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EXPERIMENTAL SIMULATION OF ANISOTROPIC DAMAGE

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1. Introducion

Internal cracks developed in materials due to straining have, as a rule, oriented character and do not represent solely sets of scalar voids distributions. A fissurated material becomes anisotropic in its response as well as anisotropy varies in the process of continuing damage. Evolution of continuously regularly distributed cracks in an originally isotropic and homogeneous material changes the overall mechanical properties of the material due to the increase in the oriented damage. A formulation of constitutive relations for a damaged material requires specification of its mechanical characteristics concerning both the stress-strain response and the damage evolution. KACHANOV [1] proposed to formulate the constitutive equation for damaged material as a tensor function including an independent variable called the crack density tensor or damage tensor accounting for the variation of the mechanical properties of the material due to the microcracks developement. According to the definition proposed by RABOTNOV [2] in the case of random distribution of the microcracks the damage tensor is the second rank symmetric tensor. When the distribution of the fissuration appears regular the damage tensor should account both for the variation of the cracked material strength and for the developement of the material anisotropy. Thus the form of the damage tensor depends not only on the microcracks density but also on their arrangement. Some of the attempts to derive the damage tensor and to formulate the constitutive equation were presented by MURAKAMI [3] and BETTEN [4] but due to the lack of the experimental results concerning the anisotropy of the damaged materials the theories available cannot be verified. Thus it seems worthwhile to determine the elastic and plastic characteristics of the material with regular array of the microcracks. Because of the complexity of the suitable analysis of the real damaged materials the special kind of modelling is proposed.

Generally the damage tensor is the function of the stress history but for the given well defined stages of the damage evolution this tensor depends on the actual mechanical properties of the damaged material only. The subject of this study is the experimental simulation of the oriented damage at such stages of the cracks evolution and the analysis of the overall behavior of the model of the cracked material, thus the homogenization of the material response within the continuum mechanics. The attention is purposely given to the experimental side of the question. This results in an experimental homogenization for the materials with internal oriented structure.

To simulate an oriented damage sets of cracks of given length, orientation, arrangement and density were cut out in flat metal specimens. The load then was applied producing an overall uniaxial stress and the material response was recorded regarding the magnitude and direction of the overall strains as well as changes in the fissuration density, orientation and evolution. Such a method of experimental simulation of and oriented damage can be applied both to the elastic and plastic response, although the behavior on the level of a particular cell is evidently non-homogeneous and elastic as well as plastic zones develop in non-homogeneous manner.

2. Experimental technique

The present paper deals in particular with the elastic characteristics of the damaged material when damage pattern and fissuration length are prescribed. To determine the elastic properties for such a material dairly simple uniaxially loaded models can be used. The preliminary tests concerning only one crack length but different crack orientations, presented in [5] enabled us to improve the experimental technique. The aim of the experiments presented in this note is to establish a modification of the material constants of the damaged material when the cracks length and arrangement are variable. The tests were made on the specimens cut out of the sheets of an aluminium alloy PA2. The overall length of the specimens was 400 mm, width 70 mm and thickness 0.7 mm. The cracks arranged in square patterns were cut out in the central part of the specimen 210 mm long by means of a precise punching device. The details of the cracks geometry is presented in Fig. 1. The crack width is 1 mm and their length varies from 2 to 7 mm. The pitch of the square pattern of cracks equals to 10 mm thus the dimensionless crack length $\lambda = l/P$ was variable ranging from 0.2 to 0.7. Two different crack arrangements thus two types of an internal orientation of the material tested were considered, namely either the lon-

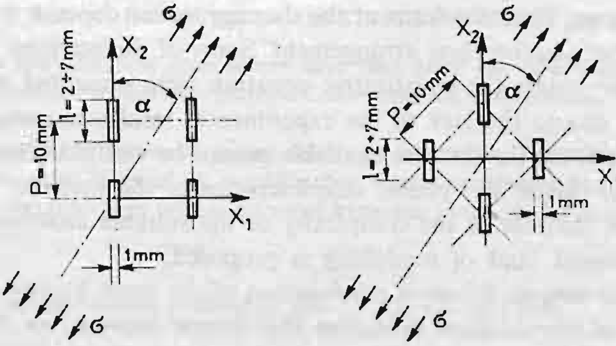

Fig. 1. Cracks arrangements.

gitudinal axis of the cracks coincides with the pitch or it makes an angle $\pi/4$ with the pitch. The specimens were cut out so as to make the overall principal stress direction and the cracks orientation variable. The specimens were subjected to axial loading but for various directions with respect to the symmetry axes of the crack pattern. The direction of loading was defined by the angles $\alpha = 0, \pi/12, \pi/6, \pi/4, \pi/3, 5\pi/12$ and $\pi/2$. In this way the oriented character of the induced damage is accounted for.

The specimens were uniaxially loaded in the testing machine and the longitudinal and lateral deformations were measured during the loading process. The strains within an elastic range were measured by means of the electric strain gauges 50 mm long. Then all the specimens were loaded to fracture and large plastic deformations were recorded employing the mechanical strain gauges. This furnished some information concerning the plasticity and fracture of the models of the damaged materials.

3. Results of the experiments

3.1. Elastic range. The overall mechanical response of the materials with the oriented damage shown in Fig. 1 corresponds to that observed for an orthotropic solid thus their elastic characteristics is described by nine material constants. According to the nomenclature employed in [6] these constants include three Young's moduli E_1, E_2, E_3 and three Poison ratios ν_{21}, ν_{32}, ν_{13} determined for loading in the directions of the Cartesian coordinate system axes x_1, x_2, x_3 and three shear moduli G_{12}, G_{23}, G_{31}. The axes x_1 and x_2 are shown in Fig. 1 and the axis x_3 is perpendicular to the surface of the specimen. As the specimens were uniaxially loaded only some of those constants can be determined employing the results of this experiments.

Restricting the analysis to the plane state of the stress four constants E_1, E_2, ν_{21} and G_{12} must be determined. The three first constants and additionally Poison ratio ν_{12} were

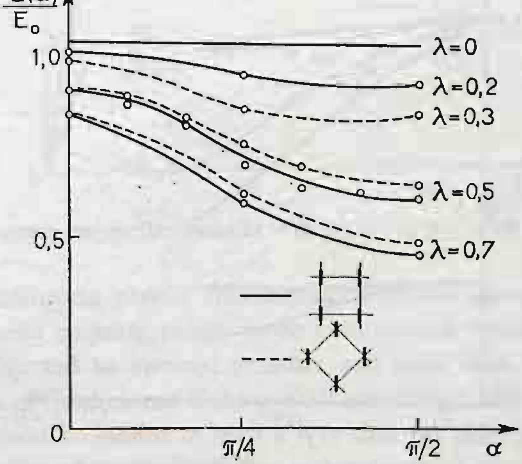

Fig. 2. Diagrams of the function $E(\alpha)$.

determined directly from the results obtained for the specimens subjected to axial load at the directions defined by the angles $\alpha = 0$ and $\pi/2$. The shear modulus G_{12} was calculated employing the effective Young's modulus measured for the specimen loaded in the direction inclined at the arbitrary angle α with respect to cracks orientation. The diagrams of the function $E(\alpha)$ for various crack length and orientation is shown in Fig. 2. The modification of the elastic constants for increasing crack length presents Fig. 3. The values of the Young's modulus E_3 shown in Fig. 3 were calculated from the relation $E_3 = \mu E$ where $\mu = 1 - \lambda/10$ is the reduction of the net area in the direction x_3 for the cracked material and $E = 67700$ MPa is the Young's modulus for the original material.

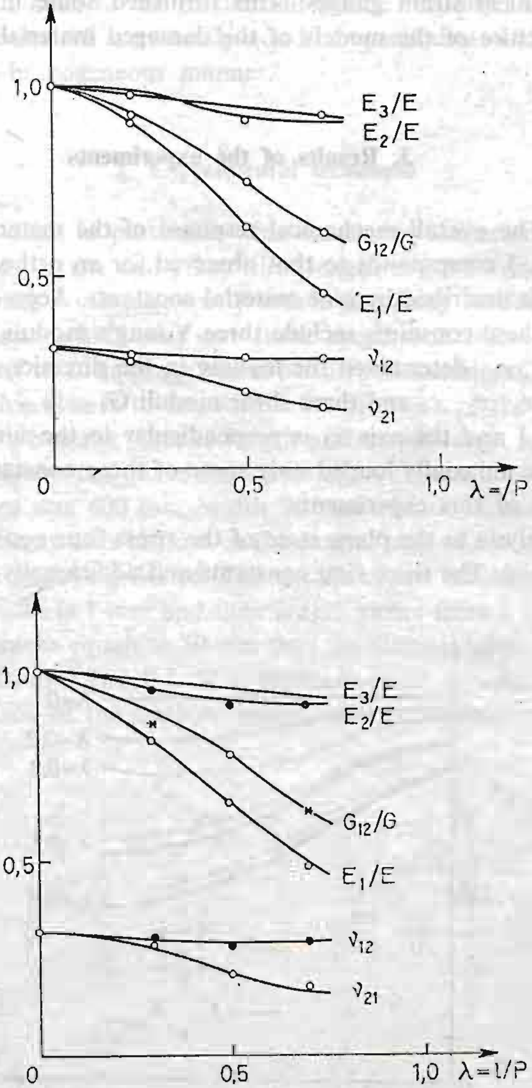

Fig. 3. Elastic constants versus the dimensionless crack length: a) cracks in the pitch direction, b) cracks in diagonal direction.

3.2. Plastic range. The plastic characteristics of the analysed models of the damaged materials includes six constants: three uniaxial yield stresses X_{11}, X_{22}, X_{33} for loading in the directions x_1, x_2, x_3 and three shear yield stresses X_{12}, X_{23}, X_{31}. The first two constants X_{11} and X_{22} can be easily determined employing the stress-strain curves for the specimens tested. These curves for various loading orientations and various dimensionless crack lengths are shown in Fig. 4 and 5. The results obtained show that the minimal strength of the cracked material models does not correspond to the loading direction defined by the angle $\alpha = \pi/2$. This is distincly shown in Fig. 6 where the diagrams of the uniaxial yield stress versus the angle α are presented. The conventional uniaxial yield streses were determined as the stress corresponding to the permanent strain equal 0.1%. The modification of the uniaxial yield stresses X_{11}, X_{22} and X_{33} for increasing crack length is shown in Fig. 7. The yield stress X_{33} was calculated similarly as E_3 from the relation $X_{33} = \mu\sigma_0$, where $\sigma_0 = 132$ MPa is the uniaxial yield stress for the original material without cracks.

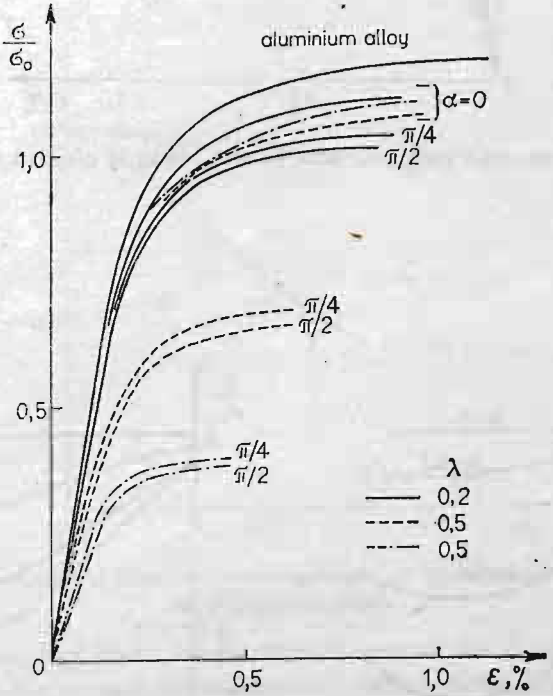

Fig. 4. Stress-strain curves for material with cracks in the pitch direction.

Determining the complete plastic characteristics of the damaged material even for the plane state of stress requires much more complicated experiments employing the tubular specimens subjected to internal pressure and axial load. The specimens should possessa given pattern of fissures and if the overall anisotropic response has to be determined the specimens should be loaded in such a way that the principal stress directions are inclined at various angles with respect to the symmetry axes of the material. Such expe-

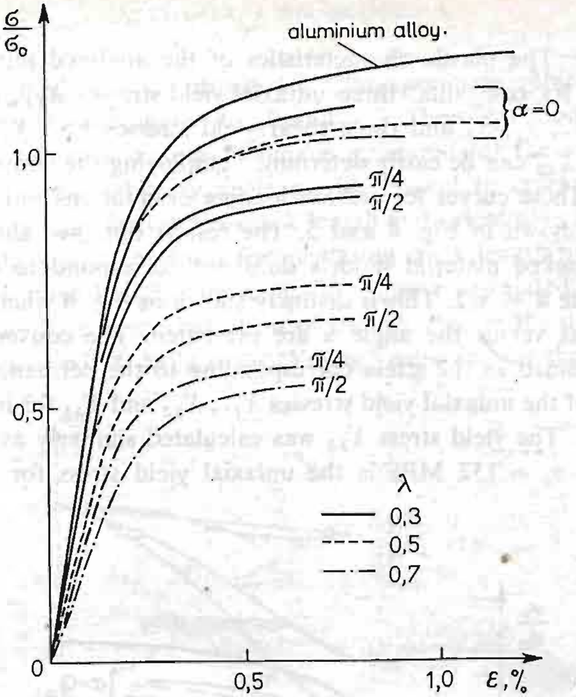

Fig. 5. Stress-strain curves for material with cracks in diagonal direction.

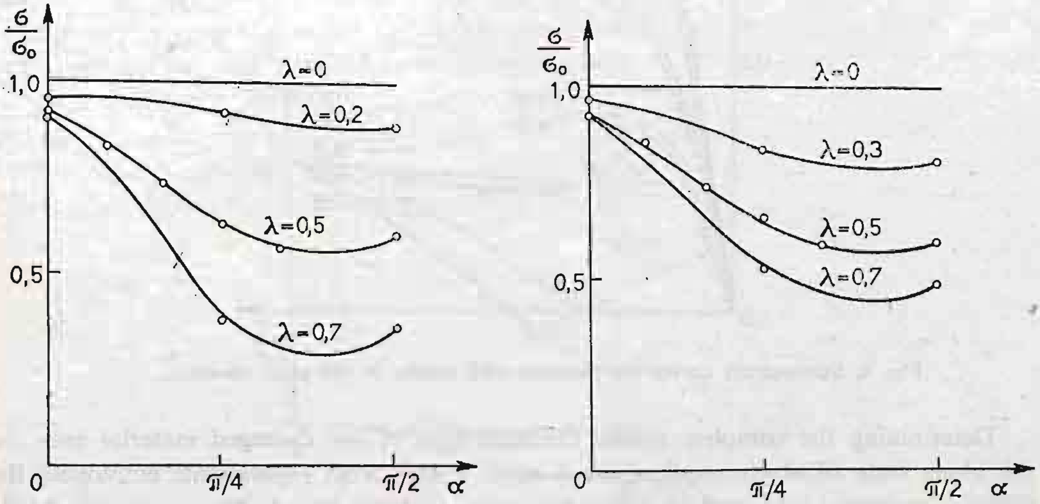

Fig. 6. Uniaxial yield stress versus the angle α: a) cracks in the pitch direction, b) cracks in diagonal direction.

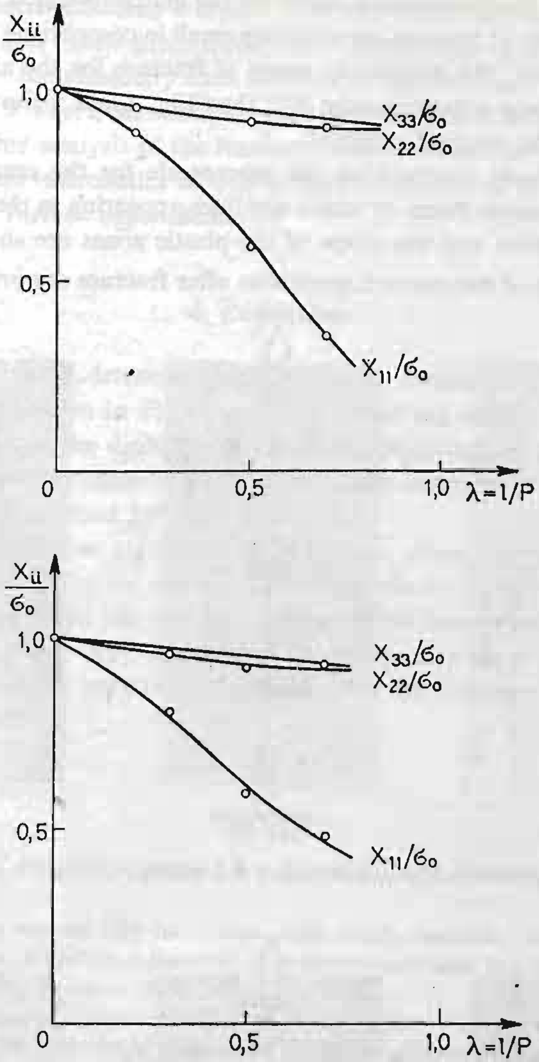

Fig. 7. Plastic constants versus the dimensionless crack length: a) cracks in the pitch direction, b) cracks in diagonal direction.

riments have not been performed for the specific case of the damaged material models but for this purpose can be used the experimental technique established when the plastic anisotropy of the perforated materials was analysed [7, 8]. When carrying the experiments as was described in [7] the material constants as well as the effective yield surfaces can be determined for various loading orientations with respect to the symmetry axes of the material.

3.3. Fracture of the specimens. All the models of the cracked materials were loaded to fracture what enabled to obtain some information concerning its behavior when large plastic strain occur in the plastic zones developed between the cracks. It is seen from

Fig. 4 and 5 presenting the stress-strain curves for the materials tested that the overall homogenized plastic strains at fracture are relatively small in comparison with those measured for the original material. The permanent strain at fracture for the aluminium alloy PA2 measured in this tests was approximately 10% thus Fig. 4 and 5 show only a part of the respective curve for the original material.

The relative brittleness observed on the macroscale for the cracked material is the result of very narrow plastic zones or rather slip lines appearing in the cells of the material structure. The localization and the shape of the plastic zones are shown in Fig. 8 and 9 where the photographs of the selected specimens after fracture are presented. Considering

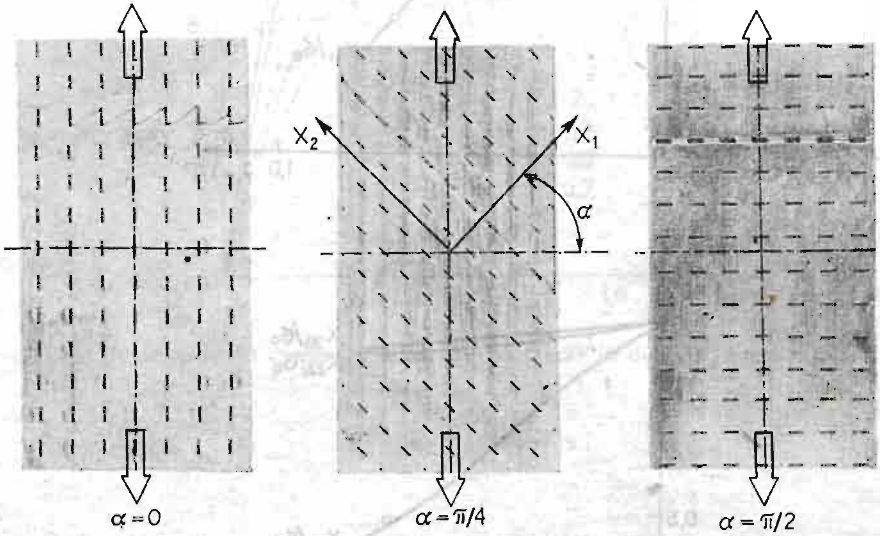

Fig. 8. Specimens after fracture (λ = 0.5 cracks in the pitch direction).

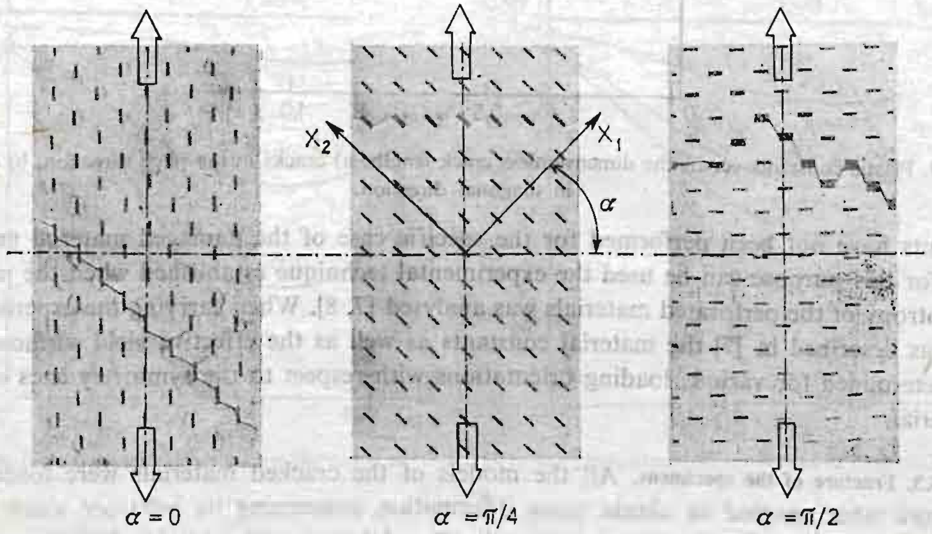

Fig. 9. Specimens after fracture (λ = 0.5, cracks in diagonal direction).

the problem on the macroscale only it is seen from Fig. 8 that the rupture of the specimens with the cracks arranged in the pitch direction is regular and occures for all the loading orientations in one of the symmetry axes of the material structure. These regularities are not observed in Fig. 9 where the models with cracks arranged in the diagonal direction are shown. The detailed analysis of the fracture mechanism for the models of the cracked materials on macro and microscales as well as the formulation of the suitable criterion of the fracture requires further experiments.

4. Conclusions

The experiments enable determining the material constants for the models of the damaged materials. It is seen in Fig. 3 and Fig. 7 that the elastic and plastic constants determined for loading in the direction of the axes x_2 and x_3 are nearly identical. Thus the materials tested can be considered as the transversaly isotropic solids with the isotropic properties in the plane defined by axes x_1 and x_3.

The mechanical properties in the elastic range are almost the same for both crack arrangements what means that the overall elastic response for given crack pattern depends mainly on the reduction of the net area of the material but not on the cracks arrangement. As it is seen in Fig. 6 and 7 this conclusion does not concern the plastic range where the yield stresses determined for both cracks arrangements are different especially for increasing crack length.

References

1. M. KACHANOV, *Continuum model of medium with cracks*, Brown Univ., Porous Media Series, Rep. no. 16, 1972.
2. Y. N. RABOTNOV, *Creep rupture*, 12th Int. Congr. Appl. Mech. Stanford, 1968. 342 - 349.
3. S. MURAKAMI, N. OHNO, *A continuum theory of creep and creep damage*, 3rd IUTAM Symp., Creep in Structures, Leicester 1980, Springer, Berlin, 1981, 422 - 443.
4. J. BETTEN, *Damage tensors in continuum mechanics*, J. Mec. Theor. Appl., 1, 2 (1983).
5. A. LITEWKA, A. SAWCZUK, *Experimental evaluation of the overall anisotropic material response at continuous damage*, in: „Mechanic of material behavior. The D. C. Drucker anniversary volume", G. J. Dvorak and R, T. Shield, ed., Elsevier Sci. Publ. Co., 1983, in press.
6. S. G. LEKHNICKIJ, *Theory of elasticity for anisotropic solid*, Nauka, Moscow 1977, (in Russian).
7. A. LITEWKA, *Plastic anisotropy of perforated materials*, Rozprawy 111, Polit. Pozn., Poznań 1980, (in Polish).
8. A. LITEWKA, A. SAWCZUK, *A yield criterion for perforated sheets*, Ing.-Archiv, 50 (1981), 393 - 400

Резюме

ЭКСПЕРИМЕНТАЛЬНОЕ МОДЕЛИРОВАНИЕ МАТЕРИАЛОВ С ТРЕЩИНАМИ

В работе представляется моделирование материалов с регулярно расположенными трещинами и экспериментальную технику для определения упругих и пластических постоянных. Получено зависимости постоянных от длины трещин для их двух конфигураций. Представляются также некоторые результаты относительно разрушения образцов материалов с трещинами.

Streszczenie

DOŚWIADCZALNA SYMULACJA ANIZOTROPOWEGO USZKODZENIA MATERIAŁU

W pracy przedstawiony został sposób modelowania materiałów z regularnie rozmieszczonymi zarysowaniami oraz omówiona jest technika badań mająca na celu wyznaczenie stałych materiałowych w zakresie sprężystym i plastycznym. W wyniku badań otrzymano zależności stałych od długości szczelin dla ich dwóch konfiguracji. Przedstawione zostały również pewne wyniki dotyczące zniszczenia modeli materiałów z uszkodzeniami.

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