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FLEXURAL–TORSIONAL VIBRATIONS OF A MULTI–SPEED GEAR TRANSMISSION

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An analysis of investigation results of work stability of a dynamic system consisting of machine tool – holder – cutting tool – workpiece shown that internal couplings between vibrations of a drive and vibrations of a frame system were an important reason for self-excited vibrations arising. This statement indicated a necessity for developing the model of flexural–torsional vibrations for drives containing gear transmissions using the same method as in the case of frame systems. This is a Rigid Finite Elements Method (RFEM) which meets this requirement. When the model of flexural–torsional vibrations was being worked out many simplifications were introduced, and their correctness had to be experimentally confirmed. Physical and mathematical models of flexural–torsional vibrations were worked out for a specially designed and built gearbox, and dynamic characteristics were calculated on the basis of it. The experimental investigation was carried out for this gearbox. The characteristics obtained experimentally were compared with the computed ones. A good agreement between the model of flexural–torsional vibrations and the real system was achieved. Correctness of accepted simplifications was proved in this way.

1. Introduction

Experimental investigation of frame systems and spindles of machine–tools [1,5] brought to a conclusion that in many cases dynamic properties of a main drive are responsible for the loss of work stability of the MHCW system. As a consequence of this it was assumed that couplings of vibrations in different directions (translation and rotary directions) occuring in these drives have an essential influence on vibration stability of the MHCW system. Former separate models for torsional vibrations and flexural vibrations having been used in practical calculations of the dynamics of drives did not allow to show reasons of a stability loss of the MHCW system arising from interaction between translation and rotary vibrations.

Computational accuracy of an analysis and evaluation of a vibration stability will be higher when the model applied in the calculations is in better agreement with the real system. Theoretical principles of formulation of the model of torsional–flexural vibrations of a drive[1] (regarding mentioned interactions) and computer programs for computing frequency characteristics of it were elaborated in the Institute of Mechanical Technology of Technical University of Szczecin. Verifying investigation of the model was made by comparing characteristics received from wheels vibrations measurement carried out in a specially built two–speed gearbox with computational characteristics of the model.

2. The principles of formulation of the model of a gear transmission

Before starting to elaborate the physical model of multi–speed gear transmission the following general assumptions were accepted

- modelling of both a gear transmission and a whole drive should be methodically correlated with the frame structures of machine–tools modelling, what gives a possibility of an easier formulation of a general dynamic model of a machine–tool,
- gear transmission is a component part of a multi–speed drive so a model of it is only an element of the model of this drive.

Careful analysis both of the working conditions of a gear transmission, and of the properties of elements of it together with the aim of a study were the base of the following simplifications and detailed modelling assumptions introduction

1. An average loading of a gear transmission is much higher than an amplitude of load oscilations so the interteeth clearance uncovering does not occur.
2. Nonlinear rigid characteristics of both the bearing and the elements mounting can be linearizated with a good approximation in a range determined by an average value of loading and amplitude of it.
3. Variable stiffness of mesh has a local importance, also has a weak effect on the dynamics of a whole drive, so can be neglected at the first approximation.

[1]Notion "torsional–flexural vibrations" means vibrations of the model, in which the motion of each mass is described by six dislocation coordinates

4. For the same reasons an error of teeth performing may not be taken into account.

5. Furthermore the gyro effect and the phenomena resulting from assembly and unbalance errors are neglected.

Introducing assumptions no.3, no.4 and no.5 is valid inside investigated here range of frequency from 40 to 300 [Hz]. In a considerated example the inertial forcings due to neglected proprieties and phenomena are of frequency below 25 Hz or above 500 Hz.

The Rigid Finite Elements Method (RFEM) having been applied for several years in the Institute of Mechanical Technology fulfils requirements resulting from the above general assumptions. Models elaborated using this method retain geometrical structure of objects. Their parameters refer to specific properties of a real system in a simple and almost natural way. Formulation of a model of a multi–speed gear transmission can be divided into two steps

1 – formulation of models of particular subsystems of shafts,

2 – formulation of models of mating of proper pair of toothed wheels and joining them to a general model.

The first step consists in proficient application of the Rigid Finite Elements Method and will not be presented in detail in this paper. In the previous paper [2] authors discussed in detail the second step of the model formulation however only theoretical considerations were carried out. Those considerations are going to be reminded now and become completed with calculation results of frequency characteristics of a two–speed gear transmission.

The physical model of a two–speed gear transmission is shown in Fig.1. Rigid Finite Elements (RFEs) no.4, 6 and p and r fit mating pairs of the toothed wheels. Proper coordinate systems $x_1x_2x_3$, and $y_1y_2y_3$, to which inertial torsional and damping parameters of the model refer are assigned to each RFE and elastic–damping element (EDE).

Orientation of these systems is shown in Fig.2. As generalized dislocations there were admitted three translation dislocations q_1, q_2, q_3, in a longitudinal direction, and three rotation dislocations q_4, q_5, q_6, around axes x_1, x_2, x_3.

The division of subsystems into RFEs and the calculation of parameters of the model are classical problems of the RFEM. These are routine acts and the first step of the model formulation is ended when components of the classical matrix equation of motion are stated

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{H}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{P} \tag{2.1}$$

i.e.

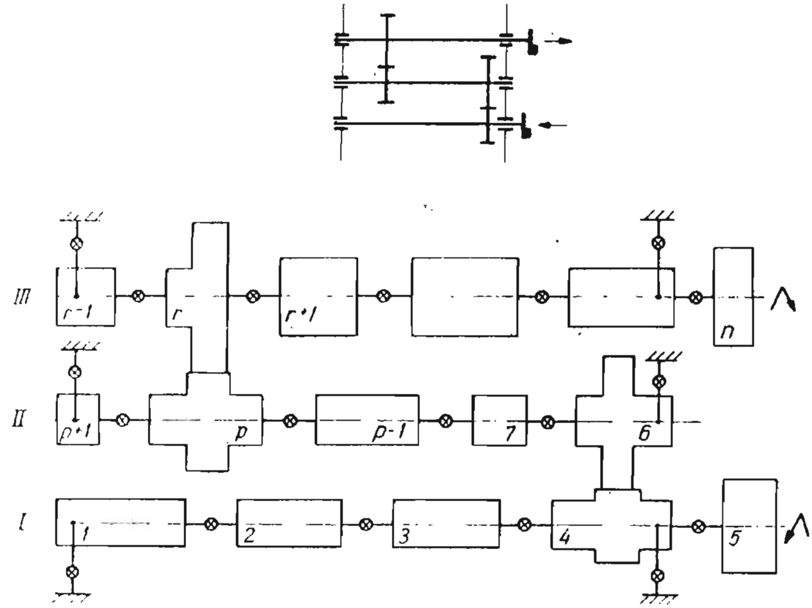

Fig. 1. The physical model of a two-speed gear transmission

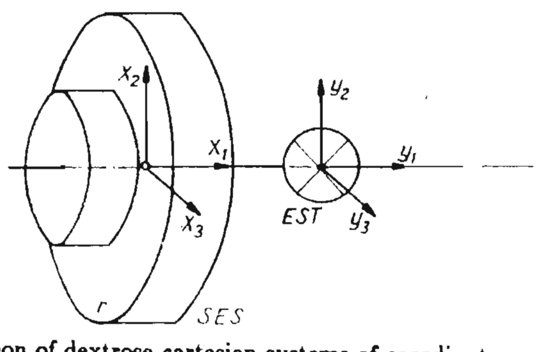

Fig. 2. The orientation of dextrose cartesian systems of coordinates connected with RFEs and EDEs

M – diagonal matrix of inertia,
H – symmetric, band matrix of damping,
K – symmetric, band matrix of rigidity,
P – column matrix of external extortions,
q – column matrix of generalized dislocations coordinates.

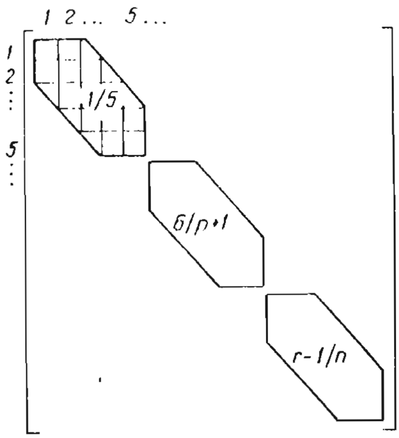

Fig. 3. The structure of matrices of rigidity and damping

When RFEs are numbered properly matrices **H** and **K** have a structure shown in Fig.3 (6 rows or 6 columns are assigned to each index, and a block of 6 × 6 rank to each pair of indices). The equations of motion can be divided into groups corresponding with the subsystems of shafts. In a range of every group equations are associated with each other. Nonzero blocks of the matrix laying outside the main diagonal of matrices **H** and **K** are responsible for that. However the groups of equations are not associated with each other. The model does not describe reciprocal interactions between subsystems of shafts.

The second step of a gear transmission model formulation consists of describing spatial force interactions of mating toothed wheels and of the proper completing of an initial model.

It is assumed that an interteeth forces are perpendicular to a surface of mating wheels at the points of their contact. Previously presented conditions of teeth mating and aforementioned modelling assumptions (especially the assumptions of the teeth clerance uncovering) describe ideal geometrical constraints imposed on the teeth dislocations. These constraints are of a shape of a surface tangential to both the outline and the line of teeth at the point of their mating. Imposing on teeth dislocation of mating wheels constraints determined in this way is equivalent to a demand that points P_p and P_r still remain on the surface which trace in Fig.4

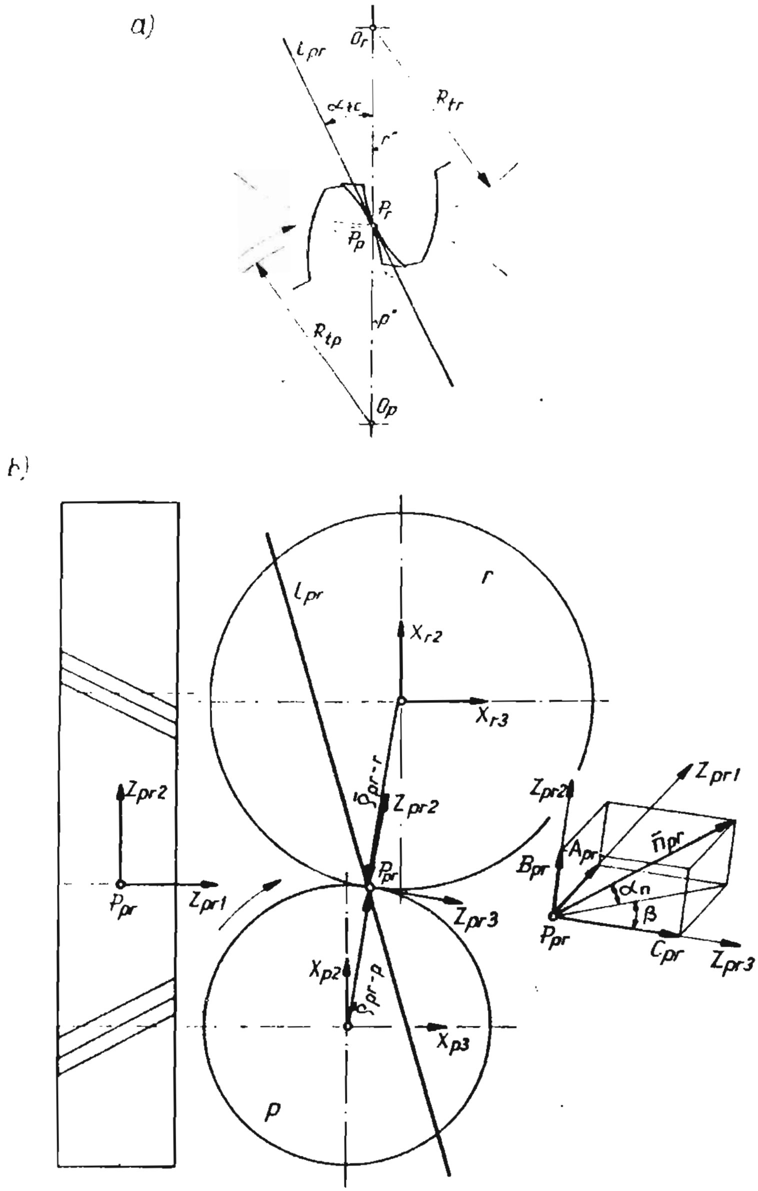

Fig. 4. The physical model of toothed wheels mating: a) – position of the surface of constraints, b) – determining of components of directional $\bar{n}_{pr}$ vector of the surface of constraints

is a straight line l_{pr}. An unit normal vector $\bar{n}_{pr}$ of the surface of constraints is determined for a statical stage of equilibrum of wheels and it does not change in spite of the wheel vibrations (the small dislocations of the teeth and the surface of constraints).

The cartesian coordinate system z_1, z_2, z_3 origin was placed at the point of wheels mating for the simplicity of the mathematical formulation of the equation of geometrical constraints. In this system the normal vector $\bar{n}_{pr}$ of the surface of constraints has A, B, C, components, and their values are functions of an α angle of teeth contact and of an β angle of teeth line inclination

$$\begin{aligned} A_{pr} &= -\cos\alpha_n \sin\beta \\ B_{pr} &= \sin\alpha_n \\ C_{pr} &= \cos\alpha_n \cos\beta \qquad (\text{right teeth} \Rightarrow \beta > 0). \end{aligned} \tag{2.2}$$

When these values are put into $\mathbf{n}_{pr}$ matrix

$$\mathbf{n}_{pr} = [A_{pr}\ B_{pr}\ C_{pr}]$$

and the dislocation coordinates of points P_p and P_r referring to the system z_1, z_2, z_3 are recorded in column matrices $\mathbf{Z}_p$ and $\mathbf{Z}_r$

$$\begin{aligned} \mathbf{Z}_p &= \mathrm{col}\{Z_{p_p-1}, Z_{p_p-2}, Z_{p_p-3}\} \\ \mathbf{Z}_r &= \mathrm{col}\{Z_{p_r-1}, Z_{p_r-2}, Z_{p_r-3}\} \end{aligned}$$

the equation of geometrical constraints is expressed by a formula like a clasical equation of surface

$$f_{pr}: \qquad \mathbf{n}_{pr}(\mathbf{Z}_p - \mathbf{Z}_r) = 0. \tag{2.3}$$

Points P_p and P_r belong to a rigid solid figure and that is why it is easy to express their dislocations by accepted generalized dislocations of these solid figures. To this end there should be formed matrices of directional factors $\mathbf{c}$ of systems x (RFEs p and r) with respect to system z (mating point P_{pr}) and matrices of coordinates of position $\mathbf{V}^*$ of the mating point P in systems x. Then

$$\begin{aligned} \mathbf{Z}_p &= \mathbf{c}_{pr-p}\mathbf{V}^*_{pr-p}\mathbf{q}_p \\ \mathbf{Z}_r &= \mathbf{c}_{pr-r}\mathbf{V}^*_{pr-r}\mathbf{q}_r. \end{aligned} \tag{2.4}$$

After putting these relations into the equation (2.3), the equation of constraints can be expressed in the following form

$$f_{pr}: \qquad \mathbf{n}_{pr}\mathbf{c}_{pr-p}\mathbf{V}^*_{pr-p}\mathbf{q}_p - \mathbf{n}_{pr}\mathbf{c}_{pr-r}\mathbf{V}^*_{pr-r}\mathbf{q}_r = 0. \tag{2.5}$$

Introducing notions

$$\begin{aligned} \mathbf{T}_{pr-p} &= \mathbf{n}_{pr}\mathbf{c}_{pr-p}\mathbf{V}^{*}_{pr-p} \\ \mathbf{T}_{pr-r} &= \mathbf{n}_{pr}\mathbf{c}_{pr-r}\mathbf{V}^{*}_{pr-r} \end{aligned} \tag{2.6}$$

the equation of constraints is written finally in the following form

$$f_{pr}: \qquad \mathbf{T}_{pr-p}\mathbf{q}_p - \mathbf{T}_{pr-r}\mathbf{q}_r = 0. \tag{2.7}$$

A formula (2.7) is a linear form of the generalized dislocation coordinates, and entries of matrices $\mathbf{T}_{pr-p}$ and $-\mathbf{T}_{pr-r}$ are derivatives of the equation of constraints with respect to the proper coordinates of generalized dislocations

$$\begin{aligned} \mathbf{T}_{pr-p} &= \left[\frac{\partial f_{pr}}{\partial q_{p1}}\ \frac{\partial f_{pr}}{\partial q_{p2}} \cdots \frac{\partial f_{pr}}{\partial q_{p6}}\right]_{1\times 6} \\ -\mathbf{T}_{pr-r} &= \left[\frac{\partial f_{pr}}{\partial q_{r1}}\ \frac{\partial f_{pr}}{\partial q_{r2}} \cdots \frac{\partial f_{pr}}{\partial q_{r6}}\right]_{1\times 6}. \end{aligned} \tag{2.8}$$

When a drive consists of several elementary gear transmissions, all equations of constraints can be written in the following form

$$f: \qquad \mathbf{g}^{\top}\mathbf{q} = \mathbf{0} \tag{2.9}$$

and

$$\mathbf{g} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{T}^{\top}_{46-4} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ -\mathbf{T}^{\top}_{46-6} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{T}^{\top}_{pr-p} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{T}^{\top}_{pr-r} \\ \mathbf{0} & \mathbf{0} \\ \vdots & \vdots \end{bmatrix}_{6n\times l} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ p-1 \\ p \\ p+1 \\ r-1 \\ r \\ r+1 \\ \\ \end{matrix} \tag{2.10}$$

where: l – number of pairs of toothed wheels being in mesh and transferring power (in the example described here $l = 2$).

The rectangle matrix $\mathbf{g}$ contains the derivatives of equations of constraints with respect to the generalized dislocation coordinates.

A full mathematical model of a system with geometrical constraints has the following form

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{H}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{P} + \mathbf{g}\boldsymbol{\lambda} \tag{2.11}$$

$$\mathbf{g}^{\top}\mathbf{q} = \mathbf{0}. \tag{2.12}$$

The column matrix $\boldsymbol{\lambda}$ contains the indeterminated Lagrange's multipliers assigned to successive equations of constraints. In the considered case

$$\boldsymbol{\lambda} = \mathrm{col}\{\lambda_{46}, \lambda_{pr}\}.$$

To eliminate the indeterminated Lagrange's multipliers from the mathematical model, authors used the algorithm proposed by Suslov [9]. The matrix of generalized accelerations is calculated from the equation of motion

$$\ddot{\mathbf{q}} = \mathbf{M}^{-1}(\mathbf{P} + \mathbf{g}\boldsymbol{\lambda} - \mathbf{H}\dot{\mathbf{q}} - \mathbf{K}\mathbf{q}) \tag{2.13}$$

and putting it to the twice differentiated with respect to time equation (2.9) one obtains

$$\mathbf{g}^{\top}\mathbf{M}^{-1}(\mathbf{P} + \mathbf{g}\boldsymbol{\lambda} - \mathbf{H}\dot{\mathbf{q}} - \mathbf{K}\mathbf{q}) = \mathbf{0}.$$

After simple transformations the formula of matrix $\boldsymbol{\lambda}$ take a form

$$\boldsymbol{\lambda} = (\mathbf{g}^{\top}\mathbf{M}^{-1}\mathbf{g})^{-1}\mathbf{g}^{\top}\mathbf{M}^{-1}(\mathbf{H}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} - \mathbf{P}). \tag{2.14}$$

An analysis of matrix $(\mathbf{g}^{\top}\mathbf{M}^{-1}\mathbf{g})$ showes that its rank is equal l what implies its non–singularity and invertibility. Putting the expression (2.14) into the equation (2.11) it is obtained

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{H}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{P} + \mathbf{g}(\mathbf{g}^{\top}\mathbf{M}^{-1}\mathbf{g})^{-1}\mathbf{g}^{\top}\mathbf{M}^{-1}(\mathbf{H}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} - \mathbf{P}).$$

Designating

$$\mathbf{F} = -\mathbf{g}(\mathbf{g}^{\top}\mathbf{M}^{-1}\mathbf{g})^{-1}\mathbf{g}^{\top}\mathbf{M}^{-1} \tag{2.15}$$

and

$$\bar{\mathbf{H}} = \mathbf{F}\mathbf{H} \tag{2.16}$$

$$\bar{\mathbf{K}} = \mathbf{F}\mathbf{K} \tag{2.17}$$

$$\bar{\mathbf{P}} = \mathbf{F}\mathbf{P} \tag{2.18}$$

the equation of motion of the model can be expressed in a form

$$\mathbf{M}\ddot{\mathbf{q}} + (\mathbf{H} + \bar{\mathbf{H}})\dot{\mathbf{q}} + (\mathbf{K} + \bar{\mathbf{K}})\mathbf{q} = \mathbf{P} + \bar{\mathbf{P}}. \tag{2.19}$$

So a task of introducing of geometrical constraints into the mathematical model of a gear transmission consists in calculating of $\bar{\mathbf{H}}$, $\bar{\mathbf{K}}$ and $\bar{\mathbf{P}}$ matrices which update formerly determined matrices of damping, rigidity and extortions. A structure of $\bar{\mathbf{H}}$ and $\bar{\mathbf{K}}$ matrices for the exemplary model is shown in Fig.5. After introducing notations

$$\hat{\mathbf{H}} = \mathbf{H} + \bar{\mathbf{H}}, \qquad \hat{\mathbf{K}} = \mathbf{K} + \bar{\mathbf{K}}, \qquad \hat{\mathbf{P}} = \mathbf{P} + \bar{\mathbf{P}}$$

the final form of the equation of motion is obtained

$$\mathbf{M}\ddot{\mathbf{q}} + \hat{\mathbf{H}}\dot{\mathbf{q}} + \hat{\mathbf{K}}\mathbf{q} = \hat{\mathbf{P}}. \tag{2.20}$$

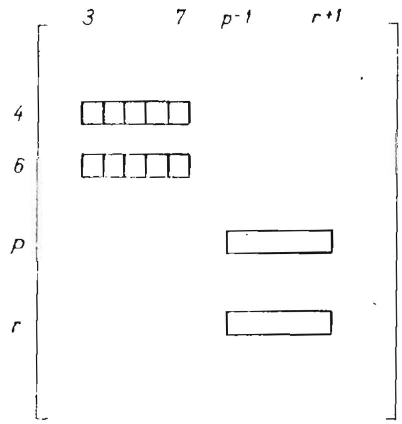

Fig. 5. The structure of correcting $\bar{\mathbf{H}}$ and $\bar{\mathbf{K}}$ matrices

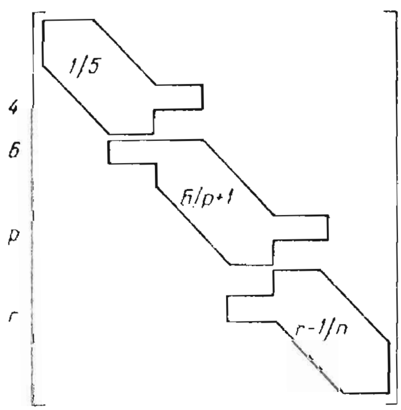

Fig. 6. The final structures of matrices of damping ($\hat{\mathbf{H}}$) and rigidity ($\hat{\mathbf{K}}$)

It nearly does not happen in practice that the generalized external extortion acts on RFE containing a toothed ring and that is why generally $\bar{\mathbf{P}} = \mathbf{0}$ and $\hat{\mathbf{P}} = \mathbf{P}$.

The final structure of matrices of damping $\hat{\mathbf{H}}$ and rigidity $\hat{\mathbf{K}}$ is shown in Fig.6. These matrices are not symmetrical. Although the linkage between groups of equations of motion modelling succeeding shaft subsystems results from the structure of these matrices.

3. The verification of the model of gear transmission

The aim of experimental verification of the formulated model containing a gear transmission was

- confirmation of the correctness of introduced shape of constraints modelling toothed wheels mating and linkage between vibrations,
- proving that the introduced modelling assumptions and simplifictions are valid.

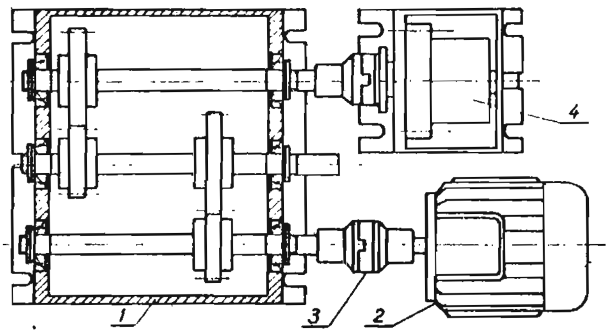

Fig. 7. The simplified constructional drawing of the transmission

A special two–speed gear transmission driven by electric motor SF 100L (2) and loaded on the output by a special disk brake (4) was designed and made of metal (1) – Fig.7. Total gear ratio $i \approx 0.5$ was received by means of two pairs of gears with skew teeth and was connected with a motor and a brake by means of Oldham's clutches 3.

The idea of a verification task consisted in comparison of the frequency characteristics computed on a base of previously presented mathematical torsional–flexural model of the vibration of gear transmission and the proper characteristics experimentally obtained on a special test stand.

Before starting formulation of the model, the torsional rigidities of clutches under nominal loading were calculated. They resulted in: for the clutch near the motor $k_{s1} = 100000$ [Nm/rad], for the clutch near the brake $k_{s2} = 180000$ [Nm/rad]. These rigidities were taken into account when values of parameters of the model were determined. Rigidities and damping coefficients of bearing were calculated according to formulae and recommendations presented in [4,8]. The coefficients of vibration energy dissipation in the material were introduced and the damping coefficients of EDEs were calculated following instructions given in paper [7].

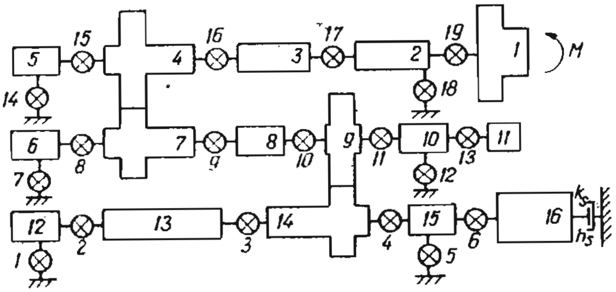

Fig. 8. The physical model of a transmission

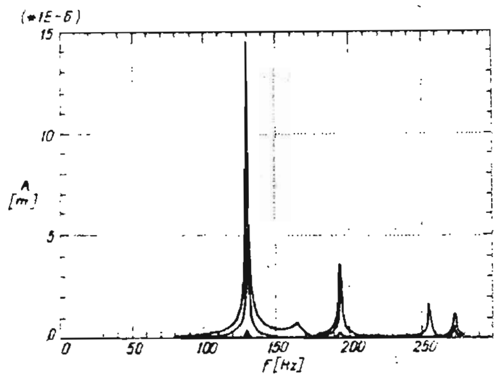

Fig. 9. RFE no.7 – q_1, q_2, q_3 coordinates – matching of $A - f$ characteristics

To the model of gear transmission (Fig.8) formed after the algorythm presented in the former part of this paper, the classical for stationary problems model of asynchronous motor, parameters of which were calculated using catalogue data, was joined. Inertial and elastic–damping parameters of the brake are taken into account adequately in RFE no.1 and EDE no.19.

Frequency characteristics were calculated. For example the amplitude–frequency characteristics of vibrations of RFE no.7 in a direction of progressive co-

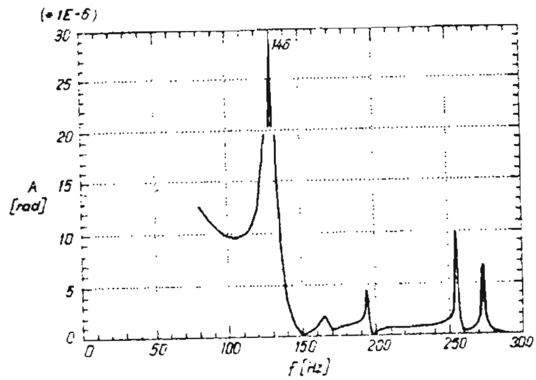

Fig. 10. RFE no.7 – q_4 coordinate – A – f characteristic

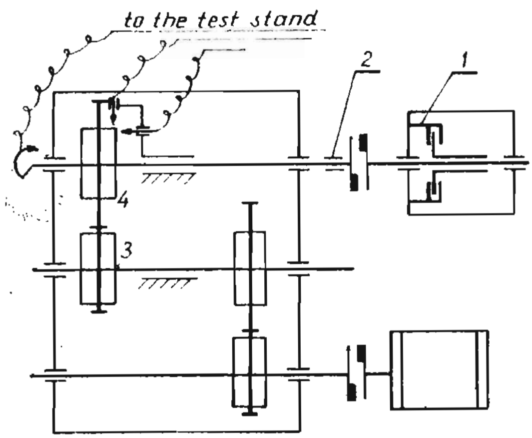

Fig. 11. The chart of a transmission during investigation in the motion

ordinates (q_1, q_2, q_3) and in a torsional direction (q_4) are shown in Fig.9 and 10. This RFE models the smaller one of the mating toothed wheels. Characteristics calculated for both mating wheels (RFE no.4 and 7) and in all directions of vibration (translation and torsional vibrations) show that resonances appear for the same frequency 130, 195, 255, and 275 [Hz] (although proportions of amplitudes between them are different). This testifies that elaborated and applied to calculations model takes into account coupled (in different directions) vibrations of mating toothed wheels. The heavy damping of resonances for a high frequency can be observed on characteristics of torsional vibrations (coordinate q_4).

Experimental investigation of dynamic characteristics of the unique two–speed gear transmission were made on a test stand which scheme is shown in Fig.11.

During dynamic investigation in motion the gear transmission was loaded by a torque moment measured by strain gauges 2. Both the constant component and the variable component of this load were imposed by disk brake 1 hydraulically operated by electro–hydraulic Moog's transducer.

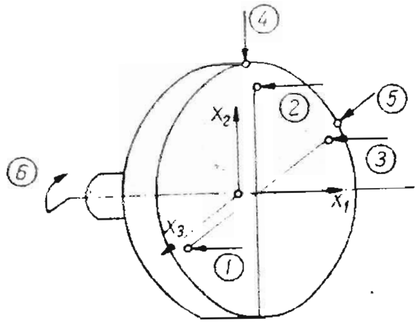

Fig. 12. Positions of sensors during dynamic investigation

Determination of spatial dislocations of toothed wheel required registration of indications of six properly arranged and directed gauges. Positions and acting directions of these gauges during measurements of vibrations of wheels are shown in Fig.12. Gauges with numbers from 1 to 5 were non–tactile, inductive sensors of dislocations, while the gauge number 6 was inductive sensor of torsional vibrations.

Frequency characteristics of spatial vibrations of mating wheels no.3 and 4 were calculated during investigation (Fig.11). Earlier introduced (for the description of the model of motion) coordinates of generalized dislocations fixed positions of axes of reference systems to which indications of gauges (characteristics) were transformed.

It is possible to isolate three units inside the matched measuring system (Fig.13)

- unit of realisation of extortions,
- main measuring unit (recording vibration of the wheels),
- auxiliary measuring unit (recording vibrations of the gauge holders).

Taking into account results of preliminary investigations, primary investigations were carried out loading the gear transmission with variable moment, which constant component $M_{avg} = 40$ [Nm], and variable component (of almost sinusoidal shape) had an amplitude $M_A = 5$ [Nm]. Frequency of its variations changed smoothly and slowly within the limits from 40 to 300 [Hz].

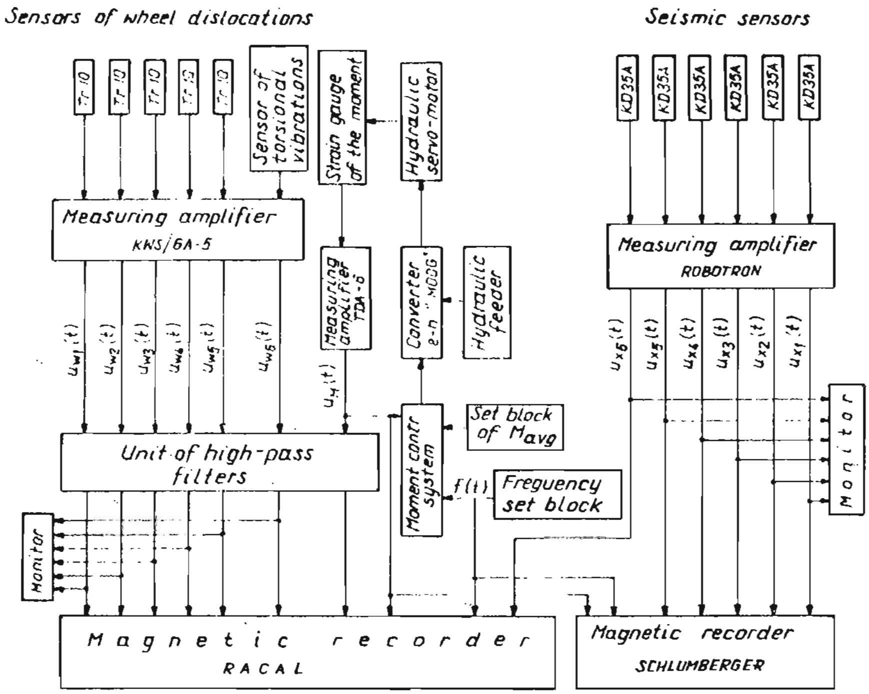

Fig. 13. The chart of measuring unit for primary investigation

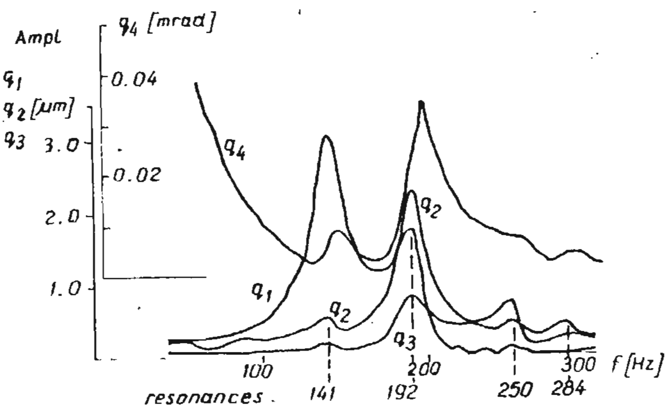

Fig. 14. Wheel no.3 (RFE no.7) – a comparison of $A - f$ characteristics of torsional and translation vibrations

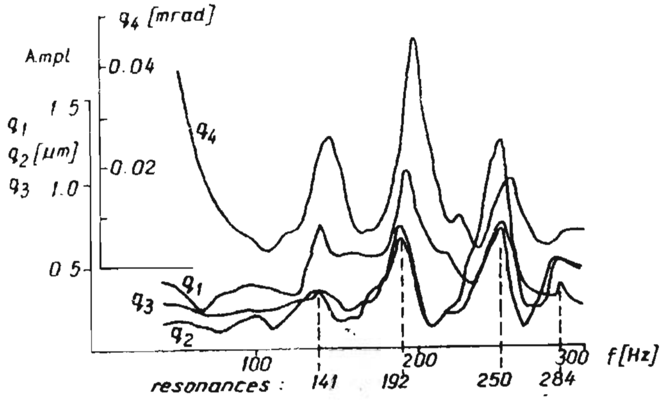

Fig. 15. Wheel no.4 (RFE no.4) – a comparison of $A - f$ characteristics of torsional and translation vibrations

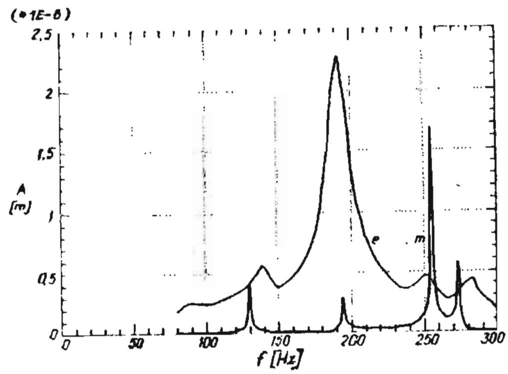

Fig. 16. The comparison of experimental (e) and modelling (m) $A - f$ characteristics for q_2 coordinate: wheel no.3 – RFE no.7

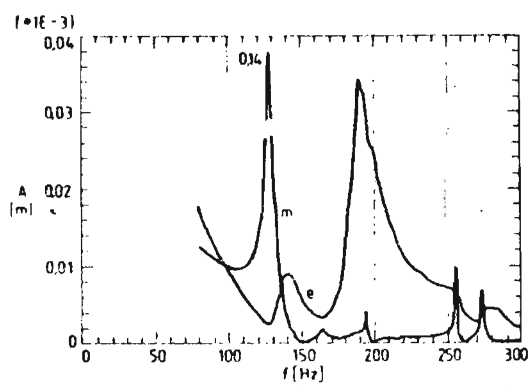

Fig. 17. The comparison of experimental (e) and modelling (m) $A - f$ characteristics for q_4 coordinate: wheel no.3 – RFE no.7

Fig.14 and Fig.15 show in the same scale three $A - f$ characteristics of vibrations of wheels no.3 and 4 (Fig.11) in direction of translation dislocations q_1, q_2, q_3 and the characteristic of torsional vibrations, i.e. in q_4 direction. All of them have their resonances for the same (or nearly the same) frequency 141, 192, 250, and 284 [Hz]. Proportions between succesive resonances are different.

Experimental showing that in all directions of vibrations of both mating wheels resonances appear for the same frequency is equal to proving the existance of couplings of vibrations in the transmission.

To assess proposed method of formulation of the model of a gear transmission, the amplitude–frequency characteristics ($A - f$) obtained experimentally (e) and those obtained from the model (m) of corresponding RFE no.7 and 4 (see Fig.8) were confronted with each other on the common charts. The characteristics e and m comparison of which is shown on these charts refer to the same point of the transmission (i.e. centre of mass of RFE) and the same direction of vibrations and they are drawn in the same scale. Examples of compared characteristics are shown in Fig.16 and Fig.17.

Both kinds of characteristics (e and m) show four resonances for a frequency band of 80 $\div$ 300 [Hz]. Resonances occurring in the characteristics obtained by solving the model are situated close enough to resonances accuring in the experimental characteristics so it is possible to assign each other without making an error. Such an assignment makes the assessment of the model of the transmission it easier. Confrontation of frequency of corresponding resonances looks as follows:

resonance no.1: experiment – 141 [Hz], model – 130 [Hz],
resonance no.2: experiment – 192 [Hz], model – 194 [Hz],
resonance no.3: experiment – 250 [Hz], model – 255 [Hz],
resonance no.4: experiment – 284 [Hz], model – 273 [Hz].

Differences between resonance frequencies are subtle in a case of resonances no.2 and 3 and rather great (11 Hz) in a case of resonances no.1 and 4. These divergences most likely do not result from the taken way of modelling of wheels mating, but rather from inaccurate stating of the values of coefficients of rigidity of elastic–damping elements of the model. This refers particularly to the rigidity of EDEs assigned to bearing supports. Theoretical rigidity of every support can considerably differ from the real one as a result of influence of flexibility of bearing mounting, clamps (clearances), quality of assembly, flexibility of frame walls of a gearbox and the like. Determination of these influences (even by experimental investigation) is a problem itself and was not the aim of this work.

In a direction of torsional vibrations in both kinds of characteristics resonances for higher frequencies (resonances no.3 and 4) have small amplitudes. Predomination of resonance no.2 results from experimental investigation, while modelling characteristics show that resonance of the largest amplitude is a resonance no.1.

Analysing each pair of the characteristics obtained experimentally and calculated on the base of the model respectively, similarities and differences between them can be indicated. Generally, it should be stated that proportions between amplitudes of vibrations in resonances are different for experimentally obtained and calculated characteristics, although in certain definite cases it is possible to find similarity. Generally, resonances obtained by calculations have larger amplitudes.

Considering on a base of results of experimental investigations and calculations usefulness of the elaborated model or ways of chansing it, at least three problems should be analysed

1. usefulness (accuracy) of the RFEM for formulation of models of shafts subsystems,
2. validity of the proposed model of toothed wheels mating, i.e. the model of couplings of their vibrations,
3. accurancy of calculations of parameters of models.

Summarizing consideration of the first problem it should be stated that constructional nature of shafts subsystems does not make any difficulties for building their models by RFEM. On the contrary in a comparision with for example frame systems it is possible to determine easier and most often more exactly the structure of models (dividing points for RFEs, position of elastic–torsional elements) and the values of parameters of their elements. So there is no reason to formulate a conclusion that RFEM is no useful for designing of dynamic models of toothed wheels of tools–machines.

Considering usefulness of the model of toothed wheels mating in calculations of characteristics of vibrations of a gear transmission by RFE method it should be indicated that

- characteristics of toothed wheels vibrations obtained from the model of a transmission show couplings of vibrations in all directions. These couplings appear both in a motion of the same mass (wheel) and as a dependence of vibrations of one wheel on vibrations of another one,
- comparission of characteristics obtained experimentally and analytically show qualitative convergences and quantitative differences (height of resonances).

The reasons for the divergence of amplitudes values of vibrations can be found as well in the part of the model of wheels mating as in an accuracy of calculations of values of parameters of the transmission model (especially values of damping parameters). Introducing the model of mating of toothed wheels it was assumed that two–sided geometrical constraints imposed on the motion of wheels are ideal.

When give up this assumption it would be possible to change values of components of the versor of a meshing surface (formula 2.7) and influence in this way on a change of the ratio of amplitudes of vibrations in different directions. It is hardly to forejudge whether acting like this would bring about more accurate model.

Changing the model of mating of wheels (its form and values of parameters) can appear as important as searching more accurate values of parameters of damping and rigidity. It especially refers to modelling such elements of a construction as bearing supports, clutches, mounting of elements and like this. In this paper to avoid disturbances in the estimation of the model proposed, the simplest and more often used way in engineering calculations was introduced – the values of coefficients necessary for the model were determined on a base of commonly approachable data presented in literature. The approach like this (no identification of the values of parameters of the model) was caused by the desire to determine the practical usefulness of the model to calculations of characteristics of drives on a step of their designing. Results of modelling calculations and experimental investigation confirm this usefulness.

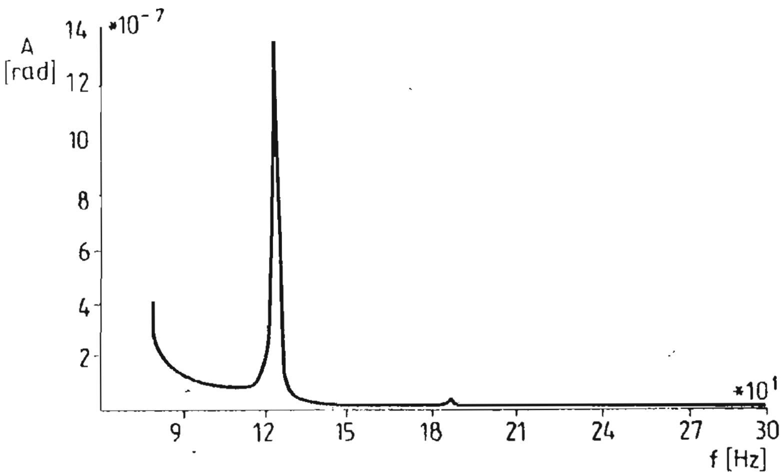

Fig. 18. The $A - f$ characteristic obtained from the classical model of torsional vibrations – wheel no.3 (RFE no.7)

To show the propriety of application of the presented model, a classical model of torsional vibrations for the considering transmission was formulated. Some parameters of the flexural–torsional model of vibrations were used for this purpose (e.i. mass moments of inertia around the axis of rotation, rigidities and torsional damping, parameters of the motor). $A - f$ characteristic of vibrations of wheel no.3 (equivalent to RFE no.7) of such a model is shown in Fig.18. From two resonances ($f = 123$ and 188 [Hz]), the first one ($f = 123$ [Hz]) dominates clearly on this characteristic. An analysis of computer printouts does not show any other resonances for a whole band of frequency (e.i. from 40 to 300 [Hz]). So the classical model of torsional vibrations shows only two resonances (resonances no.1 and no.2 on characteristics obtained from the torsional–flexural model correspond with them). This classical model does not show the two left resonances existence of which was confirmed by the experimental investigation results.

The above example confirms the assumptions which laid down foundations of taking the subject–matter of this work. The model of torsional–flexural vibrations proposed here is undoubtedly more accurate than a model of torsional vibrations. For all analysed coordinates of generalized dislocations in the investigated gear transmission $A - f$ characteristics have resonances resulting from properties of the system determined in a torsional direction (resonances no.1 and 2) and resonances resulting from properties of the system determined in left directions (resonances no.3 and 4). A whole set of these resonances appears on both experimental and obtained from the model characteristics. Usefulness of RFEM for modelling and calculations of the dynamic characteristics of a transmission (with the model of mating of toothed wheels joined to it) is proved in this way.

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Streszczenie

Analiza wyników badań stabilności pracy obrabiarek wskazała na potrzebę opracowania modelu giętno–skrętnych drgań ich napędów. W artykule przedstawiono sposób wykorzystania do tego celu metody sztywnych elementów skończonych (SES). Porównano charakterystyki doświadczalne otrzymane z badań przekładni zębatej z charakterystykami utworzonego modelu wyciągając wniosek ogólny, że proponowany sposób wykorzystania metody SES można stosować do modelowania i obliczania charakterystyk dynamicznych napędów.

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