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ANALYSIS OF DYNAMIC PARAMETERS IN A METAL CYLINDRICAL ROD STRIKING A RIGID TARGET

Edward Włodarczyk, Marcin Sarzyński
Military University of Technology, Faculty of Mechatronics and Aviation, Warsaw, Poland
e-mail: edward.wlodarczyk@wat.edu.pl; marcin.sarzynski@wat.edu.pl

The initial-boundary value problem of one-dimensional dynamical plastic deformation within the scope of large strain of a metallic cylindrical rod has been analytically solved in a closed form in this paper. The deformation of a flat-ended rod has been caused by its normal impact on a rigid target. The rod material in the deformed part is defined by an incompressible, rigid-plastic, power strain hardening model. Such a model of the material provided better correlation with experimental data than a perfectly-plastic model. Results presented in this paper have applicable values. Derived in the paper closed analytical relations, written by elementary functions, give researchers and engineers insight into interaction of the physical parameters of the rod during of the impact process and post-impact.

Key words: dynamics, rod, Taylor impact

1. Introduction

Taylor published his theory in 1948 (Taylor, 1948). Originally, one-dimensional analysis of Taylor was used by Whiffin (1948) to estimate the dynamic yield stress of specimens. There has been much interest in the impact testing and estimating the dynamic yield stress since then. A selective review of the literature with respect to the Taylor impact test is in the papers by Jones *et al.* (1987, 1997), Field *et al.* (2004) and Zuckas *et al.* (1982), and is not necessary to be discussed in this paper.

The Taylor impact test is a useful experiment for estimating material behavior at high strain rates (Meyers, 1994). The test is reproducible and is reasonably economical after the initial investment has been made.

Jones *et al.* (1987) asserted that simple engineering theories still have a considerable value. Such theories frequently give investigators insight into the interaction of physical parameters and their relationship with the outcome of the event. Most often, these interactions are difficult to ascertain from complex computer outputs. As a result, simple engineering theories often provide the basis for design of experiments and are frequently used to refine the areas in which computing is to be done.

Bearing in mind the above-mentioned opinions, the Taylor problem, for a rigid-plastic material with power strain hardening, loaded by an impact has been analytically solved in a closed form in this paper.

2. Formulation of the problem

Consider a uniform flat-ended metal rod of initial parameters, length L, cross-section area S_0 and density ρ, that normally impacts against a rigid flat target. The initial velocity of the rod is denoted by U. Assuming that U is large enough, a portion of the rod placed at the target face will deform plastically. As in other problems of plasticity, it may be permissible to neglect elastic strains in comparison with plastic strains, particularly within the scope of the large plastic strains that occur in the considered here problem.

Bearing in mind this fact, it is assumed that the rod material is rigid-plastic with power strain hardening and incompressible into the region of plastic strains. Furthermore, it is assumed that the material behavior is rate-independent, i.e. $\sigma = \sigma(\varepsilon)$.

The process of rod deformation during the impact is schematically shown in Fig. 1. Let x denote a Lagrangian coordinate aligned with the axis of the rod and having its origin at the striking end of the rod. The rigid back part of the rod is approaching to the target with absolute time-dependent velocity $u(t)$. The plastic wave front is propagated away from the impact surface with absolute time-dependent velocity $v(t)$ leaving the material behind it at rest since elastic recovery is neglected. S and σ are, respectively, the cross sectional area and nominal stress just behind the plastic wave front, and S_0 and σ_{sd} just ahead of it; σ_{sd} is the dynamic yield-point stress. The remaining symbols, shown in Fig. 1, denote: $x_p = x_p(t)$ momentary length of the plastically deformed part of the rod, $x_s = x_s(t)$ momentary length of undeformed portion of the rod, $x_t = x_t(t)$ momentary displacement of the rear end of the rod, $X_t = X_t(t)$ momentary total length of the rod, D_0 initial diameter of the rod, D_L maximum diameter of the deformed section of the rod, L initial length of the rod, x Lagrangian coordinate. Additionally, the index k denotes the final values of the quoted above parameters of the post-test, e.g. x_{pk} is the final length of the deformed part of the rod, and so on.

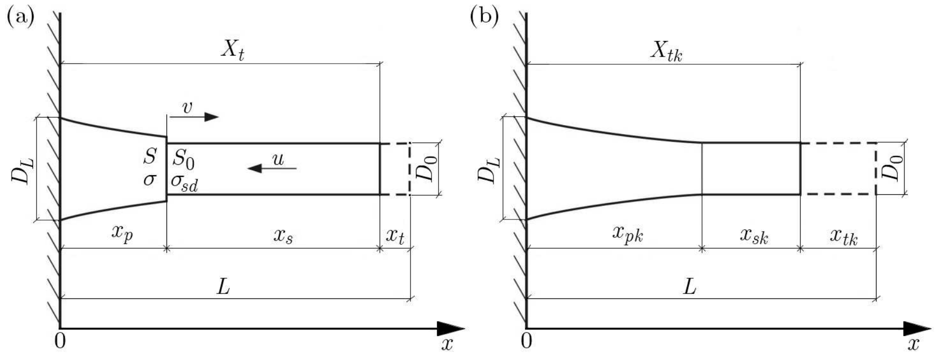

Fig. 1. Forms of deformation of the rod: (a) during the striking, (b) post-striking

Neglecting compressibility in the plastic deformed material, the equation of continuity takes the form

$$(u + v)S_0 = vS \tag{2.1}$$

This leads to the following expression for the longitudinal compressive strain behind the plastic wave front

$$\varepsilon = \frac{S - S_0}{S} = \frac{u}{u + v} \tag{2.2}$$

Defining x_s as the time-dependent undeformed length of the rod at any instant, we have

$$\frac{dx_s}{dt} = -(u + v) \tag{2.3}$$

Momentum considerations across the wave front yield the equation

$$\rho(u + v)u = \sigma - \sigma_{sd} \tag{2.4}$$

The equation of motion of the rigid part of the rod is

$$\rho x_s \frac{du}{dt} = -\sigma_{sd} \tag{2.5}$$

The stress-strain curve of the rod material is approximated by the rigid-plastic with a power strain hardening model in the following form

$$\sigma - \sigma_{sd} = E_w \varepsilon^m \tag{2.6}$$

where the coefficient E_w and exponent m are obtained experimentally for a given material.

Such a formulated problem, completed by initial and boundary conditions (see Sec. 3) has been solved in an analytical form in the next section of this paper.

3. Analytical solution of the problem

Eliminating the differential dt from Eqs. (2.3) and (2.5), and using relationship (2.2), we obtain

$$d(\rho u^2) = 2\sigma_{sd}\varepsilon \frac{dx_s}{x_s} \tag{3.1}$$

From formulae (2.2), (2.4) and (2.6) it follows that

$$\rho u^2 = E_w \varepsilon^{m+1} \tag{3.2}$$

Then, from Eqs. (3.1) and (3.2), we have

$$\frac{dx_s}{x_s} = \frac{m+1}{2}\frac{E_w}{\sigma_{sd}}\varepsilon^{m-1} d\varepsilon \tag{3.3}$$

At the moment $t = 0$, the rod is striking against the rigid target with velocity U. Then $x_s(0) = L$, and $u(0) = U$. Taking into account this fact, from expression (3.2), we obtain

$$\varepsilon(x_s = L) = \varepsilon_L = \Big(\frac{\rho U^2}{E_w}\Big)^{\frac{1}{m+1}} = \Big(\frac{U}{c}\Big)^{\frac{2}{m+1}} \qquad\qquad c = \sqrt{\frac{E_w}{\rho}} \tag{3.4}$$

Integration of Eq. (3.3) at initial conditions (3.4), gives

$$\varepsilon = \Big[\frac{2m}{m+1}\frac{\sigma_{sd}}{E_w}\ln\frac{x_s}{L} + \Big(\frac{U}{c}\Big)^{\frac{2m}{m+1}}\Big]^{\frac{1}{m}} = [y(x_s)]^{\frac{1}{m}} \tag{3.5}$$

where

$$y(x_s) = \frac{2m}{m+1}\frac{\sigma_{sd}}{E_w}\ln\frac{x_s}{L} + \Big(\frac{U}{c}\Big)^{\frac{2m}{m+1}} \tag{3.6}$$

In turn, straightforward algebraic transformations of relationships (2.2), (3.2) and (3.5), yield

$$\begin{aligned} &u(x_s) = c\varepsilon^{\frac{m+1}{2}} = c[y(x_s)]^{\frac{m+1}{2m}} \qquad\qquad u(x_s) + v(x_s) = c[y(x_s)]^{\frac{m-1}{2m}} \\ &v(x_s) = \frac{1-\varepsilon}{\varepsilon}u = c(1-\varepsilon)\varepsilon^{\frac{m-1}{2}} = c\Big\{[y(x_s)]^{\frac{m-1}{2m}} - [y(x_s)]^{\frac{m+1}{2m}}\Big\} \end{aligned} \tag{3.7}$$

In agreement with Eqs. (2.3), (3.5) and (3.7)2, differentiation of the function $\varepsilon[x_s(t)]$ with respect to t gives formulae for the strain rate, namely

$$\begin{aligned} \frac{d\varepsilon}{dt} &= \frac{2}{m+1}\frac{\sigma_{sd}}{E_w}[y(x_s)]^{\frac{1-m}{m}}\frac{1}{x_s}\frac{dx_s}{dt} = -\frac{2}{m+1}\frac{\sigma_{sd}}{E_w}[y(x_s)]^{\frac{1-m}{m}}\frac{u+v}{x_s} \\ &= -\frac{2}{m+1}\frac{\sigma_{sd}}{E_w}\frac{c}{x_s}[y(x_s)]^{\frac{1-m}{2m}} \end{aligned} \tag{3.8}$$

From Eqs. $(3.7)_2$ and (2.3) after transformation and separation of the variables x_s and t, we obtain

$$[y(x_s)]^{\frac{1-m}{2m}} d\Big(\frac{x_s}{L}\Big) = -d\Big(\frac{ct}{L}\Big) \tag{3.9}$$

The inverse function to the function $y(x_s)$, in agreement with notation (3.6), has form

$$\frac{x_s}{L} = \exp\Big[-\frac{m+1}{2m}\frac{E_w}{\sigma_{sd}}\Big(\frac{U}{c}\Big)^{\frac{2m}{m+1}}\Big]\exp\Big(\frac{m+1}{2m}\frac{E_w}{\sigma_{sd}}y\Big) \tag{3.10}$$

Differentiation of Eq. (3.10) gives

$$d\Big(\frac{x_s}{L}\Big) = \frac{m+1}{2m}\frac{E_w}{\sigma_{sd}}\exp\Big[-\frac{m+1}{2m}\frac{E_w}{\sigma_{sd}}\Big(\frac{U}{c}\Big)^{\frac{2m}{m+1}}\Big]\exp\Big(\frac{m+1}{2m}\frac{E_w}{\sigma_{sd}}y\Big)dy \tag{3.11}$$

Integrating Eq. (3.9) and using relation (3.11) as well as noting that for $t=0$ is $x_s = L$ and $y(L) = y_L$, we obtain

$$-\frac{ct}{L} = \frac{m+1}{2m}\frac{E_w}{\sigma_{sd}}\exp\Big[-\frac{m+1}{2m}\frac{E_w}{\sigma_{sd}}\Big(\frac{U}{c}\Big)^{\frac{2m}{m+1}}\Big]\int\limits_{y_L}^{y(x_s)} \xi^{\frac{1-m}{2m}}\exp\Big(\frac{m+1}{2m}\frac{E_w}{\sigma_{sd}}\xi\Big)d\xi \tag{3.12}$$

where

$$y_L = \Big(\frac{U}{c}\Big)^{\frac{2m}{m+1}} \qquad y(x_s) = \frac{2m}{m+1}\frac{\sigma_{sd}}{E_w}\ln\frac{x_s}{L} + y_L \qquad y(x_s) \leqslant \xi \leqslant y_L \tag{3.13}$$

So that the current length of the rigid part of the rod $x_s(t)$ has been obtained in form of an implicit function of t for any value of the coefficient m. In some cases, one can define the time-dependent function $x_s(t)$ explicitly (see example).

In turn, the function $x_p(t)$, in agreement with Fig. 1a, is defined by the formula

$$\frac{x_p(t)}{L} = 1 - \Big[\frac{x_s(t)}{L} + \frac{x_t(t)}{L}\Big] \qquad \text{where} \qquad \frac{x_t}{L} = \int\limits_0^{ct/L}\frac{u[x_s(\tau)]}{c}\,d\Big(\frac{c\tau}{L}\Big) \tag{3.14}$$

This quantity x_t/L, after making use of relations (3.7), (3.9) and (3.11) has been transformed to

$$\frac{x_t(x_s)}{L} = \frac{m+1}{2m}\frac{E_w}{\sigma_{sd}}\exp\Big(-\frac{m+1}{2m}\frac{E_w}{\sigma_{sd}}y_L\Big)\int\limits_{y(x_s)}^{y_L}\xi^{\frac{1}{m}}\exp\Big(\frac{m+1}{2m}\frac{E_w}{\sigma_{sd}}\xi\Big)\,d\xi \tag{3.15}$$

In agreement with expressions (2.2), $(3.7)_3$ and $(3.7)_2$ the current diameter of the plasticly deformed part of the rod D_p has been determined by the formula

$$D_p(x_s) = D_0\sqrt{\frac{u+v}{v}} = \frac{1}{\sqrt{1-[y(x_s)]^{\frac{1}{m}}}}D_0 \tag{3.16}$$

Thus have been defined all the dynamic parameters of the rigid-plastic rod with power strain hardening by means of the implicit function of x_s. The quantity x_s is a time-dependent function defined by Eqs. (3.12) and (3.13). In the example (next Section) these parameters are presented in form of an elementary time-dependent function during the striking and their final post-impact values.

4. Example

Uniform annealed (500°C, 1 h) cooper (Cu-ETP) rods of the initial dimensions: length $L = 48$ mm and diameter $D_0 = 12$ mm were used in the examinations. The mechanical parameters of cooper are: density $\rho = 8900\,\mathrm{kg/m^3}$, engineering static yield stress $R_{0.2} = 84$ MPa, Young's modulus $E = 130$ GPa. Stress-strain curves in compression of cooper for the true system $\sigma = \sigma(\varphi)$ and for the nominal system $\sigma = \sigma(\varepsilon)$ are depicted in Fig. 2, where φ is the true (logarithmic) strain, and ε is the nominal (engineering) strain, where $\varepsilon = 1 - \exp(-\varphi)$.

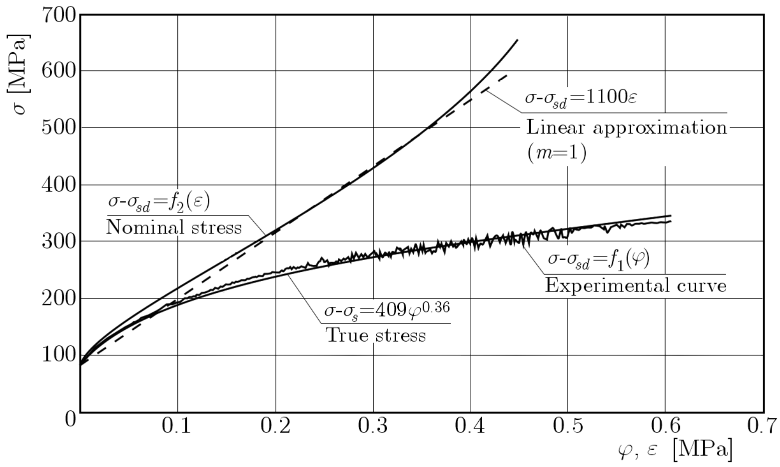

Fig. 2. Stress-strain curves in compression of annealed cooper for the true system $\sigma = \sigma(\varphi)$ and the nominal system $\sigma = \sigma(\varepsilon)$

The rods were driven by a firing gun to the initial speeds within the range from 60 m/s to 220 m/s. The rods impacted perpendicularly against the flat rigid target. The pictures of deformed rods after impact are shown in Fig. 3.

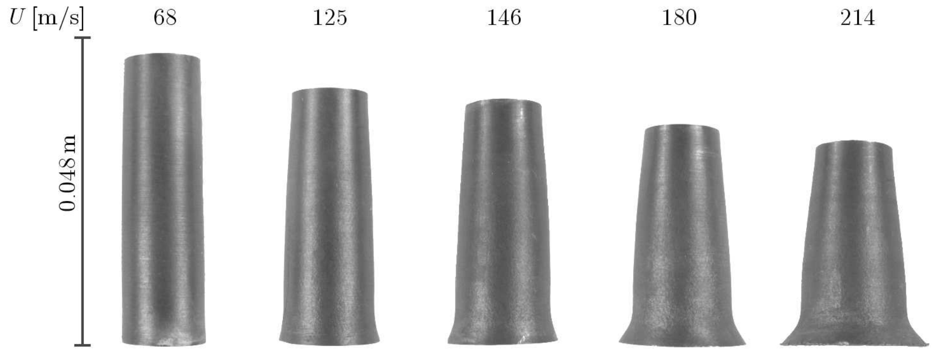

Fig. 3. Pictures of deformed rods

Figure 2 shows that the curve of the nominal stress-strain for $\varepsilon < 0.4$ can be well enough approximated by means of a straight line ($m = 1$), namely $\sigma - \sigma_{sd} = 1100\varepsilon$ [MPa] (dashed line in Fig. 2). The approximation error does not exceed several percent.

In agreement with the general solution of the problem presented in Section 3, for $m = 1$, we obtain

$$
\begin{gathered}
x = ct \qquad \frac{x_s}{L} = 1 - \frac{ct}{L} \qquad \frac{x_t}{L} = \Big(\frac{U}{c} - \frac{\sigma_{sd}}{E_w}\Big)\frac{ct}{L} - \frac{\sigma_{sd}}{E_w}\Big(1 - \frac{ct}{L}\Big)\ln\Big(1 - \frac{ct}{L}\Big) \\
\frac{x_p}{L} = \Big(1 - \frac{U}{c} + \frac{\sigma_{sd}}{E_w}\Big)\frac{ct}{L} + \frac{\sigma_{sd}}{E_w}\Big(1 - \frac{ct}{L}\Big)\ln\Big(1 - \frac{ct}{L}\Big) \qquad \frac{X_t}{L} = \frac{x_p}{L} + \frac{x_s}{L}
\end{gathered}
\tag{4.1}
$$

and

$$
\begin{aligned}
&\varepsilon = \frac{u}{c} = \frac{U}{c} + \frac{\sigma_{sd}}{E_w}\ln\Big(1-\frac{ct}{L}\Big) && \varepsilon_L = \varepsilon_{max} = \frac{U}{c}\\
&\frac{d\varepsilon}{dt} = -\frac{\sigma_{sd}}{E_w}\frac{c}{x_s} = -\frac{\sigma_{sd}}{E_w}\frac{c}{L-ct} && \sigma = \sigma_{sd} + E_w\varepsilon\\
&\sigma_L = \sigma_{max} = \sigma_{sd} + E_w\frac{U}{c} && u = U + c\frac{\sigma_{sd}}{E_w}\ln\Big(1-\frac{ct}{L}\Big)\\
&v = c - U - c\frac{\sigma_{sd}}{E_w}\ln\Big(1-\frac{ct}{L}\Big) && u + v = c = \text{const}\\
&D_p = D_0\Big[1-\frac{U}{c}-\frac{\sigma_{sd}}{E_w}\ln\Big(1-\frac{ct}{L}\Big)\Big]^{-\frac{1}{2}} && D_L = D_{p\,max} = D_0\Big(1-\frac{U}{c}\Big)^{-\frac{1}{2}}
\end{aligned}
\tag{4.2}
$$

The end of the striking process occurs at the instant when $\varepsilon(t_k) = u(t_k) = 0$. From this condition and Eq. $(4.2)_1$, it follows that

$$
t_k = \frac{L}{c}\Big[1-\exp\Big(-\frac{E_w}{\sigma_{sd}}\frac{U}{c}\Big)\Big]
\tag{4.3}
$$

Then the final values of the parameters of the rod, in agreement with the above quoted formulae, one may define by the following expressions

$$
\begin{aligned}
&\frac{x_{sk}}{L} = \exp\Big(-\frac{E_w}{\sigma_{sd}}\frac{U}{c}\Big) && \frac{x_{tk}}{L} = \frac{U}{c} - \frac{\sigma_{sd}}{E_w}\Big(1-\frac{x_{sk}}{L}\Big)\\
&\frac{x_{pk}}{L} = \Big(1-\frac{x_{sk}}{L}\Big)\Big(1+\frac{\sigma_{sd}}{E_w}\Big) - \frac{U}{c} && \frac{X_{tk}}{L} = 1 - \frac{x_{tk}}{L}
\end{aligned}
\tag{4.4}
$$

From Eq. $(4.2)_1$ and condition $\varepsilon(t_k) = 0$ as well as formula (4.3), after transformations, we obtain

$$
\sigma_{sd} = \frac{E_w(U/c)}{\ln(L/x_{sk})}
\tag{4.5}
$$

The simple Taylor formula for σ_{sd} has form (Taylor, 1948)

$$
\sigma_{sd} = \frac{L-x_{sk}}{2(L-X_{tk})}\frac{\rho U^2}{\ln(L/x_{sk})}
\tag{4.6}
$$

So that all dynamical parameters of the annealed cooper (Cu-ETP) rod loaded according to the Taylor test have been defined explicitly by elementary time-dependent functions during the striking, and by closed form formulae in post-impact. Bearing in mind this fact, we will make an analysis only for some of them.

The experimental data obtained from measurements of the deformed post-impact cooper rods and the results of theoretical calculations for several values of the impact velocity are listed in Table 1. One ought to notice that time of the impact action is very short, about 0.13 ms in the whole range of variation of the impact velocity (60 m/s – 220 m/s). It corresponds with the literature data (Whiffin, 1948; Jones *et al.*, 1987, 1997).

From analysis of the variations of quantities listed in Table 1 with respect to the values impact velocity, it follows that the parameters σ_{sd}, $\overline{X}_{tk}$, $\overline{x}_{pk}$ and ε_L vary approximately linearly with respect to an increase in the impact velocity within the range 60 m/s – 220 m/s, and one may them well enough extrapolate by means of the following expressions:

— for formula (4.5)

$$
\sigma_{sd} = 65 + 0.712(U - 68)
\tag{4.7}
$$

Table 1. Comparison of the experimental data obtained from Taylor tests with theoretical results calculated for annealed cooper (Cu-ETP)

U [m/s]		68	125	146	180	214
σ_{sd} [MPa]	Paper (4.5)	65	111	122	142	169
	Taylor (4.6)	61	89	96	112	133
$t_k \cdot 10^3$ [s]		0.13200	0.13315	0.13400	0.13457	0.13458
$\overline{x}_{sk} = x_{sk}/L$	theoretical	0.03733	0.02904	0.02325	0.01869	0.01860
	experimental	0.03752	0.02910	0.02292	0.01877	0.01873
$\lvert \varDelta x_{sk} \rvert$ [%]	difference	0.5	0.2	1.5	0.9	0.2
$\overline{X}_{tk} = X_{tk}/L$	theoretical	0.863	0.741	0.691	0.613	0.539
	experimental	0.901	0.790	0.744	0.682	0.622
$\lvert \varDelta \overline{X}_{tk} \rvert$ [%]	difference	4.3	6.2	7.2	10.1	13.3
$\overline{x}_{pk} = x_{pk}/L$	theoretical	0.825	0.712	0.668	0.594	0.521
	experimental	0.864	0.757	0.721	0.663	0.603
$\lvert \varDelta x_{pk} \rvert$ [%]	difference	4.5	5.9	7.4	10.4	13.7
D_L/D_0	theoretical	1.12	1.25	1.31	1.44	1.60
	experimental	1.11	1.31	1.43	1.65	1.97
$\lvert \varDelta (D_L/D_0) \rvert$ [%]	difference	1.06	4.97	8.58	13.03	18.48
ε_L	theoretical	0.19	0.36	0.42	0.51	0.61
	experimental	0.18	0.42	0.51	0.63	0.74
$\lvert \varDelta \varepsilon_L \rvert$ [%]	difference	5.6	14.3	17.6	19.0	17.8

— for Taylor formula (4.6)

$$\sigma_{sd} = 61 + 0.493(U - 68) \tag{4.8}$$

— theoretical expressions

$$\begin{aligned} &\overline{X}_{tk} = \frac{X_{tk}}{L} = 0.002219(214 - U) + 0.539 \qquad && \overline{x}_{pk} = \frac{x_{pk}}{L} = 0.002082(214 - U) + 0.521 \\ &\varepsilon_L = \frac{U}{c} && c = \sqrt{\frac{E_w}{\rho}} = 351\,\mathrm{m/s} \end{aligned} \tag{4.9}$$

— experimental expressions

$$\begin{aligned} &\overline{X}_{tk} = \frac{X_{tk}}{L} = 0.001911(214 - U) + 0.622 \qquad && \overline{x}_{pk} = \frac{x_{pk}}{L} = 0.001788(214 - U) + 0.603 \\ &\varepsilon_L = 0.003836(U - 68) + 0.18 \end{aligned} \tag{4.10}$$

where, in the quoted above relationships values of the impact velocity U ought to substitute in m/s.

The absolute value of the differences between the experimental and theoretical results of the studied parameters increase from several to over a dozen percent together with the increase in the impact velocity from 60 m/s to 220 m/s. The cause of this fact is the influence of the radial spreading (inertia) of the tested specimen on its longitudinal compressive strain. The spreading increases together the increasing impact velocity, especially as related to the annealed cooper (Cu-ETP), which is very sensitive to plastic deformation (Jones *et al.*, (1997). In case of alloy steels (e.g. nickel-chrome steel), the differences are considerably smaller (Włodarczyk *et al.*, 2012). The quoted above facts show that the presented here one-dimensional theory, with respect to the Taylor impact problem, is permissible for materials with strain hardening and moderate values of impact velocities.

Variations of the relative speeds v/c and u/c versus the Lagrangian coordinate $x/L = ct/L$ for selected values of the impact velocity U are presented in Fig. 4.

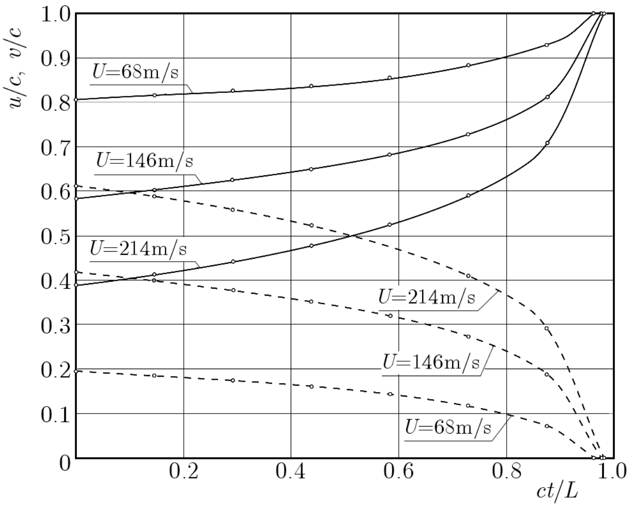

Fig. 4. Variations of relative speeds v/c (solid line) and u/c (dashed line) versus Lagrangian coordinate ct/L for selected values of the impact velocity U

As can seen, the propagation speed of the plastic wave front v intensively increases in the surroundings of the fore-front of the undeformed portion of the rod, and at $x = L - x_{sk}$ reaches the maximum value $v = v_{max} = c = \sqrt{E_w/\rho}$. On the contrary, the velocity of the undeformed part of the rod u intensively decreases here, and is equal to 0 at $x = L - x_{sk}$. The sum of these velocities $u(ct/L) + v(ct/L) \equiv c$ is constant. This fact was noticed by Lee and Tupper (1951) in their computational scheme. In Taylor (1948) and Whiffin (1948) investigations, it was assumed that $v(t) \equiv$ const and $du/dt \equiv$ const. It is an erroneous assumption which gives unreal results.

The graph of variation of the longitudinal compressive strain behind the plastic wave front ε with respect to the Lagrangian coordinate $x/L = ct/L$ is the same as the graph of u/c ($\varepsilon = u/c$ – see $(4.2)_1$).

Figure 5 shows that the absolute value of the strain rate intensively increases at the immediate surroundings of the plastic wave front from the target side at the final stage of the impact process. It reaches maximum values at the instant $t = t_k$ which is determined by the impact velocity, e.g. $[\dot{\varepsilon}(t_k)] = \Big[\dot{\varepsilon}\Big]_{max} = 11502\,\mathrm{s}^{-1}$ for $U = 68\,\mathrm{m/s}$, on the other hand $[\dot{\varepsilon}]_{max} = 59845\,\mathrm{s}^{-1}$ for $U = 214\,\mathrm{m/s}$. One ought to notice that in excess of 90%, the specimen length is deformed quasi-statically.

The absolute value of the maximum strain rate $|\dot{\varepsilon}|_{max}$ [s^{-1}], and the absolute value of the average strain rate $|\dot{\varepsilon}|_{mid}$ [s^{-1}] are determined by formulae

$$
\begin{aligned}
|\dot{\varepsilon}|_{max} &= \frac{\sigma_{sd}}{E_w}\frac{1}{1-\eta_k}\frac{c}{L} \qquad\qquad \eta_k = \frac{ct_k}{L} \\
|\dot{\varepsilon}|_{mid} &= \frac{1}{\eta_k}\frac{\sigma_{sd}}{E_w}\int\limits_0^{\eta_k}\frac{d\eta}{1-\eta} = \frac{\sigma_{sd}}{E_w}\frac{|\ln(1-\eta_k)|}{\eta_k}
\end{aligned}
\tag{4.11}
$$

From analysis of formula $(4.11)_3$, it follows that the quantity $|\dot{\varepsilon}|_{mid}$ monotonically increases from $|\dot{\varepsilon}|_{mid} = 0.202\,\mathrm{s}^{-1}$ for $U = 68\,\mathrm{m/s}$ to $|\dot{\varepsilon}|_{mid} = 0.647\,\mathrm{s}^{-1}$ for $U = 214\,\mathrm{m/s}$. Accordingly, the assumption that $\sigma = \sigma(\varepsilon)$ at the formulation of the problem is permissible.

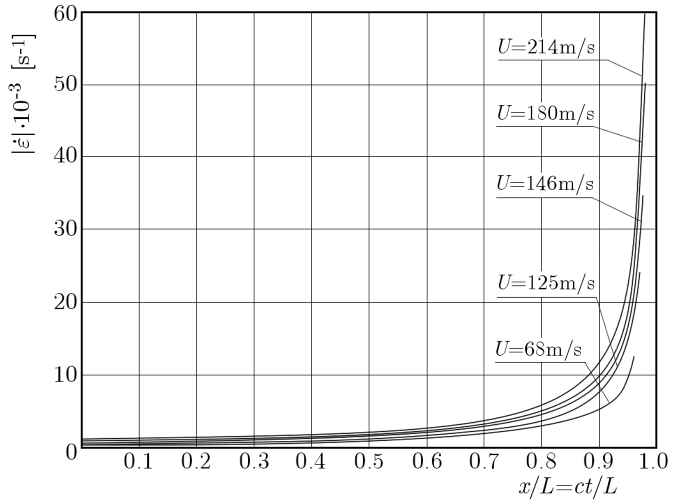

Fig. 5. Variation of the absolute values of the strain rate $|\dot{\varepsilon}|$ versus Lagrangian coordinate $x/L = ct/L$ for selected values of the impact velocity U

Figure 6 shows, among other things, graphs of the parameter $(\sigma - \sigma_{sd})/E_w$ in the deformed part of the rod. As seen, the stress is constant in the whole deformed part of the rod at the given instant t, and is equal to the stress in the fore-front of the plastic wave from the target side at the same time t. The stress decreases together with propagation of the wave front down to value σ_{sd} and suddenly decreases to 0 at the final impact process – the rod separates from the target.

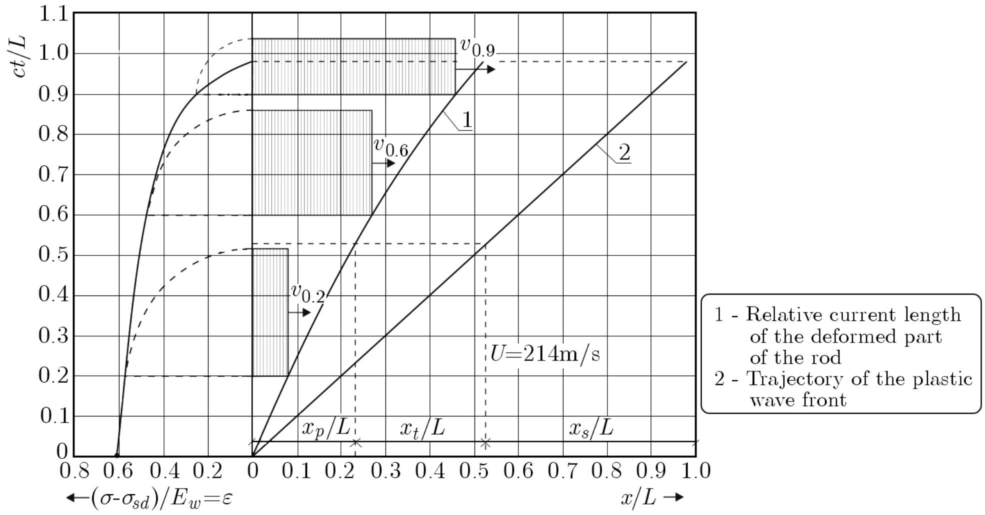

Fig. 6. Graphs of the parameter $(\sigma - \sigma_{sd})/E_w$ in the deformed portion of the rod at the instants: $t = 0.2L/c$, $0.6L/c$ and $0.9L/c$ for the impact velocity $U = 214\,\mathrm{m/s}$

5. Final conclusions

The main conclusions from the above theoretical and experimental investigations may be briefly summarized as follows:

- The analytical solution of the dynamics of the one-dimensional rigid-plastic cylindrical rod striking against a rigid target has been found in this paper. It is an extended original Taylor perfectly-plastic model for materials with power strain hardening.

- The extended theory takes into account the step change of the nominal stress across the plastic wave front in the equation of momentum balance, because this front is a source of strong discontinuity in physical parameters. In the Taylor model, it was assumed that the stress is continuous across this front.
- The dynamical parameters of the annealed cooper (Cu-ETP) rod were examined by means of the Taylor test.
- The static stress-strain curves of this cooper rod for the true system, $\sigma = \sigma(\varphi)$, and for the nominal system, $\sigma = \sigma(\varepsilon)$, were well enough approximated by the following functions $\sigma - \sigma_{sd} = 409\varphi^{0.36}$ MPa, and $\sigma - \sigma_{sd} = 1100\varepsilon$ MPa (see Fig. 2).
- The absolute velocity of propagation of the plastic wave front v and the absolute speed of displacement of the rear of the rod u are individually intensively time-dependent functions (see Fig. 4). On the other hand, their sum, $u(t) + v(t) = c$ is constant. In Taylor (1948) and Whiffin (1948) investigations, it was assumed that $v(t) \equiv$ const and $du/dt \equiv$ const. These are far-reaching simplifications contradictory to reality.
- The absolute value of the strain rate $|\dot{\varepsilon}|$ intensively increases only in the immediate surrounding of the plastic wave front at the final stage of the impact process, and reaches the maximum value at the instant $t = t_k$. On the contrary, in excess of 90%, the specimen length is deformed quasi-statically. The average absolute value of the strain rate $|\dot{\varepsilon}|_{mid}$ increases from $|\dot{\varepsilon}|_{mid} = 0.202\,\mathrm{s}^{-1}$ for $U = 68$ m/s to $|\dot{\varepsilon}|_{mid} = 0.647\,\mathrm{s}^{-1}$ for $U = 214$ m/s. Accordingly, the influence of the strain rate on the stress-strain curve for annealed cooper (Cu-ETP) is negligible.
- The stress is constant in the whole deformed section of the rod at the given moment t, and is equal to stress in the fore-front of the plastic wave from the target side at the same time t (see Fig. 7).
- All final parameters of the deformed rods post-impact, together with the dynamic yield stress for the examined cooper, are approximately linear functions of the impact velocity U, see formulae (4.7)-(4.10).
- The absolute values of differences between the experimental and the theoretical results of the studied parameters increase from several to a dozen percent with the increasing impact velocity (see Table 1). These differences are caused by the radial spreading during impact, especially with respect to annealed cooper, which is very sensitive to plastic deformation.
- The above quoted facts show that the presented here one-dimensional theory, with respect to the Taylor impact problem, is permissible for materials with strain hardening and moderate impact velocities.
- According to the authors, the results presented in this paper have applicable and cognitive effects. The derived in a closed form formulae, which are written by elementary functions, give investigators and engineers insight into the interaction of the physical parameters of the rod during the impact process and post-impact.

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