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SOME PROBLEMS OF ELASTIC/VISCOPLASTIC MULTILAYERED COMPOSITES

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For the elastic/viscoplastic multilayered composite a certain model with microlocal parameters was proposed. Some solutions for the multilayered plate and the rotating multilayered disc within the frames of this model were studied.

1. Introduction

The list of papers on modelling of composite materials is extensive. Lots of them concern elastic composites and as main textboks and monographs we can mention here those by: R.M.Christensen [5], R.Jones [6], A.Bensoussan, J.L.Lions, G.Papanicolau [7], Bachvalov, Panasenko [8] or C.T.Sun, J.D.Achenbach, G.Hermann [9]. The papers on modelling of nonelastic composite materials constitute the new trend in the mechanic of compositie; we can mention here those by: J.Aboudi [19], P.M.Suquet [20], J.J.Marigo, P.Mialon, J.C.Michel, P.Suquet [21], P.P.Castaneda, J.R.Willis [22], G.P.Tandon, G.J.Weng [23]. In [19] an analytical approach for the modelling of the thermo–elastoplastic two phase composite materials is presented. The mathematical theory of plasticity and the process of homogenization of nonelastic composites are analysed in [20,21]. The overall properties of nonlinear two phase viscous composites are described in [22]. A simple, approximate theory is developed in [23] to determine the elastoplastic behaviour of particle–reinoforced materials. A new method of the modelling of periodic composites based on some concepts of the nonstandard analysis leading to so homogenized models with microlocal parameters was proposed in [1] and then developed in the series of papers [11–17]. Within the framework of this method the model with microlocal parameters for the elastic/viscoplastic, [18], periodic composites was proposed in [13].

The aim of this paper is to present and discuss on the basis of [13]

1. the model with microlocal parameters for the multilayered elastic/viscoplastic periodic thick plates together with the pertinent approximation formulae.

2. selected solutions for elastic/viscoplastic multilayered periodic thick plates within the framework of this model.

The considerations are carried on within the small deformation gradient theory.

2. Formulation of the primary problem

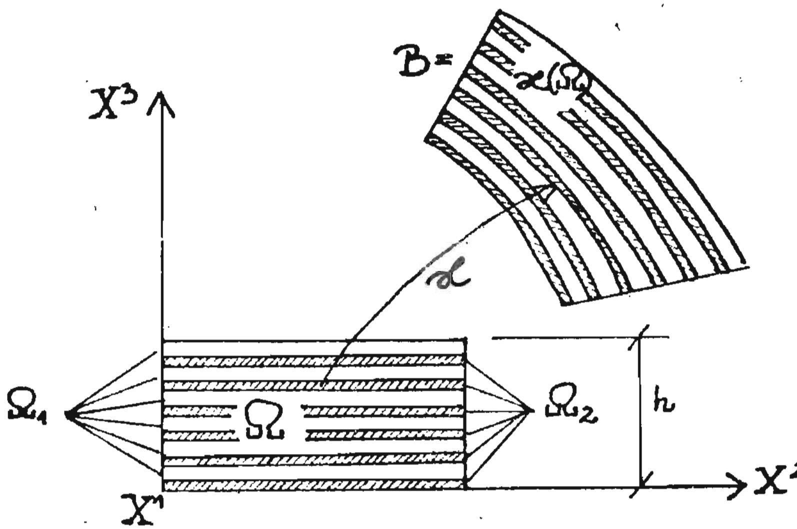

Fig. 1.

Let at a moment $t = t_0$ an elasticviscoplastic multilayered body in its natural state occupies the region B in the Euclidean 3–space of points $\boldsymbol{x} = (x^i)$, $i = 1,2,3$. By means of an invertible smooth mapping $\kappa : \Omega \rightarrow R^3$ (Fig.1), such that $\overline{\kappa(\Omega)} = \bar{B}$ we introduce in B curvilinear coordinates $\boldsymbol{X} \in \Omega$. For the simplicity we assume $\Omega = \Pi \times (0,h)$, Π being a regular region in R^2. We shall also use the denotation $\boldsymbol{X} = (\boldsymbol{X}', X^3)$, where $\boldsymbol{X}' = (X^1, X^2)$. The region Ω is assumed to be composed of thin layers with the interfaces perpendicular to the x^3–axis. The perfect bonding between the layers is assumed. Each layer under consideration has the constant thickness δ and is made of M sublayers with thicknesses δ_E respectively, $E = 1,...,M$, $\delta = \delta_1 + ... + \delta_M$, every sublayer being made of a certain homogeneous (as related to B) elastic/viscoplastic material. Hence the decomposition[1] (Fig.1) $\bar{\Omega} = \bigcup \bar{\Omega}_E$, $E = 1,...,M$, where $\Omega_E \cap \Omega_F = \emptyset$ for every $E \neq F$, Ω_E is a part of the body under consideration occupied by the E–th material.

We assume that the multilayered body is δ–periodic (as related to B) i.e. for every function $\mathcal{F}_\kappa(\cdot)$ which describes the material properties of the body there is: $\mathcal{F}_\kappa(\boldsymbol{X}) \equiv \mathcal{F}(\kappa(\boldsymbol{X})) = \mathcal{F}(\kappa(\boldsymbol{X}', X^3)) = \mathcal{F}(\kappa(\boldsymbol{X}', X^3 + \delta))$ in the whole domain of the definition of $\mathcal{F}_\kappa(\cdot)$. In the sequal functions $\mathcal{F}_\kappa(\cdot)$ are assumed to be constant in every $\kappa(\Omega_E)$, $E = 1,...,M$ and for every $\boldsymbol{X} \in \Omega_E$ their values will be denoted by $\mathcal{F}_\kappa^E$, $E = 1,...,M$.

Let $\rho(\cdot)$, $\sigma(\cdot)$, $b(\cdot)$, $v(\cdot)$, $t(\cdot)$ stand for a mass density (related to Ω), a Cauchy stress tensor, body forces, a velocity and boundary tractions, respectively.

The equations of motion can be assumed in the form of a condition

$$\begin{aligned}\int_{\Omega} \sigma^{\alpha\beta}(\boldsymbol{X},t)u_{(\alpha|\beta)}(\boldsymbol{X})dv &= \int_{\partial\Omega} t^{\alpha}(\boldsymbol{X},t)u_{\alpha}(\boldsymbol{X})dA + \\ &+ \int_{\Omega} \rho(\boldsymbol{X})b^{\alpha}(\boldsymbol{X},t)u_{\alpha}(\boldsymbol{X})dv - \int_{\partial\Omega} \rho(\boldsymbol{X})\dot{v}^{\alpha}(\boldsymbol{X},t)u_{\alpha}(\boldsymbol{X})dv, \qquad t \in [t_0, t_f]\end{aligned} \tag{2.1}$$

which has to hold for every regular enough test function $u_\alpha(\cdot)$ defined on $\bar{\Omega}$.

Let $\gamma(\cdot)$, $\gamma(\boldsymbol{X}) > 0$, $\boldsymbol{X} \in \bigcup \Omega_E$, $E = 1,...,M$, $A(\cdot)$, $k(\cdot)$ be the viscosity coefficient, the elastic modulus tensor, and the yield stress, respectively, at $\boldsymbol{X} \in \bigcup \Omega_E$.

The constitutive relations for the composite as an nonhomogeneous elastic/viscoplastic material with the Huber–Misses–Hencky criterion will be based on those proposed in [10].

Setting

$$\begin{aligned}\varepsilon(v)(\boldsymbol{X},t) &\equiv v_{(\alpha|\beta)}(\boldsymbol{X},t) \\ s^{\alpha\beta}(\boldsymbol{X},t) &\equiv \sigma^{\alpha\beta}(\boldsymbol{X},t) - \frac{1}{3}G^{\alpha\beta}(\boldsymbol{X})\sigma^{\gamma}_{\gamma}(\boldsymbol{X},t) \\ g(\boldsymbol{X},t) &\equiv \sqrt{\frac{1}{2}s^{\alpha\beta}(\boldsymbol{X},t)s_{\alpha\beta}(\boldsymbol{X},t)}\end{aligned} \tag{2.2}$$

we define: the strain rate tensor, the stress deviator and the second invariant of the stress deviator, respectively.

[1]Sub- and superscript α, β, γ, δ run over 1, 2, 3 and are related to the curvilinear coordinates X in B with the metric tensor $G(X) = \nabla\kappa^{\top}(X)\nabla\kappa(X)$, $X \in \Omega$. Subscripts E, F run over $1,...,M$ and subscript a over $1,...,M-1$. Summation convention holds with respect to α, β, γ, δ and a. A comma denotes a partial differentation, a vertical line stands for a covariant differentiation in a metric $G(X)$, $X \in \Omega$.

Then the constitutive relations can be assumed in the form (given by [10])

$$\varepsilon_{\alpha\beta}(v)(X,t) - A_{\alpha\beta\gamma\delta}(X)\dot{\sigma}^{\delta\gamma}(X,t) = \xi_{\alpha\beta}(X,t) \tag{2.3}$$

where

$$\xi_{\alpha\beta}(X,t) = \begin{cases} 0 & \text{if } \ g(X,t) < k(X) \\ \dfrac{1}{2\gamma(X)}\dfrac{g(X,t)-k(X)}{g(X,t)}s_{\alpha\beta}(X,t) & \text{if } \ g(X,t) \geq k(X) \end{cases} \tag{2.4}$$

Eqs (2.1), (2.3) have to be considered together with the following boundary and initial conditions

$$\begin{array}{lll} v(X,t) = V(X,t) & \text{for } \ X \in \partial\Omega_u & \text{and } \ t \in [t_0,t_f] \\ \sigma(X,t)n(X) = F(X,t) & \text{for } \ X \in \partial\Omega_F & \text{and } \ t \in [t_0,t_f] \\ \sigma(X,0) = \sigma_0(X) & \text{for } \ X \in \Omega & \\ v(X,0) = V_0(X) & \text{for } \ X \in \Omega & \end{array}$$

where $\partial\Omega \equiv \overline{\partial\Omega_u} \cup \overline{\partial\Omega_F}$.

The problem of finding functions $\sigma(\cdot)$ and $v(\cdot)$ satisfying eqs (2.1), (2.3) under denotations (2.2) and boundary and initial conditions will be called the primary problem and denoted by P.

3. Elastic/viscoplastic model with microlocal parameters

As it was shown in [1,12] we can obtain the approximate solution to the problem P within the framework of the model with microlocal parameters.

To formulate the equations of motion and the constitutive relation for that model we introduce

1. the basic periodicity layer $\Delta \equiv \Pi \times (0,\delta)$,
2. the family $\mathcal{L}$ of parallel planes in R^3 given by

$$\mathcal{L} \equiv \{Y \in R^3 : \ Y = (Y', n\sigma), \ Y' = (Y^1, Y^2) \in R^2, \ n = 0, \pm1, \pm2, ...\} \tag{3.1}$$

(for every δ–periodic composite there exists the decomposition of the periodicity layer Δ, $\bar{\Delta} = \bigcup \bar{\Delta}_E$, $E = 1,...,M$, $\Delta_E \cap \Delta_F = \emptyset$ for every $E \neq F$, such that the occupied by the E–th component part of the body under consideration is $\Omega_E = \Omega \cap (\mathcal{L} \oplus \Delta_E)$),

3. the real–valued functions $la(\cdot)$ $a = 1,...,M-1$, defined on $[0,\delta]$ by means of

$$la(X^3) = \begin{cases} \dfrac{\delta}{\delta_a}(X^3 - \zeta_{a-1}) & X^3 \in [\zeta_{a-1}, \zeta_a) \\ -\dfrac{\delta}{\delta_{a+1}}(X^3 - \zeta_a) + \delta & X^3 \in [\zeta_a, \zeta_{a+1}) \\ 0 & X^3 \in [0,\delta] - [\zeta_{a-1}, \zeta_{a+1}] \end{cases} \tag{3.2}$$

with

$$\zeta_a \equiv \sum_{i=0}^{a} \delta_i \qquad\qquad \delta_0 \equiv 0.$$

Functions $la(\cdot)$ can be extended from $[0,\delta]$ to R by means of $l(X^3) = l(X^3+n\delta)$ for every integer n. Functions $la(\cdot)$ are called the shape functions, [1].

Using the denotations

$$\begin{aligned}
&\eta^E = \eta_E \equiv \frac{\delta_E}{\delta} \\
&\Lambda^E_{a\beta} \equiv la_{,\beta}(X^3) \\
&\nu^E(X^3) = \nu_E(X^3) = \begin{cases} 1 & (\boldsymbol{X}', X^3) \in \Omega_E \\ 0 & (\boldsymbol{X}', X^3) \in \Omega \backslash \Omega_E \end{cases} \\
&J(\boldsymbol{X}) \equiv \det \nabla \kappa(\boldsymbol{X})
\end{aligned} \tag{3.3}$$

and applying the way of approach similar to that given in [1,13] from the problem P we pass to the homogenized problem $\tilde{P}$ which is governed by

– equations of motion

$$\begin{aligned}
&\tau^{\alpha\beta}\Big|_{\beta}(\boldsymbol{X},t) + \tilde{\rho}(\boldsymbol{X}) b^{\alpha}(\boldsymbol{X},t) = \tilde{\rho}(\boldsymbol{X}) \dot{w}^{\alpha}(\boldsymbol{X},t) \\
&S^{\alpha}_a(\boldsymbol{X},t) = 0, \qquad a = 1,...,M-1, \qquad \alpha = 1,2,3, \qquad t \in [t_0, t_f]
\end{aligned} \tag{3.4}$$

where

$$\begin{aligned}
&\tau^{\alpha\beta}(\boldsymbol{X},t) = \sum_{E=1}^{M} \eta_E \sigma^{\alpha\beta}_E(\boldsymbol{X},t) \\
&S^{\alpha}_a(\boldsymbol{X},t) = \sum_{E=1}^{M} \eta_E \sigma^{\alpha\beta}_E(\boldsymbol{X},t) \Lambda^E_{a3} \\
&\qquad\qquad a = 1,...,M-1, \qquad \alpha = 1,2,3
\end{aligned} \tag{3.5}$$

– constitutive relations

$$\begin{aligned}
\xi^E_{\alpha\beta}(\boldsymbol{X},t) &= \varepsilon_{\alpha\beta}(w)(\boldsymbol{X},t) + \varphi^E_{\alpha\beta}(q)(\boldsymbol{X},t) + \\
&\quad + A^E_{\alpha\beta\gamma\delta}(\boldsymbol{X}) \dot{\sigma}^{\delta\gamma}_E(\boldsymbol{X},t) \\
&\boldsymbol{X} \in \Omega, \qquad t \in [t_0, t_f], \qquad E = 1,...,M
\end{aligned} \tag{3.6}$$

where

$$\varphi^E_{\alpha\beta}(q)(\boldsymbol{X},t) \equiv \nu^E(\boldsymbol{X}) q^a_{(\alpha}(\boldsymbol{X},t) \Lambda^E_{a\beta)}$$

$$\xi^E_{\alpha\beta}(X,t) = \begin{cases} 0 & \text{if} \quad g_E(X,t) < k_E \\ \frac{1}{2\gamma_E}\frac{g_E(X,t)-k_E}{g_E(X,t)}S_{E\alpha\beta}(X,t) & \text{if} \quad g_E(X,t) \geq k_E \end{cases}$$

$$A^E_{\alpha\beta\gamma\delta}(X) \equiv J(X)A^E_{\kappa\alpha\beta\gamma\delta} \tag{3.7}$$

$$s^{\alpha\beta}_E(X,t) \equiv \sigma^{\alpha\beta}_E(X,t) - \frac{1}{3}G^{\alpha\beta}(X)\sigma^\gamma_\gamma(X,t)$$

$$g_E(X,t) \equiv \sqrt{\frac{1}{2}s^{\alpha\beta}_E(X,t)s_{E\alpha\beta}(X,t)}$$

– boundary and initial conditions

$$\begin{aligned} &w(X,t) = V(X,t) && \text{for} \quad X \in \partial\Omega_u, \quad t \in [t_0,t_f] \\ &\nu_E(X^3)\sigma_E(X,t)n(X) = F(X,t) && \text{for} \quad X \in \partial\Omega, \quad t \in [t_0,t_f] \\ &\sigma_E(X,0) = \sigma_0(X) && \text{for} \quad X \in \Omega \\ &w(X,0) = v_0(X) && \text{for} \quad X \in \Omega. \end{aligned} \tag{3.8}$$

Formulating the problem $\tilde{P}$ we shall mean the problem of finding so-called macrovelocities $w(\cdot)$, microlocal parameters $q_a(\cdot)$, $a = 1,...,M-1$ and partial stresses $\sigma_E(\cdot)$, $E = 1,...,M$.

Eqs (3.4) – (3.8) represent so called the model of the periodic body with micrilocal parameters. It has to be emphasized that fields $\sigma_E(\cdot,t)$, $E = 1,...,M$ are defined on Ω for every $t \in [t_0,t_f]$. If the solution to the problem $\tilde{P}$ is known we can evaluate the solution to the problem P by the following approximation formulae, [1,13]

$$\begin{aligned} &v_\alpha(X,t) \sim w_\alpha(X,t) \\ &\varepsilon_{\alpha\beta}(v)(X,t) \sim \nu_E(X^3)\Big(\varepsilon_{\alpha\beta}(w)(X,t) + \varphi^E_{\alpha\beta}(q)(X,t)\Big) \\ &\sigma^{\alpha\beta}(X,t) \sim \nu_E(X^3)\sigma^{\alpha\beta}_E(X,t) \end{aligned} \tag{3.9}$$

where the summation convention with respect to $E = 1,...,M$ holds. In the case of kinematic boundary conditions it can be shown that there exists the one and only one solution to the problem $\tilde{P}$, [13]. As the examples of application of eqs (3.4) – (3.8) we shall investigate below the 2–dimensional strain problem for the multilayered periodic elastic/viscoplastic infinite plate and the plane stress problem for multilayered periodic elastic/viscoplastic rotating disc. Both problems will be analysed as quasi–stationary ones.

4. Infinite multilayered plate

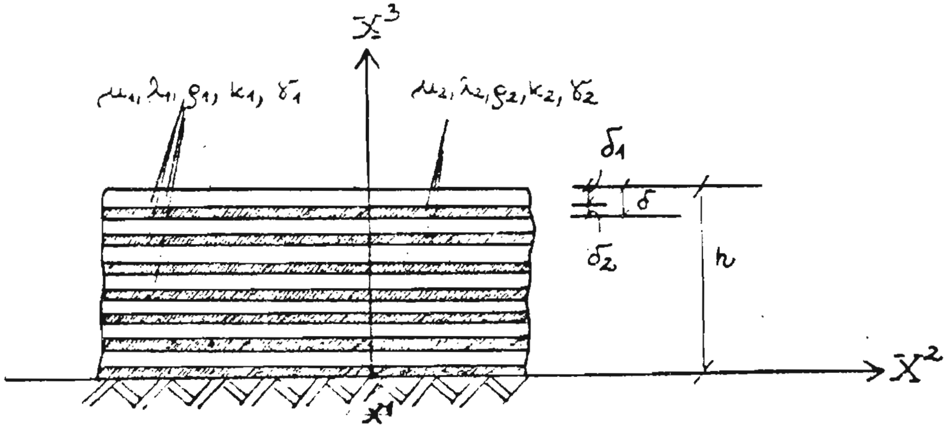

Fig. 2.

Consider an infinite multilayered periodic elastic/viscoplastic plate (Fig.2) with thickness h. Each layer has a constant thickness δ and is composed of two homogeneous, isotropic sublayers with thicknesses δ_1 and δ_2, $\delta_1 + \delta_2 = \delta$. The material properties of the sublayers are determined by the material modulae μ_E, λ_E, γ_E, k_E, ρ_E, $E = 1, 2$. We shall assume that $k_1 < k_2$. The plate is laying on the rigid foundation and its upper plane is located with the known uniformly distributed loading $- p(t)$, which is the function of time only functions. We also assume that $p(\cdot)$ is the monotone, regular function of an argument t, which at a moment $t = t_0$ is equal to zero. These conditions lead to the plane strain problem. In this problem the velocity vector has only one nonzero component $\boldsymbol{v} = (0, 0, v_3)$.

The shape function (3.2) for the problem under consideration will be introduced in the form:

$$l(x_3) = \begin{cases} \frac{x_3}{\eta_1} + \frac{\delta}{2} & x_3 \in [-\delta_1, 0) \\ \frac{x_3}{\eta_1} + \frac{\delta}{2} & x_3 \in [0, \delta_2] \end{cases} \tag{4.1}$$

for $x_3 \in [-\delta_1, \delta_2]$ and extended to R by means of $l(x_3) = l(x_3 + n\delta)$ for every integer.

The equations of motion (3.4), the constitutive relations (3.6), and the boundary and the initial conditions for the multilayered plate within the framework of the model with microlocal parameters are given by

– equations of motion

$$\eta_1 \sigma^{33}_{1,3}(x_3, t) + \eta_2 \sigma^{33}_{2,3}(x_3, t) = 0$$

$$\sigma_1^{\alpha 3}(x_3,t) = \sigma_2^{\alpha 3}(x_3,t) \tag{4.2}$$

$$x_3 \in (0,h), \qquad t \in [t_0,t_f], \qquad \alpha = 1,2,3$$

– constitutive relations

$$\begin{aligned}\xi^E_{\alpha\beta}(x_3,t) = \delta_{\alpha 3}\delta_{\beta 3}\Big(w_{,3}(x_3,t) + \varphi^E_{33}(q)(x_3,t)\Big) + \\ +A^E_{\alpha\beta\gamma\delta}\dot{\sigma}^{\delta\gamma}_E(x_3,t)\end{aligned} \tag{4.3}$$

$$E = 1,2, \qquad x_3 \in (0,h)$$

where

$$\begin{aligned}\varphi^E_{33}(q)(x_3,t) &= \frac{(-1)^{E+1}}{\eta_E} q(x_3,t) \\ \xi^E_{\alpha\beta}(x_3,t) &= \begin{cases} 0 & \text{if} \quad g_E(x_3,t) < k_E \\ \dfrac{1}{\gamma_E}\dfrac{g_E(x_3,t) - k_E}{g_E(x_3,t)} s_{E\alpha\beta}(x_3,t) & \text{if} \quad g_E(x_3,t) \geq k_E \end{cases}\end{aligned} \tag{4.4}$$

– the boundary and initial conditions

$$\begin{aligned}
&w(x',0,t) = 0 && \text{for } \ t \in [t_0,t_f], \ \ x' \in R^2 \\
&\sigma^{33}_E(x',h,t) = -p(t) && \text{for } \ t \in [t_0,t_f], \ \ x' \in R^2, \ \ E = 1,2 \\
&\sigma^{\alpha}_E(x',h,t) = 0 && \text{for } \ t \in [t_0,t_f], \ \ x' \in R^2, \ \ E = 1,2, \ \ \alpha = 1,2 \\
&w(x',x_3,0) = 0 && \text{for } \ (x',x^3) \in R^2 \times (0,h) \\
&\sigma^{33}_E(x',x_3,0) = 0 && \text{for } \ (x',x^3) \in R^2 \times (0,h), \ \ E = 1,2 \\
&\sigma^{\alpha 3}_E(x',x_3,0) = 0 && \text{for } \ (x',x^3) \in R^2 \times (0,h), \ \ E = 1,2, \ \ \alpha = 1,2.
\end{aligned} \tag{4.5}$$

The approximation formulae (3.8) will take the form

$$\begin{aligned}
&v_3(x_3,t) \sim w(x_3,t) \\
&\varepsilon_{33}(v_3)(x_3,t) \sim \nu_1(x_3)\Big(\varepsilon_{33}(w)(x_3,t) + \varphi^1_{33}(q)(x_3,t)\Big) + \\
&\qquad +\nu_2(x_3)\Big(\varepsilon_{33}(w)(x_3,t) + \varphi^2_{33}(q)(x_3,t)\Big) \\
&\sigma^{\alpha\beta}(x_3,t) \sim \nu_1(x_3)\sigma_1^{\alpha\beta}(x_3,t) + \nu_2(x_3)\sigma_2^{\alpha\beta}(x_3,t).
\end{aligned} \tag{4.6}$$

Introducing the denotations

$$
\begin{aligned}
&[\mu] \equiv \mu_1 - \mu_2 && [\lambda] \equiv \lambda_1 - \lambda_2 \\
&\tilde{\mu} \equiv \eta_1\mu_1 + \eta_2\mu_2 && \tilde{\lambda} \equiv \eta_1\lambda_1 + \eta_2\lambda_2 \\
&\hat{\mu} \equiv \frac{\mu_1}{\eta_1} + \frac{\mu_2}{\eta_2} && \hat{\lambda} \equiv \frac{\lambda_1}{\eta_1} + \frac{\lambda_2}{\eta_2}
\end{aligned}
\tag{4.7}
$$

$$
\kappa \equiv 2\tilde{\mu} + \tilde{\lambda} - \frac{(2[\mu] + [\lambda])^2}{2\hat{\mu} + \hat{\lambda}}
$$

we obtain the follwoing solution for the model with microlocal parameters in the elastic state

$$
\begin{aligned}
q(x_3, t) &= \frac{(2[\mu] + [\lambda])\dot{p}(t)}{(2\hat{\mu} + \hat{\lambda})\kappa} \\
w(x_3, t) &= \frac{-\dot{p}(t)x_3}{\kappa} \\
\sigma_E^{\alpha\alpha}(x_3, t) &= -\lambda_E\Big(1 + \frac{(-1)^E}{\eta_E}\frac{2[\mu] + [\lambda]}{2\hat{\mu} + \hat{\lambda}}\Big)\frac{p(t)}{\kappa} \qquad \text{for } \alpha = 1, 2 \\
\sigma_E^{33}(x_3, t) &= -p(t).
\end{aligned}
\tag{4.8}
$$

The second invariants of the stress deviators will take the form

$$
\begin{aligned}
g_E(x_3, t) &= \frac{\sqrt{2}}{3}|\sigma_E^{22}(x_3, t) - \sigma_E^{33}(x_3, t)| = \\
&= \frac{2p(t)}{3}\Big|1 - \lambda_E\Big(1 - \frac{(-1)^E}{\eta_E}\frac{2[\mu] + [\lambda]}{2\hat{\mu} + \hat{\lambda}}\Big)\frac{1}{\kappa}\Big|.
\end{aligned}
\tag{4.9}
$$

The form of the solution for the elastic state implies that stresses and invariants of their deviators are the time dependend functions only. They are also the monotonic functions of time.

Since invariants of the stress deviators are time dependent only the monotonic functions, then for $t = t_1$ and $x_3 \in (0, h)$ we can obtain the one from following three relations

1^o	$g_1(x_3, t_1) = k_1$	and	$g_2(x_3, t_1) < k_2$
2^o	$g_1(x_3, t_1) < k_1$	and	$g_2(x_3, t_1) = k_2$
3^o	$g_1(x_3, t_1) = k_1$	and	$g_2(x_3, t_1 = k_2$.

For the sake of simplicity in the sequal we shall discuss only the first of these relations (it can be shown by the direct calculations that this solution holds for $[\lambda] = 0$ and $[\mu] \neq 0$). Let $g_1(x_3, t_1) = k_1$ and $g_2(x_3, t_1) < k_1$ where

$$
t_1 = p^{-1}\left(\frac{3k_1\kappa}{\sqrt{2}\Big|\kappa - \lambda_1\Big(1 - \frac{1}{\eta_1}\frac{2[\mu]+[\lambda]}{2\hat{\mu}+\hat{\lambda}}\Big)\Big|}\right). \tag{4.10}
$$

The solution in the form of (4.8) for the model with microlocal parameters holds up to a moment $t = t_1$. For any moment $t > t_1$ the solution to the system of equations of motion (4.2) and constitituve relations (4.3) is more complicated. Namely, for $t > t_1$ we are not able to present the form of solution to the system of eqs (4.2), (4.3) with the boundary conditions (4.5) in the explicit form. We can find it at every moment t_{m+1}, $t_{m+1} > t_1$ using the step by step procedure i.e. if we know the value of function and its time derivative at a moment $t = t_m$ we are able to present the value of this function at a moment $t = t_{m+1}$ by the known approximation formulae

$$f(t_{m+1}) \approx f(t_m) + \dot{f}(t_m)\Delta t_m \tag{4.11}$$

where $\Delta t_m \equiv t_{m+1} - t_m$.

In the first step we know the solution at a moment $t = t_1$ ($\sigma_E(x_3, t_1)$, $w(x_3, t_1)$, $q(x_3, t_1)$) for the model with microlocal parameters. We are able to find the stress rates at the moment $t = t_1$ from the constitutive relations (4.3).

At a moment $t = t_1$ we obtain

$$\xi^E_{\alpha\beta}(x_3, t_1) = 0, \qquad E = 1,2, \qquad \alpha,\beta = 1,2,3 \tag{4.12}$$

and

$$\begin{aligned}
\dot{\sigma}_E^{\alpha\alpha}(x_3,t_1) &= -\lambda_E\Big(1 + \frac{(-1)^E}{\eta_E}\frac{2[\mu]+[\lambda]}{2\hat{\mu}+\hat{\lambda}}\Big)\frac{\dot{p}(t_1)}{\kappa}, \qquad E = 1,2\\
\sigma_E^{33}(x_3,t_1) &= -p(t_1)\\
w(x_3,t) &= \frac{-\dot{p}(t_1)x_3}{\kappa}\\
q(x_3,t) &= \frac{2[\mu]+[\lambda]}{2\hat{\mu}+\hat{\lambda}}\dot{p}(t_1).
\end{aligned} \tag{4.13}$$

From these relations and the approximation formulae (4.10) we obtain

$$\begin{aligned}
\sigma_E^{\alpha\alpha}(x_3,t_2) &\approx -\lambda_E\Big(1 + \frac{(-1)^E}{\eta_1}\frac{2[\mu]+[\lambda]}{2\hat{\mu}+\hat{\lambda}}\Big)\frac{p(t_1)+\dot{p}(t_1)\Delta t_1}{\kappa}\\
&\qquad \alpha = 1,2\\
\sigma_E^{33}(x_3,t_2) &= -p(t_2).
\end{aligned} \tag{4.14}$$

In the k–th step, $k \geq 2$ – we can determine the stress rates by the known stresses and the unknown velocities at a moment $t = t_k$ from the constitutive relations (4.3). Then using of the equation of motion (4.2) and after satisfying the boundary conditions we can find the velocities at a moment $t = t_k$. Substituting these vellocities at the moment $t = t_k$ into the constitutive relations (4.3) we can find the stress rates at a moment $t = t_k$.

If at any moment $t = t_l$, $t_1 \leq t_l \leq t_k$, $g_1(x_3, t_l) \geq k_1$ and $g_2(x_3, t_l) < k_2$, in the t–th step we obtain at a moment $t = t_k$

$$\begin{aligned} \xi^1_{\alpha\beta}(x_3, t_k) &= \xi^1_{\alpha\beta}(t_k) \neq 0 \\ \xi^2_{\alpha\beta}(x_3, t_k) &= \xi^2_{\alpha\beta}(t_k) = 0. \end{aligned} \tag{4.15}$$

Introducing the denotation

$$\begin{aligned} \alpha(t_k) \equiv \Big\{ &-\dot{p}(t_k)\eta_2(2\hat{\mu} + \hat{\lambda}) + (2\mu_2 + \lambda_2)\cdot \\ &\cdot\Big[(2\mu_1 + \lambda_1)\xi^1_{33}(t_k) + \lambda_1\Big(\xi^1_{11}(t_k) + \xi^1_{22}(t_k)\Big)\Big]\Big\}\cdot \\ &\cdot\Big[\eta_2(2\mu_2 + \lambda_2)(2\hat{\mu} + \hat{\lambda})\Big(1 + \frac{2[\mu] + [\lambda]}{\eta_2(2\hat{\mu} + \hat{\lambda})}\Big)\Big]^{-1} \end{aligned} \tag{4.16}$$

the stress rates and velocities at a moment $t = t_k$ are given by the formulae

$$\begin{aligned} \dot{\sigma}^{\alpha\alpha}_1(x_3, t_k) &= -2\mu_1\xi^1_{\alpha\alpha}(t_k) + \lambda_1\Big[\alpha(t_k)\Big(1 - \frac{2[\mu] + [\lambda]}{\eta_1(2\hat{\mu} + \hat{\lambda})}\Big) + \\ &\quad + \frac{(2\mu_1 + \lambda_1)\xi^1_{33}(t_k) + \lambda_1[\xi^1_{11}(t_k) + \xi^1_{22}(t_k)]}{(2\hat{\mu} + \hat{\lambda})\eta_1} - \\ &\quad - \xi^1_{11}(t_k) - \xi^1_{22}(t_k) - \xi^1_{33}(t_k)\Big] \\ \dot{\sigma}^{\alpha\alpha}_2(x_3, t_k) &= \lambda_2\Big[\alpha(t_k)\Big(1 + \frac{2[\mu] + [\lambda]}{(2\hat{\mu} + \hat{\lambda})\eta_2} - \\ &\quad - \frac{2(\mu_1 + \lambda_1)\xi^1_{33}(t_k) + \lambda_1[\xi^1_{11}(t_k) + \xi^1_{22}(t_k)]}{\eta_2(2\hat{\mu} + \hat{\lambda})}, \qquad \alpha = 1,2 \\ \sigma^{33}_E(x_3, t_k) &= -p(t_k), \qquad E = 1,2 \\ w(x_3, t_k) &= \alpha(t_k)x_3 \\ q(x_3, t_k) &= \frac{-(2[\mu] + [\lambda])\alpha(t_k) + (2\mu_1 + \lambda_1)\xi^1_{33}(t_k) + \lambda_1[\xi^1_{11}(t_k) + \xi^1_{22}(t_k)]}{2\hat{\mu} + \hat{\lambda}}. \end{aligned} \tag{4.17}$$

From the relations (4.17) and the approximation formulae (4.12) it follows that for a moment $t = t_{k+1}$

$$\begin{aligned} \sigma^{\alpha\alpha}_E(x_3, t_{k+1}) &\approx \sigma^{\alpha\alpha}_E(x_3, t_1) + \sum_{l=1}^{k} \dot{\sigma}^{\alpha\alpha}_E(x_3, t_l)\Delta t_l \\ \sigma^{33}_E(x_3, t_{k+1}) &= -p(t_{k+1}), \qquad \alpha = 1,2 \end{aligned} \tag{4.18}$$

where $\Delta t_l \equiv t_{l+1} - t_l$.

The representation for the approximate solution (4.17) and (4.18) holds up to the moment $t = t_m$ at which $g_1(x_3, t_m) \geq k_1$ and $g_2(x_3, t_m) > k_2$.

At any moment $t = t_n \geq t_m$ we obtain

$$\begin{aligned} \xi^1_{\alpha\beta}(x_3, t_n) &= \xi^1_{\alpha\beta}(t_n) \neq 0 \\ \xi^2_{\alpha\beta}(x_3, t_n) &= \xi^2_{\alpha\beta}(t_n) \neq 0. \end{aligned} \tag{4.19}$$

Introducing the denotation

$$\begin{aligned} \beta(t_n) = \Big\{ &- p(t_n)\eta_2(2\hat{\mu} + \hat{\lambda}) + (2\mu_2 + \lambda_2)[(2\mu_1 + \lambda_1)\xi^1_{33}(t_n) + \\ &+ \lambda_1\xi^1_{11}(t_n) + \xi^1_{22}(t_n)] - (2\mu_2 + \lambda_2)\xi^2_{33}(t_n) - \\ &- \lambda_2[\xi^2_{11}(t_n) + \xi^2_{22}(t_n)]\Big\}\Big[\eta_2(2\mu_2 + \lambda_2)(2\hat{\mu} + \hat{\lambda}) \cdot \\ &\cdot \Big(1 + \frac{1}{\eta_2}\frac{2[\mu] + [\lambda]}{2\hat{\mu} + \hat{\lambda}}\Big)\Big]^{-1} \end{aligned} \tag{4.20}$$

we can show that in the n–th step (at a moment $t = t_n$) the stress rates and the velocities are given by

$$\begin{aligned} \dot{\sigma}^{\alpha\alpha}_E(x_3, t_n) &= -2\mu_E\xi^E_{\alpha\alpha}(t_n) + \lambda_E\Big\{\beta(t_n)\Big(1 + \frac{(-1)^{E+1}}{\eta_E}\frac{2[\mu] + [\lambda]}{2\hat{\mu} + \hat{\lambda}}\Big) + \\ &+ \frac{(-1)^E}{\eta_E}\frac{(2\mu_1 + \lambda_1)\xi^1_{33}(t_n) + \lambda_1[\xi^1_{11}(t_n) + \xi^1_{22}(t_n)]}{2\hat{\mu} + \hat{\lambda}} + \\ &+ \frac{(-1)^E}{\eta_E}\frac{-(2\mu_2 + \lambda_2)\xi^2_{33}(t_n) - \lambda_2[\xi^2_{11}(t_n) + \xi^2_{22}(t_n)]}{2\hat{\mu} + \hat{\lambda}} - \\ &- \xi^E_{11}(t_n) - \xi^E_{22}(t_n) - \xi^E_{33}(t_n)\Big\}, \qquad E = 1,2, \quad \alpha = 1,2 \\ \sigma^{33}_E(x_3, t_n) &= -p(t_n), \qquad E = 1,2 \\ w(x_3, t_n) &= \beta(t_n)x_3 \\ q(x_3, t_n) &= -(2[\mu] + [\lambda])\beta(t_n) + (2\mu_1 + \lambda_1)\xi^1_{33}(t_n) + \\ &+ \lambda_1[\xi^1_{11}(t_n) + \xi^1_{22}(t_n)] - (2\mu_2 + \lambda_2)\xi^2_{33}(t_n) - \\ &- \lambda_2[\xi^1_{11}(t_n) + \xi^2_{22}(t_n)](2\hat{\mu} + \hat{\lambda})^{-1}. \end{aligned} \tag{4.21}$$

From the relations (4.21) and the approximation formulae (4.10) we obtain for any moment t_{n+1} ¿ t_m

$$\sigma^{\alpha\alpha}_E(x_3, t_{n+1}) \approx \sigma^{\alpha\alpha}_E(x_3, t_1) + \sum_{k=1}^{m-1} \dot{\sigma}^{\alpha\alpha}_E(x_3, t_k)\Delta t_k +$$

$$+\sum_{s=m}^{n}\dot{\sigma}_E^{\alpha\alpha}(x_3,t_s)\Delta t_s, \qquad \alpha=1,2, \quad E=1,2 \qquad (4.22)$$

$$\sigma_E^{33}(x_3,t_{n+1}) = -p(t_{n+1}).$$

This procedure has to be continued up to the finite final moment. Thus we have found the solution for the model with microlocal parameters at any moment t, $t \in \{[t_0,t_1],t_2,...,t_m,...,t_f\}$. These results together with the approximation formulae (4.6) give us the form of the solution for the primary problem P for the infinite multilayered periodic plate under consideration.

5. Rotating multilayered circular disc

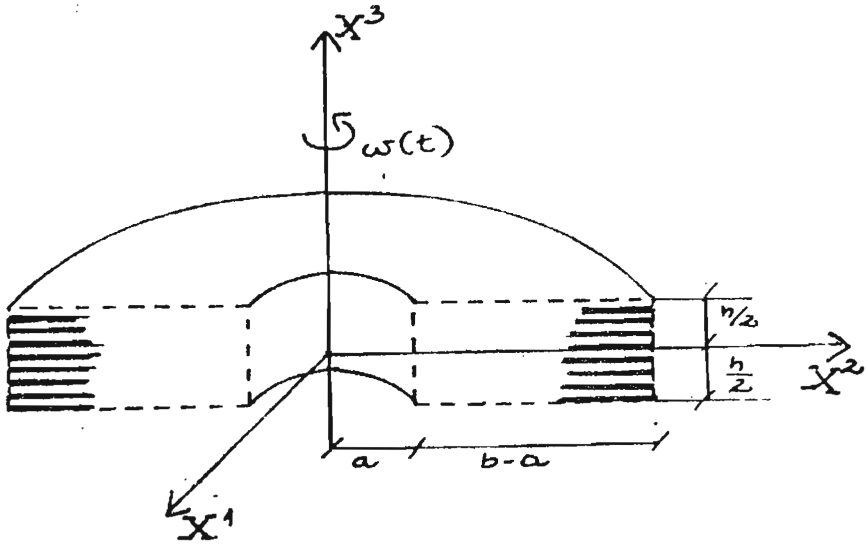

Fig. 3.

Consider the multilayered circular disc with thickness h and radii a and b (Fig.3). The disc is rotating with the angular velocity $\omega(t)$, where $\omega(\cdot)$ is a monotone, regular function of argument t (and $\omega(0)=0$). Each layer has a thickness δ and is composed of two homogeneous isotropic sublayers with thicknesses δ_1 and δ_2, $\delta_1+\delta_2=\delta$. The material properties of the sublayers are determined by the functions $\mu(z)$, $\lambda(z)$, $\gamma(z)$, $k(z)$ which are constant in every sublayer, taking values of μ_E, λ_E, γ_E, k_E, $E=1,2$, respectively. We shall also assume that $k_1 < k_2$.

The lateral surfaces of the rotating circular disc are loaded by the shear stresses $\sigma^{rz}(a,0,z,t) = \frac{c(t)z}{a}$, $\sigma^{rz}(b,0,z,t) = \frac{c(t)z}{b}$. The upper and lower boundary planes (for $z = \pm\frac{h}{2}$) are loaded with the shear stresses $\sigma^{rz}(r,0,\pm\frac{h}{2},t) = \pm\frac{c(t)h}{2r}$, where $c(\cdot)$ is a monotone regular function of argument $t \in [t_0, t_f]$ and $c(t_0) = 0$. These conditions lead to the two dimensional plane strain problem and $\sigma^{zz}(r,z,t) = 0$. The shape function (3.2) for this problem will be introduced in the form

$$l(z) = \begin{cases} \frac{z}{\eta_1} + \frac{\delta}{2} & z \in [-\delta_1, 0) \\ \frac{-z}{\eta_1} + \frac{\delta}{2} & z \in [0, \delta_2] \end{cases} \tag{5.1}$$

for $z \in [-\delta_1, \delta_2]$ and extended to R by means of $l(z) = l(z + n\delta)$ for every integer $n \in N$. Within the framework of the model with microlocal parameters the equation of motion (3.3), the constitutive relations (3.5) and the boundary and initial conditions (3.7) for the multilayered rotating circular disc under consideration are given in the form

– equation of motion

$$\begin{aligned} &\tau^{rr}_{,r}(r,z,t) + \tau^{rz}_{,z}(r,z,t) + \frac{\tau^{rr}(r,z,t) - \tau^{\theta\theta}(r,z,t)}{r} + \tilde{\rho}\omega^2(t)r = 0 \\ &\tau^{r\theta}_{,r}(r,z,t) + \tau^{\theta z}_{,z}(r,z,t) + \frac{2}{r}\tau^{r\theta}(r,z,t) + \tilde{\rho}\dot{\omega}(t)r = 0 \\ &\tau^{rz}_{,r}(r,z,t) + \frac{\tau^{rz}(r,z,t)}{r} = 0 \\ &\sigma^{1}_{rz}(r,z,t) = \sigma^{rz}_2(r,z,t) \\ &\sigma^{\theta z}_1(r,z,t) = \sigma^{\theta z}_2(r,z,t) \\ &\sigma^{zz}_1(r,z,t) = \sigma^{zz}_2(r,z,t) = 0 \end{aligned} \tag{5.2}$$

where

$$\tau^{\alpha\beta}(r,z,t) = \eta_1\sigma^{\alpha\beta}_1(r,z,t) + \eta_2\sigma^{\alpha\beta}_2(r,z,t) \tag{5.3}$$

– constitutive relations

$$\begin{aligned} \dot{\sigma}^{rr}_E(r,z,t) &= 2\mu_E[w_{r,r}(r,t) - \xi^{rr}_E(r,z,t)] + \lambda_E\Big(w_{r,r}(r,t) + \\ &+ \frac{w_r(r,t)}{r} + \varepsilon_{zz}(v_z)(r,z,t) - \xi^{rr}_E(r,z,t) - \\ &- \xi^{\theta\theta}_E(r,z,t) - \xi^{zz}_E(r,z,t)\Big) \\ \dot{\sigma}^{r\theta}_E(r,z,t) &= \mu_E\Big(w_{\theta,r}(r,t) - \frac{w_\theta(r,t)}{r} - \xi^{r\theta}_E(r,z,t)\Big) \\ \dot{\sigma}^{rz}_E(r,z,t) &= \mu_E\Big(w_{z,r}(r,z,t) + \frac{(-1)^{E+1}}{\eta_E}q_r(r,t) - \xi^{rz}_E(r,z,t)\Big) \end{aligned}$$

$$\begin{aligned}
\dot\sigma_E^{\theta\theta}(r,z,t) &= 2\mu_E\Big(\frac{w_r(r,t)}{r} - \xi_E^{\theta\theta}(r,z,t)\Big) + \lambda_E\Big(w_{r,r}(r,t) + \\
&+ \frac{w_r(r,t)}{r} + \varepsilon_{zz}(v_z)(r,z,t) - \xi_E^{rr}(r,z,t) - \\
&- \xi_E^{\theta\theta}(r,z,t) - \xi_E^{zz}(r,z,t)\Big) \\
\dot\sigma_E^{\theta z}(r,z,t) &= \mu_E\Big((-1)^{E+1}\frac{1}{\eta_E}q_\theta(r,t) - \xi_E^{\theta z}(r,z,t)\Big) \\
\dot\sigma_E^{zz}(r,z,t) &= (2\mu_E+\lambda_E)[\varepsilon_{zz}(v_z)(r,z,t) - \xi_E^{zz}(r,z,t)] + \\
&+ \lambda_E\Big(w_{r,r}(r,t) + \frac{w_r(r,t)}{r} - \xi_E^{rr}(r,z,t) - \\
&- \xi_E^{\theta\theta}(r,z,t)\Big) = 0, \qquad E = 1,2
\end{aligned} \tag{5.4}$$

where

$$\xi_E^{\alpha\beta}(r,z,t) = \begin{cases} 0 & \text{if} \quad g_E(r,zt) < k_E \\ \dfrac{g_E(r,z,t)-k_E}{2\gamma_E g_E(r,z,t)} s_E^{\alpha\beta}(r,z,t) & \text{if} \quad g_E(r,z,t) \geq k_E \end{cases} \tag{5.5}$$

with the boundary conditions in the form

$$\begin{aligned}
& w_r(a,z,t) = w_r(b,z,t) = 0 && \text{for} \quad z \in \Big[-\frac{h}{z},\frac{h}{z}\Big], \quad t \in [t_0,t_f] \\
& w_\theta(a,z,t) = w_\theta(b,z,t) = 0 && \text{for} \quad z \in \Big[-\frac{h}{z},\frac{h}{z}\Big], \quad t \in [t_0,t_f] \\
& \sigma_E^{rz}(a,z,t) = \frac{c(t)z}{a} && \text{for} \quad z \in \Big[-\frac{h}{z},\frac{h}{z}\Big], \quad t \in [t_0,t_f] \\
& \sigma_E^{rz}(b,z,t) = \frac{c(t)z}{b} && \text{for} \quad z \in \Big[-\frac{h}{z},\frac{h}{z}\Big], \quad t \in [t_0,t_f] \\
& \sigma_E^{\theta z}(r,\pm\frac{n}{z},t) = \pm\frac{c(t)h}{2r} && \text{for} \quad z \in [a,b], \quad t \in [t_0,t_f] \\
& \sigma_E^{\theta z}(r,\pm\frac{h}{z},t) = 0 && \text{for} \quad z \in [a,b], \quad t \in [t_0,t_f]
\end{aligned} \tag{5.6}$$

and the initial conditions in the homogeneous form.

The approximation formulae (3.7) will take the form

$$\begin{aligned}
& v_r(r,z,t) \sim w_r(r,t) \\
& v_\theta(r,z,t) \sim w_\theta(r,t) \\
& \varepsilon_{\alpha\beta}(v)(r,zt,) \sim \nu_1(z)[\varepsilon_{\alpha\beta}(w)(r,z,t) + \varphi_{\alpha\beta}^1(q)(r,z,t)] + \\
& \qquad + \nu_2(z)[\varepsilon_{\alpha\beta}(w)(r,z,t) + \varphi_{\alpha\beta}^2(q)(r,z,t)], \quad \alpha,\beta = 1,2 \\
& \sigma^{\alpha\beta}(r,z,t) \sim \nu_1(z)\sigma_1^{\alpha\beta}(r,z,t) + \nu_2(z)\sigma_2^{\alpha\beta}(r,z,t), \quad \alpha,\beta = 1,2,3.
\end{aligned} \tag{5.7}$$

Introducing the denotations

$$
\begin{aligned}
&[\mu] \equiv \mu_1 - \mu_2 && [\lambda] \equiv \lambda_1 - \lambda_2 \\
&\bar{\mu} \equiv \eta_1\mu_1 + \eta_2\mu_2 && \bar{\lambda} \equiv \eta_1\lambda_1 + \eta_2\lambda_2 \\
&\hat{\mu} \equiv \frac{\mu_1}{\eta_1} + \frac{\mu_2}{\eta_2} && \hat{\lambda} \equiv \frac{\lambda_1}{\eta_1} + \frac{\lambda_2}{\eta_2} \\
&\alpha \equiv \frac{2\mu_1\lambda_1\eta_1}{2\mu_1 + \lambda_1} + \frac{2\mu_2\lambda_2\eta_2}{2\mu_2 + \lambda_2}
\end{aligned}
\tag{5.8}
$$

we obtain the following solution for the model with microlocal parameters in the elastic state

$$
\begin{aligned}
w_r(r,t) &= \frac{\dot{D}(t)}{r} + \dot{H}(t)r - \frac{1}{4}\frac{\tilde{\rho}\omega(t)\dot{\omega}(t)r^3}{2\tilde{\mu} + \alpha} - \frac{1}{2(2\tilde{\mu} + \lambda)}\dot{c}(t)\ln r \\
w_\theta(r,t) &= \frac{\dot{B}(t)}{2r} + \dot{E}(t)r - \frac{\tilde{\rho}\tilde{\omega}(t)r^3}{8\tilde{\mu}} \\
q_r(r,t) &= -\frac{[\mu]}{\hat{\mu}\tilde{\mu} - [\mu]^2}\frac{\dot{c}(t)z}{r} \\
q_\theta(r,t) &= 0 \\
\sigma_E^{rr}(r,z,t) &= 2\mu_E\Big(-\frac{D(t)}{r^2} + H(t) - \frac{3}{8}\frac{\tilde{\rho}\omega^2(t)r^2}{2\tilde{\mu} + \alpha} - \frac{c(t)\ln(r+1)}{2(2\tilde{\mu} + \alpha)}\Big) + \\
&\quad + \frac{2\mu_E\lambda_E}{2\mu_E + \lambda_E}\Big(2H(t) - \frac{1}{2}\frac{\tilde{\rho}\omega^2(t)r^2}{2\tilde{\mu} + \alpha} - \frac{c(t)}{2\tilde{\mu} + \alpha}\ln r - \frac{c(t)}{2(2\tilde{\mu} + \alpha)}\Big) \\
\sigma_E^{\theta\theta}(r,z,t) &= 2\mu_E\Big(\frac{D(t)}{r^2} + H(t) - \frac{1}{8}\frac{\tilde{\rho}\omega^2(t)r^2}{2\tilde{\mu} + \alpha} - \frac{c(t)\ln r}{2(2\tilde{\mu} + \alpha)}\Big) + \\
&\quad + \frac{2\mu_E\lambda_E}{2\mu_E + \lambda_E}\Big(2H(t) - \frac{1}{8}\frac{\tilde{\rho}\omega^2(t)r^2}{2\tilde{\mu} + \alpha} - \frac{c(t)}{2\tilde{\mu} + \alpha}\ln r - \frac{c(t)}{2(2\tilde{\mu} + \alpha)}\Big) \\
\sigma_E^{rz}(r,z,t) &= \frac{c(t)z}{r} \\
\sigma_E^{r\theta}(r,z,t) &= \mu_E\Big(-\frac{B(t)}{r^2} - \frac{1}{4}\frac{\tilde{\rho}}{\tilde{\mu}}\dot{\omega}(t)r^2\Big) \\
\sigma_E^{\theta z}(r,z,t) &= 0
\end{aligned}
\tag{5.9}
$$

where $D(t)$, $H(t)$, $E(t)$, $F(t)$ can be found from the boundary and the initial conditions (5.6).

From the form of the solution for the model with microlocal parameters in the elastic state it is easy to find that

- if the rotating circular disc is made of the two materials with the same value of μ we shall not get any microlocal effects,

- if $c(t) = 0$ for $t \in [t_0, t_f]$ the distribution of stresses does not depend on z, and we do not get any miccrolocal effects too.

The squares of the second invariants of the stress deviators take the form

$$g_E^2(r,z,t) = \frac{2}{3}\Big(\sigma_E^{rr}(r,z,t) - \sigma_E^{\theta\theta}(r,z,t)\Big)^2 + \frac{2}{3}\sigma_E^{rr}(r,z,t)\cdot$$
$$\cdot\sigma_E^{\theta\theta}(r,z,t) + 2\Big(\sigma_E^{rz}(r,z,t)\Big)^2 + \Big(\sigma_E^{r\theta}(r,z,t)\Big)^2. \tag{5.10}$$

After substituting into (5.10) the RHS of (5.9) we obtain very complicated formula which depends on three arguments r, z and t. The solution for the model with microlocal parameters holds up to a moment $t = t_1$. Ffor that moment there exists a point (r_0, z_0) for which $g_1^2(r_0, z_0, t_1) = k_1$ or $g_2^2(r_0, z_0, t_1) = k_2$. For any moment t, $t > t_1$ we are not able to find the explicit form of solution. We have to apply the step by step procedure which was described in the first example.

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Streszczenie

Dla periodycznego wielowarstwowego sprężysto/lepkoplastycznego kompozytu zaproponowano równania modelu z parametrami mikrolokalnymi. Podano rozwiązania w ramach tego modelu dla wielowarstwowej płyty oraz dla wielowarstwowej wirującej tarczy.

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