MECHANIKA TEORETYCZNA
I STOSOWANA
Journal of Theoretical
and Applied Mechanics
3, 31, 1993

INVOLUTE GEARS WITH VARIABLE PRESSURE ANGLE PROFILE FOR MECHATRONIC APPLICATION

WALDEMAR OLEKSIUK

Faculty of Fine Mechanics
Warsaw University of Technology

Involute toothed gears for mechatronic application are discussed with particular attention paid to the gears withvariably modified tooth profiles.

1. Introduction

This issue of our quarterly "Mechanika Teoretyczna i Stosowana * Journal of Theoretical and Applied Mechanics" is devoted to mechatronics. Mechatronics is a relatively new notion, which has been used in science and technology for some time, although it is still waiting for a satisfactory definition. However, it is widely accepted, that mechatronics is at least indicating the direction to be followed by the contemporary precision engineering.

Being far from trying to define the concepts and aware of many gear specialists reading this magazine, I should like to present my views on gear transmissions for application in mechatronic devices and, in particular, to indicate those designed exclusively for mechatronics. So perhaps it could provide some explanation of a certain part of mechatronics.

It is commonly presumed, that mechatronic devices are those, designed for production, processing or storage of information data. Thus, the related transmissions, which operate under low load conditions, should be as small as possible with small modules and low numbers of teeth, and perform with extreme accuracy and low resistance. Generally, modules are related in these gears to specifications of the generating machines and to practicable means of measurement of the tolerances rather, than to the loads.

2. Typical mechatronic gear transmissions

The involute toothed gears applied in mechatronic devices can be split into two groups: one, gears with standard tooth profiles and two, those with other, or so called, modified profiles. Group one covers the well known, small module transmissions, which do not need any explanation. The gears of group two are sought, when the transmission specification can not be met by the standard profile gears. These may be constant speed ratio or constant torque (or rather minimum torque variation) transmission gears. The latter type can effectively replace the clockwork mesh gears, which require relatively complex generating methods.

It has Tryliński (1978) found, that the constant speed ratio gears may not be at the same time the constant torque gears. Thus, it would be more convenient to discuss the two types separately.

2.1. Constant speed ratio gears

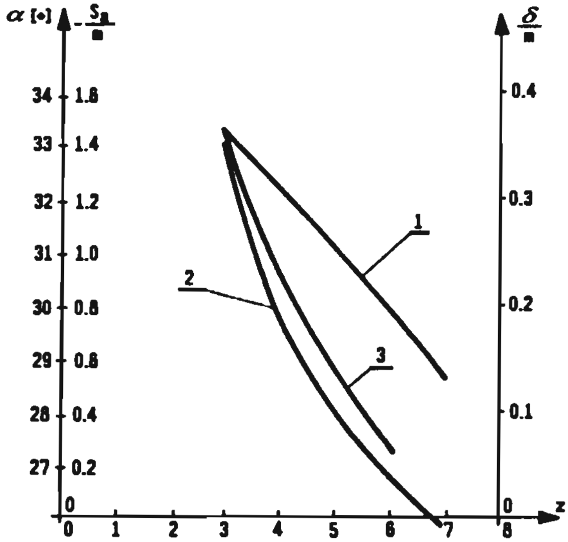

Fig. 1. Effect of wider nominal pressure angles on minimum number of teeth in pinion, relative tip thickness and tooth height modification depth (Curves 1, 2 and 3, respectively)

The gear transmissions are very often required to be as small as possible. Reducing the number of teeth is the simplest way to meet this requirement. It can be readily seen Oleksiuk (1983), that wider nominal pressure angles are very helpful in arriving at the minimum number of teeth. See the diagrams in Fig.1, where Curve 1 represents the optimum value of nominal pressure angle α respective to the pinion number of teeth z $(\alpha = f(z))$, Curve 2 shows relative tip surface width S_a (against module m) of a pinion tooth at the optimum nominal pressure angle $(S_a/m = f(z))$, Curve 3 shows relative tip modification depth δ (reduction of tooth height) in pinion $(\delta/m = f(z))$. The diagrams indicate, that at wider nominal pressure angles, pinions with seven teeth could be generated without height reduction or, if six or even five teeth were designed, the tooth height reduction would be negligible (smaller than that resulting from standard nominal pressure angle).

If prerequisite of the correct mesh as applicable in our case was similar to that in standard gear transmissions (that pinion and wheel had equal modules and nominal pressure angles), the wheel should also have the nominal pressure angle widened to a value applied in the pinion. However, this would deteriorate operating conditions of the pair, since wider nominal pressure angles result in respectively wider operating pressure angle and also, smaller engagement factor. This is particularly severe when adding to low number of teeth and presumably lower height of teeth in pinion. In extreme circumstances the engagement factor may prove too small for correct operation of the gear transmission.

However, this problem could be relatively easy to overcome by using narrower nominal pressure angle α_k in the mating wheel than that in the pinion. It seems reasonable, that nominal pressure angle α_k should be selected so as to arrive at operating pressure angle $\alpha_{tw} = 20°$. Having this condition in mind, we can calculate α_k from the formula

$$\alpha_k = \arccos \frac{(Z + z)\cos 20° - z\cos\alpha_z}{Z} \tag{2.1}$$

where

Z – number of teeth in wheel

z – number of teeth inpinion

α_z – nominal pressure angle in pinion.

However, if such transmission gear (or any other pair of mating gears with different nominal pressure angles) was to operate at constant speed ratio, base pitch values of the wheel and pinion should be made equal, Oleksiuk (1983). Thus, the following requirement should be met

$$m_1 \cos\alpha_1 = m_2 \cos\alpha_2 \tag{2.2}$$

where

m_1, m_2 – modules in the driving and driven gear, respectively

α_1, α_2 – nominal pressure angles in the driving and driven gear, respectively.

The above relation is of more general nature, than the commonly applied principle of equal modules and nominal pressure angles, since in this case we can match gears that have different modules and nominal pressure angles. It is much more practicable to start working from changes in nominal pressure angles, for the module values would then be dependent on the nominal pressure angles only.

2.2. Low variation momentary torque gears

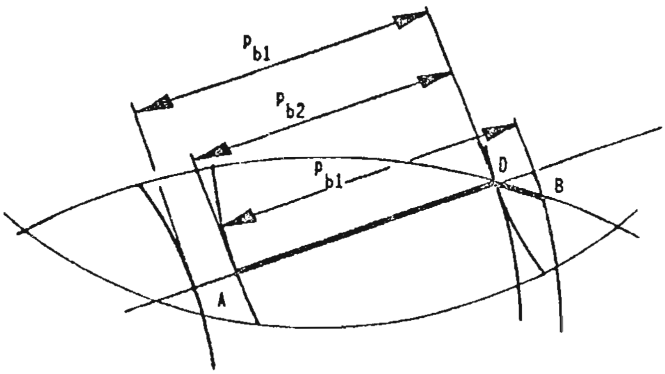

Fig. 2. Mating gears with different basic pitch values

To reduce momentary torque variations in an involute gear transmission, the working section of the path of contact should be moved towards recess. Thus, the base pitch in driving gear p_{b1} should be larger than that in driven gear p_{b2} (Fig.2), Vulgakov and Vasina (1978). It can be seen from the drawing, that the teeth are meshed not only over straight section AD of path of contact, but also over arc DB of the tip circle of driving gear to the effect that geometric criterion of the constant speed ratio is not fulfilled. However, constant speed is not critical in this type of transmission gears, since they operate at extremely low speeds and very often by steps. Thus, the following relation shall apply

$$m_1 \cos \alpha_1 > m_2 \cos \alpha_2 \tag{2.3}$$

but with equal modules applied in both gears of the pair, which is the most common case, the above relation (2.3) will reduce to inequality

$$\alpha_1 < \alpha_2 \tag{2.4}$$

Coming to conclusions, the above discussed transmission gears are characteristic in that the two gears of the matching pair have different nominal pressure angles.

3. Asymmetrical tooth profile gears

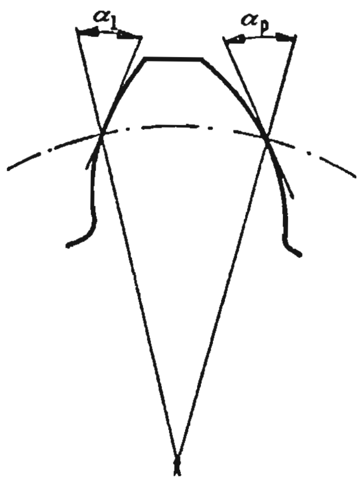

Fig. 3. Asymmetrical profile tooth $(\alpha_p \neq \alpha_n)$

Should the two flanks of a tooth be always generated from identical nominal pressure angle? So far, it has been a common practice. However, it is possible to apply different nominal pressure angles and obtain an asymmetrical tooth profile, see Fig.3. This could be called simple asymmetrical profile (the form of which is still described by the conventional method of defining reference profile, as in any standard profile, but with nominal pressure angle $\alpha \neq$ const). Qualifying this asymmetrical profile as "simple" results from the fact, that a suggestion was made before in literature, Oleksiuk (1976), that asymmetrical profiles could be designed to different principles, where relative tooth thickness and nominal pressure angle at the tooth tip were specified. It is an important change in description of the tooth form which radically affects the profile generating process. Generating tools should be designed to quite new concept, which would involve manufacture of different tool versions to cope with changes in individual gear parameters. Such gears, especially in unit production, should be very expensive.

There are several cases, Oleksiuk (1991) where application of asymmetrical profile gears in mechatronic devices could be advantageous and in particular when it is required

- to increase break-off strength of tooth; this can be obtained by applying wider nominal pressure angle for the non-working flank (for exemplary strength calculations see Fig.4, Curve 1);

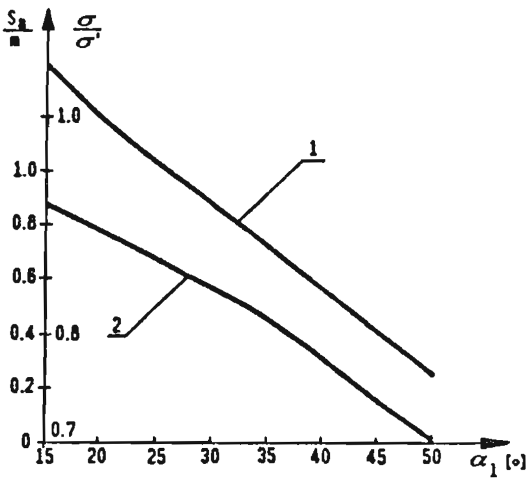

Fig. 4. Example of the effects of changes in nominal pressure angle α_l of the non-working flank of tooth on stress condition at the tooth base (Curve 1) and on tip thickness S_a in a 60 teeth, 1 mm module gear a pressure angle $\alpha_p = 20°$: σ_p – stress in a standard profile tooth, σ' – stress in an asymmetrical profile tooth

- to minimize number of teeth; this can be obtained by applying wider nominal pressure angle for the working flank of tooth (which will reduce the minimum limit number of teeth) and by additionally applying narrower nominal pressure angle for the non-working flank of tooth (which will eliminate, or at least reduce tip modification depth when the number of teeth in pinion is very small). An example is provided in which a pinion with six teeth is designed without tip reduction, where working flange has nominal pressure angle $\alpha_p = 27°$ (which is less than the optimum value for symmetrical profiles), but the non-working flange has nominal pressure angle $\alpha_p = 12°36'$;

- to increase addendum height h_a (or the tooth height coefficient $y^* = h_a/m$) in order to increase the engagement ratio or increase permissible tolerance of centre distance of matching gears; this can be obtained by applying narrower nominal pressure angle for the non-working flange of tooth. For illustration see diagram in Fig.5;

- to increase tip thickness S_a in order to reduce stresses which are likely to occur in narrow tips of teeth during heat treatment operations; this can be obtained by applying narrower nominal pressure angle for the non-working flange of tooth (for exemplary calculations see Fig.4, Curve 2);

- to keep resistance to motion irrespective of direction of drive in transmissions, where backlash is taken-up by the swinging gear (Fig.6); this is obtained

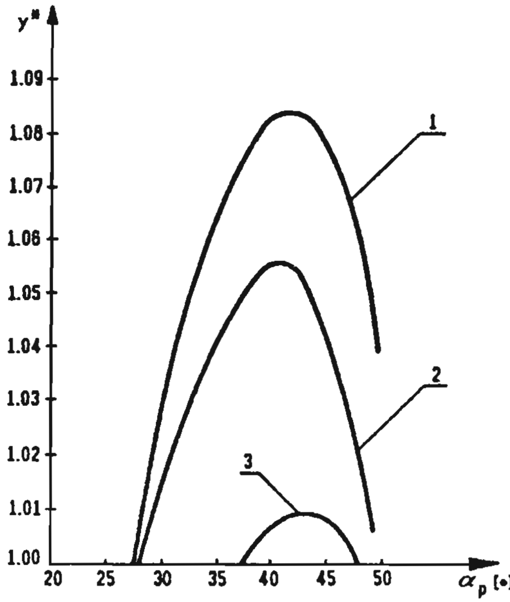

Fig. 5. Effect of changes in nominal pressure angle α_p on maximum tooth height factor y^*: Curve 1: $z = 6$, $\alpha_l = 15°$; Curve 2: $z = 6$, $\alpha_l = 20°$; Curve 3: $z = 5$, $\alpha_l = 15°$

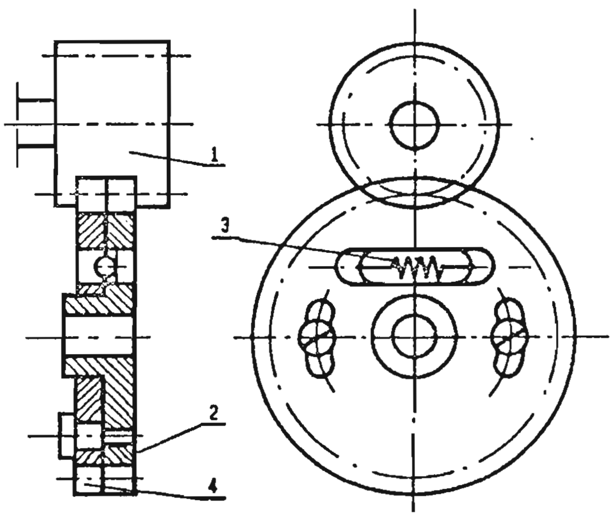

Fig. 6. Swinging gear to take-up backlash

by applying wider nominal pressure angle in pinion 1 for the flank, which is mating with the wheel 2 running on rigid shaft, than that for the other flank, which is mating with the wheel 4 loaded with the backlash take-up spring (3). This was verified on two transmission gears with identical modules $m = 1$ mm and numbers of teethes $z = 20$ and $Z = 100$. One of the transmissions was standard, while the other had asymmetrical profile teeth ($\alpha_p = 20°$, $\alpha_n = 30°$). Both pinions and wheels were made from brass MO63. The transmissions were operated without lubrication at angular speed of $\omega = \pi$ rad/s, under constant torque of $M = 1$ cNm. The load applied by the backlash take-up spring was sufficient for the gear to transmit specified torque with safety factor $\alpha = 2$. Average ratios of torques as transmitted through the symmetrical and asymmetrical profile gears were 0.90 and 0.98 respectively (while theoretical ratios, considering friction coefficient $\mu = 0.2$, should amount to 0.895 and 0.976 respectively), Oleksiuk (1991);

- to design an irreversible transmission gear to be operable in one direction only; this can be obtained in a reduction gear, in the case when driving torque would be applied at the wheel with larger number of teeth. This would require that either much wider nominal pressure angles were applied for those flanges, which should resist reverse transmission of torque, or else, different nominal pressure angles were applied for the matching flanges of teeth (for explanation of this principle see Fig.7). Transmission gear will not operate with the torque applied at the wheel, since force P acting on the pinion tooth could be displaced below centre O_1 of the pinion to be driven.

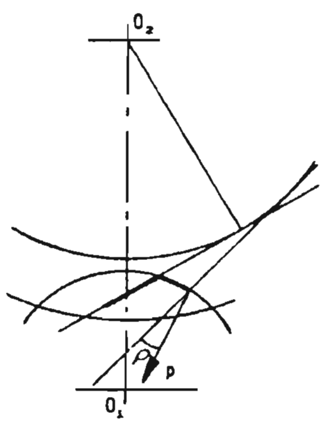

Fig. 7. Schematic diagram of irreversible transmission gear with different pressure angle flanks; ρ – tooth bearing friction angle

Beside the above mentioned examples, the gears with asymmetrical teeth produced by application of varying nominal pressure angles could be used to design irreversible transmissions of minimum torque variation (as already mentioned under 2.2 above). The driving and the driven gears of identical asymmetrical profiles (being generated by the same tool) are assembled in such manner, that narrow pressure angle active flank of driving gear would mate with wide pressure angle active flank of driven gear.

4. Conclusion

It should be expected, that in all cases of the discussed constant speed ratio and minimum torque variation transmission gears used in mechatronic devices, necessity will arise some time in future, that standard nominal pressure angles would have to be suitably modified to the requirements.

Nominal pressure angles of tooth profiles can be modified to suit the application. Such modifications can be used to improve the gear design or performance, and in particular

- to reduce size of the transmission, to reduce momentary torque variations, to reduce variations in momentary efficiency of the gear transmission or to increase its average efficiency.

It should be noted, that involute tooth profiles are still used in all these gear transmissions and are still machinable by means of generating methods.

Presuming the above discussion and the examples quoted were reasonable, the following two conclusions could be drawn

1. An important group of transmission gears used in mechatronic devices is that comprising gears with tooth profiles generated to different nominal pressure angles. One or both tooth flanks can be made different, or nominal pressure angle can deviate from standard value. It can be said, that the transmission gears of this class are specific in that they have teeth profiles varying in nominal pressure angles.

2. To have these transmission gears commonly applied, the manufacturing methods should be developed. Large series or mass production of gears with arbitrarily selected pressure angle profiles is a relatively simple problem to solve. The cost of a special tool, to be provided for production, will spread over immense number of the components without significant effect on their price. However, in unit production the process becomes a complex problem, since conventional generating methods would require a number of cutters to be made to different nominal pressure angles for each module size This is

impracticable, as the cost of a modified profile gear would be many times higher than the cost of standard profile gear, if price of the component had to include cost of tooling. It is therefore imperative, that process methods must be developed to suit unit production requirements by which different nominal pressure angle profiles could be generated for individual flanks of teeth, using as few cutters as possible. This is one of the problems on which the research work is carried out in Warsaw University of Technology, Institute of Design of Precise and Optical Instruments.

References

1. OLEKSIUK W., 1976, *Théorie d'un engrenage à développante et à angles inégaux du profil des dents de roues coopérants*, IET 72
2. OLEKSIUK W., 1983, *Verfahren zur Minimierung der Baugröße feinmechanischer einstufiger Zahnradgetribe*, Feingerätetechnik, 5
3. OLEKSIUK W., 1991, *Drobnomodułowe przekładnie zębate o niesymetrycznym uzębieniu*, Sprawozdanie z finansowego przez MEN projektu badawczego, IKPPiO PW, Warszawa
4. TRYLIŃSKI W., 1978, *Drobne mechanizmy i przyrządy precyzyjne. Podstawy konstrukcji*, WNT Warszawa
5. VULGAKOV E.B., VASINA L.M., 1978, *Evolventnye zubchatye peredachi v obobshchayushchikh parametrakh*, Spravochnik po geometricheskomu raschëtu, Mashinostroenie Moskwa

Ewolwentowe przekładnie zębate o zróżnicowanych kątach zarysu jako przykład przekładni stosowanych w urządzeniach mechatroniki

Streszczenie

W artykule omówiono ewolwentowe przekładnie zębate znajdujące zastosowanie w urządzeniach mechatroniki. Wskazano na istotne znaczenie przekładni zmodyfikowanych, których wspólną cechą jest zróżnicowanie kątów zarysu.

Manuscript received March 10, 1993; accepted for print March 29, 1993