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A COMPARISON BETWEEN THE EXPONENTIAL AND LIMITING FIBER EXTENSIBILITY PSEUDO-ELASTIC MODEL FOR THE MULLINS EFFECT IN ARTERIAL TISSUE

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This study compares the capability of two different mathematical forms of the so-called softening variable to describe strain-induced stress softening observed within cyclic uniaxial tension of the human thoracic aorta. Specifically, the softening variable, which serves as the stress reduction factor, was considered to be tangent hyperbolic-based and error function-based. The mechanical response of the aorta was assumed to be pseudo-hyperelastic, incompressible and anisotropic. The strain energy density function was employed in a classical exponential form and in a not well-known limiting fiber extensibility model. This study revealed that both the limiting fiber extensibility and exponential models of the strain energy describe mechanical the response of the material with similar results. It was found that it is not a matter which kind of the softening variable is employed. It was concluded that such an approach can fit the Mullins effect in the human aorta, however the question of the best fitting model still remains.

Key words: aorta, limiting fiber extensibility, Mullins effect

1. Introduction

Due to cardiac cycle, arteries are subjected to cyclic loading and unloading in their physiological conditions. In vitro, the mechanical response of arteries is mostly studied within the cyclic inflation-extension and the tensile test. One of the irreversible effects observed during these experiments is the Mullins effect. This strain-induced softening phenomenon is well known in elastomer mechanics. It is characterized by the following features: when a so-called virgin material (previously undeformed) is loaded to a certain value of the deformation, the stress-strain curve follows the so-called primary loading curve. Subsequent unloading exhibits the stress softening. Next reloading follows the former unloading curve until the previous maximum strain is reached. At this moment, when the previous deformation maximum is exceeded, the stress-strain path starts to trace the primary loading curve (Diani *et al.*, 2009). This definition describes quite ideal material behavior. Within the real experiment the unloading and the reloading may not match exactly due to the hysteresis. The comparison between the ideal and true cyclic softening is depicted in Fig. 1.

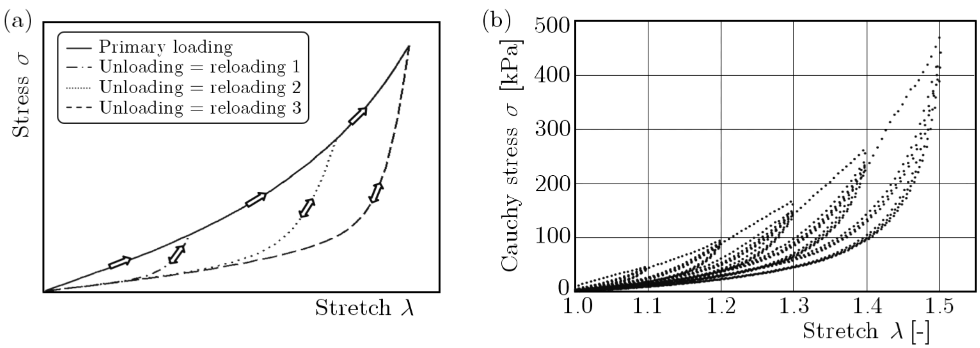

Fig. 1. (a) Idealized Mullines effect, (b) true experimental data

The elastic response of blood vessel walls is significantly nonlinear, anisotropic and requires large strains to be incorporated (Holzapfel *et al.*, 2000; Humphrey, 2003). Such behavior is usually modeled within the framework of hyperelasticity which presumes existence of an elastic potential (strain energy density function, SEDF), and constitutive equations are obtained by differentiation of the elastic potential with respect to the strain tensor.

Although existence of the stress softening within in vitro cyclic loading of blood vessels has been known for long time only few attempts have been made to develop new theories. Fung *et al.* (1979) proposed to model the mechanical response of arteries as pseudo-elastic. In this concept, an artery wall is considered to be elastic, however loading and unloading responses are defined with different constitutive equations. Nowadays, models capture the Mullins effect using either the Continuum Damage Mechanics (CDM) or the pseudo-elasticity theory.

Damage models describe the Mullins effect incorporating a damage parameter which serves as a reduction factor in the strain energy density function. The damage parameter is considered as an internal variable (Simo, 1987) and can be applied in a more general manner for an arbitrary irreversible process (Holzapfel, 2000, chap. 6.9-11). The damage parameter can be full-strain-history dependent (continuous damage) or maximum-value dependent (discontinuous damage).

Another concept describing the Mullins effect is the theory of the pseudo-elasticity developed by Ogden and Roxburgh (1999). They suggested the introduction of the a variable (softening variable) into the strain energy function which is thereafter called the pseudo-strain energy density function. The softening variable then governs the energy density and switch on and off between the primary and softened response of a material. The particular model of the pseudo-energy function suggested by Ogden and Roxburgh (1999) results in similar symbolic form as in CDM. Successful application of the pseudo-elasticity in rubber mechanics were reported by Dorfmann and Ogden (2003, 2004) and Elías-Zúñiga (2005). Peña and Doblaré (2009) proposed the application of this theory for anisotropic biological materials considering different softening variables for an extracellular matrix and fibers. This model successfully described the softening behavior of sheep vena cava.

The aim of this paper is to compare the pseudo-elastic models for the Mullins effect using different forms of the SEDF. In the first case, the mechanical response of the artery wall is described with the strain energy function adopted in the exponential form (Holzapfel *et al.*, 2000). In the second case, not well-known the limiting fiber extensibility form of the SEDF is applied. The comparison is shown by fitting the experimental data recorded within uniaxial tension of human thoracic aorta. The artery is considered to be nonlinear, incompressible and anisotropic continuum. Here we focus only on the strain-induced softening. Temperature, heat and strain-rate effects are not concerned as well as the active response (smooth muscle fibers) of arterial tissue.

2. Methods

2.1. Experiment – uniaxial tension

In order to illustrate the Mullins effect in human aorta, cyclic uniaxial tension tests were performed on MTS Mini Bionix testing machine (MTS, Eden Prairie, USA). Samples of healthy human thoracic aorta were resected from cadaveric donor (male, 47 years old) with the approval of the Ethic Committee of the University Hospital Na Kralovskych Vinohradech in Prague. Respecting the anisotropy of an aorta, samples were resected in the circumferential and longitudinal direction. The total number of samples was eight (five oriented longitudinally, three oriented circumferentially).

Five levels of maximum stretch were performed during the tests: $\lambda_m = 1.1$, 1.2, 1.3, 1.4 and $\lambda_m = 1.5$, where λ_m is the maximum ratio between the current length l and the referential length L. The representative of the recorded data is shown in Fig. 1b. Each λ_m level was preformed as four-cycle of the loading and unloading. Considering the incompressibility of the tissue, the loading stress was obtained according to the following relation

$$\sigma = \frac{F}{s} = \frac{Fl}{LBH} \tag{2.1}$$

Here F denotes the applied force and s the current cross-section. B and H denote the width and the thickness of the sample in the reference (zero-stress) configuration. Dimensions of the samples in the reference configurations were determined within the analysis of digital photographs (thickness) and by a caliper (length and width). Strain rate was 120 mm/min.

2.2. Constitutive modeling – pseudo-elasticity

A deformation is considered as the homeomorphic mapping φ between material particles embedded in the reference Cartesian coordinate system $\{O; X_1X_2X_3\}$ and the same particles embedded in the spatial Cartesian coordinate system $\{O; x_1x_2x_3\}$. The reference position vector $\boldsymbol{X}$ is mapped onto $\boldsymbol{x}$ according to $\boldsymbol{x} = \boldsymbol{\varphi}(\boldsymbol{X})$. The deformation is then described with the deformation gradient $\mathbf{F} = \partial\boldsymbol{\varphi}/\partial\boldsymbol{X}$ which generates right Cauchy-Green strain tensor $\mathbf{C} = \mathbf{F}^\top\mathbf{F}$. Within the modeling of the uniaxial tension we restrict the attention only to pure homogenous strains $x_i = \lambda_i X_i$ $(i = 1, 2, 3)$. Thus the deformation gradient is of the form $\mathbf{F} = \operatorname{diag}[\lambda_1, \lambda_2, \lambda_3]$.

The response of the artery during the primary loading by uniaxial tension is modeled as incompressible, hyperelastic and anisotropic. The anisotropy is generated with two preferred directions in continuum which are perfectly aligned with β and $-\beta$ angles. These angles lay in the X_1X_2-plane of the sample. It is assumed that both preferred directions are mechanically equivalent and hence the resulting degree of anisotropy is called local orthotropy (for details see p. 272 in Holzapfel, 2000; or in Holzapfel *et al.*, 2000). The hyperelastic behavior of the material is determined by the strain energy density function W_0. Here index 0 denotes the primary loading response. The stored energy is additively decoupled into isotropic and anisotropic part

$$W_0 = W_0^{ISO}(I_1) + W_0^{ANISO}(I_4, I_6) \tag{2.2}$$

I_1, I_4 and I_6 denote the invariants of the Right Cauchy-Green deformation tensor $\mathbf{C}$. The first invariant of $\mathbf{C}$ can be expressed as $I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$. The invariants I_4 and I_6 arise from material anisotropy. Due to the mechanical equivalency between the preferred directions $I_4 = I_6$. Therefore W_0 is considered to be $W_0 = W_0(I_1) + 2W_0(I_4)$.

Two specific forms of strain energy (2.2) were incorporated. The first one corresponds to the exponential function proposed by Holzapfel *et al.* (2000)

$$W_0^{HGO} = \frac{c}{2}(I_1 - 3) + \frac{k_1}{k_2}\Big[\exp\Big(k_2(I_4 - 1)^2\Big) - 1\Big] \tag{2.3}$$

where c and k_1 denote stress-like material parameters and k_2 is the dimensionless parameter. Such a model is invariant-based modification of the classical Fung-type exponential model which was many times successfully applied in soft tissue biomechanics. The second form of the SEDF was incorporated via the limiting fiber extensibility model proposed by Horgan and Saccomandi (2005), see equation (2.4)

$$W_0^{HS} = \frac{c}{2}(I_1 - 3) - \mu J_f \ln\Big(1 - \frac{(I_4 - 1)^2}{J_f^2}\Big) \tag{2.4}$$

Here c and μ are stress-like material parameters and J_f is the dimensionless, so-called limiting extensibility parameter. Both W_0^{HGO} and W_0^{HS} include the isotropic Neo-Hookean term linked with the matrix of biological composite. The material nonlinearity, related to the presence of wavy collagen fibers in the soft tissue is, however in (2.3) and (2.4), captured in significantly different manners. Exponential function (2.3) reflects the famous result of Y.C. Fung that the stiffness is proportional to the applied stress. In contrast to (2.3) the limiting extensibility model restricts admissible deformation to a certain maximum value under which the stored energy approaches infinity. The deformation admissible in (2.4) has to satisfy condition

$$\frac{(I_4 - 1)^2}{J_f^2} < 1 \tag{2.5}$$

which implies that $I_4 < J_f + 1$. When the unit vector aligned with the preferred direction $\boldsymbol{M} = \cos(\beta)\boldsymbol{E}_1 + \sin(\beta)\boldsymbol{E}_2$ is considered then the additional invariant I_4 can be expressed as $I_4 = \boldsymbol{M}(\mathbf{C}\boldsymbol{M})$. Combining equations $\mathbf{F} = \operatorname{diag}[\lambda_1, \lambda_2, \lambda_3]\mathbf{C} = \mathbf{F}^\top\mathbf{F}$, and $I_4 = \boldsymbol{M}(\mathbf{C}\boldsymbol{M})$ we arrive at

$$I_4 = \lambda_1^2 \cos^2\beta + \lambda_2^2 \sin^2\beta \tag{2.6}$$

Now it is clear that I_4 can be considered as the square of the stretch in the preferred direction that must be invariant under a change of the frame of reference. Here introduced the limiting fiber extensibility model was proposed by Horgan and Saccomandi (2005) and is adopted with minor modification in the square of J_f. It may be regarded as the extension of the simple phenomenological model originally proposed by Gent (1996) which is suitable to capture large-strain stiffening behavior of isotropic materials. The applicability of such a class of models in soft tissue mechanics was pointed out by Horgan and Saccomandi (2003) and Ogden and Saccomandi (2007). It is worth noting that limiting fiber extensibility model (2.4) offers some advantages. Incorporating SEDF in form (2.4) one can obtain closed analytical solutions of some boundary-value problems important in blood vessel biomechanics like e.g. an inflation-extension of a thick-walled tube (Horny *et al.*, 2008). This is in contrast to classical (Fung-type) exponential models. Constitutive equations for the primary response of the hyperelastic incompressible material are now obtained as expressed in (2.7). Here the principal stresses are denoted by σ_{0i}, and p_0 denotes a Lagrange multipiler associated with the incompressibility constraint $\lambda_1\lambda_2\lambda_3 = 1$

$$\sigma_{0i} = \lambda_i \frac{\partial W_0}{\partial \lambda_i} - p_0 \qquad i = 1,2,3 \tag{2.7}$$

In order to reproduce softened behavior during unloading and reloading, we introduce the softening factors η into constitutive equations (2.7). Thus the same form of the strain energy W_0 still takes place here

$$\sigma_i = \eta\lambda_i \frac{\partial W_0}{\partial \lambda_i} - p \qquad i = 1,2,3 \tag{2.8}$$

The stress is reduced by the factor $\eta \in [0;1]$. In this study we only concern with the idealized Mullins effect, thus unloading and reloading paths match exactly. Now the softening factors must govern the constitutive equations into expression (2.7) for the primary loading and into (2.8) for the unloading/reloading. Within uniaxial tension of the sample in the direction j, it is satisfied by definition

$$\eta = \begin{cases} 1 & \text{for} \quad \lambda_j = \lambda_{j\,max} \\ \eta(\lambda_1, \lambda_2, \lambda_3) & \text{for} \quad \lambda_j < \lambda_{j\,max} \end{cases} \tag{2.9}$$

Definition (2.9) says that if the sustained stretch in the direction of the loading is maximal then there is no softening. And when the sustained stretch in the direction of the loading is smaller than the maximum value in the history of the loading then the softening occurs.

A particular form of the mathematical expression for η must be defined. We adopt forms for softening factors as was originally introduced by Ogden and Roxburgh (1999) and Dorfmann and Ogden (2003). Hence, let η be of the form

$$\eta = 1 - \frac{1}{r} f\Big(\frac{W_m - W_0(\lambda_1,\lambda_2,\lambda_3)}{m}\Big) \tag{2.10}$$

where $f(t)$ is $\operatorname{erf}(t)$ or $\tanh(t)$. W_m denotes the maximum value of W_0 reached within the loading history. r and m are real positive parameters. The resulting model, regardless if $\operatorname{erf}(t)$ or $\tanh(t)$ is operative in (2.10), contains six material parameters. It is explicitly c, μ, J_f, β, r and m when the limiting fiber extensibility model W_0^{HS} is applied, and c, k_1, k_2, β, r and m in the case of the exponential model W_0^{HGO}.

3. Results – fitting the model

The capability of the introduced models was tested within the regression analysis of uniaxial tension experimental data. The total number of tested samples was eight and they exhibited similar results. Only one pair of sample (one strip in the circumferential and one in the longitudinal direction of the artery) was considered for the regression. The selected samples were obtained from one donor and resected in the same region of the thoracic aorta.

With respect to anisotropy exhibited by human arteries, W_0 has to be considered as a function of λ_1, λ_2 and λ_3. Incorporating the incompressibility assumption, the out-of-plane stretch λ_3 is eliminated. However, our experimental equipment does not allow one to measure transversal stretches. In order to overcome this drawback we employed the boundary condition $\sigma_{transversal} = 0$ which is used to calculate transversal stretches upon the uniaxial state of stress. The data from the circumferential and longitudinal experiment were optimized simultaneously to find the minimum of objective function

$$Q = \frac{1}{Mean^2(\sigma_{i,j}^{EXP})} \sum_{i,j=1}^{2} \sum_{k} \Big(\sigma_{i,j}^{EXP} - \sigma_{i,j}^{MOD}\Big)_k^2 \tag{3.1}$$

Here upper indices EXP and MOD indicate the experimental observation and the model prediction, respectively. The observed stresses were calculated according to (2.1). Model predictions were based on equation (2.8) incorporating the definition of stress reduction factors (2.9) and (2.10). The lower indices i and j are operative representing the direction of demanding stress and the direction of the uniaxial tensile test, respectively. It means that $\sigma_{i,j}$ denotes the stress in the direction i during the loading in the direction j. Only the data from the first two cycle-levels were included in the regression ($\lambda_m = 1.1$ and 1.2). The regression was performed with the optimization package in Maple 13 (Maplesoft, Waterloo, Canada).

Both W_0^{HGO} and W_0^{HS} were used in order to compare their suitability. The softening factor η was employed in both mathematical forms; $\eta = 1 - r^{-1}\tanh(t)$ and $\eta = 1 - r^{-1}\,\mathrm{erf}\,(t)$. The estimated material parameters are listed in Table 1. The model predictions are compared with the experiment in Fig. 2. The regression results were also checked on the condition $I_4 > 1$. Because I_4 models reinforcement with collagen fibers, they may contribute to the stored energy only in tensile strains. It was found that this condition was satisfied in all data points. Figure 3 shows the stress ratio σ_0/σ computed from the experimental data which can be considered as the observation of η.

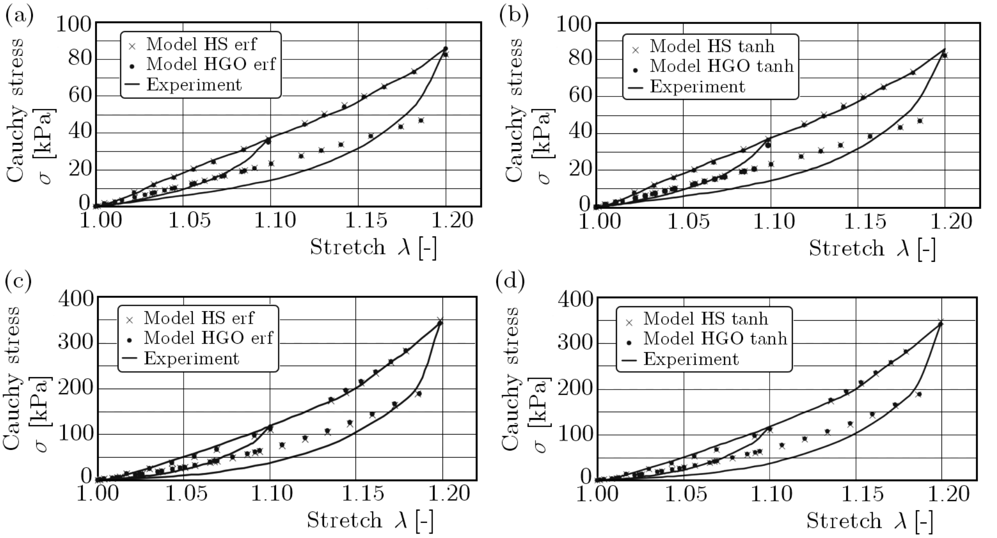

Fig. 2. (a), (b) The Mullins effect – circumferential strip, (c), (d) the Mullins effect – longitudinal strip

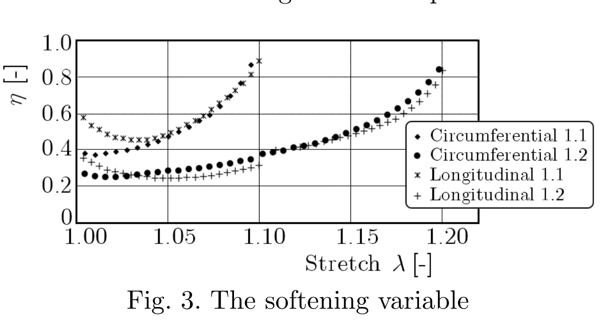

Fig. 3. The softening variable

4. Discussion

This study presents the comparison between two (primary) strain energy density functions, W_0^{HS} and W_0^{HGO}, used in the pseudo-elastic model for the Mullins effect observed within the periodic uniaxial tension of arterial tissue. The strain-induced stress softening has been described by means of the stress-reduction factors η which can be simply considered as the stress ratio σ_{0i}/σ_i, where σ_{0i} takes place during the primary loading and σ_i corresponds to the softened behavior. A particular mathematical form of η has been adopted from the pseudo-elasticity theory introduced by Ogden and Roxburgh (1999) and Dorfmann and Ogden (2003, 2004). These reduction factors, $\eta = 1 - r^{-1}\tanh(t)$ and $\eta = 1 - r^{-1}\,\mathrm{erf}\,(t)$ were reported to be successful in the description of the Mullins effect observed in particle-reinforced rubber, and herein were used for healthy human thoracic aorta. Based on the comparison in Fig. 2, one can conclude that the primary response of the aorta can be successfully modeled by both W_0^{HS} and W_0^{HGO} strain energy functions. But the results of predictions obtained for the softening behavior are not quite satisfactory. The data suggests that it is not a matter if $\tanh(t)$ or $\mathrm{erf}\,(t)$ is employed in the model for the softening variable (reduction factor). Almost the same predictions are also obtained by incorporating W_0^{HS} and W_0^{HGO} into the softening model. Nevertheless, the quality of the models is controversial. They can mimic the softening behavior in principle, however the character seems to be almost piecewise linear which is in contrast to the experimental data. It was tried to find parameters with better approximation ability but the optimization procedure always converged to the presented parameters.

Model parameters obtained within the minimization of objective function (3.1) are listed in Table 1. Numeric values of parameters obtained for W_0^{HS} and W_0^{HGO} are similar, which confirms the graphically displayed results in Fig. 2. The shear modulus related with the Neo-Hookean term, which is usually linked to the response of isotropic matrix, was found to be almost the same in every model (~ 110 kPa). This is slightly higher than usually reported values around tens of kPa. Stress-like parameters in the nonlinear terms of W_0^{HS} and W_0^{HGO}, μ and k_1, were obtained in hundreds of kPa, which is in accordance with some values summarized in Holzapfel (2009).

There are only few papers reporting the limiting extensibility parameter J_f. In our previous studies, values of J_f ranging from 0.1 up to 1.044 were found ($J_f = 0.1202$ for thoracic aorta in Horny *et al.* (2010); $J_f = 1.044$ for abdominal aorta in Horny *et al.* (2008); $J_f = 0.7498$ for saphenous vein coronary artery bypass graft after 35 months of remodeling in Horny *et al.* (2009); and $J_f \approx 0.3$ for human vena cava Horny *et al.* (2011)). Ogden and Saccomandi used $J_f = 0.422$ in their simulation (Ogden and Saccomandi, 2007).

The parameter β is interpreted as the orientation of reinforcement fibers. The estimated value is around $52°$. However, the artery wall is significantly heterogeneous and fibers show dispersed alignment (Gasser *et al.*, 2006). Thus, this parameter without histological observation is rather phenomenological.

The softening parameter r (dimensionless) was reported to be 1.05 for soft-bodied arthropod (Dorfmann *et al.*, 2007) and 1.105 for vaginal tissue and sheep vena cava (Peña *et al.*, 2009). Our values are also of the order of unity (approx. $r = 2.6$). The parameter m was obtained as $m = 0.00725$ for W_0^{HGO} and $\eta = 1-r^{-1}\tanh(t)$; $m = 0.0082$ for W_0^{HGO} and $\eta = 1-r^{-1}\,\mathrm{erf}\,(t)$; $m = 0.735$ for W_0^{HS} and $\eta = 1 - r^{-1}\tanh(t)$; and $m = 0.937$ for W_0^{HS} and $\eta = 1-r^{-1}\,\mathrm{erf}\,(t)$. It can be compared with Ogden and Dorfmann (2003) who found it to be 0.3 (for particle reinforced rubber), and Dorfmann *et al.* (2007) whok reported 0.0038 (for muscle of soft-bodied arthropod).

For the sake of completeness, we have to note that the primary response of the material was fitted at first (parameters c, k_1, k_2, β in W_0^{HGO}; and c, μ, J_f, β in W_0^{HS}). Subsequently, the regression of r and m was performed with fixed values of the parameters in W_0. Such a way of the fitting procedure was established due to still remaining lack of clear (physical) interpretation of the softening parameters.

Finally, the employed model for the softening variable η was isotropic. It means that the stress ratio σ_{0i}/σ_i is independent of the direction in which the tension was applied. There is no explicit dependence of r and m on the direction of the stress. There is only implicit anisotropy generated with relation $\eta = \eta(W_0)$ because W_0 is naturally anisotropic. It was justified by the observation presented in Fig. 3.

We conclude that the exponential and the limiting extensibility strain energy functions are both suitable for the description of the primary response within uniaxial tension of the thoracic aorta. They can be coupled with the pseudo-elastic softening variable η in order to capture the idealized Mullins effect. Nevertheless, the model predictions suggested that specific forms of the softening variable may not be quite appropriate.

Conflict of interest

None of the authors have a conflict of interest related to the research described in this manuscript.

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References

1. Chagnon G., Verron E., Marckmann G., Gornet L., 2006, Development of new constitutive equations for the Mullins effect in rubber using the network alteration theory, *International Journal of Solids and Structures*, **43**, 6817-6831

2. Diani J., Fayolle B., Gilormini P., 2009, A review on the Mullins effect, *European Polymer Journal*, **45**, 601-612

3. Dorfmann A., Ogden R.W., 2003, A pseudo-elastic model for loading, partial unloading and reloading of particle-reinforced rubber, *International Journal of Solids and Structures*, **40**, 2699-2714

4. Dorfmann A., Ogden R.W., 2004, A constitutive model for the Mullins effect with permanent set in particle-reinforced rubber, *International Journal of Solids and Structures*, **41**, 1855-1878

5. Dorfmann A., Trimmer B.A., Woods J. W.A., 2007, A constitutive model for muscle properties in a soft-bodied arthropod, *Journal of the Royal Society Interface*, **4**, 257-269

6. Elías-Zúñiga A., 2005, A phenomenological energy-based model to characterize stress-softening effect in elastomers, *Polymer*, **46**, 10, 3496-3506

7. Fung Y.C., Fronek K., Patitucci P., 1979, Pseudoelasticity of arteries and the choice of its mathematical expression, *The American Journal of Physiology*, **237**, H620-H631

8. Gasser, T.C., Ogden R.W., Holzapfel G.A., 2006, Hyperelastic modelling of arterial layers with distributed collagen fibre orientations, *Journal of the Royal Society Interface*, **3**, 6, 15-35

9. Gent A.N., 1996, A new constitutive relation for rubber, *Rubber Chemistry and Technology*, **69**, 819-832

10. Gracia L.A., Peña E., Royo J.M., Pelegay J.L., Calvo B., 2009, A comparison between pseudo-elastic and damage models for modelling the Mullins effect in industrial rubber components, *Mechanics Research Communications*, **36**, 769-776

11. Gregersen H., Emery J.L., McCulloch A.D., 1998, History-dependent mechanical behavior of guinea-pig small intestine, *Annals of Biomedical Engineering*, **26**, 850-858

12. Holzapfel G.A., 2000, *Nonlinear Solid Mechanics: A Continuum Approach for Engineering*, John Wiley & Sons, Chichester

13. Holzapfel G.A., 2009, Arterial tissue in health and diseases: experimental data, collagen-based modeling and simulation, including aortic dissection, *Biomechanical Modelling at the Molecular, Cellular and Tissue Levels*, 259-344

14. Holzapfel G.A., Gasser T.C., Ogden R.W., 2000, A new constitutive framework for arterial wall mechanics and a comparative study of material models, *Journal of Elasticity*, **61**, 1-48

15. Holzapfel G.A., Ogden R.W., 2009, On planar biaxial tests for anisotropic nonlinearly elastic solids. A continuum mechanical framework, *Mathematics and Mechanics of Solids*, **14**, 474-489

16. Horgan C.O., Ogden R.W., Saccomandi G., 2004, A theory of stress softening of elastomers based on finite chain extensibility, *Mathematics and Mechanics of Solids A*, **460**, 1737-1754

17. Horgan C.O., Saccomandi G., 2002, A molecular-statistical basis for the Gent constitutive model of rubber elasticity, *Journal of Elasticity*, **68**, 167-176

18. Horgan C.O., Saccomandi G., 2003, A description of arterial wall mechanics using limiting chain extensibility constitutive models, *Biomechanics and Modeling in Mechanobiology*, **1**, 251-266

19. Horgan C.O., Saccomandi G., 2005, A new constitutive theory for fiber-reinforced incompressible nonlinearly elastic solids, *Journal of the Mechanics and Physics of Solids*, **53**, 1985-2015

20. Horny L., Chlup H., Zitny R., Konvickova S., Adamek T., 2009, Constitutive behavior of coronary artery bypass graft, *IFMBE Proceedings*, **25**, 4, 181-184

21. Horny L., Gultova E., Chlup H., Sedlacek R., Kronek J., Vesely J., Zitny R., 2010, Mullins effect in an aorta and limiting extensibility evolution, *Bulletin of Applied Mechanics*, **6**, 1-5

22. Horny L., Zitny R., Chlup H., 2008, Strain energy function for arterial walls based on limiting fiber extensibility, *IFMBE Proceedings*, **22**, 1910-1913

23. Horny L., Zitny R., Chlup H., Adamek T., Sara M., 2011, Limiting fiber extensibility as parameter for damage in venous wall, *International Journal of Biological and Life Sciences*, **7**, 196-199

24. Humphrey J.D., 2003, Continuum biomechanics of soft biological tissues, *Proc. R. Soc. Lond.*, **A 459**, 3-46

25. Liao D., Zhao J., Kunwald P., Gregersen H., 2009, Tissue softening of guinea pig oesophagus tested by the tri-axial test machine, *Journal of Biomechanics*, **42**, 804-810

26. Marckmann G., Verron E., Gornet L., Chagnon G., Charrier P., Fort P., 2002, A theory of network alteration for the Mullins effect, *Journal of the Mechanics and Physics of Solids*, **50**, 2011-2028

27. Miehe C., 1995, Discontinuous and continuous damage evolution in Ogden-type large-strain elastic materials, *European Journal of Mechanics – A/Solids*, **14**, 697-720

28. Muñoz M.J., Bea J.A., Rodríguez J.F., Ochoa I., Grasa J., Pérez del Palomar A., Zaragoza P., Osta R., Doblaré M., 2008, An experimental study of the mouse skin behaviour: Damage and inelastic aspects, *Journal of Biomechanics*, **41**, 93-99

29. Ogden R.W., Roxburgh D.G., 1999, A pseudo-elastic model for the Mullins effect in filled rubber, *Proceedings of the Royal Society A*, **455**, 2861-2877

30. Ogden R.W., Saccomandi G., 2007, Introducing mesoscopic information into constitutive equations for arterial walls, *Biomechanics and Modeling in Mechanobiology*, **6**, 333-344

31. Peña E., Alasturé V., Laborda A., Martínez M.A., Doblaré M., 2010, A constitutive formulation of vascular tissue mechanics including viscoelasticity and softening behavior, *Journal of Biomechanics*, **43**, 984-989

32. Peña E., Doblare M., 2009, An anisotropic pseudo-elastic approach for modelling Mullins effect in fibrous biological materials, *Mechanics Research Communications*, **36**, 784-790

33. Peña E., Pena J.A., Doblare M., 2009, On the Mullins effect and hysteresis of fibered biological materials: A comparison between continuous and discontinuous damage models, *International Journal of Solids and Structures*, **46**, 1727-1735

34. Simo J.C., 1987, On a fully three-dimensional finite-strain viscoelastic damage model: Formulation and computational aspects, *Computer Methods in Applied Mechanics and Engineering*, **60**, 153-173

Porównanie modelu wykładniczego z pseudo-sprężystym modelem o ograniczonej rozszerzalności włókien przy opisie efektu Mullinsa w tkance tętniczej

Streszczenie

Praca zawiera analizę porównawczą poprawności dwóch różnych matematycznych sformułowań tzw. zmiennej osłabienia przy opisie zjawiska osłabienia naprężeń indukowanych odkształceniem obserwowanym podczas cyklicznego jednoosiowego rozciągania aorty piersiowej. W szczególności, zmienną osłabienia jako czynnika redukującego poziom naprężeń opisano funkcją typu tangens hiperboliczny oraz funkcją błędu. Założono, że mechaniczne właściwości aorty odpowiadają modelowi pseudohipersprężystemu, nieściśliwemu i anizotropowemu. Funkcję gęstości energii odkształcenia przyjęto w klasycznej formie wykładniczej i mało rozpoznanej postaci, która ogranicza zakres rozszerzalności włókien. Badania wykazały, że obydwa podejścia opisują właściwości mechaniczne tkanki z podobnym skutkiem. Pokazano, że rodzaj przyjętej zmiennej osłabienia nie ma wpływu na rezultaty badań. W konkluzji podkreślono, że obydwa modele nadają się do analizy efektu Mullinsa w aorcie, jakkolwiek nadal otwartą kwestią pozostaje problem znalezienia najlepiej dopasowanego modelu do opisu tego zjawiska.

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