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NUMERICAL SIMULATION OF THE EFFECT OF WIND ON THE MISSILE MOTION

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The paper presents a model of the missile dynamics and the impact of the wind field thereon. Sample results of numerical simulation of the missile flight across the wind field are given and conclusions drawn.

Keywords: unguided missile, modeling, simulation, external ballistics

1. Introduction

A missile should be launched in such a way that the target is hit with the maximum accuracy. Launch conditions can vary. Differences result from different initial trajectory angles (horizontal jump angles). Depending on these parameters, a different point on the Earth's surface is reached. Wind is another factor which can affect the missile trajectory. Direction of the wind and its velocity may be different, so the impact of these factors on the missile trajectory and the fall point is also different. Therefore, even if initial conditions (i.e. initial trajectory angles) are the same, flight trajectories are different.

The aim of this study is to estimate the effect of the wind field on the missile flight. A series of numerical simulations have been carried out with a model of motion with six degrees of freedom (6 DOF) used to describe the missile flight in 3D space. The model has been adopted from the study on the aircraft flight dynamics (Gacek, 1998) with necessary modifications included (Awrejcewicz and Koruba, 2012; Baranowski, 2006).

2. Mathematical description of the missile motion

2.1. Assumptions for a physical model

To analyse the missile flight dynamics, the following assumptions have been made to formulate of the mathematical description of the missile motion:

1. A missile is a rigid body but the mass and moments of inertia change during the initial, active-flight portion of the trajectory, and
2. The missile has two symmetry planes. These are the Oxz and Oxy planes (Fig. 1), which are planes of geometric, mass and aerodynamic symmetries.

2.2. Systems of coordinates

To determine a mathematical model of a missile, the following orthogonal systems of coordinates are used:

$Oxyz$ – the missile-fixed system with the origin at the centre of mass of the missile,

$Ox_ay_az_a$ – the air-trajectory reference system,

$Ox_gy_gz_g$ – the Earth-fixed system with the origin at the centre of mass of the missile.

These systems are related by the following angles:

– the systems $Oxyz$ and $Ox_gy_gz_g$ are interrelated by the yaw angle Ψ, the pitch angle Θ, and the bank angle Φ,
– the systems $Oxyz$ and $Ox_ay_az_a$ are linked by the sideslip angle β and the angle of attack α.

Performing a sequence of rotations of the angles Ψ, Θ and Φ about the coordinate axes, the matrix of transformations from $Ox_gy_gz_g$ to $Oxyz$ can be determined

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{L}_{s/g} \begin{bmatrix} x_g \\ y_g \\ z_g \end{bmatrix} \tag{2.1}$$

where the matrix $\mathbf{L}_{s/g}$ is

$$\mathbf{L}_{s/g} = \begin{bmatrix} \cos\Psi\cos\Theta & \sin\Psi\cos\Theta & -\sin\Theta \\ \cos\Psi\sin\Theta\sin\Phi - \sin\Psi\cos\Phi & \sin\Psi\sin\Theta\sin\Phi + \cos\Psi\cos\Phi & \cos\Theta\sin\Phi \\ \cos\Psi\sin\Theta\cos\Phi + \sin\Psi\sin\Phi & \sin\Psi\sin\Theta\cos\Phi - \cos\Psi\sin\Phi & \cos\Theta\cos\Phi \end{bmatrix} \tag{2.2}$$

Performing rotations one by one with angles β and α, the matrix of transformations from $Ox_ay_az_a$ to $Oxyz$ can be determined

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{L}_{s/a} \begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix} \tag{2.3}$$

where the matrix $\mathbf{L}_{s/a}$ is

$$\mathbf{L}_{s/a} = \begin{bmatrix} \cos\alpha\cos\beta & -\cos\alpha\sin\beta & -\sin\alpha \\ \sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & -\sin\alpha\sin\beta & \cos\alpha \end{bmatrix} \tag{2.4}$$

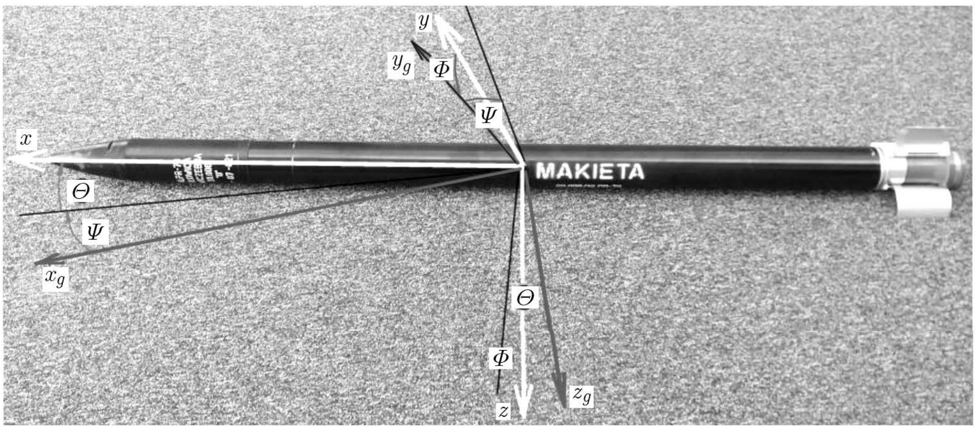

Fig. 1. $Ox_gy_gz_g$ and $Oxyz$ systems of coordinates and coordinate transformation angles

2.3. Equation of the missile motion

2.3.1. A general form of the equation of motion

Taking into account that tunnel measurements of aerodynamic forces are usually taken in the air-trajectory reference frame $Ox_ay_az_a$, equations of equilibrium of forces will be determined in this system. However, equations of equilibrium of moments will be determined in the missile-fixed coordinate system $Oxyz$, because in this system the tensor of moments of inertia is independent of time.

The vector equation of motion of the centre of mass of the missile is

$$\frac{d(m\mathbf{V})}{dt} = \frac{\partial(m\mathbf{V})}{\partial t} + \boldsymbol{\Omega} \times (m\mathbf{V}) = \mathbf{F} \tag{2.5}$$

and can be described as three scalar equations in any rectangular moving coordinate system

$$m(\dot{U} + QW - RV) = X \qquad m(\dot{V} + RU - PW) = Y \qquad m(\dot{W} + PV - QU) = Z \tag{2.6}$$

where m is mass of the missile, $\mathbf{V}$ – velocity vector with components $\mathbf{V} = [U, V, W]^{\mathrm{T}}$ in any moving coordinate system, $\boldsymbol{\Omega}$ – angular velocity vector of a moving system as related to the inertial reference frame with components $\boldsymbol{\Omega} = [P, Q, R]^{\mathrm{T}}$ in the moving coordinate system, $\mathbf{F}$ – resultant vector of forces acting on the missile with components $[X, Y, Z]^{\mathrm{T}}$ in the moving coordinate system.

In the air-trajectory reference frame $Ox_a y_a z_a$, the velocity vector has only one component $U_a = V$ (which should not be mistaken for the second component of the vector $\mathbf{V}$, according to the designation above).

Equations (2.6) have the following forms

$$m\dot{V} = X_a \qquad mR_a V = Y_a \qquad -mQ_a V = Z_a \tag{2.7}$$

Assuming that we know the angular velocity of the system $Oxyz$ as related to the inertial reference frame $\boldsymbol{\Omega}_s$ and velocity of the system $Oxyz$ as related to the $Ox_a y_a z_a$ frame, the angular-velocity vector of the system $Ox_a y_a z_a$ as related to the inertial reference frame can be determined as

$$\boldsymbol{\Omega}_a = \boldsymbol{\Omega}_s + \boldsymbol{\Omega}_{s/a} = \boldsymbol{\Omega}_s + \dot{\boldsymbol{\beta}} - \dot{\boldsymbol{\alpha}} \tag{2.8}$$

In the frame $Oxyz$, the vector $\boldsymbol{\Omega}_s$ has the following components: $\boldsymbol{\Omega}_s = [P, Q, R]^{\mathrm{T}}$, in the coordinate system $Ox_a y_a z_a$, the vector $\dot{\boldsymbol{\beta}}$ has the following components: $\dot{\boldsymbol{\beta}} = [0, 0, \dot{\beta}]^{\mathrm{T}}$, and in the frame $Oxyz$, the vector $\dot{\boldsymbol{\alpha}}$ vector has the components: $\dot{\boldsymbol{\alpha}} = [0, \dot{\alpha}, 0]^{\mathrm{T}}$. Taking the above into account and using transformation matrix (2.4), on the basis of (2.8), we receive

$$\begin{aligned} P_a &= P\cos\alpha\cos\beta + (Q - \dot{\alpha})\sin\beta + R\sin\alpha\cos\beta \\ Q_a &= -P\cos\alpha\sin\beta + (Q - \dot{\alpha})\cos\beta - R\sin\alpha\sin\beta \\ R_a &= -P\sin\alpha + R\cos\alpha + \dot{\beta} \end{aligned} \tag{2.9}$$

Applying equations (2.9) to equations (2.7), after transformations, we get the following set of equations

$$\begin{aligned} &\dot{V} = \frac{1}{m}X_a \qquad \dot{\beta} = \frac{1}{mV}Y_a + P\sin\alpha - R\cos\alpha \\ &\dot{\alpha} = \frac{1}{\cos\beta}\Big[\frac{Z_a}{mV} + Q\cos\beta - (P\cos\alpha + R\sin\alpha)\sin\beta\Big] \end{aligned} \tag{2.10}$$

The vector equation for equilibrium of moments of forces has the following form

$$\frac{d(\mathbf{K})}{dt} = \frac{\partial(\mathbf{K})}{\partial t} + \boldsymbol{\Omega} \times \mathbf{K} = \mathbf{M} \tag{2.11}$$

where $\mathbf{M}$ is the resultant moment of forces acting on the missile with the components $\mathbf{M} = [L, M, N]^{\mathrm{T}}$ in a moving system of coordinates.

The missile angular-momentum vector is

$$\mathbf{K} = \mathbf{I}\boldsymbol{\Omega} \tag{2.12}$$

where the tensor of moments and products of inertia $\mathbf{I}$ is determined as

$$\mathbf{I} = \begin{bmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{yx} & I_y & -I_{yz} \\ -I_{zx} & -I_{zx} & I_z \end{bmatrix} \tag{2.13}$$

As said above, equation (2.11) will be written in the system $Oxyz$ fixed to the missile. The missile mass characteristics being time independent in this system imply that all derivatives of components of the moment of inertia tensor as related to time are zero. It means that

$$\frac{\partial \mathbf{K}}{\partial t} = \frac{\partial (\mathbf{I}\boldsymbol{\Omega}_s)}{\partial t} = \frac{\partial \mathbf{I}}{\partial t}\boldsymbol{\Omega}_s + \mathbf{I}\frac{\partial \boldsymbol{\Omega}_s}{\partial t} = \mathbf{I}\frac{\partial \boldsymbol{\Omega}_s}{\partial t} \tag{2.14}$$

After transformations, on the basis of Eq. (2.11) and using Eq. (2.14), one receives a set of three scalar equations describing rotational motion of the missile in the moving system of coordinates $Oxyz$ fixed to the missile. The set has the following form

$$\begin{aligned} &I_x\dot{P} - I_{yz}(Q^2 - R^2) - I_{zx}(\dot{R} + PQ) - I_{xy}(\dot{Q} - RP) - (I_y - I_z)QR = L \\ &I_y\dot{Q} - I_{zx}(R^2 - P^2) - I_{xy}(\dot{P} + QR) - I_{yz}(\dot{R} - PQ) - (I_z - I_x)RP = M \\ &I_z\dot{R} - I_{xy}(P^2 - Q^2) - I_{yz}(\dot{Q} + RP) - I_{zx}(\dot{P} - QR) - (I_x - I_y)PQ = N \end{aligned} \tag{2.15}$$

However, with the fact that taken into account the planes Oxz and Oxy are the missile planes of symmetry, the following equalities may be written

$$I_{xy}, I_{yx}, I_{zy}, I_{yz} = 0 \tag{2.16}$$

Hence, the last set of equations reduces to

$$I_x\dot{P} - (I_y - I_z)QR = L \qquad I_y\dot{Q} - (I_z - I_x)RP = M \qquad I_z\dot{R} - (I_x - I_y)PQ = N \tag{2.17}$$

Finally, after some elementary transformations, system (2.17) takes the form

$$\dot{P} = \frac{1}{I_x}[L + (I_y - I_z)QR] \qquad \dot{Q} = \frac{1}{I_y}[M + (I_z - I_x)RP] \qquad \dot{R} = \frac{1}{I_x I_z}[L + (I_y - I_z)QR] \tag{2.18}$$

Complementary to systems (2.10) and (2.18) are kinematic relations allowing us to determine the rates of changes in angles Ψ, Θ and Φ using angular velocities

$$\begin{aligned} &\dot{\Phi} = P + (R\cos\Phi + Q\sin\Phi)\tan\Theta \qquad \dot{\Theta} = Q\cos\Phi - R\sin\Phi \\ &\dot{\Psi} = \frac{1}{\cos\Theta}(R\cos\Phi + Q\sin\Phi) \end{aligned} \tag{2.19}$$

Furthermore, with relationships (2.1) and (2.3) applied, the velocity vector of the centre of mass of the missile in the $Ox_gy_gz_g$ reference frame can be determined

$$\begin{bmatrix} U_g \\ V_g \\ W_g \end{bmatrix} = \begin{bmatrix} \dot{x}_g \\ \dot{y}_g \\ \dot{z}_g \end{bmatrix} = \mathbf{L}_{s/g}^{-1}\mathbf{L}_{s/a}\begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix} \tag{2.20}$$

where

$$\begin{aligned} \dot{x}_g &= V[\cos\alpha\cos\beta\cos\Theta\cos\Psi + \sin\beta(\sin\Phi\sin\Theta\cos\Psi - \cos\Phi\sin\Psi) \\ &\quad + \sin\alpha\cos\beta(\cos\Phi\sin\Theta\cos\Psi + \sin\Phi\sin\Psi)] \\ \dot{y}_g &= V[\cos\alpha\cos\beta\cos\Theta\sin\Psi + \sin\beta(\sin\Phi\sin\Theta\sin\Psi + \cos\Phi\cos\Psi) \\ &\quad + \sin\alpha\cos\beta(\cos\Phi\sin\Theta\sin\Psi + \sin\Phi\cos\Psi)] \\ \dot{z}_g &= V[-\cos\alpha\cos\beta\sin\Theta + \sin\beta\sin\Phi\cos\Theta + \sin\alpha\cos\beta\cos\Phi\sin\Theta] \end{aligned} \tag{2.21}$$

Equations (2.10), (2.18), (2.19) and (2.21) make a set of 12 differential equations that describe the missile motion in 3D space, the missile being treated as a rigid body. It can be written down in the following form

$$\frac{d\mathbf{X}}{dt} = \mathbf{F}(t, \mathbf{X}, \mathbf{S}) \tag{2.22}$$

$\mathbf{X}$ is a twelve-component vector of the missile flight parameters

$$\mathbf{X} = [V, \alpha, \beta, P, Q, R, \Phi, \Theta, \Psi, x_g, y_g, z_g]^{\mathrm{T}}$$

where V is the missile flight velocity (the absolute value of the flight velocity vector), α – angle of attack, β – sideslip angle, P, Q, R – roll, pitch, and yaw angular velocities in the system of coordinates $Oxyz$, Θ, Φ, Ψ – angles of pitch, roll and yaw, respectively.

2.3.2. General expressions that describe forces and moments acting on the missile

Forces acting on the missile

The right side of equation (2.5) represents forces acting on the missile

$$\mathbf{F} = \mathbf{Q} + \mathbf{T} + \mathbf{R} \tag{2.23}$$

According to designations in equations (2.7), there are the following components

$$X_a = Q_{x_a} + T_{x_a} + R_{x_a} \qquad Y_a = Q_{y_a} + T_{y_a} + R_{y_a} \qquad Z_a = Q_{z_a} + T_{z_a} + R_{z_a} \tag{2.24}$$

Particular components in expression (2.24) are determined below:
— The missile weight $\mathbf{Q}$, which has only one component $\mathbf{Q} = [0, 0, mg]^{\mathrm{T}}$ in the system $Ox_gy_gz_g$. Using relations between (2.1) and (2.3), we can calculate components of the vector $\mathbf{Q}$ in the system $Ox_ay_az_a$

$$\begin{bmatrix} Q_{x_a} \\ Q_{y_a} \\ Q_{z_a} \end{bmatrix} = \mathbf{L}_{s/a}^{-1}\mathbf{L}_{s/g} \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} \tag{2.25}$$

we get

$$\begin{aligned} Q_{x_a} &= mg(-\cos\alpha\cos\beta\sin\Theta + \sin\beta\cos\Theta\sin\Phi + \sin\alpha\cos\beta\cos\Theta\cos\Phi) \\ Q_{y_a} &= mg(\cos\alpha\sin\beta\sin\Theta + \cos\beta\cos\Theta\sin\Phi - \sin\alpha\sin\beta\cos\Theta\cos\Phi) \\ Q_{z_a} &= mg(\sin\alpha\sin\Theta + \cos\alpha\cos\Theta\cos\Phi) \end{aligned} \tag{2.26}$$

— The aerodynamic force $\mathbf{R}$, which has the following components in the system $Ox_ay_az_a$

$$\begin{aligned} R_{x_a} &= -P_{xa} = -C_{xa}\frac{\rho V_*^2}{2}S \qquad R_{y_a} = P_{ya} = -C_{ya}\frac{\rho V_*^2}{2}S \\ R_{z_a} &= -P_{za} = -C_{za}\frac{\rho V_*^2}{2}S \end{aligned} \tag{2.27}$$

where C_{xa}, C_{ya}, C_{za} are coefficients of aerodynamic drag, side and lift forces, respectively, S – cross section of the missile, ρ – air density, V_* – missile air speed calculated in the following way

$$\mathbf{V}_* = \mathbf{V} - \mathbf{V}_w \tag{2.28}$$

where $\mathbf{V}_w$ is the wind vector. Its influence on the sideslip angle and the angle of attack is shown in Fig. 2.

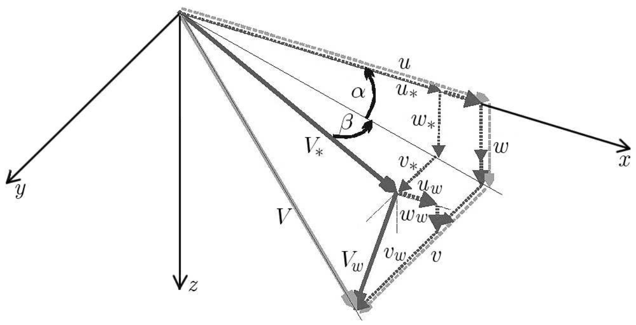

Fig. 2. The sideslip angle and the angle of attack

Moments of forces acting on the missile

The right side of the set of equations (2.17) has a vector $\mathbf{M} = [L, M, N]^{\mathrm{T}}$ which is a resultant vector of moments of forces acting on the missile. Taking into account that equations (2.18) are determined in the system of the missile principal axes of inertia with their origins in the centre of mass of the missile, the only moments acting on the missile are aerodynamic moments. Therefore, the components are

$$L = C_l \frac{\rho V_*^2}{2} S d \qquad M = C_m \frac{\rho V_*^2}{2} S d \qquad N = C_n \frac{\rho V_*^2}{2} S d \tag{2.29}$$

where C_l, C_m, C_n are coefficients of rolling, pitching and yawing moments, respectively, d – diameter of the missile.

2.4. Aerodynamic coefficients

Aerodynamic forces and moments acting on the missile described with expressions (2.27), (2.29) are determined according to their aerodynamic coefficients. These coefficients depend on many factors, such as the missile shape, angle of attack, sideslip angle, Mach number, Reynolds number and angular velocities. There are no general methods of determining these characteristics for any attitude of the missile. Therefore, various methods are used depending on the problem discussed and availability of the missile source data. Because of high velocity of the missile, the most important is the effect of Mach number on aerodynamic characteristics.

Sample formulae describing coefficients of aerodynamic forces and moments that have been taken into account are as follows (Dmitrevskiί, 1979; Kowaleczko, 2003; McCoy, 1999)

$$\begin{aligned} C_{xa} &= C_{xa0} + C_{xa2}\sin^2\alpha & C_{za} &= C_{za1}\sin\alpha + C_{za3}\sin^3\alpha \\ C_m &= C_{m1}\sin\alpha + C_{mQ}\frac{Qd}{2V_*} & C_l &= C_{l\delta}\delta + C_{lP}\frac{Pd}{2V_*} \end{aligned} \tag{2.30}$$

where C_{mQ} and C_{lP} are coefficients of damping moments, $C_{l\delta}\delta$ is the spin driving moment coefficient that depends on the fin cant angle δ.

The coefficients C_{ya} and C_n can be determined in a similar way as the C_{za} and C_m ones, with the sideslip angle β taken into account. Basic aerodynamic coefficients are shown in Figs. 3-5.

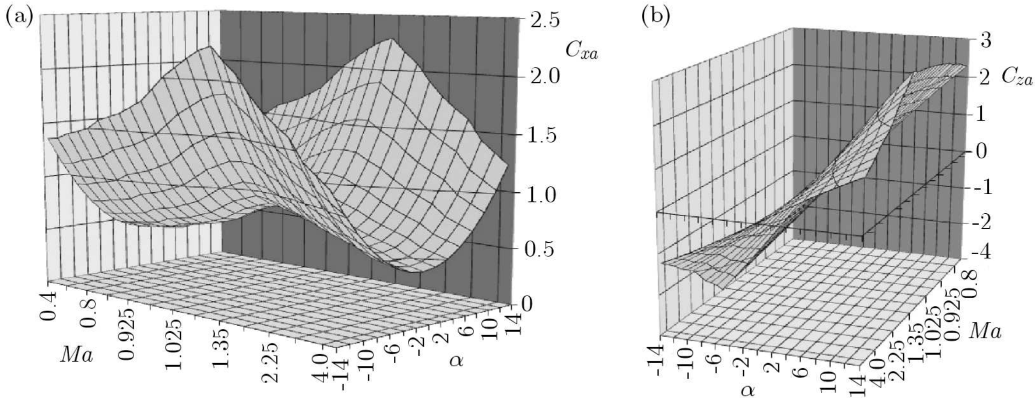

Fig. 3. Drag (a) and lift (b) force coefficient

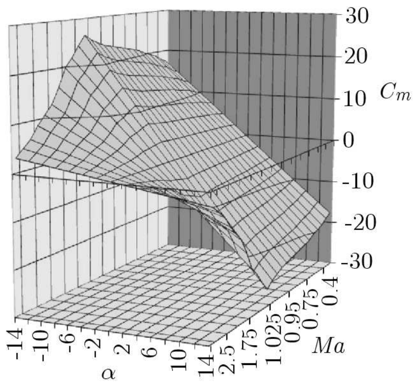

Fig. 4. Pitch moment coefficient

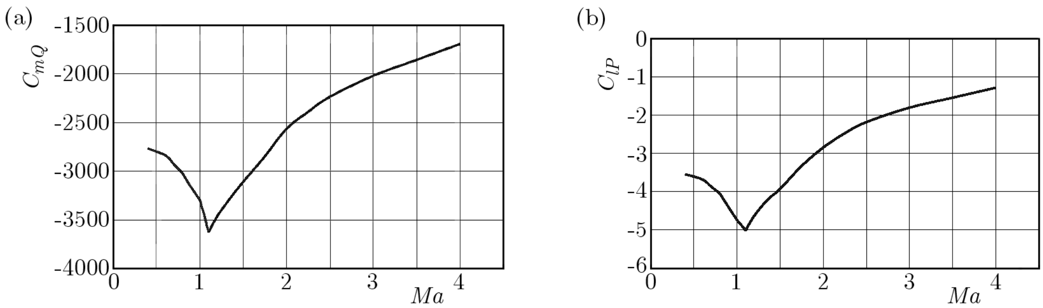

Fig. 5. Pitch (a) and roll (b) damping moment coefficient

3. Analysis of missile flight in calm atmosphere

To start the intended analyses, numerical simulations based on set (2.22) have been performed for the case with no wind. The initial trajectory angle has been changed. The value of this angle, which gives the maximum range, is $42°$. It is illustrated in Fig. 6.

From Figure 4 one can find that the missile is statistically stable – the derivative $\partial C_m/\partial\alpha = C_{m1}$ is negative in the whole range of Mach number. All simulation results show that the missile is also dynamically stable. This has been confirmed by plots of all parameters shown in Figs. 7 and 8. The first figure (Fig. 7) presents velocity of the missile. During the initial (active) phase of flight, when the engine works, acceleration of the missile is observed. Then the missile velocity decreases because of the aerodynamic drag force. During the active phase, the mass and inertia moments of the missile change linearly from initial to final values. Now, the missile follows the ballistic trajectory.

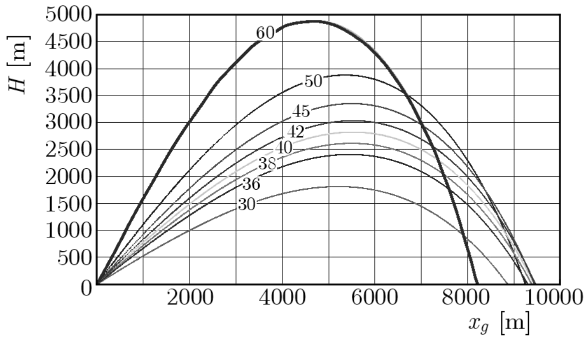

Fig. 6. Missile trajectories for various initial trajectory angles

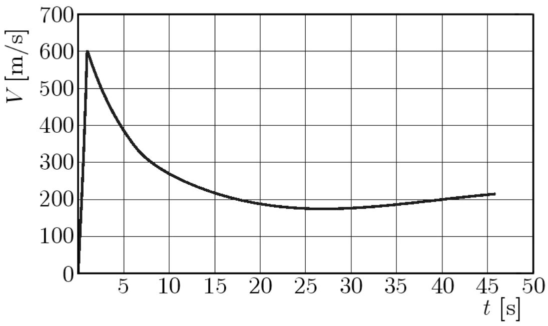

Fig. 7. Missile velocity

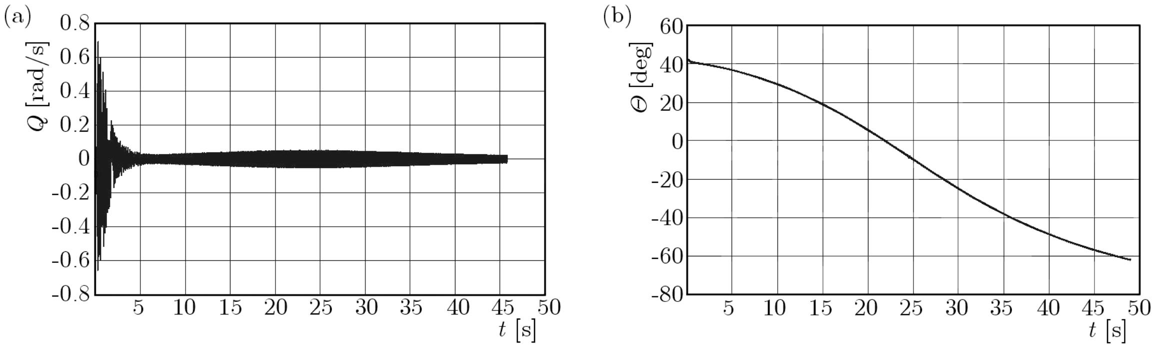

Fig. 8. Pitch angular velocity Q (a) and pitch angle Θ (b)

During the whole flight, the missile rotates about the Ox axis. At the beginning, the rolling moment is produced by the engine, and then the fins force the missile to rotate in the opposite direction. Because of this rotation, the pitching and yawing motion is observed but both angular velocities are kept in the limited range, see Fig. 8a. The pitching moment keeps decreasing from 42° at the initial stage of flight to -60° in the final portion of flight – Fig. 8b.

4. Missile flight with longitudinal wind influence

For the optimal elevation angle (42°), the effect of longitudinal and lateral wind has been investigated. The value of longitudinal wind was changing over the range of $-10\,\mathrm{m/s}$ to $+10\,\mathrm{m/s}$, with a step of $2.5\,\mathrm{m/s}$. Figure 9a shows that the longitudinal wind changes the range of the missile. One can find that the relation is linear and the following formula can be written

$$\mathrm{range} = \mathrm{range}_0 + 41.56\,\mathrm{wind}$$

Similar results have also been obtained for the derivation. Figure 9b shows this derivation versus the longitudinal wind. The relation is approximately linear and has the following form

$$\text{derivation} = \text{derivation}_0 + 1.78\,\text{wind}$$

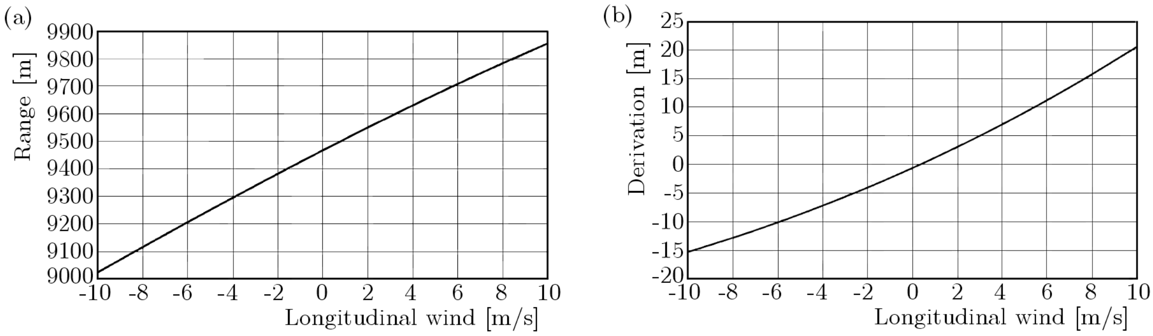

Fig. 9. Missile range (a) and derivation (b) versus longitudinal wind

The same values of lateral wind were tested (from -10 m/s to $+10$ m/s). Its influence on the missile trajectory is more complicated. The range (measured along the initial direction of the missile to the target) decreases at any lateral wind. This is presented in Fig. 10a. The relationship between the range and the lateral wind is nonlinear. In Fig. 10b, very crucial changes of the derivation are observed. The wind equal to 10 m/s produces more than 800 meters derivation. This is effected by rapid changes in the angle of yaw at the active part of the trajectory, see Fig. 11. Since the missile is stable, it changes the direction of the Ox axis against the wind to minimize the angle of attack. Therefore, the wind from the right (left) causes the right (left) derivation – Fig. 12.

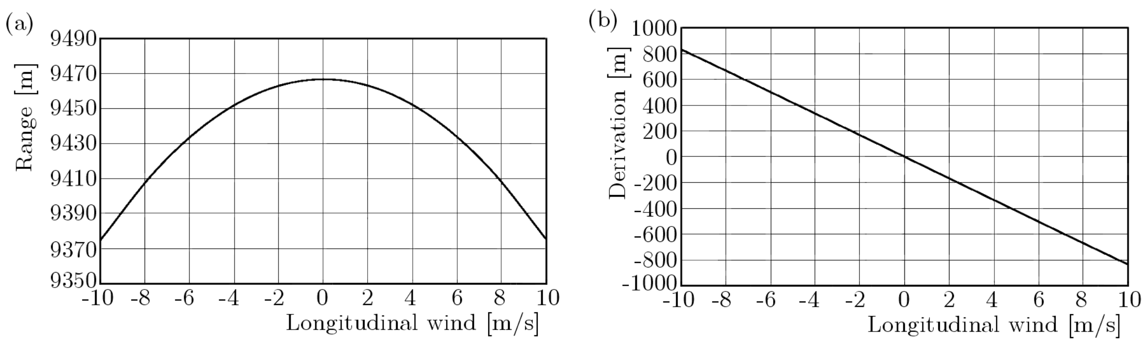

Fig. 10. Missile range (a) and derivation (b) versus lateral wind

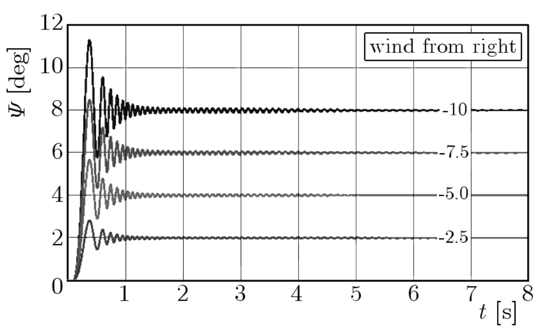

Fig. 11. Yaw angle produced by lateral wind

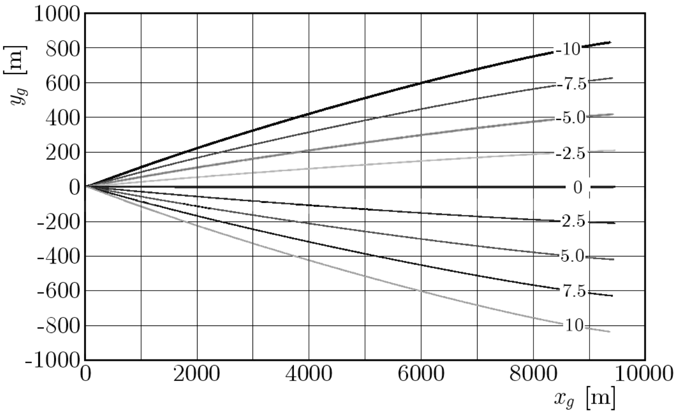

Fig. 12. Horizontal projection of the missile trajectory

5. Conclusions

The conducted analyses have shown that the effect of wind on the accuracy of the missile launch is essential and must be taken into account when planning the use of the missiles. The longitudinal wind first of all affects the range, whereas the lateral wind produces derivation of the trajectory. If the lateral wind forces the missile at the active-flight portion of the trajectory, the missile changes the direction of its trajectory to the side against the wind direction.

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