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INTEGRAL FATIGUE CRITERIA EVALUATION FOR LIFE ESTIMATION UNDER UNIAXIAL COMBINED PROPORTIONAL AND NON-PROPORTIONAL LOADINGS

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The paper presents a review and verification of integral fatigue criteria. The review signals the key assumptions and criteria structure elements. The verification has been developed drawing on the experimental data reported in literature containing fatigue life for uniaxial, combined proportional and non-proportional loads. The verification involves a comparison of computational fatigue life with the experimental one. To determine the quality of the results generated, statistical parameters were used. As a result of the analysis the best and the worst criteria were pointed to.

Key words: multiaxial fatigue, fatigue life, fatigue criteria, integral approach

1. Introduction

A continuous attempt at cutting down machinery manufacturing and operation costs can be seen in the changes in the engineering design strategy from *Infinite-Life Design* through *Safe-Life Design to Damage-Tolerant Design*. The need to minimize the costs results in successive design strategies, with more and more precise calculation models at their disposal, demonstrating lower and lower safety coefficient values. Bearing that in mind, a change in the machinery design strategy can trigger structure damage not found earlier. It surely concerns the effects of the non-proportional fatigue load. What is characteristic for that kind of load is rotation of the main axes of stresses and deformations throughout the fatigue process. The rotation of the main axes activates many slip systems and can have an essential effect on fatigue properties. Depending on the material type and the degree of load non-proportionality, this type of forced behaviour can result in even a 10-fold decrease in fatigue life (Ellyin *et al.*, 1991; Socie, 1987) and a 25% decrease in fatigue limit (McDiarmid, 1987; Nishara and Kawamoto, 1945).

It is assumed that the right approach to defining the fatigue criteria under non-proportional load conditions can be the integral approach (Weber *et al.*, 2004). It is based on the assumption that for the right fatigue behaviour evaluation it is necessary to integrate the value of the damage parameter in all the planes going through the material point considered.

The aim of this paper is to evaluate the possibility to evaluate fatigue life with the use of fatigue criteria. The analysis was made applying the three most frequent integral criteria: the Zenner criterion (Zenner, 1983; Zenner *et al.*, 2000) and the two Papadopoulos criteria (Papadopoulos, 1994, 2001). The results were compared with the McDiarmid fatigue criterion (McDiarmid, 1992), based on the competitive to the integral approach to critical plane and, commonly applied in many fields of material fatigue, namely the Huber-Mises-Hencky criterion.

Interestingly, there are many reports offering the analysis or computational verification of fatigue criteria. The most essential reports of that type include e.g., the report by Garud (1981) with an extensive description of the computational models developed until 1981 and the paper by You and Lee (1996), with a presentation of the criteria developed 1980 through 1995. There are also papers available on specific groups of criteria, e.g. the reports by Macha and Sonsino (1999) on the energy criteria and the report by Karolczuk and Macha (2005), being a discussion of criteria based on the critical plane idea. A high study value is provided by the comparative studies of multiaxial criteria including their computation verification, e.g. the paper by Papadopoulos *et al.* (1997), Wang and Yao (2004), Niesłony and Sonsino (2008), Walat *et al.* (2012) as well as by Łagoda and Ogonowski (2005). None of the above studies, however, focuses on integral criteria.

The criteria covered by this analysis have been verified drawing on the results of the experimental tests of 7075-T651 aluminium alloy (Mamiya *et al.*, 2011), 1045 steel – for the data reported in McDiarmid (1992) as well as Verreman and Guo (2007), and the tests of X2CrNiMo17-12-2 steel (Skibicki *et al.*, 2012). The types of materials have been selected in terms of their various sensitivity to non-proportional load; the lowest value for aluminium, average for carbon steels and the highest value for austenitic steels (Socie and Marquis, 2000).

The experimental data derived from those papers provide fatigue life values for sinusoidally variable loads: uniaxial, namely tensile-compressive (marked with R) as well as torsion (S), proportional combined loads, namely compliant at the phase of tensile-compressive and torsion (P) and non-proportional combined loads obtained as a result of a simultaneous tensile-compressive and torsion with the phase shift equal $90°$ (N). For combined loads P and N, the ratio of the amplitudes of shear to normal stress is an important load-defining parameter

$$\lambda = \frac{\tau_{xya}}{\sigma_{xa}} \tag{1.1}$$

Further in this paper, a description of the criteria analysed, the method of analysis of the calculation results, analysis of the load results and conclusions are to be found.

2. Description of the criteria analysed

2.1. McDiarmid criterion

The McDiarmid criterion involved the use of the critical plane approach. In the case of that criterion, it is the plane determined by the tangent stress of the highest value τ_{max}. To calculate the limit state, besides τ_{max}, the effect of normal stress in the same plane σ_{max} is considered (McDiarmid, 1992). The mathematical criterion can take the following form

$$\frac{\tau_{max}}{\tau_{af\,A,B}} + \frac{\sigma_{max}}{2\sigma_u} = 1 \tag{2.1}$$

where $\tau_{af\,A}$ and $\tau_{af\,B}$ are torsion fatigue limits, for the case of an increase in cracking type A or B (Socie and Marquis, 2000), and σ_u is a monotonic tensile strength. By transforming formula (2.1), we obtain a relationship defining the equivalent stress

$$\sigma_{MD} = \tau_{max} + k\sigma_{max} \leqslant \tau_{af\,A,B} \tag{2.2}$$

where

$$k = \frac{\tau_{af\,A,B}}{2\sigma_u} \tag{2.3}$$

2.2. Criterion according to Huber-Mises-Hencky

The criterion according to the hypothesis by Huber-Mises-Hencky (abbreviated to HMH) for fatigue loads can be given as follows

$$\sigma_{HMH} = \sqrt{3J_2} \leqslant \sigma_{af} \tag{2.4}$$

where: σ_{HMH} is the value of equivalent stress, J_2 – the second invariant of the deviator of stress state, and σ_{af} – tensile-compressive fatigue limit. For axial load and torsion, J_2 is expressed by the formula

$$J_2 = \frac{1}{3}\sigma_x^2 + \tau_{xy}^2 \tag{2.5}$$

where: σ_x and τ_{xy} are sinusoidally variable patterns of normal and shear stresses, respectively. In this paper, the criterion has been used to calculate the equivalent stress in two ways. The first approach assumes that the parameters representing the cycle of fatigue load are amplitudes of sinusoidal patterns, and then the equivalent stress can be calculated as follows

$$\sigma_{HMH}^a = \sqrt{\sigma_{xa}^2 + 3\tau_{xya}^2} \tag{2.6}$$

The second approach involves the occurrence of the in-phase displacement between load components, and so the mathematical formula expresses the cycle-maximum value of the equivalent stress

$$\sigma_{HMH}^{max} = \max_t\Big(\sqrt{\sigma_x^2 + 3\tau_{xy}^2}\Big) \tag{2.7}$$

Among a few physical interpretations of the second invariant of deviator J_2, there is an integral interpretation proposed by Novozhilow (in Zenner *et al.*, 2000). It equates J_2 with the root mean square of tangent stresses $\tau_{\gamma\varphi}$ calculated for all the possible planes passing through the neighbourhood of the point considered (Fig. 1). Using that interpretation the idea of integral criteria presented further in this paper is given with the HMH criterion as an example.

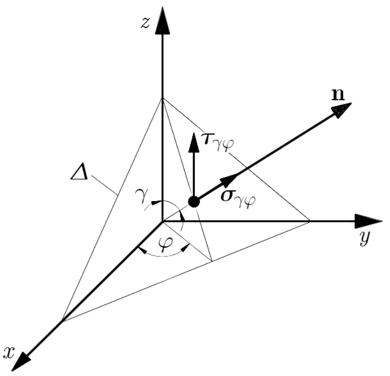

Fig. 1. Tangent stress in the plane

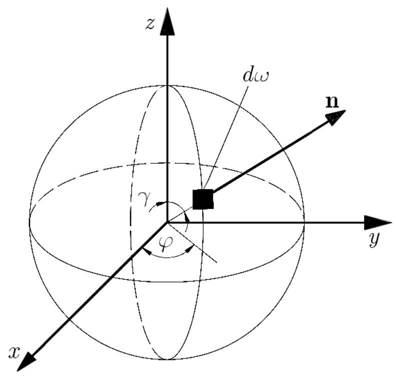

Fig. 2. Coordinates of the normal line in the spherical coordinate system

For the purpose of integration, it is convenient to define the position of plane Δ as tangent to the sphere with unitary radius. In the contact point of the plane and the sphere, there is found a unitary normal vector $\mathbf{n}$, the direction and the sense of which in the spherical system are described by angles φ and γ (Fig. 2).

The square of the root mean square of all tangent stresses can be expressed as (Zenner *et al.*, 2000)

$$\tau_{rms}^2 = \frac{1}{\Omega}\int\limits_{\Omega} \tau_{\gamma\varphi}^2 \, d\Omega \tag{2.8}$$

where $\tau_{\gamma\varphi}$ is the tangent stress, Ω – unitary-radius sphere surface area

$$\Omega = 4\pi \tag{2.9}$$

and $d\Omega$ is an elementary plane according to the following formula

$$d\Omega = \sin\gamma \, d\varphi \, d\gamma \tag{2.10}$$

Substituting (2.9) and (2.10) to (2.8), we receive

$$\tau_{rms} = \sqrt{\frac{1}{4\pi} \int\limits_{\gamma=0}^{\pi} \int\limits_{\varphi=0}^{2\pi} \tau_{\gamma\varphi}^2 \sin\gamma \, d\varphi \, d\gamma} \tag{2.11}$$

In the case of the state of stress in two dimensions, the square of the tangent stress in the plane, the position of which is determined in the spherical coordinate system, is defined by the formula (Zenner and Richter, 1977)

$$\begin{aligned} \tau_{\gamma\varphi}^2 &= \sin^2\gamma[(\sigma_x^2 + \tau_{xy}^2)\cos^2\varphi + \tau_{xy}^2 \sin^2\varphi + 2\sigma_x\tau_{xy}\sin\varphi\cos\varphi] \\ &\quad - \sin^4\gamma[\sigma_x^2\cos^4\varphi + 4\sigma_x\tau_{xy}\sin\varphi\cos^3\varphi + 4\tau_{xy}^2\sin^2\varphi\cos^2\varphi] \end{aligned} \tag{2.12}$$

Having substituted $\tau_{\gamma\varphi}^2$ according to (2.12) to formula (2.11) and integrated, the following is obtained

$$\tau_{rms} = \sqrt{\frac{2}{15}(\sigma_x^2 + 3\tau_{xy}^2)} = \sqrt{\frac{2}{15}}\sigma_{HMH} \tag{2.13}$$

It can be noted that the term in round brackets is the square of the equivalent stress according to the HMH hypothesis for the state of stress in two dimensions. A comparison of equations (2.11) and (2.13) provides

$$\sqrt{\frac{2}{15}}\sigma_{HMH} = \sqrt{\frac{1}{4\pi} \int\limits_{\gamma=0}^{\pi} \int\limits_{\varphi=0}^{2\pi} \tau_{\gamma\varphi}^2 \sin\gamma \, d\varphi \, d\gamma} \tag{2.14}$$

After transformations, we obtain a formula for the integral form of the HMH criterion

$$\sigma_{HMH} = \sqrt{\frac{15}{8\pi} \int\limits_{\gamma=0}^{\pi} \int\limits_{\varphi=0}^{2\pi} \tau_{\gamma\varphi}^2 \sin\gamma \, d\varphi \, d\gamma} \leqslant \sigma_{af} \tag{2.15}$$

2.3. Zenner criterion

The general form of the Zenner criterion is identical with notation (2.15). Zenner, however, considers the observation that besides the tangent stress, the fatigue life of the material is also affected by normal stress (Zenner, 1983). The author factors in that fact by generalising quantity $\tau_{\gamma\varphi}$ in a form of

$$\tau_{\gamma\varphi} = a\tau_{\gamma\varphi a}^2 + b\sigma_{\gamma\varphi a}^2 \tag{2.16}$$

where the coefficients of the effect of the tangent stress $\tau_{\gamma\varphi}$ and normal stress $\sigma_{\gamma\varphi}$ can be calculated as

$$a = \frac{1}{5}\Big[3\Big(\frac{\sigma_{af}}{\tau_{af}}\Big)^2 - 4\Big] \qquad\qquad b = \frac{2}{5}\Big[3 - \Big(\frac{\sigma_{af}}{\tau_{af}}\Big)^2\Big] \tag{2.17}$$

For the purpose of this paper, the effect of mean stress values, which are also considered in the Zenner criterion, has been disregarded. Thanks to coefficients a and b, the criterion can be applied for a greater group of materials. The HMH criterion is applied in the case of materials for which $\tau_{af}/\sigma_{af} = 1/\sqrt{3}$, whereas the Zenner criterion can be used for ductile materials for which the ratio of fatigue limits falls within the range $0.5 < \tau_{af}/\sigma_{af} < 0.8$.

Finally, the mathematical formula describing the equivalent stress according to Zenner assumes the form of (Karolczuk and Macha, 2005; Mamiya *et al.*, 2011)

$$\sigma_Z = \sqrt{\frac{15}{/8\pi}\int\limits_{\gamma=0}^{\pi}\int\limits_{\varphi=0}^{2\pi}(a\tau_{\gamma\varphi a}^2 + b\sigma_{\gamma\varphi a}^2)\sin\gamma\,d\gamma\,d\varphi} \leqslant \sigma_{af} \tag{2.18}$$

2.4. Papadopoulos criterion 1 (1997)

Papadopoulos based his criterion on the statement that plastic microdeformation along the slip direction in the plane of crystal slip is proportional to the tangent stress T_a acting in the slip direction (Papadopoulos, 1994). He notes, at the same time, that cracking of single plastically-flowing crystals is not the most critical event since, in the engineering approach, the initiation of cracking occurs upon breaking of a few material grains and successive coalescence of the emerging microcracks (Papadopoulos, 1994). The author states that the useful criterion in the engineering approach should consider an elementary volume V. That volume is defined by Papadopoulos as a cubic neighbourhood of the point investigated the size of which in the statistical sense ensures that grains of a various crystallographic orientation are equally represented. Besides the tangent stress, fatigue life is also affected by the normal stress. To sum up, the criterion considers averaged values of the shear stress acting in the direction of slip T_a and the maximum values of normal stress N

$$\sigma_{P1} = \sqrt{\langle T_N^2\rangle} + \alpha(\max_t\langle N\rangle) \leqslant \tau_{af} \tag{2.19}$$

(20) where $\sqrt{\langle T_N^2\rangle}$ stands for the root mean square of the amplitude of the tangent stress acting in the slip direction, $\max_t\langle N\rangle$ is the maximum value of the mean for the normal stress, reported during the load cycle, while α is the quantity calculated based on material constants in the following way

$$\alpha = \frac{\frac{\sigma_{af}-\tau_{af}}{\sqrt{3}}}{\frac{\tau_{af}}{3}} \tag{2.20}$$

The value of the amplitude of stress T_a depends not only on the position of plane Δ but also on the direction of slip L, defined with angle χ (Fig. 3). To simplify the calculations, the author introduces auxiliary quantities

$$\begin{aligned}
&a = \tau_a\cos\gamma\cos\varphi\cos\theta && b = -\tau_a\cos\gamma\cos\varphi\sin\vartheta\\
&c = \sigma_a\sin\gamma\cos\theta - \tau_a\cos(2\gamma)\sin\varphi\cos\theta && d = \tau_a\cos(2\gamma)\sin\varphi\sin\theta\\
&C_{a,b} = \sqrt{\frac{a^2+b^2+c^2+d^2}{2}\sqrt{\Big(\frac{a^2+b^2+c^2+d^2}{2}\Big)^2 - (ad-bc)^2)}}
\end{aligned} \tag{2.21}$$

The symbol θ in the above notations stands for the phase shift angle. Using the above auxiliary quantities, the equation for root mean square $\sqrt{\langle T_N^2\rangle}$ can assume the following form

$$\sqrt{\langle T_N^2\rangle} = \sqrt{5}\sqrt{\frac{1}{4\pi}\int\limits_{\gamma=0}^{\pi}\int\limits_{\varphi=0}^{2\pi}\sqrt{\frac{1}{2\pi}\int\limits_{\chi=0}^{2\pi}(C_a^2\cos^2\chi + C_b^2\sin^2\chi)\,d\chi)}\sin\gamma\,d\gamma\,d\varphi} \tag{2.22}$$

Finally, the notation can be then simplified to

$$\sqrt{\langle T_N^2\rangle} = \sqrt{\frac{5}{8\pi^2}\int\limits_{\gamma=0}^{\pi}\int\limits_{\varphi=0}^{2\pi}\int\limits_{\chi=0}^{2\pi}(C_a^2\cos^2\chi + C_b^2\sin^2\chi)\sin\gamma\,d\gamma\,d\varphi\,d\chi} \tag{2.23}$$

The mean value of the normal stress has been defined as the mean of normal stresses in all possible positions of the plane Δ passing through the elementary volume V, namely

$$\langle N\rangle = \frac{1}{4\pi}\int\limits_{\gamma=0}^{\pi}\int\limits_{\varphi=0}^{2\pi} N\sin\gamma\,d\gamma\,d\varphi \tag{2.24}$$

2.5. Papadopoulos criterion 2 (2001)

In his second criterion, Papadopoulos (2001) gives up the considerations over microdamage in theelementary volume V. The criterion is further based on the integral approach and also relates the effect of shear and normal stresses to each other, but remains greatly simplified to the form of

$$\sigma_{p2} = \max T_a + \alpha_\infty\sigma_{H,max} \leqslant \gamma_\infty \tag{2.25}$$

where ,$\max T_a$ is denoted by the author as the value of generalised shear stress, while $\sigma_{H,max}$ stands for the cycle-maximum hydrostatic stress.

The quantity $\max T_a$ is a function of the position of plane Δ in a spherical coordinate system, described with angles γ and φ (Fig. 2). The walue T_a is determined from the formula

$$T_a = \sqrt{\frac{1}{\pi}\int\limits_{\chi=0}^{2\pi}\tau_a^2\,d\chi} \tag{2.26}$$

where τ_a is the amplitude of the tangent stress τ acting along the slip direction. The quantity $\boldsymbol{\tau}$ is the projection of the vector of stress acting in the plane Δ on the slip direction, represented by the vector $\mathbf{m}$. The location of the vector $\mathbf{m}$ is described with the angle χ which is formed by it together with the unitary vector $\mathbf{l}$. In the plane Δ, the vectors $\mathbf{l}$ and $\mathbf{r}$ form an orthogonal frame of reference (Fig. 4). The coordinates of the vectors $\mathbf{n}$ and $\mathbf{m}$, needed to determine $\boldsymbol{\tau}$, are as follows

$$\mathbf{l} = \begin{bmatrix} -\sin\varphi \\ \cos\gamma \\ 0 \end{bmatrix} \qquad \mathbf{m} = \begin{bmatrix} -\sin\varphi\cos\chi - cos\gamma\cos\varphi\sin\chi \\ \cos\varphi\cos\chi - cos\gamma\sin\varphi\sin\chi \quad \sin\gamma\sin\chi \end{bmatrix} \tag{2.27}$$

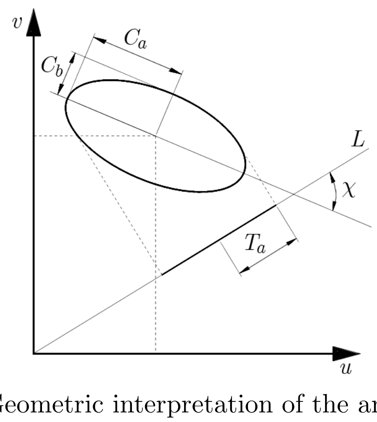

Fig. 3. Geometric interpretation of the amplitude of tangent stress T_a acting in the slip direction

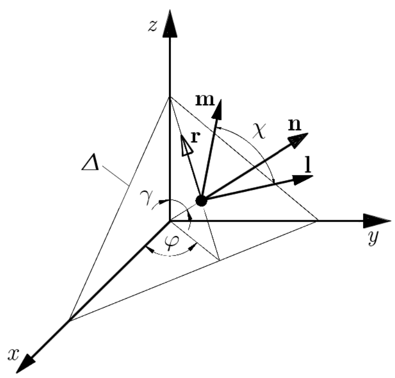

Fig. 4. Description of the slip direction

Stress $\boldsymbol{\tau}$ can assume the following form

$$\boldsymbol{\tau} = \mathbf{n}\boldsymbol{\sigma}\mathbf{m} \tag{2.28}$$

where $\boldsymbol{\sigma}$ stands for the stress state tensor. The value of amplitude τ_a is determined based on the maximum and minimum value reached by the vector $\boldsymbol{\tau}$ in the time of cycle, which can be given as follows

$$\tau_a = \frac{1}{2}(\max\tau - \min\tau) \tag{2.29}$$

The quantities α_∞ and γ_∞ are material parameters. The quantity γ_∞ equals torsional fatigue limit τ_{af}, and α_∞ is defined from the following formula

$$\alpha_\infty = 3\Big(\frac{\tau_{af}}{\sigma_{af}} - \frac{1}{2}\Big) \tag{2.30}$$

The method of parameters determination method is described in Papadopoulos (2001).

3. Method of analysis of calculations results

The equivalent stresses calculated with the criteria analysed, similarly as in papers by McDiarmid (1992) and Papadopoulos (2001), can become related with computational life by means of the Basquin equation (Stephens *et al.*, 2001)

$$\sigma_{eq} = AN_{cal}^B \tag{3.1}$$

where A and B are coefficients of the Basquin equation, and N_{cal} is the number of cycles calculated. The coefficients A and B are obtained from the approximation of the results of uniaxial sample tensile-compressive or torsion life testing. The choice which uniaxial samples should be used comes from nature of the equivalent stress. As for the HMH and Zenner criteria, the coefficients A and B have been calculated based on tensile-compressive fatigue life, and for the McDiarmid and Papadopoulos criteria, based on torsion life. By transforming equation (3.1), we obtain a relationship which allows determination of computational fatigue life

$$N_{cal} = \Big(\frac{\sigma_{eq}}{A}\Big)^{\frac{1}{B}} \tag{3.2}$$

The criteria of analysis made in the present paper involve the comparison of experimental life N_{exp} with life N_{cal} calculated according to formula (3.2). The comparison was made using two statistical parameters described in paper by Walat and Łagoda (2011). The first of them is the mean statistical dispersion of life

$$T_N = 10^{\overline{E}} \tag{3.3}$$

where $\overline{E}$ is calculated from the formula

$$\overline{E} = \frac{1}{n}\sum_{i=1}^{n}\log\frac{N_{exp,i}}{N_{cal,i}} \tag{3.4}$$

where n stands for the number of the results compared.

The second parameter used is the life estimation mean-squared error

$$T_{RMS} = 10^{E_{RMS}} \tag{3.5}$$

where

$$E_{RMS} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\log^2\frac{N_{exp,i}}{N_{cal,i}}} \tag{3.6}$$

The measure T_N assumes the following values: 1 in the case where the mean experimental and computational life are equal; more than 1, when the experimental life values are higher than the computational ones; lower than 1, when the experimental life values are lower than the computational ones. The measure T_N is insensitive to the statistical dispersion of life. It can assume the same value for the results with a low and high statistical dispersion. The quantity T_{RMS} is a measure of statistical dispersion. It assumes the value equal to 1 when the mean and the statistical dispersion of experimental and computational life are identical as well as values higher than 1 in other cases. Unlike T_N, based on T_{RMS}, however, we have no information on whether the computational life values are higher or lower than the experimental ones.

With the above properties of measures in mind, it seems that to make a complete evaluation of the results, both measures must be applied.

4. Analysis of the results

The results of calculations have been presented in comparative computational and experimental life plots (Figs. 5, 6, 7 and 8). For each material, plots have been made for equivalent stress formulas: σ_{MD}, σ^a_{HMH}, σ^{max}_{HMH}, σ_Z, σ_{P1}, σ_{P2}. The points of the plot were marked compliant with the nature of the load, namely R, S, P and N. The number after the letter symbol stands for the value of coefficient λ. Solid lines mark the scatter band of factor 2, and dashed lines – the scatter band of factor 3.

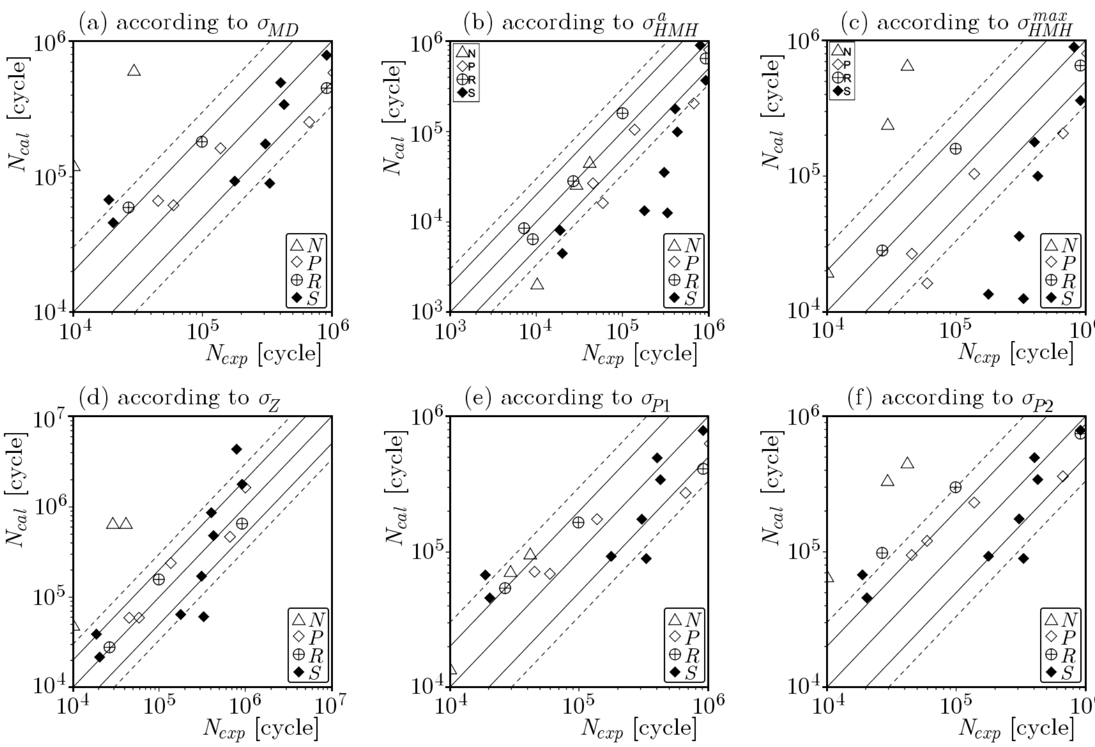

Fig. 5. Comparison of the experimental life values with the calculated ones for 7075-T651 (Mamiya *et al.*, 2011)

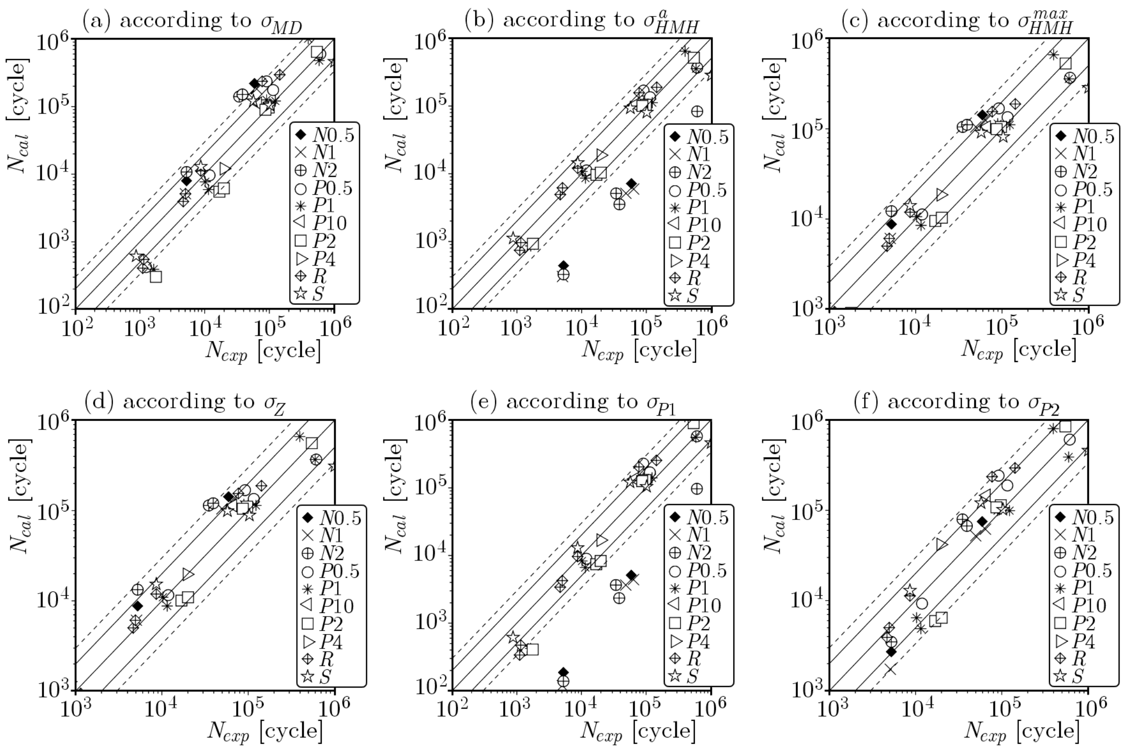

Fig. 6. Comparison of the experimental life values with those calculated for 1045 steel (McDiarmid, 1992)

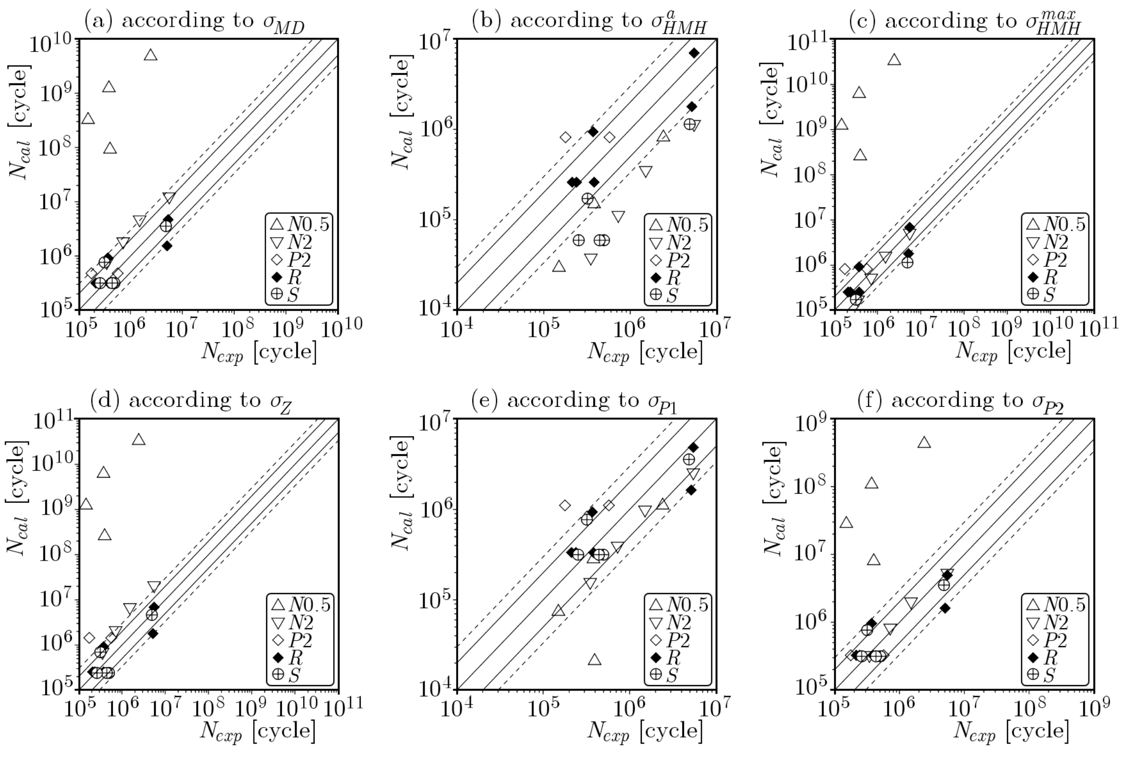

Fig. 7. Comparison of the experimental life values with the ones calculated for 1045 steel (Verreman and Guo, 2007)

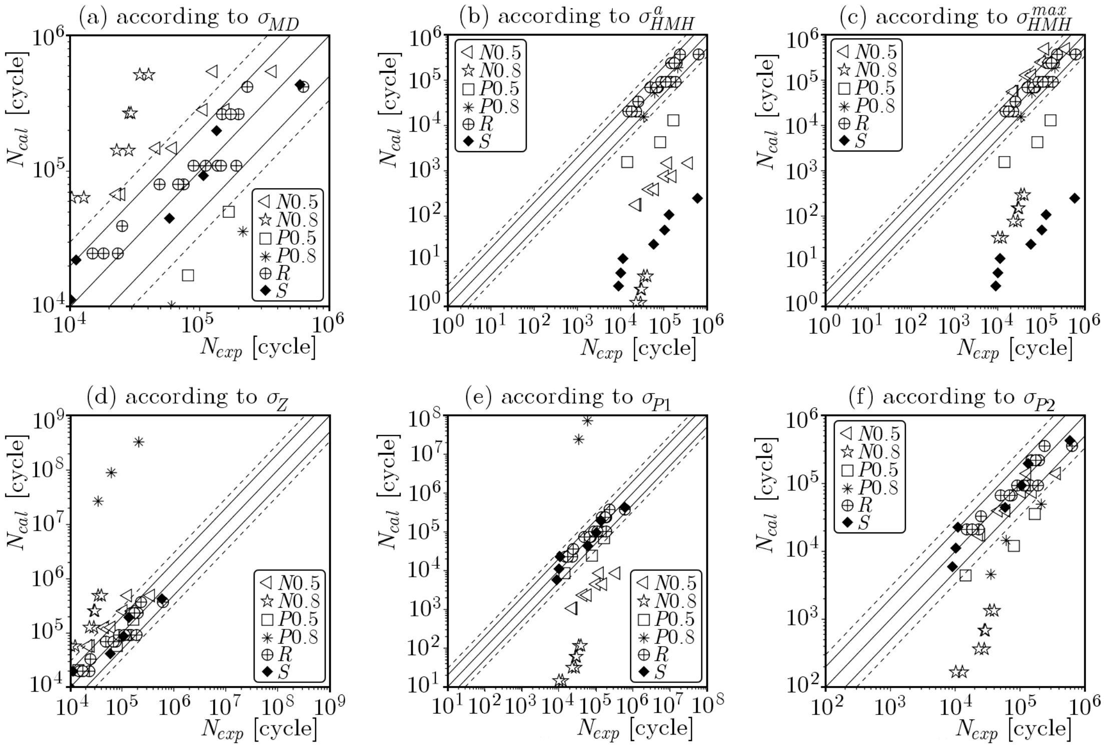

Fig. 8. Comparison of the experimental life values with the computational ones for X2CrNiMo17-12-2

For each material, criterion and the type of a sample, the measures T_N and T_{RMS} have been calculated. The results are broken down in Tables 1, 2, 3 and 4. For better understanding, the results for which the computational life falls within the scatter band of factor 2, namely for value T_N falling in the range 0.5-2, and value T_{RMS} in the range 1-2, are marked with a double underscore. The results for which the computational life falls within the scatter band of factor between 2 and 3, namely for value T_N falling in the range 0.3(3)-0.5 as well as 2-3 and values T_{RMS} in the range 2-3, are marked with a single underscore.

As for uniaxial loads R and S for aluminium alloy 7075-T651, the results most frequently fall within the scatter band of factor 2, for the Zenner criterion, σ_{P1} and HMH according to σ^a_{HMH}. As for the proportional load, the evaluation of the criteria by McDiarmid and HMH according to σ^{max}_{HMH} are relatively worst. For the non-proportional load, the best results were reported for the McDiarmid criterion and the first Papadopoulos criterion. For that material, the greatest errors are about 20-fold higher. Most often the Zenner and Papadopoulos criterion according to σ_{P1} gives the results which fall within the scatter band of factor 2.

Table 1. Values T_N and T_{RMS} for 7075-T651 aluminium alloy (Mamiya *et al.*, 2011)

	σ_{MD}		σ^a_{HMH}		σ^{max}_{HMH}		σ_Z		σ_{P1}		σ_{P2}	
	T_N	T_{RMS}	T_N	T_{RMS}	T_N	T_{RMS}	T_N	T_{RMS}	T_N	T_{RMS}	T_N	T_{RMS}
R	1.00	1.36	0.56	1.70	1.00	1.36	1.00	1.36	0.60	2.36	0.33	3.67
S	4.45	5.95	1.00	2.13	4.45	5.95	0.92	2.66	1.00	2.13	1.00	2.13
P	2.01	2.29	1.21	1.70	2.01	2.29	0.83	1.46	1.11	1.67	0.80	1.80
N	1.79	2.59	0.05	19.06	0.16	7.50	0.09	12.56	0.52	2.03	0.11	9.20

As for uniaxial loads of 1045 steel (Verreman and Guo, 2007), only the life values predicted based on the McDiarmid and HMH criteria according to σ^{max}_{HMH} do not fall within the scatter band of factor 3. For the proportional load, the results acceptable were only reported for the Papadopoulos criterion. As for the non-proportional load for which $\lambda = 2$, satisfactory results were recorded based on the HMH criteria according to σ_{HMH}^{max} and both criteria by Papadopoulos. For the loads demonstrating the highest degree of non-proportionality, namely N0.5, none of the criteria gives the results falling within the assumed scatter bands. For that group of data, the biggest errors reach about 6-thousand. For that material, the second criterion by Papadopoulos most frequently gives the results in the scatter band of factor 2.

Table 2. Values T_N and T_{RMS} for 1045 steel (Verreman and Guo, 2007)

	σ_{MD}		σ_{HMH}^{a}		σ_{HMH}^{max}		σ_Z		σ_{P1}		σ_{P2}	
	T_N	T_{RMS}	T_N	T_{RMS}	T_N	T_{RMS}	T_N	T_{RMS}	T_N	T_{RMS}	T_N	T_{RMS}
R	1.00	1.84	0.98	1.91	1.00	1.84	1.00	1.84	0.95	1.92	0.95	1.92
S	4.62	5.06	1.00	1.64	4.62	5.06	1.12	1.73	1.00	1.64	1.00	1.64
$P2$	0.39	3.02	0.65	2.06	0.39	3.02	0.22	5.15	0.25	4.00	0.99	1.80
$N2$	5.85	6.00	0.39	2.56	1.28	1.47	0.31	3.34	1.90	1.93	0.91	1.19
$N0.5$	6.99	10.34	0.00	1439.35	0.00	6509.40	0.00	6509.40	3.24	4.78	0.01	130.31

For the experimental data for 1045 steel reported in paper by McDiarmid (1992), for uniaxial and proportional loads all the results fall within the scatter band of factor 3. For the non-proportional load, the satisfactory results in each case are reported by applying the second criterion by Papadopoulos. For that group of data, the greatest errors are about 20-folds higher. The best results were recorded for the HMH criteria according to σ_{HMH}^{max}, the Zenner and the Papadopoulos criteria according to σ_{P2}.

Table 3. Values T_N and T_{RMS} for 1045 steel (McDiarmid, 1992)

	σ_{MD}		σ_{HMH}^{a}		σ_{HMH}^{max}		σ_Z		σ_{P1}		σ_{P2}	
	T_N	T_{RMS}	T_N	T_{RMS}	T_N	T_{RMS}	T_N	T_{RMS}	T_N	T_{RMS}	T_N	T_{RMS}
R	1.00	1.60	1.00	1.93	1.00	1.60	1.00	1.60	1.18	1.97	1.00	1.93
S	1.06	1.92	1.00	1.71	1.06	1.92	0.98	1.92	1.00	1.71	1.00	1.71
$P0.5$	1.04	1.50	0.98	1.97	1.04	1.50	1.03	1.50	1.04	1.99	0.98	2.01
$P1$	1.11	1.47	1.22	2.05	1.11	1.47	1.09	1.47	1.08	2.03	1.48	2.19
$P10$	0.64	1.56	0.59	1.68	0.64	1.56	0.59	1.70	0.49	2.05	0.45	2.19
$P2$	1.33	1.57	1.91	2.67	1.33	1.57	1.29	1.56	1.40	2.27	1.66	2.58
$P4$	1.07	1.07	1.65	1.65	1.07	1.07	1.00	1.00	1.16	1.16	0.48	2.09
$N2$	9.63	9.88	0.28	3.73	0.38	2.63	0.35	2.86	13.94	15.18	0.60	2.14
$N1$	14.44	14.86	0.50	2.27	0.71	1.68	0.71	1.68	21.08	21.79	1.47	1.81
$N0.5$	10.03	10.10	0.42	2.64	0.50	2.07	0.49	2.07	17.72	18.36	1.24	1.65

The results for uniaxial loads for X2CrNiMo17-12-2 steel, for the McDiarmid and the HMH criteria according to σ_{HMH}^{max} are unacceptable. As for the proportional load with coefficient $\lambda = 0.5$, only the Zenner criterion gave acceptable results. The first criterion by Papadopoulos slightly exceeded the admissible values. Unfortunately, for the proportional loads demonstrating a greater share of tangible stresses, namely for $\lambda = 0.8$, both criteria give very bad results. Here, in turn, life values for HMH according to σ_{HMH}^{max} fall within the acceptable limits. As for the non-proportional loads with $\lambda = 0.5$, only the second criterion by Papadopoulos generated satisfactory results. For the non-proportional loads with $\lambda = 0.8$, none of the criteria gave results falling within the scatter bands of factors 2 and 3. The results obtained according to the Zenner criterion slightly exceeded that limit. The greatest errors for the non-proportional loads are in the range of 14-thousand times. For most of the results of that group of data, most frequently the Zenner criterion and the second criterion by Papadopoulos give the results falling within the scatter band of factor 2.

Table 4. Values T_N and T_{RMS} for X2CrNiMo17-12-2 steel (Skibicki *et al.*, 2012)

	σ_{MD}		σ^a_{HMH}		σ^{max}_{HMH}		σ_Z		σ_{P1}		σ_{P2}	
	T_N	T_{RMS}	T_N	T_{RMS}	T_N	T_{RMS}	T_N	T_{RMS}	T_N	T_{RMS}	T_N	T_{RMS}
R	1.00	1.41	0.86	1.46	1.00	1.41	1.00	1.41	0.94	1.42	1.02	1.41
S	1903.4	1922.1	1.00	1.46	1903.4	1922.1	1.09	1.48	1.00	1.46	1.00	1.46
$P0.5$	13.00	13.21	3.31	3.44	13.00	13.21	0.99	1.33	2.38	2.50	4.74	4.88
$P0.8$	1.44	1.62	7.34	7.49	1.44	1.62	0.00	1214.8	0.00	1013.6	5.24	5.37
$N0.5$	146.64	148.04	0.39	2.68	0.44	2.38	0.44	2.38	24.89	25.27	1.48	1.65
$N0.8$	14075.7	14203.3	0.13	8.23	228.31	231.99	0.17	2.38	568.2	575.13	48.66	49.63

As for uniaxial loads, in most cases ,all the criteria analysed give satisfactory results. The worst results are reported by applying the HMH criterion for torsion, which must be due to the fact that this criterion is applied for a small group of materials resulting from a constant ratio of fatigue limits $\tau_{af}/\sigma_{af} = 1/3$.

A similar situation is reported for proportional loads. The life values estimated according to the HMH criterion are most often encumbered with the greatest error, which is due to the failure in considering variable material properties expressed with the ratio τ_{af}/σ_{af}.

As for the non-proportional loads for materials sensitive to non-proportionality, namely 1045 and X2CrNiMo17-12-2 steels, the results can show a very high error.

Most frequently, the best results were reported for the second criterion by Papadopoulos, and the worst results, on the other hand, for the McDiarmid and the HMH criteria. Even though the Papadopoulos criterion is an integral criterion, however, it does not allow making the statement that the integral approach is the most adequate one to describe non-proportional loads; first of all, since the HMH criterion can be also considered as the integral criterion and, second of all, since it is the Papadopulos criterion, which for non-proportional loads often gave results demonstrating the statistical dispersion much greater than desired scatter bands of factors 2 or 3.

5. Conclusions

For the experimental fatigue life data used, one can claim that:

- Relatively the best results were reported by applying the second criterion by Papadopoulos and the Zenner criterion, and the worst – according to the criterion by McDairmid and by Huber-Mises-Hencky.
- None of the criteria analysed can be applied to estimate fatigue life when exposed to non-proportional loads.
- The integral approach can be effective under the non-proportional loads conditions, however, it does not always guarantee acceptable results.

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Ocena zmęczeniowych kryteriów całkowych w zakresie szacowania trwałości w warunkach obciążeń jednoosiowych, złożonych proporcjonalnych i nieproporcjonalnych

Streszczenie

Celem niniejszej pracy jest ocena możliwości szacowania trwałości zmęczeniowej za pomocą całkowych kryteriów zmęczeniowych. Podejście całkowe bazuje na założeniu, że dla prawidłowej oceny zachowań zmęczeniowych konieczne jest zsumowanie (scałkowanie) wartości parametru zniszczenia na wszystkich płaszczyznach przechodzących przez rozpatrywany punkt materiału. Analizę przeprowadzono dla trzech najczęściej spotykanych kryteriów całkowych: kryterium Zennera i dwóch kryteriów Papadopoulosa. Uzyskane wyniki porównano z kryterium zmęczeniowym McDiarmida, bazującym na konkurencyjnym w stosunku do całkowego podejściu płaszczyzny krytycznej, oraz powszechnie stosowanym w wielu obszarach wytrzymałości materiałów kryterium Hubera-Misesa-Hencky'ego.

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