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DYNAMIC RESPONSE OF LADDER TRACK RESTED ON STOCHASTIC FOUNDATION UNDER OSCILLATING MOVING LOAD

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The ladder track is a new type of an elastically supported vibration-reduction track system that has been applied to several urban railways. This paper is devoted to the investigation of dynamic behavior of a ladder track under an oscillating moving load. The track is represented by an infinite Timoshenko beam supported by a random elastic foundation. In this regard, equations of motion for the ladder track are developed in a moving frame of reference. In continuation, by employing perturbation theory and contour integration, the response of the ladder track is obtained analytically and its results are verified using the stochastic finite element method. Finally, using the verified model, a series of sensitivity analyses are accomplished on effecting parameters including velocity and load frequency.

Keywords: ladder track, moving load, stochastic stiffness, perturbation theory

1. Introduction

In the 1940s to 1960s, weakness caused by resistance to lateral movement of cross-ties prompted studies on longitudinal sleepers laid in parallel pairs under the rails. The aim was to produce a railway track requiring a minimum of maintenance. Ladder sleepers were subsequently developed having parallel longitudinal concrete beams held together by transverse steel pipes (Wakui *et al.*, 1997). Ladder sleepers provide continuous support to the rails assuring train safety, decreasing maintenance and promising an increase in railway efficiency.

In recent years, a floating ladder track (Fig. 1a) has been developed to decrease vibration in a structure and withstand noise. Younesian *et al.* (2006) studied the dynamic performance of a ballasted ladder track. The rail and ladder units were simulated using a Timoshenko beam and the governing equations were solved using the Galerkin method. Figure 1b shows the ballasted ladder track.

(a) (b)

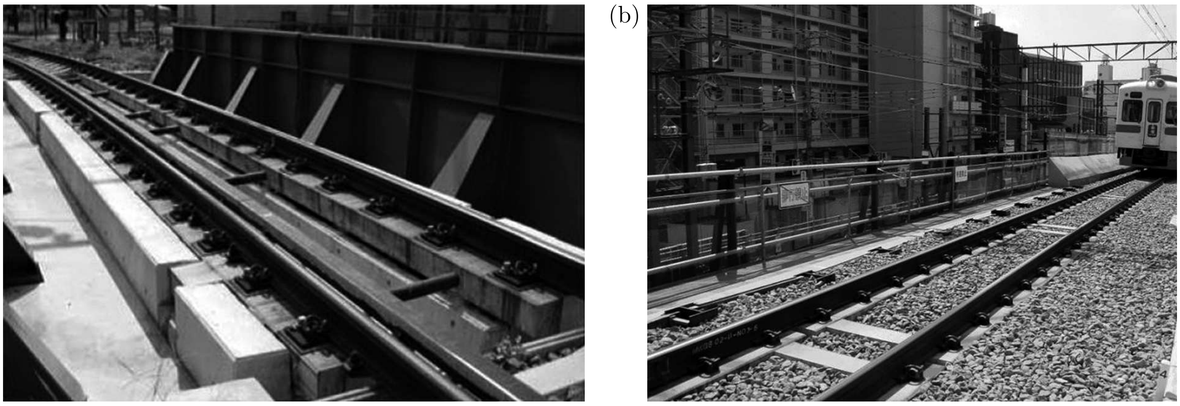

Fig. 1. (a) Floating ladder track; (b) ballasted ladder track

Hosking and Millinazzo (2007) developed a mathematical method for a floating ladder track under a moving oscillating load in which the track was simulated using an Euler-Bernoulli beam on periodic discrete elastic supports. They were able to predict the frequency and critical speed for design purposes. Xia *et al.* (2009) dynamically simulated an elevated bridge with a ladder track under a moving train and measured its dynamic response. Xia *et al.* (2010) carried out a field experiment at the trial section of an elevated bridge on Beijing Metro line where the ladder track was installed and investigatd the vibration reduction characteristics of the track.

Yan *et al.* (2014) developed dynamic models of the vehicle and the ladder track to analyze the track vibration behavior. They optimized the mechanical properties of the ladder track to reduce or eliminate the track vibrations at the corrugation frequency and ultimately to reduce the chance of rail corrugation. Ma *et al.* (2016) investigated the effect of ballasted ladder tracks and the vibration reduction effect. The results show that the ballasted ladder track can effectively decrease the peak value in the time domain and has the potential effect to control environmental vibration in low frequencies.

Analysis of beams subjected to moving loads is of substantial practical importance. Many researchers have studied the vibration of beams subjected to various types of moving loads. Since parameters such as loading, rail defection and nature of the substructure are stochastic, the dynamic response of the track is assumed to be stochastic. Table 1 lists the major studies in this area. Thus far, no study has been carried out on ladder tracks using a stochastic approach.

Table 1. Major research on stochastic approach in railway engineering

Author(s)	Subject	Loading	Year
Fryba *et al.*	Euler-Bernoulli beam resting on a Winkler random foundation	Harmonic moving load	1993
Anderson and Nielsen	Beam on a random modified Kelvin foundation	Moving vehicle	2003
Kargarnovin *et al.*	Infinite Timoshenko beams supported by nonlinear foundations	Harmonic moving loads	2005
Younesian *et al.*	Timoshenko beam on a random foundation under	Harmonic moving load	2005
Younesian and Kargarnovin	Infinite Timoshenko beam supported by a random Pasternak foundation	Harmonic moving loads	2009
Mohammadzadeh and Ghahremani	Risk of derailment using a numerical method	Railway vehicle	2010
Mohammadzadeh *et al.*	Probability of derailment where irregularity of the track is random	Railway vehicle	2011
Mohammadzadeh *et al.*	Double Euler-Bernoulli beam resting on a random foundation	Harmonic moving loads	2013
Mehrali *et al.*	Double Euler-Bernoulli beam resting on a random foundation	Railway vehicle	2014
Mohammadzadeh *et al.*	Reliability analysis of the rail fastening where load and velocity are random	Moving train	2014
Pouryousef and Mohammadzadeh	Reliability evaluation of design codes applied for railway bridges	Live load (LM71)	2014

Engineering experience has revealed that uncertainties occur in the assessment of loading as well as in the material and geometric properties of engineering systems. The logical behavior of these uncertainties in probability theory and statistics cannot be obtained accurately using the deterministic method. This approach is based on extremes (minimum, maximum) and mean values of system parameters (Stefanou, 2009). More detail on the random behavior of a structure can be found in Lutes and Sarkani (2004).

The Taylor series expansion of the stochastic finite element matrix of a physical system is known in the literature as the perturbation method. This method is used to solve probabilistic problems (Kleiber and Hein, 1992; Liu *et al.*, 1986). Another method is the Karhunen-Loeve expansion technique (Ghanem and Spanos, 1991a,b). The main initiative of the perturbation method is to formulate an analytical expansion of an input parameter around its mean value using a series representation (Jeulin and Ostoja-Starzewski, 2001; Nayfeh and Mook 1979).

A novel analytical method is presented for the analysis of the governing equations of motion for an infinite Timoshenko ladder track on a viscoelastic foundation with random stiffness under a harmonic moving load. For the stationary analysis of the response of the beam to variations in stiffness in the support, it is useful to describe it in a local moving coordinate system subjected to a harmonic moving load. Furthermore, by applying the perturbation method and complex Fourier transformation, the mean and variance of the response of the beam can be calculated analytically in an integral form. Sensitivity analysis is run using the residue theorem and key parameters are introduced.

2. Theory

Assume a harmonic load moves uniformly along a ladder track at velocity v. The ladder track is modeled using two parallel Timoshenko beams. The connection of the two beams is described using a series of springs and dashpots. In addition, the lower beam rests on a viscoelastic foundation. The vertical stiffness of the support is described by a stochastic variable along the beam with a mean of $\overline{k}$ and a stochastic component of $k_s(x)$ (Mohammadzadeh *et al.*, 2013). Here, $\kappa(x)$ is a random stationary ergodic function with zero mean value and ϕ is a small constant parameter

$$k_B(x) = \overline{k} + \phi\kappa(x) = \overline{k} + k_s(x) \tag{2.1}$$

2.1. Equation of motion

The equations of motion for the rail and ladder units are

$$\begin{aligned} &\rho_1 A_1 \frac{\partial^2 w_1}{\partial t^2} + k_1 A_1 G_1\Big(\frac{\partial \psi_1}{\partial x} - \frac{\partial^2 w_1}{\partial x}\Big) + k_p(w_1 - w_2) + c_p\Big(\frac{\partial w_1}{\partial t} - \frac{\partial w_2}{\partial t}\Big) \\ &\qquad = P\mathrm{e}^{\mathrm{i}\Omega t}\delta(x - vt) \\ &EI_1 \frac{\partial^2 \psi_1}{\partial x^2} - k_1 A_1 G_1\Big(\psi_1 - \frac{\partial w_1}{\partial x}\Big) = \rho_1 I_1 \frac{\partial^2 \psi_1}{\partial t^2} \end{aligned} \tag{2.2}$$

and

$$\begin{aligned} &\rho_2 A_2 \frac{\partial^2 w_2}{\partial t^2} + k_2 A_2 G_2\Big(\frac{\partial \psi_2}{\partial x} - \frac{\partial^2 w_2}{\partial x}\Big) + k_B w_2 - k_p(w_1 - w_2) \\ &\qquad - c_p\Big(\frac{\partial w_1}{\partial t} - \frac{\partial w_2}{\partial t}\Big) + c_B \frac{\partial w_2}{\partial t} = 0 \\ &EI_2 \frac{\partial^2 \psi_2}{\partial x^2} - k_2 A_2 G_2\Big(\psi_2 - \frac{\partial w_2}{\partial x}\Big) = \rho_2 I_2 \frac{\partial^2 \psi_2}{\partial t^2} \end{aligned} \tag{2.3}$$

where $w_1(x,t)$ is the upper beam deflection, $w_2(x,t)$ is the lower deflection, $\delta(x)$ is the Dirac delta function, and v and Ω are the speed and frequency of the load, respectively. A, E, G, I, k and ρ are the cross-sectional areas of the beams, modulus of elasticity, shear modulus, second moment of area, sectional shear coefficient, and beam material density, respectively. Figure 2 is a flowchart of the solution of the governing equation for the ladder track.

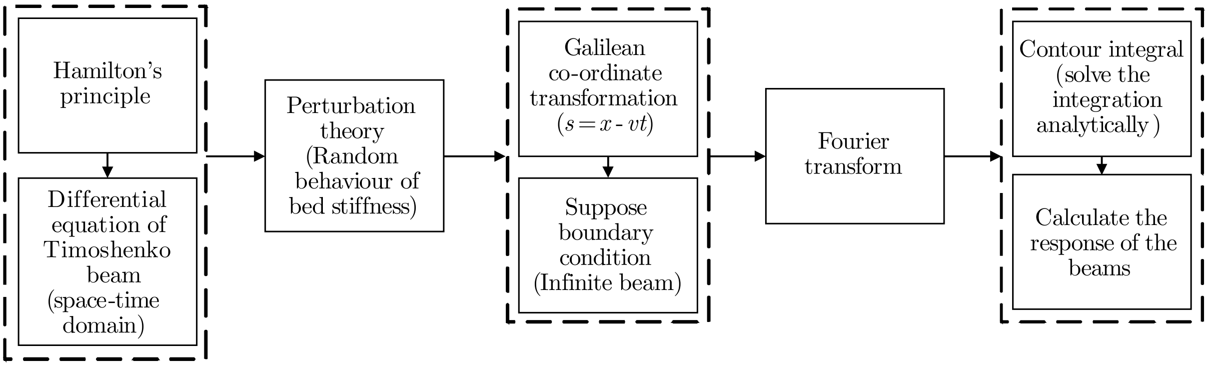

Fig. 2. Solving the governing differential equation

2.2. First-order perturbation approach

The perturbation method is proposed to compute the response of the beams to a harmonic moving load. The responses of the ladder track (rail and ladder unit) are decomposed to zero and first-order terms

$$\begin{aligned} w(x,t) &= w_0^i(x,t) + \phi w_1^i(x,t) \\ \psi(x,t) &= \psi_0^i(x,t) + \phi\psi_1^i(x,t) \end{aligned} \qquad i = 1,2 \tag{2.4}$$

where $i = 1$ for the rail and $i = 2$ for the ladder unit.

2.3. Solution

Equations (2.2) and (2.3) are solved using Eqs. (2.4) and equating terms with the same powers of ϕ. The Galilean coordinate transformation is

$$s = x - vt \tag{2.5}$$

The boundary conditions of deflection, velocity, and acceleration of the beams are assumed to be zero in positive and negative infinity. Using the state variable transformation and applying the complex Fourier transform results in

$$\begin{aligned} w_0^1(q) &= \frac{P(\beta_7 q^2 - \beta_8 q + \beta_9)D_4}{H(q)} & w_1^1(q) &= \frac{D_4 P + D_2 w_0^2}{H(q)} \\ w_0^2(q) &= \frac{P(\beta_7 q^2 - \beta_8 q + \beta_9)(-D\beta_3)}{H(q)} & w_1^2(q) &= \frac{-D_3 P - D\beta_1 w_0^2}{H(q)} \end{aligned} \tag{2.6}$$

D_1, D_2, D_3 and D_4 are described in Appendix 1. $H(q)$ is the determinant of the matrix

$$\mathbf{h} = \begin{bmatrix} D_1 & D_2 \\ D_3 & D_4 \end{bmatrix} \tag{2.7}$$

and ψ_0^1, ψ_0^2, ψ_1^1, and ψ_1^2 are equal to

$$\begin{aligned} \psi_0^1(q) &= \frac{-\beta_3 P q D_4}{H(q)} & \psi_1^1(q) &= \frac{-\beta_3 q(PD_4 + D_2\kappa w_0^2)}{(\beta_7 q^2 - \beta_8 q + \beta_9)H(q)} \\ \psi_0^2(q) &= \frac{\beta_{12} Pq(\beta_7 q^2 - \beta_8 q + \beta_9)D_3}{(\beta_{15} q^2 - \beta_{16} q + \beta_{17})H(q)} & \psi_1^2(q) &= \frac{\beta_{12} q(D_3 P + D_1\kappa w_0^2)}{(\beta_{15} q^2 - \beta_{16} q + \beta_{17})H(q)} \end{aligned} \tag{2.8}$$

General definitions for all coefficients are listed in Table 2. The response of the beams can be calculated by applying the inverse Fourier transform and using contour integrals (Mohammadzadeh *et al.*, 2014). The mean values for the beam deflection and bending moment and the covariance of a random function can be calculated as described by Mohammadzadeh *et al.* (2013) and Solnes (1997).

Table 2. Definitions of coefficients

Parameter	Definition	Parameter	Definition
β_1	$k_1A_1G_1 - \rho_1A_1v^2$	β_{10}	$k_2A_2G_2 - \rho_2A_2v^2$
β_2	$2\rho_1A_1\Omega v$	β_{11}	$2\rho_2A_2\Omega v$
β_3	$\mathrm{i}k_1A_1G_1$	β_{12}	$\mathrm{i}k_2A_2G_2$
β_4	$\mathrm{i}c_pv$	β_{13}	$\mathrm{i}c_Bv$
β_5	$-\rho_1A_1\Omega^2 + k_p - \mathrm{i}c_p\Omega$	β_{14}	$-\rho_2A_2\Omega^2 + \overline{k} + k_p + \mathrm{i}c_p\Omega + \mathrm{i}c_B\Omega$
β_6	$k_p + \mathrm{i}c_p\Omega$	β_{15}	$\rho_2I_2v^2 - EI_2$
β_7	$\rho_1I_1v^2 - EI_1$	β_{16}	$2\rho_2I_2\Omega v$
β_8	$2\rho_1I_1\Omega v$	β_{17}	$\rho_2I_2\Omega^2 - k_2A_2G_2$
β_9	$\rho_1I_1\Omega^2 - k_1A_1G_1$	β_{18}	$k_p + \mathrm{i}c_p\Omega$

3. Model validation of ladder track

The stochastic simulation of the ladder track foundation has been validated as described below.

3.1. Validation using the stochastic finite element method

The response of a beam resting on a stochastic foundation is obtained using the stochastic finite element method (SFEM) as suggested by Fryba *et al.* (1993). Consider the second beam as a rigid component and evaluate the behavior of the upper beam assuming stochastic behavior for the foundation. Then, the random behavior of the system is calculated and validated using the results of Fryba *et al.* (1993). Figure 3 shows that the results calculated in current study are in good agreement with those reported by Fryba *et al.* (1993).

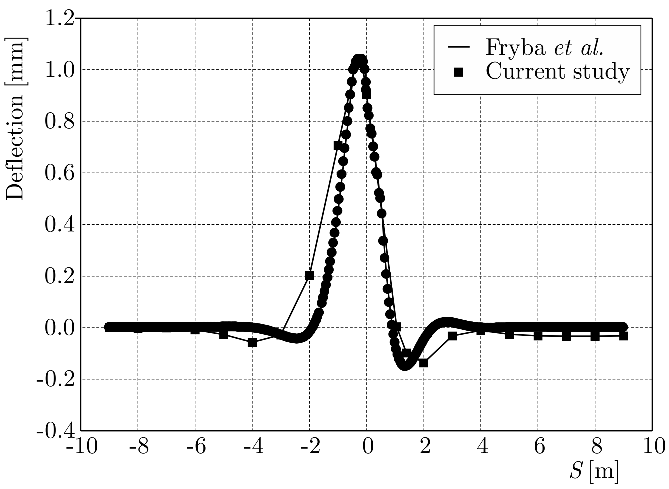

Fig. 3. Comparison between the current modeling and results by Fryba *et al.* (1993)

3.2. Validation by a deterministic model

Next, the deterministic behavior of the ladder track is verified using the results of Younesian *et al.* (2006). They investigated the dynamic behavior of a ladder track of finite length. The ladder track is simulated using a Timoshenko beam and the track is subjected to a moving load. The results of verification are illustrated in Table 3. The results of the current study are in good agreement with those reported by Younesian *et al.* (2006).

Table 3. Comparison of the current study and results by Younesian *et al.* (2006)

S [m]	Current study	Younesian *et al.*
−8	−4.13E-09	7.86E-05
−6	5.87E-08	−1.7E-05
−4	−3.11E-07	−0.00017
−2	−1.7E-05	−0.00031
0	−0.00083	−0.00037
2	−3E-05	−0.00032
4	1.06E-06	−0.00015
6	−3.24E-08	7.61E-05
8	7.16E-10	0.00018

4. Response of the ladder track

The response of the simulated ladder track is next investigated under a harmonic moving load. The railway substructure should be constructed and confirmed using adequate ground stiffness and standards (Younesian *et al.*, 2005). It is not possible to provide a track bed with absolutely uniform specifications, and there are many factors that influence the subgrade (Phoon, 2008; Griffiths and Fenton, 2007; Fenton and Griffiths, 2008; Baecher and Chrsitian, 2003). The finite distance correlation can be assumed using bed stiffness as a random field. A parametric study was done on the key parameters of solution derived using the track bed stiffness from the field data by Berggren (2009). The physical and geometrical properties of the track are listed in Table 4.

Table 4. Parameters used in the model

Rail		Ladder	
Parameters	Value	Parameters	Value
Young's modulus E_1	210 GPa	Young's modulus E_2	28.2 GPa
Shear modulus G_1	77 GPa	Shear modulus G_2	11.75 GPa
Mass density ρ_1	7850 kg/m^3	Mass density ρ_2	3954.7 kg/m^3
Cross-sectional area A_1	$7.69 \cdot 10^{-3}$ m^2	Cross sectional area A_2	$31 \cdot 10^{-3}$ m^2
Second moment of inertia I_1	$30.55 \cdot 10^{-6}$ m^4	Second moment of inertia I_2	$98.3 \cdot 10^{-6}$ m^4
Shear coefficient k_1	0.4	Shear coefficient k_2	0.43
Rail pad		**Foundation**	
Parameters	Value	Parameters	Value
Stiffness k_p	$40 \cdot 10^6$ Nm^{-2}	Mean value of stiffness k_B	$50 \cdot 10^6$ Nm^{-2}
Viscous damping c_p	$6.3 \cdot 10^3$ Nm^{-2}	Variance of stiffness σ_{kB}^2	$4.4186 \cdot 10^{13}$ N^{2}m^{-4}
		Viscous damping c_B	$41.8 \cdot 10^3$ Nsm^{-2}

4.1. Load frequency influence

The velocity of the moving load is assumed to be 100 km/h. Figure 4 shows that, by increasing the load frequency, the mean value and standard deviation of the response of the upper beam (rail) initially decreases and then increases. In addition, the distribution widens as the oscillations increase along the rail.

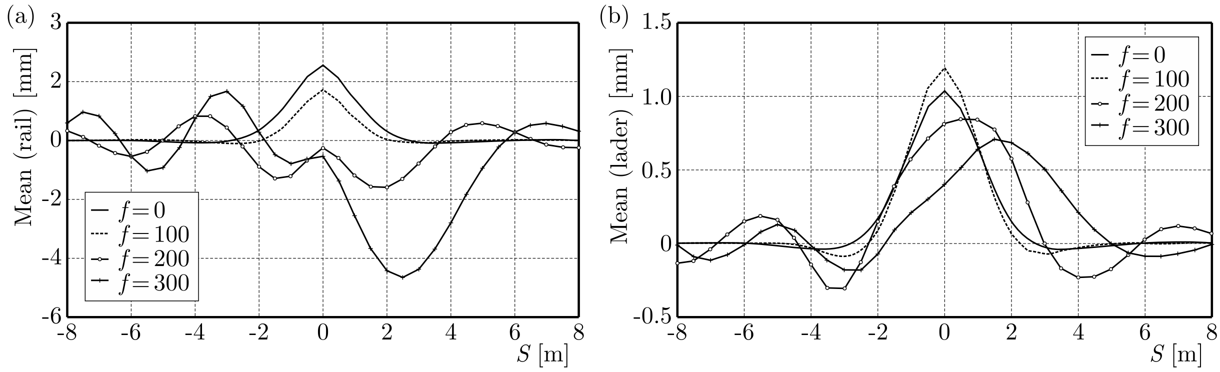

Fig. 4. Effect of load frequency on track deflection (mean value)

An increase in the load frequency decreases the response of the lower beam (ladder unit), indicating that both the mean value and standard deviation of the ladder unit show decreasing trends. Figure 5 shows the wider distribution with the increase in fluctuations along the beam.

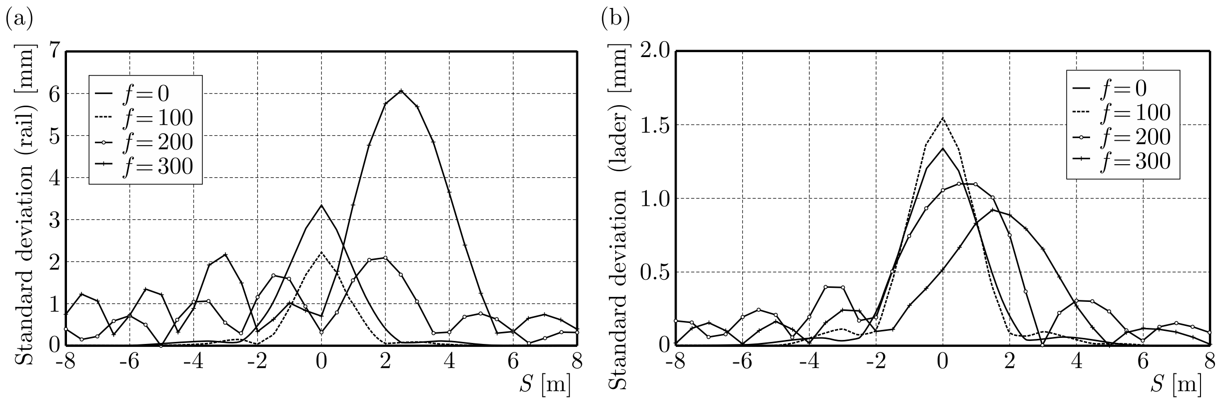

Fig. 5. Effect of load frequency on track deflection (standard deviation)

Figures 6 and 7 show the mean value and standard deviation of the rail and ladder bending moments, respectively. As the load frequency increases, the response of the rail first decreases and then increases. The velocity of the moving load is assumed to be 100 km/h.

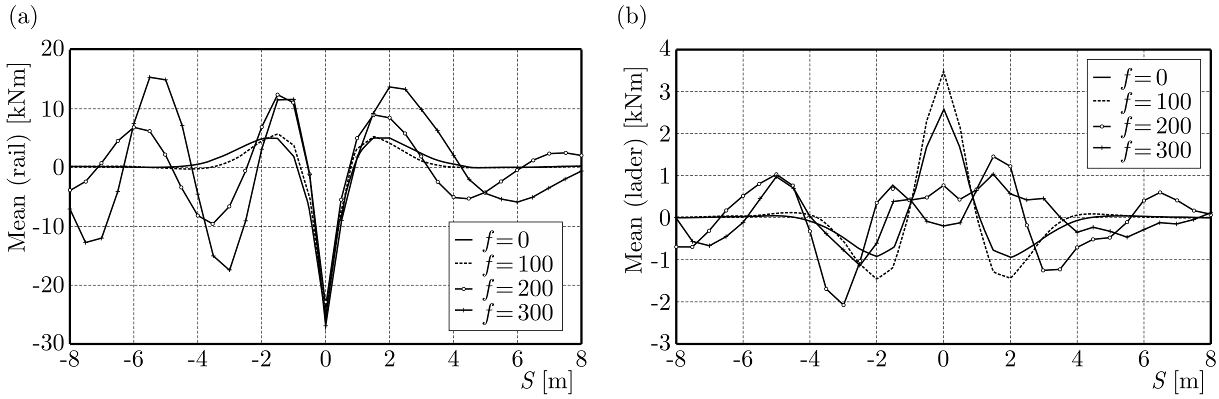

Fig. 6. Effect of load frequency on track bending moment (mean value)

The mean value and standard deviation of the ladder unit decreased as the load frequency increased. As shown, the fluctuation of the ladder first increased and then decreased.

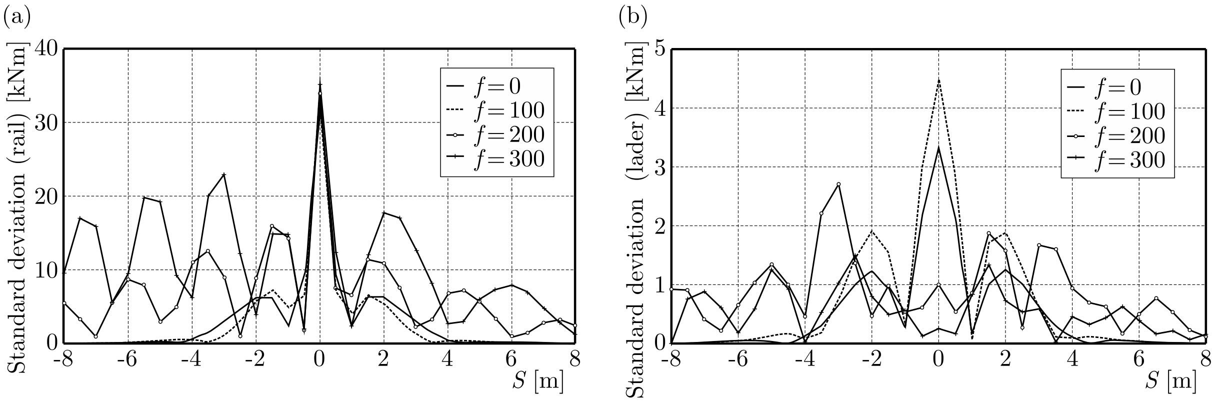

Fig. 7. Effect of load frequency on track bending moment (standard deviation)

4.2. Load velocity influence

The variation in load velocity versus the behavior of the double beam is shown in Figs. 8 and 9 for the response of the ladder track. The figures include the deflection and bending moment of both beams. As shown, the maximum response of the rail versus loading frequency have been attained and employed as design criteria. An increase in the velocity of the moving load decreased the value of this frequency.

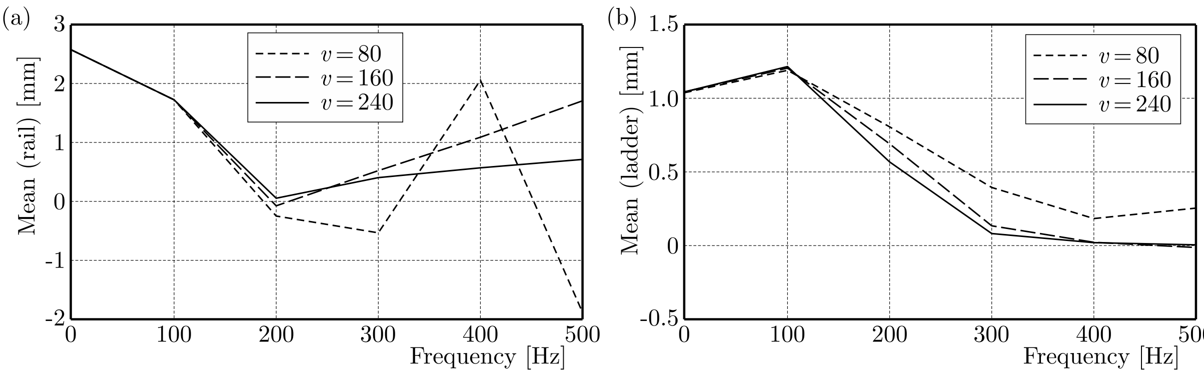

Fig. 8. Effect of velocity on the ladder track (mean value)

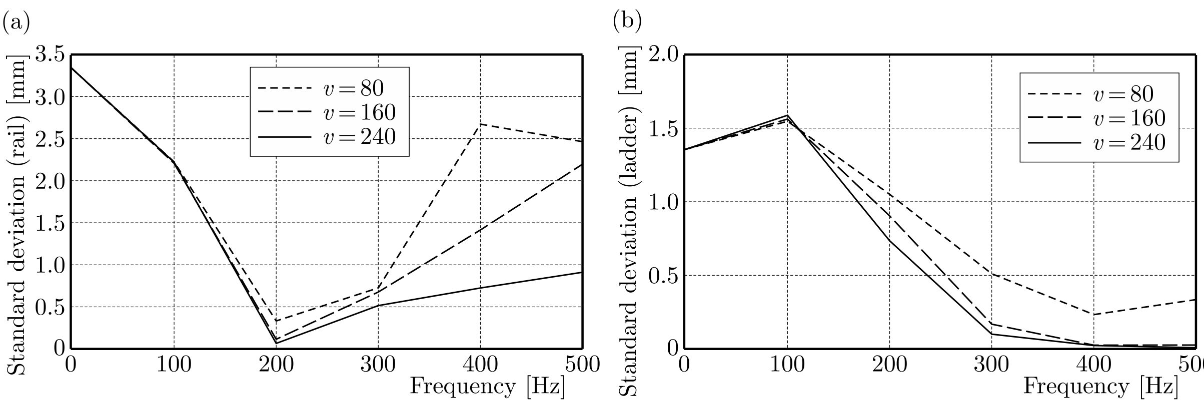

Fig. 9. Effect of velocity on the ladder track (standard deviation)

4.3. Effect of the coefficient of variation of bed stiffness

The coefficient of variation (C_V) of the stiffness of the bed is varied to assess its effect on the track bed (Figs. 10 and 11).It can be observed that increasing the C_V increases the standard deviation of the rail and ladder.

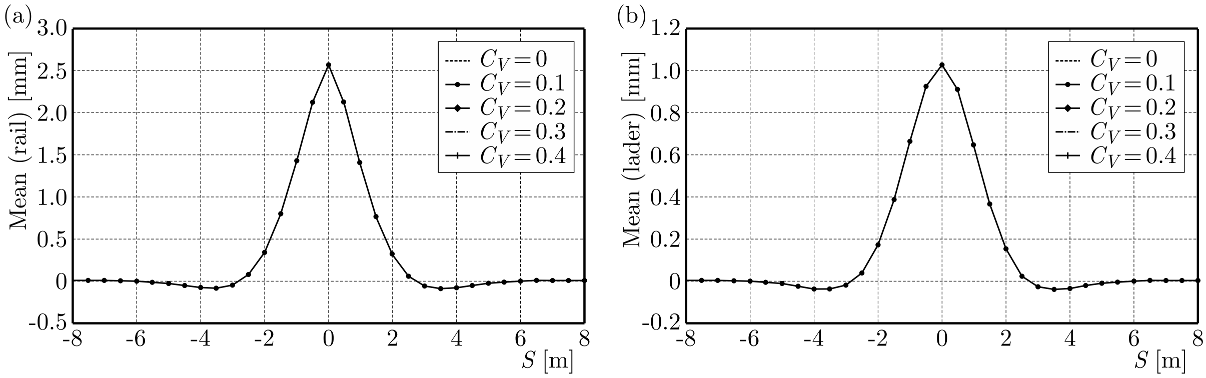

Fig. 10. Effect of C_V on the ladder track (mean value)

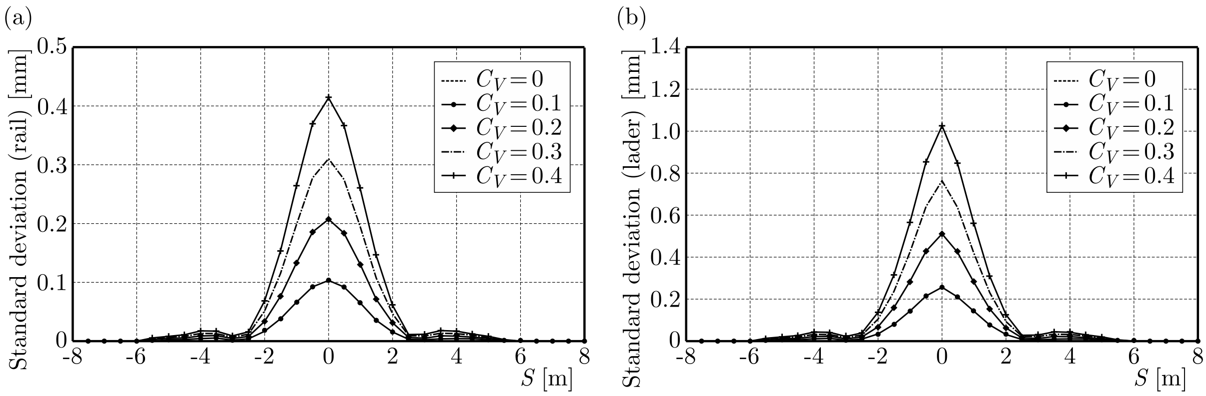

Fig. 11. Effect of C_V on the ladder track (standard deviation)

5. Conclusion

The dynamic behavior of the ladder track has been investigated in the present study. The ladder track has been simulated using an analytical model with a double Timoshenko beam. The upper beam simulated the rail and the lower beam simulated the ladder unit. A series of springs and dashpots represent the rail pad and foundation. The foundation stiffness of the system has been assumed to exhibit stochastic behavior as simulated by field tests. The first-order perturbation method has been applied and the responses, including the deflection and bending moment, are shown in form of the mean value and standard deviation. It has been found that increasing the load frequency decreased and then increased the response of the track. The peak frequency is the point at which all responses are at maximum value. It was found that the peak frequency increases as the velocity of the load velocity increases.

Appendix 1

The parameters in Equations (2.6) and (2.8) are described below

$$D_1 = \beta_1\beta_7 q^4 + (-\beta_1\beta_8 + \beta_2\beta_7 - \beta_4\beta_7)q^3 + (\beta_1\beta_9 - \beta_2\beta_8 - \beta_3^2 + \beta_4\beta_8 + \beta_5\beta_7)q^2 + (\beta_2\beta_9 - \beta_4\beta_9 - \beta_5\beta_8)q + \beta_5\beta_9$$

$$D_2 = \beta_4\beta_7 q^3 - (\beta_4\beta_8 + \beta_6\beta_7)q^2 + (\beta_6\beta_8 + \beta_4\beta_9)q - \beta_6\beta_9$$
$$D_3 = \beta_4\beta_{15} q^3 - (\beta_4\beta_{16} + \beta_{15}\beta_{18})q^2 + (\beta_4\beta_{17} + \beta_{16}\beta_{18})q - \beta_{17}\beta_{18}$$
$$\begin{aligned} D_4 = \beta_{10}\beta_{15} q^4 + (-\beta_{10}\beta_{16} + \beta_{11}\beta_{15} - \beta_4\beta_{15} - \beta_{13}\beta_{15})q^3 + (\beta_{10}\beta_{17} - \beta_{11}\beta_{16} - \beta_{12}^2 \\ + \beta_4\beta_{16} + \beta_{13}\beta_{16} + \beta_{14}\beta_{15})q^2 + (\beta_{11}\beta_{17} - \beta_4\beta_{17} - \beta_{13}\beta_{17} - \beta_{14}\beta_{16})q + \beta_{14}\beta_{17} \end{aligned}$$

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