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FILTRATION RESISTANCE OF A SYSTEM OF PARALLEL CYLINDERS AT A TRANSVERSE CREEPING FLOW

JAN A. KOŁODZIEJ

Politechnika Poznańska

Abstract

The subject matter of the present considerations is the problem of resistance of a system of cylindrical bars under perpendiculal creeping flow. This system, which has the form of a bundle of parallel bars, is treated as an antisotropic porous medium, the flow through is described by the Darcy equation of filtration. The resistance of filtration is represented as a dimensionless permeability function $F_{\perp}(\varphi)$, where φ is the ratio of the volume of the bars that of the entire bundle.

The paper contains a survey of theoretical and experimental works concerning the problem under considerations.

1. Introduction

The problem of force acting on a single circular cylinder placed in a transverse uniform stream is a standard problem of fluid mechanics. At small Reynolds numbers Navier — Stokes equations governing the flow become Stokes equations {[1], p. 436}. It should be pointed out, however, that Stokes paradox occurs when the fluid flow is perpendicular to the cylinder axis and when the boundary conditions on the cylinder surface and at infinity cannot be satisfied simultaneusly.

Oseen [2] has suggested the linearization of Navier — Stokes equations. This has not led to such paradox. Now, there are many proposals of solving the boundary value problems including Oseen equations concerning the transverse fluid flow uniform at infinity around the circular cylinder {see e. g. references in [3]}.

The problem of filtration resistance of a system of parallel cylinders is also of great importance considering the technical applications. The review of papers regarding the resistance of a system of parallel cylinders at the longitudinal laminar flow is given in [4] emphasizing the resistance dependence on the volume fraction of the solids and on the arrangement of the cylinders.

The purpose of this paper is to make a survey of the theoretical and experimental results given by various authors concerning the resistance of a system of cylinders at the transverse flow. Our considerations will be restricted to small Reynolds numbers.

It should be pointed out that in the case of the relative flow to the infinite system of cylinders when Stokes paradox does not exist, almost all the theoretical considerations regarding the transverse flow in such a system are based on the solutions of Stokes equations.

There are many different methods of representing the resistance of a system of cylinders. Usually a drag coefficient of a single cylinder of this system is given. In this paper a bundle of cylinders is considered as a porous medium, and permeability is presented as a proportionality coefficient occuring in Darcy filtration equations. This permeability presents the above mentioned resistance.

2. A bundle of parallel circular cylinders as an anisotropic porous medium

In some theoretical considerations it is convenient to treat a system of parallel circular cylinders as a porous medium. Such a model, for instance, has been used in describing the phenomena which occur in the production technology of man-made fibres [5], [6] as well as in the theory of fibrous filtres [7, 8]. This enables one to apply an appropriate filtration law for the flow phenomena through a bundle. Since we restrict our considerations to of small Reynolds numbers, then Darcy law governing of the fluid flow through a porous medium can be used {[9], p. 400}.

Taking into account the fact that the flow resistance depends on the flow directions through a bundle, then Darcy law for an anisotropic porous medium should be applied: that is

$$\vec{q} = -\frac{\overline{K}}{\mu}\operatorname{grad} P \tag{1}$$

where $\vec{q}$ is the filtration velocity, P is the pressure, μ is the viscosity of the fluid and $\overline{K}$ is the permeability tensor (the second order tensor).

It has been proved in [10 - 12] that the permeability tensor is a symmetric tensor. Therefore, in a general case of anisotropy and of any selection of the coordinate system, the permeability tensor has six independent components.

In our case, when a bundle of parallel circular cylinders is considered, the filtration properties along the direction perpendicular to the bundle axes are the same and the number of tensor permeability components reduces to two, [4], which can be described by

$$K_{||} = SF_{||}(\varphi) \tag{2}$$

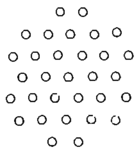
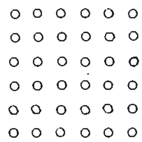
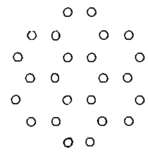
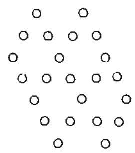
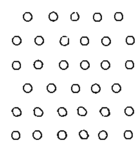
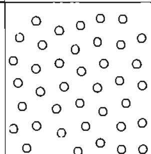
and

$$K_{\perp} = SF_{\perp}(\varphi). \tag{3}$$

In these relations $K_{||}$ and $K_{\perp}$ are the components of the permeability at the parallel and perpendicular fluid flow towards a bundle, respectively, S stands for an area of a cross-section corresponding to one cylinder at the average, $F_{||}(\varphi)$ and $F_{\perp}(\varphi)$ are the nondimensional functions characterizing the permeability at the fluid flow parallel and perpendicular towards a bundle, respectively, and φ represents a volume fraction of the solids. The parameter φ is defined as a ratio of volume of the cylinders to the bulk volume of a system.

For regular arrays of the cylinders the volume fraction φ can be expressed by means of the distance b between the adjoining cylinders and by the radius a of the cylinders. In Table 1 there are formulae of φ, S values and other parametrs for some regular arrays of the cylinders.

Table 1.

	triangular array	square array	hexagonal array	no 4	no. 5	no. 6
						
φ	$0.90690\left(\frac{2a}{b}\right)^2$	$0.785398\left(\frac{2a}{b}\right)^2$	$0.604600\left(\frac{2a}{b}\right)^2$	$0.777301\left(\frac{2a}{b}\right)^2$	$0.841810\left(\frac{2a}{b}\right)^2$	$0.841810\left(\frac{2a}{b}\right)^2$
$\frac{S}{b}$	0.8660	1	1.1299	1.0104	0.9330	0.9330
E	0.06979	0.10004	0.08093	0.04301	0.07311	0.07311

In the present paper the results of other authors regarding the flow resistance at perpendicular flow with respect to the cylinders are shown and compared by means of the nondimensional permeability functions $F_{\perp}(\varphi)$. Since some of the results are given by means of the drag coefficient of a single cylinder of the system, we assume in our calculations that the pressure drop at the fluid flow through the system is balanced by the drag forces of the cylinders.

3. Functions $F_{\perp}(\varphi)$ at any values of φ and at $\left(1-\frac{\varphi}{\varphi_{max}}\right) \ll 1$ for some regular arrays of the cylinders

The theoretical study of the flow of viscous fluid through the regular arrays of the cylinders (especially square and triangular array), continues to attract the interest because of the importance of regular configurations in the design of many heat and mass transfer equipments. The creeping flow around the circular cylinders in the square and triangular arrays was discussed in [13], making use of the analitical-numerical methods. Those considerations are based on two-dimensional Stokes equations, and boundary value problems are formulated for the recurrent cells of these systems (Fig. 1). The governing equations of the fluid flow and a part of the boundary conditions are satisfied exactly by the assumed solutions (boundary conditions on the continuous lines of a boundary in Fig. 1). For the remaining part of the boundary conditions the collocation method is applied i.e. the boudary conditions are satisfied exactly in the finite number of points on the boundary (on the doted lines of the boundary in Fig. 1). The calculations have been done practically for the range of the volume fraction, when $0 < \varphi < \varphi_{max}$. The graphs of the function $F_{\perp}(\varphi)$ obtained on the base of the results from [13] are shown in Fig. 2.

When φ approaches φ_{max} the function $F_{\perp}$ can be calculated in the approximate manner basing on Keller paper [14]; in which the flow through a narrow gap between two adjacent cylinders was analysed. Keller assumed that at amall Reynolds numbers the flow through

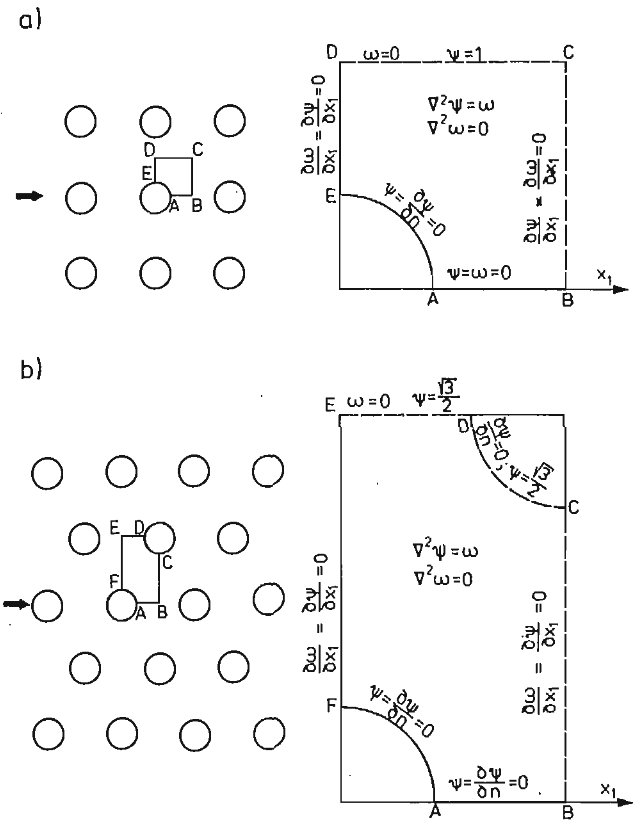


Fig. 1. The formulation of the boundary value problems in recurrent cells for the square and triangular array in the paper [13] where $A\ B\ C\ D$ is the reccurrent cell for the square array, $A\ B\ C\ D\ E\ F$ is the recurrent cell for the traingular array, ω is the vorticity, ψ is the stream function, ∇^2 is the two-dimensional Laplace operator

the narrow gap can be described using the hydrodynamic lubrication theory. His assumption and the results have been tested by Huston [15] numerically. From [14] results that the pressure difference Δp at the perpendicular flow through the gap is

$$\Delta p = \frac{9\pi\mu Q a^{1/2}}{8\sqrt{2}\,(b/2-a)^{5/2}}, \tag{4}$$

where Q is the total volume flux per unit lenght of the gap. Substituting $\Delta p = b\ \mathrm{grad}_\perp P$ and $Q = qb$ to (4) Keller obtained the filter velocity q for the square array in the form

$$q = \frac{8\sqrt{2}\,(b/2-a)^{5/2}}{9\pi\mu a^{1/2}}\ \mathrm{grad}_\perp P. \tag{5}$$

According to this result the nondimensional permeability function for the square array gets the form

$$F_\perp = E\left[1-\left(\frac{\varphi}{\varphi_{max}}\right)^{1/2}\right]^{5/2}, \tag{6}$$

where $E = 2\sqrt{2}/9\pi = 0.100036$.

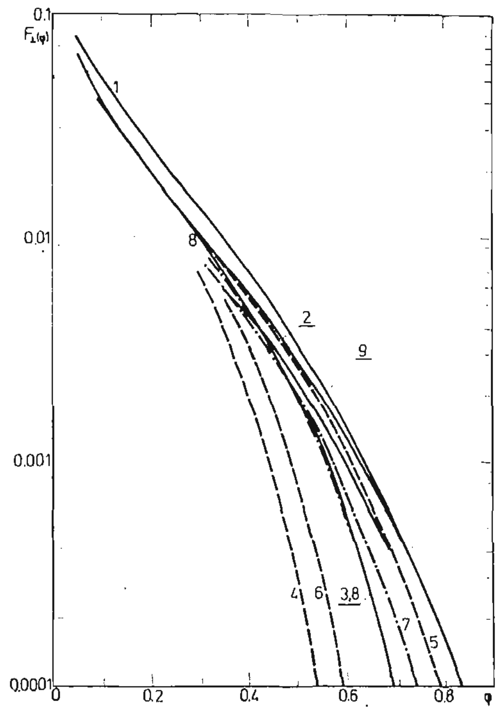


Fig. 2. The theoretical nondimensional permeability function $F_{\perp}$ versus the volume fraction φ; 1 — Kuwabara cell model [17] {eq. (7)}, 2 — Happel cell model [16] {eq. (8)}, 3 — the square array — eq. (6), 4 — the hexogonal array — eg. (6), 5 — the triangular array — eq. (6), 6 — the array no 4 in tab. 1 — eq. (6), 7 — the array no 5 and 6 in tab. 1 — eq. (6), 8 — the square array [13], 9 — the triangular array [13]

Sangani and Acrivos [13] applaying the same concept as Keller probably not knowing his paper, have got the same result for the square array. Moreover, their results give the possibility of determination of value of the constant E in (6) for the triangular array.

In this paper for the remaining arrangments of the cylinders given in Table 1, the constant E in (6) basing on Keller concept is specified. The graphs of functions $F_{\perp}$, according to (6), are shown in Fig. 2.

4. Functions $F_{\perp}(\varphi)$ at $\varphi \ll 1$ for the random and regular arrays of the cylinders

It should be pointed out that for the regular arrays as well as for the random arrangement of the cylinders the case when the volume fraction φ is small ($\varphi \ll 1$) has been exhaustively examined. This arises from the fact that-there is a wide possibility of the use of the asymptotic methods in the theoretical considerations.

For the first time the problem of the resistance of fluid flow through a parallel bundle of the cylinders for their random arrangement was considered by Happel [16] and Kuwabara [17]. They used a cell model in which every cylinder of a radius a was enclosed in thought, by a coaxial cylinder whose radius was determined by the relation $c = a/\sqrt{\varphi}$. In such a ring-shape zone these authors solved the boundary value problems with two-dimensional Stokes equation but they employed different conditions on the outer cell boundary, however. Specifically, Kuwabara assumed the vanishing of vorticity while Happel the vanishing of viscous stresses on the cylinder surface of radius c. That is why the formulae defining the function $F_{\perp}(\varphi)$ on the ground of their results, fiffer in form.

According to Kuwabara results the function $F_{\perp}(\varphi)$ is

$$F_{\perp} = \frac{1}{8\pi}\left(\ln\frac{1}{\varphi} - \frac{3}{2} + 2\varphi - \frac{\varphi^2}{2}\right), \tag{7}$$

while on the base of Happel results we have

$$F_{\perp} = \frac{1}{8\pi}\left(\ln\frac{1}{\varphi} + \frac{\varphi^2-1}{\varphi^2+1}\right). \tag{8}$$

Kirsch and Fuchs [18] tested experimentaly Happel and Kuwabara formulae using the regular triangular array and found a good agreement especially with Kuwabara results. The experiment consisted in taking the photographs of the streamlines near one of the cylinders of a bundle.

Spielman and Goren [7], using the concept of Brinkman [19] proposed a competitive cell model. They suggested that instead of considering the flow throught a system of the cylinders, the flow around a single cylinder could be analysed, taking into account the influence of the remaining cylinders by an additional drag term in Stokes equations. Using the results of these authors we obtain

$$F_{\perp} = \frac{xK_1(x)}{4\pi K_0(x)}, \tag{9}$$

where K_0 and K_1 are modified Bessel functions and x is the function of φ, determined by

$$x^2 = 4\varphi\left[\frac{x^2}{2} + x\frac{K_1(x)}{K_0(x)}\right]. \tag{10}$$

Hasimoto [20] considered, among other things, the problem of the transverse flow through a biperiodical system ogf the cylinders. He made use of Fourier series in order to solve Stokes equations. For the square array his results give

$$F_{\perp} = \frac{1}{8\pi}\left[\ln\frac{1}{\varphi} - 1.476 + 2\varphi + 0(\varphi^2)\right]. \tag{11}$$

Golowin and Lopatin [21] using the theory of the elliptic functions gave the solution describing the transverse flow for two-dimensional regular system of cyrcular cylinders. With the help of Miyagi method [22] they found the exact solution of two-dimensional Stokes equations, but boundary conditions were satisfied in the approximate manner. According to their results functions $F_{\perp}$ are:

— for the square array

$$F_{\perp} = \frac{1}{8\pi}\left[\ln\frac{1}{\varphi} - 1.478 + 2\varphi + 0(\varphi^2)\right], \tag{12}$$

— for the triangular array

$$F_{\perp} = \frac{1}{8\pi}\left[\ln\frac{1}{\varphi} - 1.508 + 2\varphi + 0(\varphi^2)\right]. \tag{13}$$

In [23], using Hasimoto method [20], the force acting on one cylinder of the square and triangular array of the cylinders was calculated preserving the higher order of terms as compared with [20] and [21]. On the ground of these results functions $F_\perp$ have a form:

— for the square array

$$F_\perp = \frac{1}{8\pi}\left[\ln\frac{1}{\varphi} - 1.476 + 2\varphi - 1.774\varphi^2 + 4.076\varphi^3 + 0(\varphi^4)\right], \tag{14}$$

— for the triangular array

$$F_\perp = \frac{1}{8\pi}\left[\ln\frac{1}{\varphi} - 1.49 + 2\varphi - 0.5\varphi^2 + 0(\varphi^4)\right]. \tag{15}$$

In the paper [24] the method of singularities [25 - 26] was adopted to biharmonic equations which has produces some rigorous and reasonably accurate formulae for the square array

$$F_\perp = \frac{1}{8\pi}\left[\ln\frac{1}{\varphi} - 1.47633597 + 2\varphi - 1.77428264\varphi^2 + 4.0770444\varphi^3 - 4.84227402\varphi^4 + 0(\varphi^5)\right], \tag{16}$$

and the triangular array

$$F_\perp = \frac{1}{8\pi}\left[\ln\frac{1}{\varphi} - 1.497504972 + 2\varphi - 0.5\varphi^2 - 0.739137296\varphi^4 + \frac{2.5341450 18\varphi^5}{1 + 1.275793652\varphi} + 0(\varphi^6)\right]. \tag{17}$$

The comparison of the theoretical results $F_\perp(\varphi)$ at $\varphi \ll 1$ for the square and the triangular array proposed by the above mentioned authors with the results accurate at any values φ is given in Tab. 2 and 3. This comparison indicates that the most rigorous formula (16) for the square array agrees with Sangani and Acrivos

Table 2

	triangular array			
φ	[13]	[24] for. (17)	[23] for. (15)	[21] for. (13)
0.05	0.6353240 (−1)*	0.6354171 (−1)	0.6384032 (−1)	0.6317386 (−1)
0.10	0.3974563 (−1)	0.3978988 (−1)	0.4009054 (−1)	0.3957280 (−1)
0.20	0.1955034 (−1)	0.1955205 (−1)	0.2065982 (−1)	0.1995158 (−1)
0.30	0.1033165 (−1)	0.1034246 (−1)	0.1070209 (−1)	0.1177638 (−1)
0.40	0.5383000 (−2)	0.5452870 (−2)	0.5820720 (−2)	0.8287620 (−2)
0.50	0.261643 (−2)	0.289646 (−2)	0.310938 (−2)	0.736677 (−2)
0.60	0.110913 (−2)	0.195553 (−2)	0.162440 (−2)	0.807018 (−2)
0.70	0.36101 (−3)		0.86242 (−3)	
0.80	0.61728 (−4)			
0.85	0.13151 (−4)			

* 0,6353240 (−1) = 0,6353240 $\times 10^{-1}$, etc

Table 3

	square array				
φ	[13]	[21] for. (12)	[20] for. (11)	[23] for. (14)	(24) for. (16)
0.05	0.6426735 (−1)	0.6436752 (−1)	0.6444710 (−1)	0.6429091 (−1)	0.6427752 (−1)
0.10	0.4027386 (−1)	0.4076695 (−1)	0.4084652 (−1)	0.4030285 (−1)	0.4027017 (−1)
0.20	0.1940617 (−1)	0.2114524 (−1)	0.2122482 (−1)	0.1969884 (−1)	0.1937730 (−1)
0.30	0.9718170 (−2)	0.1297005 (−2)	0.1304962 (−1)	0.1107578 (−1)	0.950263 (−2)
0.40	0.458947 (−2)	0.948129 (−2)	0.956086 (−2)	0.862503 (−2)	0.370187 (−2)
0.50	0.187776 (−2)	0.856043 (−2)	0.864001 (−2)	0.112661 (−1)	
0.60	0.56721 (−3)	0.926384 (−2)	0.934341 (−2)		
0.70	0.73964 (−4)	1.108812 (−2)	1.11677 (−2)		
0.75	0.79177 (−5)				

collocation calculations within 0.3% when $\varphi = 0.2$, 4% when $\varphi = 0.3$ and deteriorates rapidly thereafter. The most rigorous formula (17) for the traingular array agrees with Sangani and Acrivos calculations within 0.1% when $\varphi = 0.3$, 1.3% when $\varphi = 0.4$, 10% when $\varphi = 0.5$ and deteriorates rapidly thereafter.

The experimental investigations for small values of φ have been carried out by Sullivan and Hertel [27], Boumstart [28] and Billing [29]. Sullivan and Hertel applied Kozeny-Carman hydraulic radius theory [30] from which one could obtain

$$F_{\perp} = \frac{(1-\varphi)^3}{4\pi p\varphi}, \tag{18}$$

where constant p needs further experimental investigation. Mentioned authors assumed $p = 6$ on basis of their first experimental observations. In futher experiments [31 - 32] Sullivan found out that at small values of φ, p depends on changes of φ and formula (18) loses its mesning. Sullivan results induced desistance from futher application of the hydraulic radius theory to the filtration flow through the system of the cylinders.

In the literature concerning the air filters [33] the following empirical formula is used

$$F_{\perp} = \frac{\varphi}{16\pi\varphi^{1.5}(1+56\varphi^3)} \tag{19}$$

that is the approximation to Dowson experimental results [34] which concern small packing densities for the random cylinder arrangment in a parallel bumdle.

The results of some authors for $F_{\perp}(\varphi)$ at $\varphi \ll 1$ are shown in Fig. 3 the comparison of the theoretical and experimental indicates that experimentally established permeability is greater than the theoretical one. This can be explained by the difficulties in preserving perfectly uniform volume fraction in experiment such uniformity is assumed in the theoretical considerations. Then, in the experiment, greater quantity of the fluid can flow through the regions of less packing density producing greater permeability.

In order to get the theoretical model more approximating to the experimental results, Yu and Soong [8] have proposed a generalization of Happel and Kuwabara model. Contrary to Happel and Kuwabara model in which the distorsion of the flow due to the interaction was replaced concentrically to each single cylindrical rod, Yo and Soong proposed the

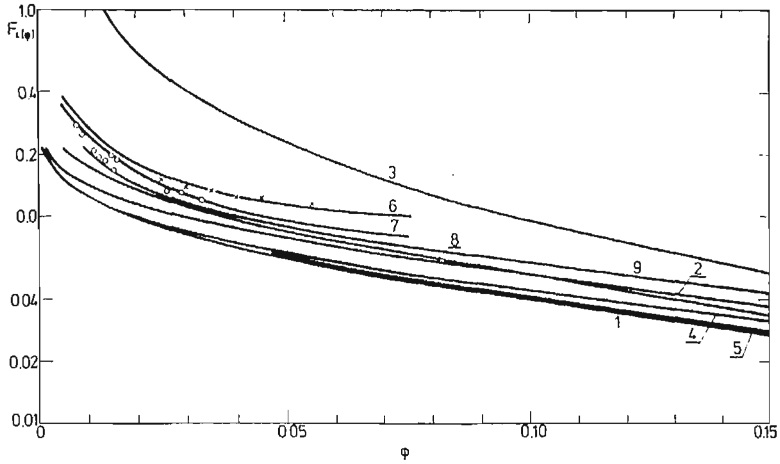


Fig. 3. The theoretical and empirical nondimensional permeability function $F_\perp$ for $\varphi \ll 1$ versus φ; 1 — Kuwabara cell model [17] {eq. (7)}, 2 — Happel cell model [16] {eq. (8)}, 3 — Sulivan and Hertel [27] {eq. (18)}, 4 — the square array obtained by: Golovin and Lopatin [12] {eq. (12)}, Hasimoto {[20] eq. (11)}, Sangani and Acrivos [23] {eq. (14)}, Drummond and Tahir [24] {eq. (16)}, 5 — the triangular array obtained by Golovin and Lopatin [21] {eq. (13)}, Sangani and Acrivos [23] {eq. (15)}, Drummond and Tahir [24] {eq. (17)}, 6 — Happel cell model improved by Yu and Soong [8], 7 — Kuwabara cell model improved by Yu and Soong [8], 8 — Spielman and Goren [7] {eq. (9)}, 9 — the empirical formula of Dowson {eq. (19)}, O — the experiments of Billing [29], × — the experiments of Baumstart [28]

filtration model consisting of the random distribution of the parallel circular cylinders. The average pressure drop through the filtration region was determined by the random cell model of hydrodynamics. Assuming the particular probability distribution they obtained good agrrement of their theoretical results with the experimental ones [28 - 29].

5. Functions $F_\perp(\varphi)$ at values of Knudsen numbers in a transition region

The classical theory of the filtration is built upon the hydrodynamics of the crreeping flow (Stokes equations) and upon the boundary conditions of the velocity field. The boundary conditions refer to both the radial and tangential velocity components. These components are equal zero on the surface of pores. Fibres in some of the types of the air filtres are made to a size which is not far from the mean free path of gas molecules. Then Knudsen number $\mathrm{Kn} = 2\lambda/a$ (where λ is the mean free path of gas molecules), may reach relatively high values, particularly when filtration takes place at the reduced pressures. For $10^{-3} < \mathrm{Kn} < 0.25$, i. e. in the transition region, the calculation of the pressure drop requires the use of the slip boundary condition for the tangential c omponent of the velocity. With the use of this boundary condition and Kuwabara cell model, Pich [35] found out that the nondimensional permeability at $\varphi \ll 1$ gives the form

$$F_\perp = \frac{\ln\dfrac{1}{\varphi} - 1.5 - 0.5\varphi^2 + 1.998\mathrm{Kn}}{8\pi(1+1.996\ \mathrm{Kn})} + \frac{\varphi}{\pi}\left(\ln\frac{1}{\varphi} + 0.5 - 0.5\varphi^2\right). \tag{20}$$

Other, more complicated considerations for the flow at Knudsen numbers in transition region have been done by Spielman and Goren [7] as well as by Yu and Soong [8]. The experimental investigations concerning the transition region have been carried out by Robinson and Franklin [36]. Their results referring to nondimensional permeability of a system of the cylinders at the reduced pressure are shown in Fig. 4.

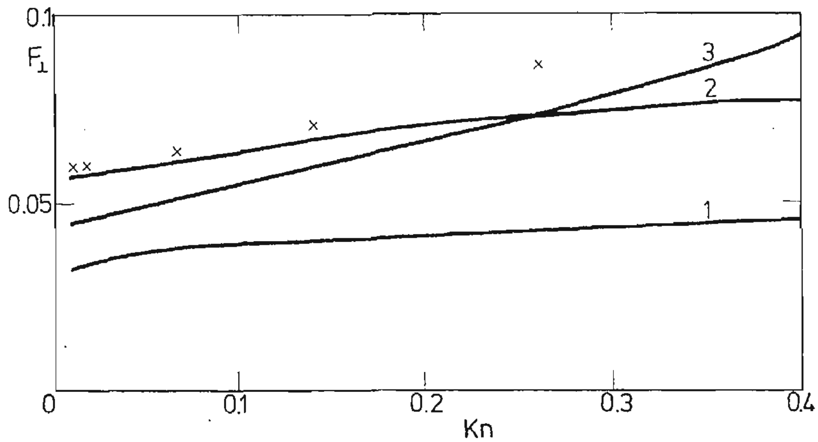


Fig. 4. The theoretical and empirical nondimensional permeability function $F_\perp$ versus Knudsen numbers; 1 — Pich cell model [35] {eq. (20)}, 2 — Yu and Soong [8], 3 — Spielman and Goren [7], × — the experiments of Robinson and Franklin [36]

6. Conclusions

The analysis of the presented results leads to the following general conclusions:

a) The theoretical nondimensional permeability for the transverse flow through a system for the regular and random arrays of the cylinders at very small values of valume fraction weekly depends on the arrangment of the cylinders. This approximately the following formula is valid:

$$F_\perp = \frac{1}{8\pi}\left(\ln\frac{1}{\varphi} - 1.5\right). \tag{21}$$

b) At very low values of the volume fraction the theoretical nondimensional permeability of the transverse flow is two times less than that of the longitudinal flow {see Eq. (24) in [3]}, and it results in

$$F_{\parallel} = 2F_\perp \tag{22}$$

c) For the square array of the cylinders the following approximate formulae are valid with an error less than 5%

$$F_\perp = \frac{1}{8\pi}\left[\ln\frac{1}{\varphi} - 1.47633597 + 2\varphi - 1.77428264\varphi^2 + \right.$$
$$\left. + 4.07770444\varphi^3 - 4.84227402\varphi^4\right], \quad \text{for } 0 < \varphi \leqslant 0.3, \tag{23}$$

$$F_\perp = \frac{2\sqrt{2}}{9\pi}\left[1 - \left(\frac{4\varphi}{\pi}\right)^{1/2}\right]^{5/2}, \quad \text{for } 0.4 \leqslant \varphi < \frac{\pi}{4}. \tag{24}$$

d) For the triangular array of the cylinders the approximate following formulae are valid with an error less than 10%

$$F_{\perp} = \frac{1}{8\pi}\left[\ln\frac{1}{\varphi} - 1.497504972 + 2\varphi - 0.5\varphi^2 - 0.739137296\varphi^4 + \right.$$
$$\left. + \frac{2.534145018\varphi^5}{1+1.275793632\varphi}\right], \quad \text{for } 0 < \varphi \leqslant 0.5, \tag{25}$$

$$F_{\perp} = \frac{4\sqrt{2}}{27\pi}\left[1-\left(\frac{2\sqrt{3}\,\varphi}{\pi}\right)^{1/2}\right]^{5/2}, \quad \text{for } 0.6 \leqslant \varphi < \frac{\pi}{2\sqrt{3}}, \tag{26}$$

e) A "disturbance" of the uniformity of the volume fraction of a system has a large influence on the filtration resistance. This is observed in the experiments, where the filtration resistance is approximately smaller by a half than obtained from the theoretical considerations. This may be due to the difficulties of preserving the uniformity of porosity in the experiments. Therefore the empirical relations are of great importance.

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Резюме

ФИЛЬТРАЦИОННОЕ СОПРОТИВЛЕНИЕ СИСТЕМЫ ПАРАЛЛЕЛЬНЫХ ЦЫЛИНДРОВ ПРИ ПОПЕРЕЧНОМ ПОЛЗУЧИМ ОБТЕКАНИИ

Предметом работы является проблема сопротивления цилиндрических стержней при поперечном ползучим обтекании. Система в виде параллельного пучка стержений, считается анизотропной пористой средой, в которой течение описывается уравнением фильтрации Дарси. Упомянутое сопротивление представлено при помощи безразмерной функции проницаемости $F_{\perp}(\varphi)$, где φ — отношение обьема стержней к общему обьему пука.

В работе приведен обзор теоретических и экспериментальных работ, касающихся сопротивления системы стержней упомянутом обтекании.

Streszczenie

OPÓR FILTRACYJNY UKŁADU RÓWNOLEGŁYCH CYLINDRÓW PRZY POPRZECZNYM OPŁYWIE PEŁZAJĄCYM

Przedmiotem pracy jest zagadnienie oporu układu prętów cylindrycznych przy poprzecznym opływie pełzającym. Układ w postaci równoległej wiązki prętów potraktowano jako anizotropowy ośrodek poro-

waty, w którym przepływ opisany jest równaniem filtracji Darcy. Wspomniany opór przedstawia się przy-pomocy bezwymiarowej funkcji przepuszczalności $F_{\perp}(\varphi)$, gdzie φ jest stosunkiem objętości prętów do objętości całkowitej wiązki.

W pracy dokonano przeglądu prac teoretycznych i doświadczalnych dotyczących zagadnienia oporu układu prętów przy wspomnianym opływie.

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