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STATIC AND DYNAMIC INTERACTIVE BUCKLING OF COMPOSITE COLUMNS

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The static and dynamic problem of interaction of global buckling modes in compressed columns with complex open and closed cross-sections was considered in the paper. The columns made of laminate composites were assumed to be simply supported at both loaded ends. A plate model was adopted in the analysis. The equations of motion of individual plates (Schokker *et al.*, 1996; Sridharan and Benito, 1984) were obtained from Hamilton's Principle, taking into account all components of inertia forces (Teter and Kołakowski, 2005). Within the frame of the first order nonlinear approximation, the dynamic problem of modal interactive buckling was solved by the transition matrix using a perturbation method. Distortions of cross-sections and the shear-lag phenomenon were taken into consideration in the problem solution. A modification of the quasi-bifurcation dynamic Kleiber-Kotula-Saran criterion (Kleiber *et al.*, 1987) was proposed. A comparison of the proposed modification to the Budiansky-Hutchinson criterion (Budiansky and Hutchinson, 1966; Hutchinson and Budiansky, 1966) was presented for a rectangular pulse loading.

Key words: lamina, composite, buckling, pulse loading, interaction

1. Introduction

Thin-walled structures, composed of plate elements, have many different buckling modes that vary in quantitative (e.g. the number of halfwaves) and qualitative (e.g. global and local buckling) aspects. In these cases, the postcritical behaviour cannot be described any more by a single generalized displacement. When the postcritical behaviour of each individual mode is stable, their interaction can lead to unstable behaviour, and thus to an increase in the imperfection sensitivity (Byskov, 1987-8; Camotion and Prola, 1996; Kołakowski, 1993; Kołakowski and Kowal-Michalska, 1999; Kołakowski and Teter, 2000; Kołakowski, 1993; Kołakowski and Kowal-Michalska, 1999; Kołakowski and Teter, 2000; Teter and Kołakowski, 2005). A nonlinear stability theory should describe all modes of global, local, distortional and interactive buckling, taking into consideration the structure imperfection.

The theory of interactive buckling of thin-walled structures subjected to static and dynamic loading has been already widely developed for over forty years. Although the problem of static coupled buckling can be treated as well recognized, the analysis of dynamic interactive buckling is limited in practice to columns (adopting their beam model), single plates and shells. In the world literature, a substantial lack of the nonlinear analysis of dynamic stability of thin-walled structures with complex cross-sections can be felt.

In this study, an analysis of static and dynamic stability of composite structures with complex cross-sections is presented. Special attention is focused on coupled buckling of various global buckling modes.

1.1. Static interactive buckling

A plate model of the column has been adopted in the study to describe global buckling, which leads to lowering the theoretical value of the limit load. In this case, buckling characteristics for the independent global mode are nonsymmetrical, and thus the equilibrium is unsteady (Kołakowski, 1993; Kołakowski and Kowal-Michalska, 1999; Kołakowski and Teter, 2000; Teter and Kołakowski, 2005). When components of the displacement state for the first order approximation are taken in account, it can be followed by a decrease in values of global loads. In the case, the critical values corresponding to global buckling modes are significantly lower than local modes, then their interaction can be considered within the first nonlinear approximation (Kołakowski, 1993; Kołakowski and Kowal-Michalska, 1999; Kołakowski and Teter, 2000; Teter and Kołakowski, 2005). It is possible as the post-buckling coefficient for uncoupled buckling is equal to zero for the second order global mode in the Euler column model, and in the case of an exact solution it is very often of little significance. The theoretical static load-carrying capacity, obtained within the frame of the asymptotic theory of the nonlinear first order approximation, is always lower than the minimum value of critical load for the linear problem. The solution method assumed in this study allows for analysing interactions of all buckling modes.

Dubina (1996) paid special attention to the interaction of global modes of buckling (flexural, flexural-torsional, lateral) with a distortional and/or localized (Byskov, 1987-8) mode.

1.2. Dynamic interactive buckling

The dynamic pulse load of thin-walled structures can be divided into three categories, namely: *impact* with accompanying perturbation propagation (a phenomenon that occurs with the sound wave propagation speed in the structure), *dynamic load* of a mean amplitude and a pulse duration comparable to the fundamental flexural vibration period, and *quasi-static load* of a low amplitude and a load pulse duration approximately twice as long as the period of fundamental natural vibrations. As for *dynamic load*, effects of damping can be neglected in practice. This study is devoted to the stability problem of a rectangular dynamic pulse load.

Dynamic buckling of a column can be treated as reinforcement of imperfections, initial displacements or stresses in the column through dynamic loading in such a manner that the level of the dynamic response becomes very high. When the load is low, the column vibrates around the static equilibrium position. On the other hand, when the load is sufficiently high, then the column can vibrate very strongly or can move divergently, which is caused by dynamic buckling.

In the literature on this problem, various criteria concerning dynamic stability have been adopted. One of the simplest is the criterion suggested by Volmir (1972). The most widely used is the Budiansky-Hutchinson criterion (Budiansky and Hutchinson, 1966; Hutchinson and Budiansky, 1966), in which it is assumed that the loss of dynamic stability occurs when the velocity with which displacements grow is the highest for a certain force amplitude. Other criteria were discussed in papers Ari-Gur and Simonetta (1997), Cui *et al.*, 2000, 2001, 2002; Gantes *et al.*, 2001; Hao *et al.*, 2000; Huyan and Simitses, 1997; Kleiber *et al.*, 1987; Kowal-Michalska *et al.*, 2004; Papazoglou and Tsouvalis (1995), Petry and Fahlbusch, 2000; Schokker *et al.*, 1996; Sridharan and Benito, 1984; Weller *et al.*, 1989; Zhang and Taheri, 2002), for instance.

A diversity of dynamic stability loss criteria follows from a lack of a generally assumed, accurate, explicit mathematical definition. One of a few exceptions, known to the author [e.g. Budiansky and Hutchinson, 1966; Gantes *et al.*, 2001; Hutchinson and Budiansky, 1966), is the quasi-bifurcation criterion of dynamic buckling for a step-like load (Heaviside's function) and that one concerning the critical pulse duration (Kleiber-Kotula-Saran criterion (Kleiber *et al.*, 1987)). This criterion is based on the condition that the tangent matrix of the system stiffness is zero, that is to say, all the Jacobian matrices are equal to zero. Eigenvalues of this matrix have to be computed.

In this study, the following modification of the Kleiber-Kotula-Saran criterion (Kleiber *et al.*, 1987) as the criterion of dynamic stability loss for a pulse loading of finite duration has been proposed:

> *Dynamic stability loss occurs when during the time* $(0, t_0)$ *of the pulse load and in its vicinity* $0 \leqslant t \leqslant 1.3t_0$, *the minimum eigenvalue of the tangent stiffness matrix (Jacobian matrix) is lower than, for example,* "-1" *(i.e.* $\rho_{min} < -1$*).*

A dynamic response to the rectangular pulse load with the duration time corresponding to the fundamental period of flexural and flexural-distortional free vibrations of the unloaded column (i.e. $t_0 = T_1$) and for $t_0 = T_1/2$ has been analysed.

2. Formulation of the problem

Prismatic thin-walled columns with open and closed cross-sections, subjected to axial compression, have been considered. Cross-sections of the elements under analysis are built of rectangular plates interconnected along longitudinal edges and simply supported at both ends. All component plates are made of the same laminate composite subject to Hooke's law.

The attention has been drawn to the necessity of considering the full strain tensor and all the components of inertial forces in order to carry out a proper dynamic analysis in the whole range of length of the structures.

For thin-walled structures with initial deflections, Lagrange's equations of motion for the case of interaction of N eigenmodes can be written as (Schokker *et al.*, 1996; Sridharan and Benito, 1984; Teter and Kołakowski, 2005)

$$\frac{1}{\omega_r^2}\zeta_{r,tt} + \Big(1 - \frac{\sigma}{\sigma_r}\Big)\zeta_r + a_{ijr}\zeta_i\zeta_j - \zeta_r^*\frac{\sigma}{\sigma_r} + \ldots = 0 \tag{2.1}$$

for $r = 1, \ldots, N$, where ζ_r is the dimensionless amplitude of the r-th buckling mode (maximum deflection referred to the thickness of the first plate), σ_r, ω_r, ζ_r^* – critical stress, circular frequency of free vibrations and dimensionless amplitude of the initial deflection corresponding to the r-th buckling mode, respectively.

The expressions for a_{ijr} are to be found in Byskov (1987-8), Byskov and Hutchinson (1977), Kołakowski and Kowal-Michalska (1999), Teter and Kołakowski (2005). In equations of motion (2.1), inertia forces of the pre-buckling state and second order approximations have been neglected (Schokker *et al.*, 1996; Sridharan and Benito, 1984; Teter and Kołakowski, 2005). The initial conditions have been assumed in the following form

$$\zeta_r(t=0) = 0 \qquad\qquad \zeta_{r,t}(t=0) = 0 \tag{2.2}$$

The static problem of interactive buckling of thin-walled multilayer columns (i.e. for $\zeta_{r,tt} = 0$ in (2.1)) has been solved with the method presented in Kołakowski and Kowal-Michalska (1999), Teter and Kołakowski (2005), the frequencies of free vibrations have been determined analogously as in Teter and Kołakowski (2005), whereas the problem of interactive dynamic buckling (2.1) have been solved by means of the Runge-Kutta numerical method modified by Hairer and Wanner (with differentiation formulas of a variable order and automatic time stepping).

3. Analysis of the calculation results

3.1. Open section columns

3.1.1. Static buckling

A detailed analysis of the calculations was conducted for compressed columns with the following dimensions of open cross-sections (Fig. 1)

$$b_1 = 100\,\text{mm} \qquad b_2 = 50\,\text{mm} \qquad b_3 = 15\,\text{mm}$$
$$b_S = 15\,\text{mm} \qquad h_1 = h_2 = h_3 = 12h_{lay} = 1.5\,\text{mm}$$

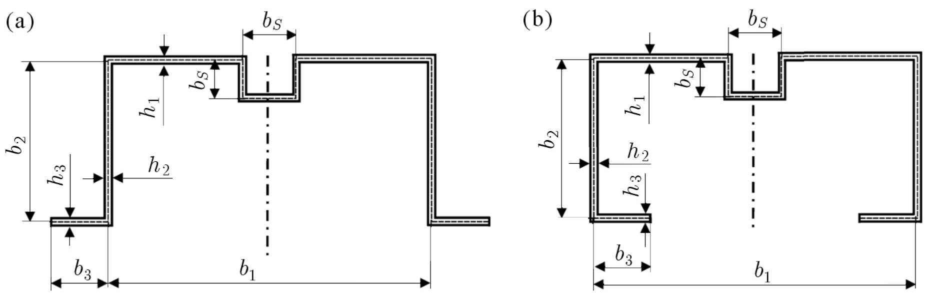

Fig. 1. Open cross-sections of columns with a central intermediate stiffener: (a) outer omega, (b) inner omega

Each plate is made of a twelve-layer composite with the symmetric ply alignment $[45/-45/0/0/0/0]_S$. Each layer of the thickness $h_{lay} = 0.125\,\text{mm}$

is characterized by the following mechanical properties (Teter and Kołakowski, 2005)

$$E_1 = 140\,\text{GPa} \qquad E_2 = 10.3\,\text{GPa} \qquad G = 5.15\,\text{GPa}$$
$$\nu_{12} = 0.29 \qquad \rho = 1600\,\text{kg/m}^3$$

The geometrical dimensions of intermediate stiffeners and the ply alignment were selected in such a way that the critical values of local loads were considerably higher than the critical values of global loads within the variability range of the column length ℓ under consideration.

For global buckling modes, the imposing of the symmetry conditions of the buckling mode corresponds to flexural or flexural-distortional buckling, whereas the antisymmetry conditions entail flexural-torsional or flexural-torsional-distortional buckling (Camotion and Prola, 1996; Dubina, 1996; Kołakowski and Kowal-Michalska, 1999). Local buckling modes correspond to short columns.

The interactive buckling of the columns with open cross-sections shown in Fig. 1 was analysed within the first order approximation for global buckling modes and five various lengths $\ell = 2500$, 2000, 1500 and 1000 mm. The interaction of buckling modes can occur among several buckling modes symmetric with respect to the symmetry axis of the cross-section and also between a symmetric mode and pairs of antisymmetric modes (Byskov, 1987-8; Kołakowski, 1993; Kołakowski and Kowal-Michalska, 1999; Kołakowski and Teter, 2000; Teter and Kołakowski, 2005). In order to consider a possible effect of the localized buckling (Byskov, 1987-8; Dubina, 1996; Kołakowski and Kowal-Michalska, 1999) for the assumed length ℓ, a global flexural-distortional mode ($m = 1$), a flexural-torsional-distortional mode ($m = 1$) and higher global modes: flexural-distortional and flexural-torsional-distortional modes, respectively, with the halfwave number $m = 3$, were analysed. The following index symbols were introduced: 1 – flexural-distortional mode for $m = 1$; 2 – flexural-torsional-distortional mode for $m = 1$; 3 – flexural-distortional mode for $m = 3$; 4 – flexural-torsional-distortional mode for $m = 3$.

Buckling modes of the open section columns (Fig. 1) for two lengths $\ell = 2500$ and 1000 mm are shown in Figs. 2 and 3, respectively. The adoption of the plate model allowed us to account for all buckling modes for columns of different shapes and flexural rigidities. This can help in their rational designing.

Detailed results of the interactive buckling analysis are presented in Table 1 for the columns with the cross-section shown in Fig. 1a and Fig. 1b. The following imperfections were assumed: $\zeta_1^* = \zeta_2^* = |\ell/(1000h)|$,

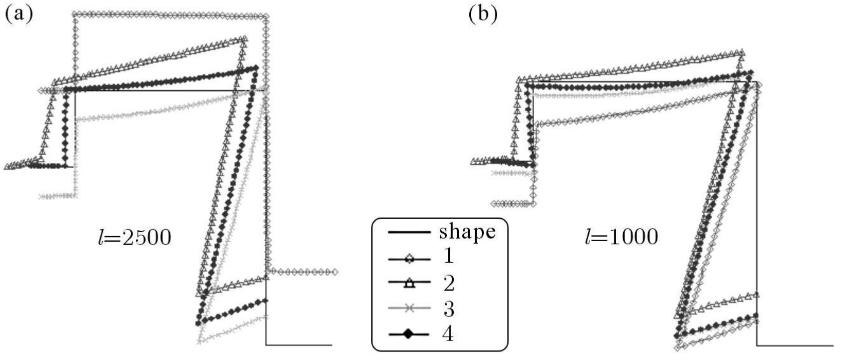

Fig. 2. Buckling modes of open section columns (Fig. 1a) for two lengths: (a) $\ell = 2500\,\mathrm{mm}$, (b) $\ell = 1000\,\mathrm{mm}$

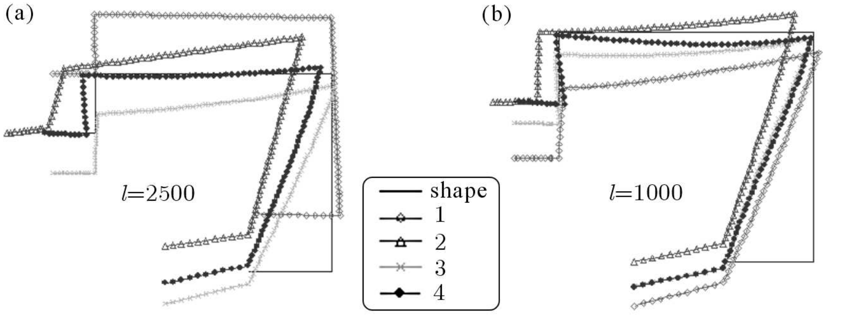

Fig. 3. Buckling modes of open section columns (Fig. 1b) for two lengths: (a) $\ell = 2500\,\mathrm{mm}$, (b) $\ell = 1000\,\mathrm{mm}$

$\zeta_3^* = \zeta_4^* = |\ell/(2000h)|$. In each case, the signs of the imperfection ζ_i^* (where $i = 1, \dots, 4$) were selected in the most unfavourable way, that is to say, as to obtain the lowest theoretical limit load-carrying capability σ_S for the given level of imperfection when the interaction of buckling modes is accounted for. The modes under consideration can be identified according to the adopted index symbols. It was demonstrated in each case under analysis that the most dangerous interaction of buckling modes occurred between global modes for $m = 1$ and $m = 3$. A decrease in the limit load capacity σ_S/σ_{min} (where $\sigma_{min} = \min(\sigma_i)$; $i = 1, \dots, 4$) did not exceed 30% practically (see Table 1).

Owing to the above-mentioned reasons, the further analysis was limited to the interaction of four global buckling modes. The signs of imperfections, defined in the static analysis, were next employed in the dynamic stability analysis.

Table 1. Critical loads and theoretical limit load-carrying capacity for the columns shown in Fig. 1

ℓ	σ_1 $(m=1)$	σ_2 $(m=1)$	σ_3 $(m=3)$	σ_1 $(m=3)$	σ_S/σ_{min}
[mm]	[MPa]	[MPa]	[MPa]	[MPa]	[–]
Fig. 1a (outer omega)					
2500	52.2	29.3	131.9	119.0	0.699
2000	79.5	42.1	127.0	112.8	0.708
1500	129.0	66.9	138.7	136.7	0.719
1000	143.9	109.2	208.1	210.2	0.785
Fig. 1b (inner omega)					
2500	50.9	42.2	106.3	189.1	0.768
2000	74.5	62.5	112.2	175.7	0.701
1500	104.9	103.5	135.7	174.2	0.685
1000	107.7	180.4	219.3	242.1	0.765

3.1.2. Linear dynamic analysis

Values of the natural frequencies of vibrations corresponding to the global buckling modes under analysis for different column lengths ℓ are presented in Table 2. The same index symbols were adopted as for the interactive static buckling. Vibration frequencies were determined, taking into account all components of inertia forces (Teter and Kołakowski, 2005) (in-plane $\rho u_{,tt}$, $\rho v_{,tt}$ and out-of-plane $\rho w_{,tt}$).

Table 2. Natural frequency of the columns shown in Fig. 1

ℓ	Fig. 1a (outer omega)				Fig. 1b (inner omega)			
	ω_1	ω_2	ω_3	ω_4	ω_1	ω_2	ω_3	ω_4
[mm]	[1/s]	[1/s]	[1/s]	[1/s]	[1/s]	[1/s]	[1/s]	[1/s]
2500	227	170	1082	1027	224	204	971	1295
2000	350	254	1327	1305	339	310	1247	1561
1500	594	428	1849	1835	536	532	1828	2072
1000	942	820	3396	3412	814	1054	3485	3662

3.1.3. Dynamic stability

Further on, an analysis of dynamic interactive buckling of the columns under consideration was conducted. Identically as in the static analysis, the interaction of the same global buckling modes was considered. A detailed analysis was conducted for a rectangular load pulse

$$\sigma(t) = \begin{cases} \sigma_D & \text{for} \quad 0 \leqslant t \leqslant t_0 \\ 0 & \text{for} \quad t_0 < t \end{cases} \tag{3.1}$$

and two cases of its duration. The first case corresponds to the duration equal to the fundamental period of flexural free vibrations $t_0 = T_1 = 2\pi/\omega_1$, whereas the second one refers to the duration equal to $t_0 = T_1/2$.

In the dynamic stability analysis, the level of imperfections was assumed to be three times lower than that one for the static load, i.e. $\zeta_1^* = \zeta_2^* = |\ell/(3000h)|$, $\zeta_3^* = \zeta_4^* = |\ell/(6000h)|$. The pulse duration T_1 assumed in the analysis corresponds to the quasi-static load of columns for higher global buckling modes (i.e. for $m = 3$). The time of tracking eigenvalues of the tangent stiffness matrix (Jacobian matrix) was assumed to be $1.3t_0$.

Table 3. Results of dynamic calculations for the columns shown in Fig. 1

ℓ [mm]	Fig. 1a (outer omega)				Fig. 1b (inner omega)			
	$t_0 = T_1$		$t_0 = T_1/2$		$t_0 = T_1$		$t_0 = T_1/2$	
	$\frac{\sigma_D^{BH}}{\sigma_{min}}$	$\frac{\sigma_D^{K}}{\sigma_{min}}$	$\frac{\sigma_D^{BH}}{\sigma_{min}}$	$\frac{\sigma_D^{K}}{\sigma_{min}}$	$\frac{\sigma_D^{BH}}{\sigma_{min}}$	$\frac{\sigma_D^{K}}{\sigma_{min}}$	$\frac{\sigma_D^{BH}}{\sigma_{min}}$	$\frac{\sigma_D^{K}}{\sigma_{min}}$
2500	1.69	1.51	2.82	1.97	1.27	1.22	2.14	1.88
2000	1.59	1.51	2.50	2.06	1.19	1.18	1.65	1.59
1500	1.42	1.37	1.93	1.70	1.06	1.04	1.35	1.30
1000	1.52	1.43	2.46	1.80	1.48	1.38	2.00	1.84

In Table 3, values of the dynamic load factors $\sigma_D^{BH}/\sigma_{min}$ and $\sigma_D^{K}/\sigma_{min}$ for two rectangular pulse load durations and for various column lengths ℓ, for two column cross-sections under analysis (Fig. 1), respectively, are given. The following notations are applied in the tables: σ_D^{BH} denotes the value of dynamic stress determined from the Budiansky-Hutchinson criterion, whereas σ_D^{K} refers to the modification of the Kleiber-Kotula-Saran criterion (Kleiber *et al.*, 1987) for $\rho_{min} < -1$, postulated in this study. The values of σ_D^{BH} presented in Table 3 correspond with some accuracy to the maximum values of deflections $\zeta_{r\,max}$ within the applicability of the assumed theory (i.e. the total maximum deflection of the column is at least fifty times as high as the cross-section wall thickness) (Byskov, 1987-8; Byskov and Hutchinson, 1977; Kołakowski, 1993; Kołakowski and Kowal-Michalska, 1999; Kołakowski and Teter, 2000; Kowal-Michalska *et al.*, 2004; Teter and Kołakowski, 2003, 2005), and not to asymptotic values (Budinsky and Kutchinson, 1966). The main limitation that results from the adopted theory is the assumption of a linear dependence between curvatures and second order derivatives of the displacement w, i.e. $\kappa_x = -w_{,xx}$, $\kappa_y = -w_{,yy}$, $\kappa_{xy} = -w_{,xy}$ (cf. Opoka and Pietraszkiewicz, 2004; Pietraszkiewicz, 1989).

The values obtained on the basis of the postulated criterion modification are lower than the values obtained from the Budiansky-Hutchinson criterion, and these differences grow for the shorter pulse duration $t_0 = T_1/2$. Therefore, results for the shorter pulse duration, that is for $t_0 = T_1/2$, are shown in Table 4, but for two minimum values of eigenvalues of the tangent stiffness matrix (Jacobian matrix) are equal to: $\rho_{min} < -1$ and $\rho_{min} < -2$, respectively. The second value was assumed arbitrarily.

Table 4. Results of dynamic calculations for $t_0 = T_1/2$ and for the columns shown in Fig. 1

ℓ [mm]	Fig. 1a (outer omega)		Fig. 1b (inner omega)	
	σ_D^K/σ_{min}			
	$\rho_{min} < -1$	$\rho_{min} < -2$	$\rho_{min} < -1$	$\rho_{min} < -2$
2500	1.97	2.64	1.88	2.18
2000	2.06	2.41	1.59	1.69
1500	1.70	1.88	1.30	1.31
1000	1.80	2.38	1.84	1.99

For the cases corresponding to the lowest eigenvalue of the Jacobian matrix, i.e. for $\rho_{min} < -2$, the differences between the Budiansky-Hutchinson criterion and the postulated modification of the Kleiber-Kotula-Saran criterion (Kleiber *et al.*, 1987) become smaller. While comparing the results for the two cases of the pulse duration, it seems that a further thorough analysis of the proposed Kleiber-Kotula-Saran criterion modification so that it will assume $|\rho_{min}|t_0 = \text{const}$ is advisable.

3.2. Closed column

3.2.1. Static buckling

Subsequently, a compressed column of the following geometrical dimensions (Fig. 4): $b_1 = 100$ mm, $b_2 = b_3 = 50$ mm, $h_1 = h_2 = 12h_{lay} = 1.5$ mm, $b_S = 15$ mm, $h_3 = h_4 = 24h_{lay} = 3.0$ mm, was analysed.

The plates, whose thickness was $h_1 = h_2 = 1.5$ mm, were made of the identical material as the open cross-sections, i.e. a twelve-layer composite with the symmetric ply alignment $[45/-45/0/0/0/0]_S$, whereas plates, whose thick-

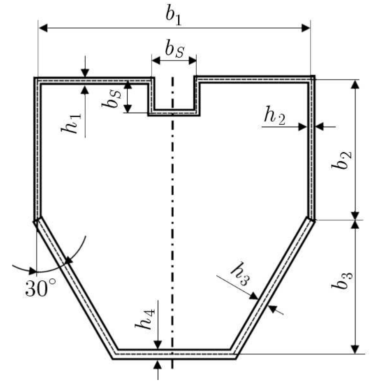

Fig. 4. Closed cross-sections of columns with a central intermediate stiffener

ness is $h_3 = h_4 = 3.0\,\mathrm{mm}$, were made of the same material but with a 24-layer composite with the symmetric ply alignment $[45_2/-45_2/0_2/0_2/0_2/0_2]_S = [45_2/-45_2/0_8]_S$. Each layer of the thickness $h_{lay} = 0.125\,\mathrm{mm}$ was characterized by the mechanical properties, identical as in the open section columns.

In the case of the closed cross-section, only static interactive buckling of the column of the length $\ell = 5000\,\mathrm{mm}$ within the first order approximation was considered. The following index notations were introduced: 1 – symmetric buckling mode for $m = 1$, 2 – antisymmetric mode for $m = 1$, 3 – symmetric mode for $m = 3$. Figure 5 presents three buckling modes for the closed column.

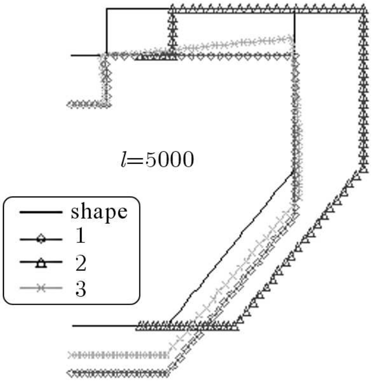

Fig. 5. Buckling modes for the closed column

In Table 5, values of global critical stresses and the dimensionless theoretical limit load carrying capacity for the first order of approximation, on the assumption of the imperfection $\zeta_1^* = \zeta_2^* = |\ell/(1000h_1)| = |3.333|$, $\zeta_3^* = |\ell/(2000h_1)| = |1.666|$, have been listed.

Table 5. Critical stresses, theoretical load-carrying capacity and frequencies for the closed column

ℓ	σ_1 $(m=1)$	σ_2 $(m=1)$	σ_3 $(m=3)$	σ_S/σ_1	σ_S/σ_2	ω_1	ω_2	ω_3
[mm]	[MPa]	[MPa]	[MPa]	[–]	[–]	[1/s]	[1/s]	[1/s]
500	55.3	47.2	405	0.953	0.963	116 (175)	108 (144)	946 (1339)

In the case of the column with a closed cross-section with one axis of symmetry, the nonlinear coefficients a_{ijk} for the symmetric mode of global buckling with respect to the axis of symmetry are low and considerably lower than for open cross-sections, whereas they are equal to zero for the antisymmteric buckling mode.

The above-mentioned issues have resulted in a slight lowering of the theoretical load carrying capacity of the column if compared to the critical load (by approximately 5%).

3.2.2. Dynamic stability

In Table 5, values of the natural frequencies, taking into account all components of inertia forces (Teter and Kołakowski, 2005), are listed as well, whereas in brackets there are values of the frequencies when the in-plane terms of inertia forces are equal to zero.

In the dynamic stability analysis, the level of imperfections was assumed to be three times lower that the one for the static load identical as for open columns.

Table 6 presents values of the dynamic load factors σ_D^{BH}/σ_1 and σ_D^K/σ_1 for two rectangular pulse load durations ($t_0 = T_1$ and $t_0 = T_1/2$), taking into account the interactions of two global modes of buckling and, respectively, σ_D^{BH}/σ_2 and σ_D^K/σ_2 for the antisymmetric mode of buckling, including only one global mode of buckling. For $t_0 = T_1$ and $t_0 = T_1/2$, the critical values of σ_D^K have been determined for two values: $\rho_{min} < -1$ and $\rho_{min} < -2$.

The values obtained on the basis of the proposed criterion modification in comparison with the values obtained from the Budiansky-Hutchinson criterion are higher for $t_0 = T_1$ and comparable for $t_0 = T_1/2$ and $\rho_{min} < -2$.

The columns with closed cross-sections are less sensitive to interactions of global modes of buckling (i.e., interactive buckling) than the columns with open cross-sections.

Table 6. Results of dynamic calculations for the columns shown in Fig. 2

ℓ [mm]	$t_0 = T_1$			
	σ_D^{BH}/σ_1	σ_D^K/σ_1	σ_D^{BH}/σ_2	σ_D^K/σ_2
		$\rho_{min} < -1$		$\rho_{min} < -1$
5000	1.311	1.872	1.316	2.017

ℓ [mm]	$t_0 = T_1/2$					
	σ_D^{BH}/σ_1	σ_D^K/σ_1		σ_D^{BH}/σ_2	σ_D^K/σ_2	
		$\rho_{min} < -1$	$\rho_{min} < -2$		$\rho_{min} < -1$	$\rho_{min} < -2$
5000	2.81	2.105	2.996	3.46	2.105	3.026

4. Conclusion

A modification of the Kleiber-Kotula-Saran criterion of dynamic stability (Kleiber *et al.*, 1987) is postulated for the case of the finite duration of a pulse load. As far as dynamic stability of bar structures is concerned, we obtain lower values than with the Budiansky-Hutchinson criterion. The criterion modification proposed allows for an explicit evaluation of the dynamic stability loss.

According to the author's opinion, a further analysis of the postulated criterion modification should tackle the following issues: the minimum eigenvalue of the tangent stiffness matrix, the tracking time of eigenvalues, and the dependence between the minimum eigenvalue of the Jacobian matrix and the pulse duration. Therefore, the postulated criterion modification should be further analysed and comprehensively and thoroughly discussed.

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Statyczne i dynamiczne interakcyjne wyboczenie słupów kompozytowych

Streszczenie

W pracy rozpatrzono zagadnienie statycznej i dynamicznej interakcji globalnych postaci wyboczenia ściskanych słupów o złożonych otwartych i zamkniętych przekrojach poprzecznych. Założono, że cienkościenne słupy wykonane z kompozytów są

przegubowo podparte na obu końcach. Przyjęto płytowy model słupa. Równania ruchu pojedynczej płyty (Schokker *et al.*, 1996; Sridharan i Benito, 1984) otrzymano z zasady Hamiltona, uwzględniając wszystkie składowe sił bezwładności (Teter i Kołakowski, 2005). Zagadnienie dynamicznego modalnego interakcyjnego wyboczenia rozwiązano metodą macierzy przejścia, wykorzystując metodę perturbacyjną w ramach pierwszego rzędu nieliniowego przybliżenia. W rozwiązaniu uwzględniono dystorsje przekrojów poprzecznych słupów oraz zjawisko *shear lag*. Zastosowano zmodyfikowane quasi-bifurkacyjne dynamiczne kryterium Kleibera-Kotuli-Sarana (Kleiber *et al.*, 1987). Przeprowadzono porównanie wyników otrzymanych w ramach proponowanego zmodyfikowanego kryterium z kryterium Budiansky'ego-Hutchinsona (Budiansky i Hutchinson, 1966; Hutchinson i Budiansky, 1966) dla impulsowego obciążenia o kształcie prostokątnym.

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