with randomly generated joint positioning errors. Theoretical ellipsoid


y
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2
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35.32 mm
13.48 mm
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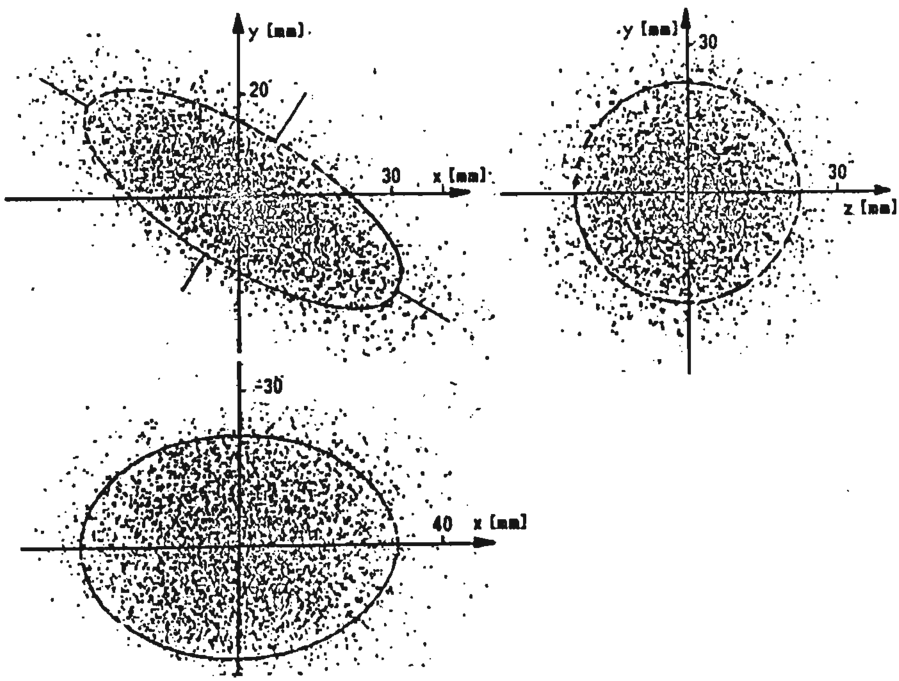
Fig. 8.


y [mm]
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z [mm]
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x [mm]

from Fig.8 is also shown in Fig.9. It can be seen that the ellipsoid obtained theoretically coincides well with the assembly of points obtained during the numerical experiment.

3.1.2. Example 2

In this example the same manipulator as shown in Fig.6 is considered. However, now the desired position of the hand is different than that analysed in the previous example. It is now determined by the following nominal values of joint positioning angles

$$\Theta_1 = \Theta_2 = 0 \qquad \Theta_3 = \frac{3}{4}\pi \tag{3.11}$$

Repeating the procedure described in the example 1 we obtain the following values of the principal radii of the ellipsoid of probability concentration of the positioning accuracy

$$a = 23.7 \text{ mm} \qquad b = 14.0 \text{ mm} \qquad c = 6.4 \text{ mm}$$

The principal axis 1 makes the angle $\gamma = 28°30'$ with the direction of the x-axis.

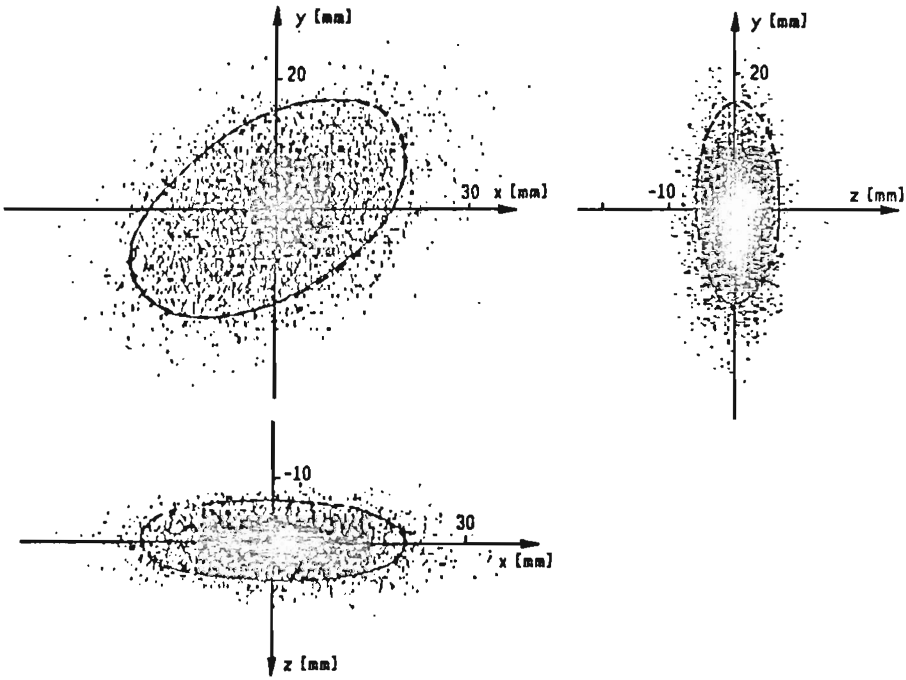

Fig. 10.

As in the Example 1 theoretical ellipsoid has been compared with the results of a numerical experiment analogous to that described above. Random positioning errors of the hand calculated by the computer are shown in Fig.10 together with the projections of the theoretical ellipsoid.

4. Concluding remarks

All the examples presented above demontrate that the theoretical ellipses and ellipsoids of probability concentration with the use of the proposed simple approximate mehod coincide well with the results of numerical simulation. Thus this approximate method seems to be of practical significance for the analysis of the positioning accuracy of manipulators. The examples show moreover how strongly the positioning accuracy depends on the particular position in which the manipulator is operating.

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O pewnych metodach wyznaczania elips i elipsoid dokładności pozycjonowania manipulatorów

Streszczenie

Przy analizowaniu dokładności pozycjonowania manipulatorów istotne jest określenie jakie mogą być przypadkowe odchylenia pozycji uchwytu od żądanej pozycji, jeśli błędy

ustawiania par kinematycznych mają rozklad normalny. Zaproponowano dwie proste metody wyznaczania elips i elipsoid koncentracji prawdopodobieństwa. Pierwsza z nich polega na uprzednim wyznaczeniu wieloboków dla wielościanów dokładności pozycjonowania i następnym wyznaczeniu elips lub elpsoid o osiach głównych i momentach drugiego rzędu takich samych jakie mają te wieloboki lub wielościany. W drugiej metodzie generuje się za pomocą komputera przypadkowe Gaussowskie odchylenia od żądanych pozycji par kinematycznych. Przy dużej liczbie wygenerowanych błędów tworzą one układy punktów dobrze pokrywające się z teoretycznymi elipsami lub elipsoidami.

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