JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
50, 2, pp. 473-485, Warsaw 2012
50th Anniversary of JTAM

MODEL OF THE FINAL SECTION OF NAVIGATION OF A SELF-GUIDED MISSILE STEERED BY A GYROSCOPE

Edyta Ładyżyńska-Kozdraś
Faculty of Mechatronics, Warsaw University of Technology, Warsaw, Poland
e-mail: lady@mchtr.pw.edu.pl

Zbigniew Koruba
Faculty of Mechatronics and Machine Building, Kielce University of Technology, Kielce, Poland
e-mail ksmzko@tu.kielce.pl

This paper presents the modelling of dynamics of a self-guided missile steered using a gyroscope. In such kinds of missiles, the main element is a self-guiding head, which is operated by a steered gyroscope. The paper presents the dynamics and the method of steering such a missile. Correctness of the developed mathematical model was confirmed by digital simulation conducted for a Maverick missile equipped with a gyroscope being an executive element of the system scanning the earth's surface and following the detected target. Both the dynamics of the gyroscope and the missile during the process of scanning and following the detected target were subject to digital analysis. The results were presented in a graphic form.

Key words: self-guiding, dynamics and steering, steered gyroscope, missile

1. Introduction

In the case of automatic steering of self-guided missiles, kinematic equations of spins were related to missile equations of dynamics, using the Boltzmann-Hamel equations (Ładyżyńska-Kozdraś *et al.*, 2008), which were developed in a relative frame of reference $Oxyz$, rigidly connected with the missile (Żyluk, 2009).

At the moment of detecting the target, it was assumed that the missile automatically passes from the flight on the programmed trajectory to tracking flight of the target according to the assumed algorithm, in this case –

the method of proportional navigation. Controlling the motion of the missile is carried out by the deflection of control surfaces, i.e. direction steer and height steer at the angles δ_V and δ_H respectively. The control laws constitute kinematic relations of deviations of set and realised flight parameters, stabilising the movement of the missile in channels of inclination and deflection. The realisation of the desired flight path of the missile is carried out by the autopilot, which generates control signals based on the compounds derived for the executive system of steering.

In the final section of navigation, various types of disturbances may affect the missile, such as wind or shock waves from shells exploding nearby. Therefore, additional stabilisation is necessary, in this case performed by the gyroscope. When searching for a ground target, the gyroscope axis, facing down, strictly outlines defined lines on the earth's surface with its extension. The optic system positioned in the axis of the gyroscope, with a specific angle of view, can thus find the light or infrared signal emitted by a moving object. Therefore, kinematic parameters of reciprocal movement of the missile head and gyroscope axis should be selected to detect the target at the highest probability possible. After locating the target (receiving the signal by the infrared detector), the gyroscope goes into the tracking mode, i.e. from this point its axis takes a specific position in space, being directed onto the target.

2. The general equations of missile dynamics

Dynamic equations of missile motion in flight were derived in quasi-coordinates φ, θ, ψ and quasi-velocities U, V, W, P, Q, R (Fig. 1) using the Boltzmann-Hamel equations true for mechanical systems in the system associated with the object.

The following correlation expresses them in a general form

$$\frac{d}{dt}\frac{\partial T^*}{\partial \omega_\mu} - \frac{\partial T^*}{\partial \pi_\mu} + \sum_{r=1}^{k}\sum_{\alpha=1}^{k} \gamma^r_{\mu\alpha}\frac{\partial T^*}{\partial \omega_r}\omega_\alpha = Q^*_\mu \tag{2.1}$$

where: $\alpha,\mu,r = 1,2,\ldots,k$, k – number of degrees of freedom, ω_μ – quasi-velocities, T^* – kinetic energy expressed in quasi-velocities, π_μ – quasi-coordinates, Q^*_μ – generalised forces, $\gamma^r_{\alpha\mu}$ – three-index Boltzmann factors, expressed by the following correlation

$$\gamma^r_{\alpha\mu} = \sum_{\delta=1}^{k}\sum_{\lambda=1}^{k}\Big(\frac{\partial a_{r\delta}}{\partial q_\lambda} - \frac{\partial a_{r\lambda}}{\partial q_\delta}\Big) b_{\delta\mu} b_{\lambda\alpha} \tag{2.2}$$

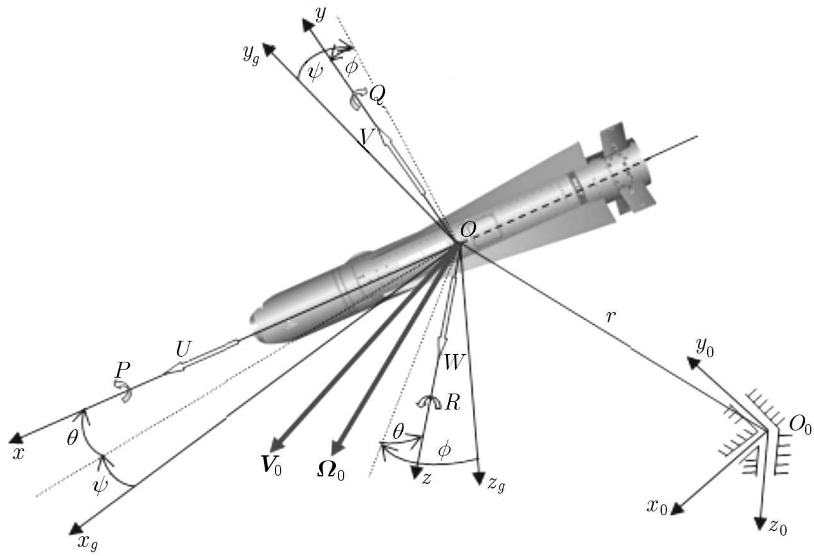

Fig. 1. Assumed reference system and parameters of the missile in the course of guidance

Relations between quasi-velocities and generalised velocities are

$$\omega_\delta = \sum_{\alpha=1}^{k} a_{\delta\alpha}\dot{q}_\alpha \dot{q}_\delta = \sum_{\mu-1}^{k} b_{\delta\mu}\omega_\mu \tag{2.3}$$

where: $\dot{q}_\delta$ – generalised velocities, q_k – generalised coordinates, $a_{\delta\alpha} = a_{\delta\alpha}(q_1, q_2, \ldots, q_k)$ and $b_{\delta\alpha} = b_{\delta\alpha}(q_1, q_2, \ldots, q_k)$ – coordinates being functions of generalised coordinates, while the following matrix correlation exists: $[a_{\delta\mu}] = [b_{\delta\mu}]^{-1}$.

The Boltzmann-Hamel equations, after calculating the values of Boltzmann factors and indicating kinetic energy in quasi-velocities, a system of ordinary differential equations of the second order was received which describes the behaviour of the missile on the track during guidance.

In the frame of reference associated with the moving object $Oxyz$, they have the following form

$$\mathbf{M}\dot{\boldsymbol{V}} + \mathbf{KM}\boldsymbol{V} = \boldsymbol{Q} \tag{2.4}$$

where: $\mathbf{M}$ is the inertia matrix, $\mathbf{K}$ – kinematic relations matrix, $\boldsymbol{V}$ – velocity vector and

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 & 0 & S_z & -S_y \\ 0 & m & 0 & -S_z & 0 & S_x \\ 0 & 0 & m & S_y & -S_x & 0 \\ 0 & -S_z & S_y & I_x & -I_{xy} & -I_{xz} \\ S_z & 0 & -S_x & -I_{yx} & I_y & -I_{yz} \\ -S_y & S_x & 0 & -I_{zx} & -I_{zy} & I_z \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} 0 & -R & Q & 0 & 0 & 0 \\ R & 0 & -P & 0 & 0 & 0 \\ -Q & P & 0 & 0 & 0 & 0 \\ 0 & -W & V & 0 & -R & Q \\ W & 0 & -U & R & 0 & -P \\ -V & U & 0 & -Q & P & 0 \end{bmatrix}$$

$$\boldsymbol{V} = \text{col}\,[U, V, W, P, Q, R]$$

The vector of forces and moments of external forces $\boldsymbol{Q}$ affecting the moving missile is the sum of interactions of the centre, in which it is moving. This vector consists of forces: aerodynamic $\boldsymbol{Q}_a$, gravitational $\boldsymbol{Q}_g$, steering $\boldsymbol{Q}_\delta$ and thrust $\boldsymbol{Q}_T$. The flying missile is steered automatically. The steering is done in two channels: inclination by tilting the height steer by δ_H and deflection by tilting the direction steer by δ_V

$$\boldsymbol{Q} = \boldsymbol{Q}_g + \boldsymbol{Q}_a + \boldsymbol{Q}_\delta + \boldsymbol{Q}_T = \text{col}\,[X, Y, Z, L, M, N] \tag{2.5}$$

where

$$\begin{aligned} X &= -mg\sin\theta + T - \frac{1}{2}\rho S V_0^2 (C_{xa}\cos\beta\cos\alpha + C_{ya}\sin\beta\cos\alpha - C_{za}\sin\alpha) \\ &\quad + X_Q Q + X_{\delta H}\delta_H + X_{\delta V}\delta_V \\ Y &= mg\cos\theta\sin\phi - \frac{1}{2}\rho S V_0^2 (C_{xa}\sin\beta - C_{ya}\sin\beta) + Y_P P + Y_R R + Y_{\delta V}\delta_V \\ Z &= mg\cos\theta\cos\phi - \frac{1}{2}\rho S V_0^2 (C_{xa}\cos\beta\sin\alpha + C_{ya}\sin\beta\sin\alpha + C_{za}\cos\alpha) \\ &\quad + Z_Q Q + Z_{\delta H}\delta_H \\ L &= -\frac{1}{2}\rho S V_0^2 l (C_{mxa}\cos\beta\cos\alpha + C_{mya}\sin\beta\sin\alpha - C_{mza}\sin\alpha) + L_P P \\ &\quad + L_R R + L_{\delta V}\delta_V \\ M &= -mgx_c\cos\theta\cos\phi - \frac{1}{2}\rho S V_0^2 l (C_{mxa}\sin\beta + C_{mza}\cos\beta) + M_Q Q \\ &\quad + M_W W + M_{\delta H}\delta_H \\ N &= mgx_c\cos\theta\sin\phi - \frac{1}{2}\rho S V_0^2 l (C_{mxa}\cos\beta\sin\alpha + C_{mya}\sin\beta\sin\alpha \\ &\quad + C_{mza}\cos\alpha) + N_P P + N_R R + N_{\delta V}\delta_V \end{aligned}$$

while: m – missile mass, T – missile engine thrust vector (Fig. 2), $\rho(H)$ – air density at a given flight altitude, l – characteristic dimension (total length of the missile body), S – area of reference surface (cross-section of rocket body), $V_0 = \sqrt{U^2 + V^2 + W^2}$ – velocity of the missile flight, C_{xa}, C_{ya}, C_{za}, C_{mxa}, C_{mya}, C_{mza} – dimensionless coefficients of aerodynamic component forces, respectively: resistance P_{xa}, lateral P_{ya} and bearing P_{za} as well as the moment of tilting M_{xa}, inclination M_{ya} and deflection M_{za} (Fig. 2), X_Q, Y_P, Y_R, Z_Q, L_P, L_R, M_Q, N_P, N_R – derivatives of aerodynamic forces and moments with respect to components of linear and angular velocities.

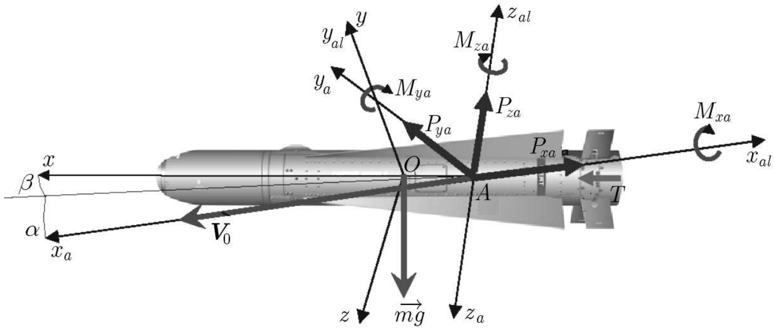

Fig. 2. Forces and moments of forces acting on the missile in flight

3. Layout of gyroscopic self-guidance of missiles

Figures 3 presents a simplified diagram of the layout of gyroscopic self-guidance of missiles onto a ground target emitting infrared radiation (e.g. a tank or a combat vehicle).

Figure 4 shows the general view of the missile used in the scanning and tracking gyroscope, i.e. one which can perform programmed movements while searching for the target and tracking movements after detecting the ground target through an adequate steering mounted on its frames.

The equations expressing dynamics of this kind of gyroscope steered by omitting the moments of inertia of its frames, have the following form

$$\begin{aligned} &J_{gk}\frac{d\omega_{yg_2}}{dt}\cos\vartheta_g + J_{gk}\omega_{gx_2}(\omega_{gz_2} + \omega_{gy_2}\sin\vartheta_g) + M_k\sin\vartheta_g \\ &\quad -J_{go}\Big(\omega_{gz_2} + \frac{d\Phi_g}{dt}\Big)\omega_{gx_2}\cos\vartheta_g + \eta_c\frac{d\psi_g}{dt} = M_c \end{aligned}$$

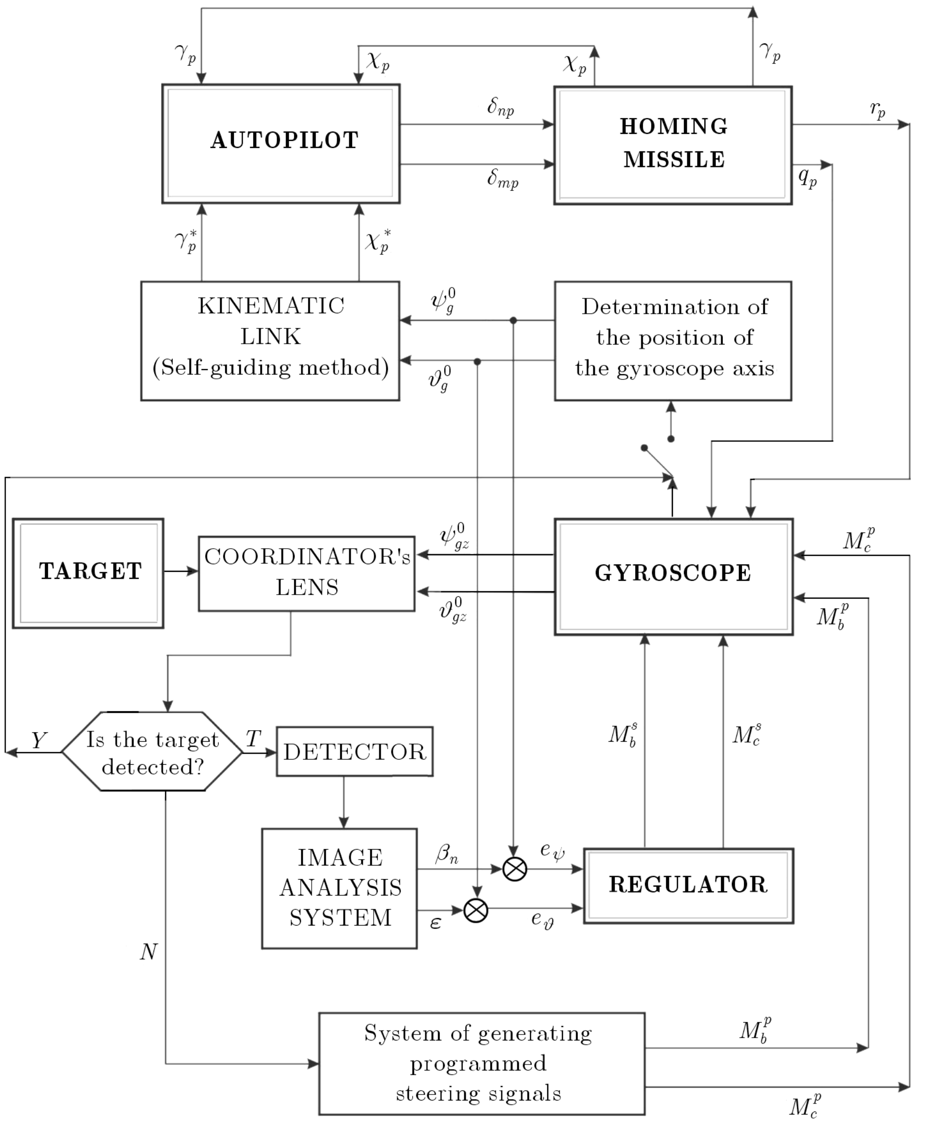

Fig. 3. Diagram of the process of self-guiding a missile on a target

$$J_{gk}\frac{d\omega_{gx_2}}{dt} - J_{gk}\omega_{gy_2}\omega_{gz_2} + J_{go}\Big(\omega_{gz_2} + \frac{d\Phi_g}{dt}\Big)\omega_{gy_2} + \eta_b\frac{d\vartheta_g}{dt} = M_b \qquad (3.1)$$

$$J_{go}\frac{d}{dt}\Big(\omega_{gz_2} + \frac{d\Phi_g}{dt}\Big) = M_k - M_{rk}$$

where

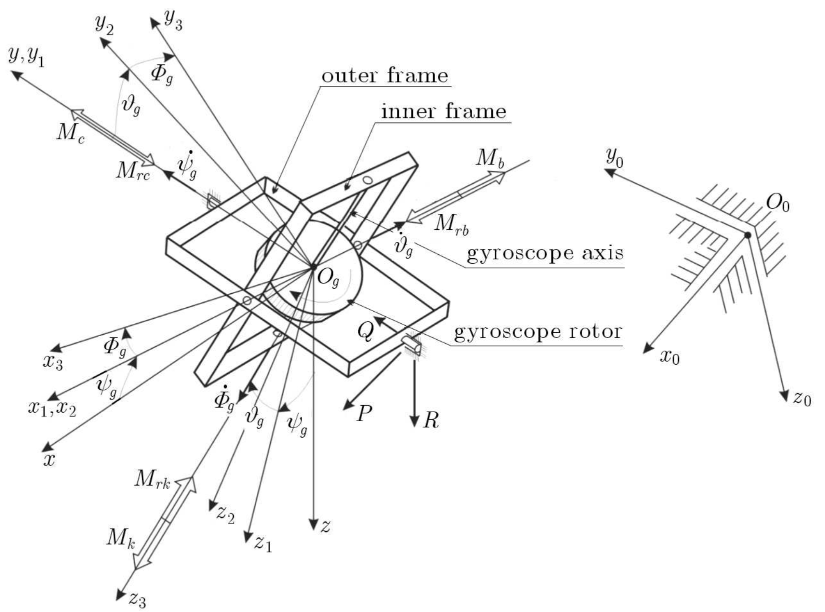

Fig. 4. General view of the gyroscope and assumed systems of coordinates

$$
\begin{aligned}
\omega_{gx_2} &= P\cos\psi_g - R\sin\psi_g + \frac{d\vartheta_g}{dt} \\
\omega_{gy_2} &= (P\cos\psi_g + R\sin\psi_g)\sin\vartheta_g + \Big(\frac{d\psi_g}{dt} + Q\Big)\cos\vartheta_g \\
\omega_{gz_2} &= (P\cos\psi_g + R\sin\psi_g)\cos\vartheta_g - \Big(\frac{d\psi_g}{dt} + Q\Big)\sin\vartheta_g
\end{aligned}
$$

and J_{go}, J_{gk} – moments of inertia of the gyroscope rotor in terms of its longitudinal axis and precession axis, respectively, ϑ_g, ψ_g – angles of rotation of internal and external frames of the gyroscope, respectively, M_k, M_{rk} – torques driving the rotor of the gyroscope and friction forces in the rotor bearing in the frame, respectively.

The steering moments M_b, M_c affecting the gyroscope expressed by Eqs. (3.1), found on the PR board, we shall present as follows

$$
\begin{aligned}
M_b &= \Pi(t_o, t_w)M_b^p(t) + \Pi(t_s, t_k)M_b^s \\
M_c &= \Pi(t_o, t_w)M_c^p(t) + \Pi(t_s, t_k)M_c^s
\end{aligned}
\tag{3.2}
$$

where: $\Pi(\cdot)$ are functions of the rectangular impulse, t_o – time moment of the beginning of spatial scanning, t_w – moment of detecting the target, t_s – moment of the beginning of target tracking, t_k – moment of completing the process of penetration, tracking and laser lighting of the target.

The program steering moments $M_b^p(t)$ and $M_c^p(t)$ put the axis of the gyroscope in the required motion and are found by the method of solving the inverse problem of dynamics (Osiecki and Stefański, 2008)

$$\begin{aligned}
M_b^p(\tau) &= \Pi(\tau_o,\tau_w)\Big[\frac{d^2\vartheta_{gz}}{d\tau^2} + b_b\frac{d\vartheta_{gz}}{d\tau} - \frac{1}{2}\Big(\frac{d\psi_{gz}}{d\tau}\Big)^2 \sin 2\vartheta_{gz} - \frac{d\psi_{gz}}{d\tau}\cos\vartheta_{gz}\Big]\frac{1}{c_b}\\
M_c^p(\tau) &= \Pi(\tau_o,\tau_w)\Big(\frac{d^2\psi_{gz}}{d\tau^2}\cos^2\vartheta_{gz}^p + b_c\frac{d\psi_{gz}}{d\tau} + \frac{d\psi_{gz}}{d\tau}\frac{d\vartheta_{gz}}{d\tau}\sin 2\vartheta_{gz}\\
&\quad + \frac{d\vartheta_{gz}}{d\tau}\cos\vartheta_{gz}\Big)\frac{1}{c_c}
\end{aligned} \tag{3.3}$$

where

$$\tau = t\Omega \qquad \Omega = \frac{J_{go}n_g}{J_{gk}} \qquad c_b = c_c = \frac{1}{J_{gk}\Omega^2} \qquad b_b = b_c = \frac{\eta_b}{J_{gk}\Omega}$$

and ϑ_{gz}, ψ_{gz} are the angles determining the position of the gyroscope axis in space.

For the target tracking status, values of angles determining the given position of the gyroscope axis are equal to

$$\vartheta_{gz} = \varepsilon \qquad \psi_{gz} = \sigma \tag{3.4}$$

where: ε, σ are the angles determining a given position of the target observation line (TOL).

The angles ε, σ are defined by the following relationships constituting kinematic equations TOL (Mishin, 1990)

$$\begin{aligned}
&\frac{dr_e}{dt} = V_{pxe} - V_{cxe} \qquad -\frac{d\varepsilon}{dt}r_e\cos\varepsilon = V_{pye} - V_{cye}\\
&\frac{d\sigma}{dt}r_e = V_{pze} - V_{cze}
\end{aligned} \tag{3.5}$$

where

$$\begin{aligned}
V_{pxe} &= V_0[\cos(\varepsilon - \chi_p)\cos\varepsilon\cos\gamma_p - \sin\varepsilon\sin\gamma_p]\\
V_{pye} &= -V_0\sin(\varepsilon - \chi_p)\cos\gamma_p\\
V_{pze} &= V_0[\cos(\varepsilon - \chi_p)\sin\varepsilon\cos\gamma_p - \cos\varepsilon\sin\gamma_p]\\
V_{cxe} &= V_c[\cos(\varepsilon - \chi_c)\cos\varepsilon\cos\gamma_c - \sin\varepsilon\sin\gamma_c]\\
V_{cye} &= -V_c\sin(\varepsilon - \chi_c)\cos\gamma_c\\
V_{cze} &= V_c[\cos(\varepsilon - \chi_c)\sin\varepsilon\cos\gamma_c - \cos\varepsilon\sin\gamma_c]
\end{aligned}$$

and r_e – distance between the centre of gravity mass of PR and the ground target, V_0, V_c – velocities of PR and the ground target, $\gamma_p = \theta - \alpha$, $\chi_p = \psi - \beta$ – position angles of the PR velocity vector, γ_c, χ_c – position angles of the velocity vector of the ground target.

If angular deviations between the real angles ϑ_g and ψ_g and required angles ϑ_{gz} and ψ_{gz} are denoted as follows

$$e_{\psi} = \psi_g - \psi_{gz} \qquad\qquad e_{\psi} = \psi_g - \psi_{gz} \tag{3.6}$$

then the tracking steering moments of the gyroscope shall be expressed as follows

$$\begin{aligned} M_b^s(\tau) &= \Pi(\tau_s,\tau_k)\Big(\overline{k}_b e_{\vartheta} - \overline{k}_c e_{\psi} + \overline{h}_g \frac{de_{\vartheta}}{d\tau}\Big) \\ M_c^s(\tau) &= \Pi(\tau_s,\tau_k)\Big(\overline{k}_b e_{\psi} + \overline{k}_c e_{\vartheta} + \overline{h}_g \frac{de_{\psi}}{d\tau}\Big) \end{aligned} \tag{3.7}$$

where

$$\overline{k}_b = \frac{k_b}{J_{gk}\varOmega^2} \qquad\qquad \overline{k}_c = \frac{k_c}{J_{gk}\varOmega^2} \qquad\qquad \overline{h}_g = \frac{h_g}{J_{gk}\varOmega}$$

Thus, the steering law for the autopilot, taking into account the dynamics of inclination of steers, shall be expressed as follows

$$\begin{aligned} \frac{d^2\delta_m}{dt^2} + h_{mp}\frac{d\delta_m}{dt} + k_{mp}\delta_m &= k_m(\gamma_p - \gamma_p^*) + h_m\Big(\frac{d\gamma_p}{dt} - \frac{d\gamma_p^*}{dt}\Big) \\ \frac{d^2\delta_n}{dt^2} + h_{np}\frac{d\delta_n}{dt} + k_{np}\delta_n &= k_n(\chi_p - \chi_p^*) + h_m\Big(\frac{d\chi_p}{dt} - \frac{d\chi_p^*}{dt}\Big) \end{aligned} \tag{3.8}$$

where: b_m, b_n are the coefficients of stabilising steers, k_{mp}, k_{np} – coefficients of amplifications of steer drives, h_{mp}, h_{np} – coefficients of suppressions of steer drives, k_m, k_n – coefficients of amplifications of the autopilot regulator, h_m, h_n – coefficients of suppressions of the autopilot regulator.

The required angles of position γ_p^*, χ_p^* of the PR velocity vector are determined by the method of proportional navigation (Koruba, 2001)

$$\frac{d\gamma_p^*}{dt} = a_{\gamma}\frac{d\vartheta_g}{dt} \qquad\qquad \frac{d\chi_p^*}{dt} = a_{\chi}\frac{d\psi_g}{dt} \tag{3.9}$$

where: a_{γ}, a_{χ} are the required coefficients of proportional navigation.

4. Obtained results and final conclusions

The tested model of navigation and the steering of the self-guiding missile describes the fully autonomous motion of the Maverick combat vessel, which is to directly attack and destroy a ground target after being detected and identified.

Figures 5-8 show selected results of digital simulation of the dynamics of the missile during self-guidance on a detected ground target. It was assumed that the missile was launched from an aircraft-carrier moving at a speed of 200 m/s at a height of 400 m. The target was moving along an arc of a circle at the speed of 10m/s. The parameters of the steered gyroscope were as follows

$$J_{gk} = 2.5 \cdot 10^{-4}\,\mathrm{kgm}^2 \qquad J_{go} = 5.0 \cdot 10^{-4}\,\mathrm{kgm}^2$$
$$n_g = 600\,\frac{\mathrm{rad}}{\mathrm{s}} \qquad \eta_b = \eta_c = 0.01\,\frac{\mathrm{Nms}}{\mathrm{rad}}$$

while the coefficients of its regulator had the values

$$k_b = 31.480\,\frac{\mathrm{Nm}}{\mathrm{rad}} \qquad k_c = 2.986\,\frac{\mathrm{Nm}}{\mathrm{rad}} \qquad h_g = 31.525\,\frac{\mathrm{Nms}}{\mathrm{rad}}$$

The coefficients of proportional navigation and parameters of the regulator of PR autopilot were as follows

$$a_\gamma = 3.5 \qquad a_\chi = 3.5 \qquad k_m = 2.703\,\frac{\mathrm{Nm}}{\mathrm{rad}}$$
$$k_n = 11.439\,\frac{\mathrm{Nm}}{\mathrm{rad}} \qquad h_m = 9.887\,\frac{\mathrm{Nms}}{\mathrm{rad}}$$

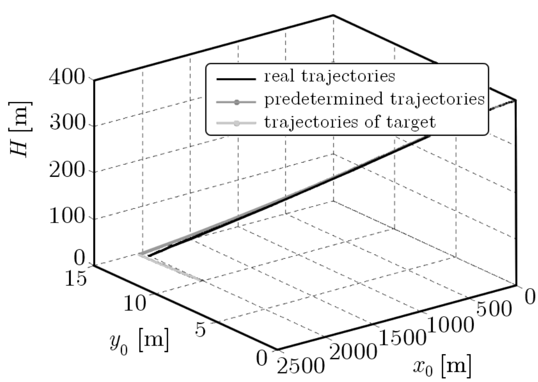

Fig. 5. Spatial trajectory of a self-guiding missile

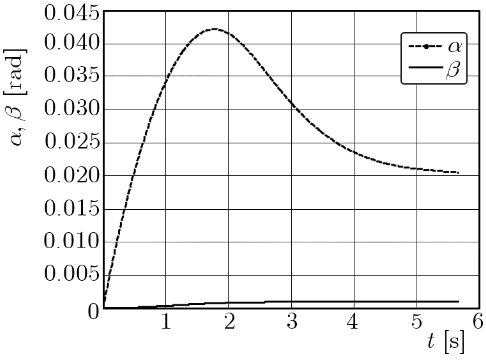

Fig. 6. Change of angles of attack and glide angles in function of time

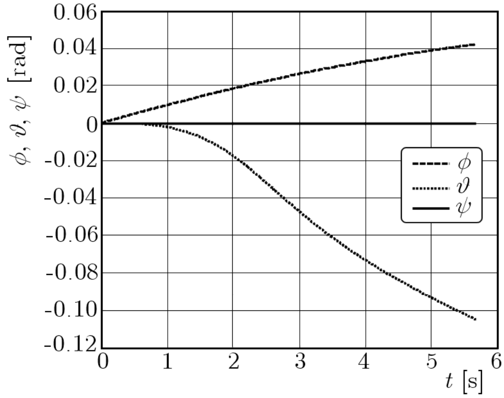

Fig. 7. Angular position of the missile in function of time during guidance

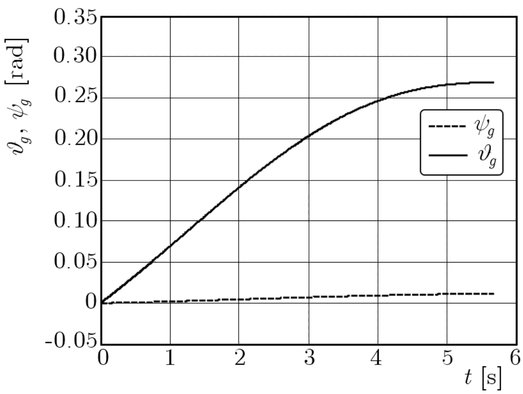

Fig. 8. Angular position of the gyroscope axis in function of time

With the parameters selected as in the above, the target was destroyed in 6 s of the flight.

It should be emphasised that the gyroscope scanning and tracking layout proposed in this paper improves the stability of the system of missile self-guidance and increases resistance to vibrations born from the board of the missile itself.

References

1. Koruba Z., 2001, *Dynamics and Control of a Gyroscope on Board of an Flying Vehicle*, Monographs, Studies, Dissertations No. 25, Kielce University of Technology, Kielce [in Polish]
2. Koruba Z., Ładyżyńska-Kozdraś E., 2010, The dynamic model of combat target homing system of the unmanned aerial vehicle, *Journal of Theoretical and Applied Mechanics*, **48**, 3, 551-566
3. Ładyżyńska-Kozdraś E., 2008, Dynamical analysis of a 3D missile motion under automatic control, [In:] *Mechanika w Lotnictwie ML-XIII 2008*, J. Maryniak (Edit.), PTMTS, Warszawa
4. Ładyżyńska-Kozdraś E., 2009, The control laws having a form of kinematic relations between deviations in the automatic control of a flying object, **Journal of Theoretical and Applied Mechanics**, **47**, 2, 363-381
5. Ładyżyńska-Kozdraś E., Maryniak J., Żyluk A., Cichoń M., 2008, Modeling and numeric simulation of guided aircraft bomb with preset surface target, *Scientific Papers of the Polish Naval Academy*, **XLIX**, 172B, 101-113
6. Mishin V.P., 1990, *Missile Dynamics*, Mashinostroenie, Moskva, pp. 463
7. Osiecki J., Stefański K., 2008, On a method of target detection and tracking used in air defence, **Journal of Theoretical and Applied Mechanics**, **46**, 4, 909-916
8. Żyluk A., 2009, Sensitivity of a bomb to wind turbulence, **Journal of Theoretical and Applied Mechanics**, **47**, 4, 815-828

Model końcowego odcinka nawigacji samonaprowadzającego pocisku rakietowego sterowanego giroskopem

Streszczenie

W pracy zaprezentowano modelowanie dynamiki samonaprowadzającego pocisku rakietowego sterowanego przy użyciu giroskopu. W tego rodzaju pociskach rakietowych atakujących samodzielnie wykryte cele głównym elementem jest samonaprowadzająca głowica, której napęd stanowi giroskop sterowany. W pracy przedstawiona została dynamika i sposób sterowania takiego pocisku. Poprawność opracowanego modelu matematycznego potwierdziła symulacja numeryczna przeprowadzona dla pocisku klasy „Maverick" wyposażonego w giroskop będący elementem wykonawczym skanowania powierzchni ziemi i śledzenia wykrytego na niej celu. Analizie numerycznej poddana została zarówno dynamika giroskopu, jak i pocisku podczas procesu skanowania i śledzenia wykrytego celu. Wyniki przedstawione zostały w postaci graficznej.

Manuscript received July 11, 2011; accepted for print September 12, 2011