JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
44, 3, pp. 691-712, Warsaw 2006

NUMERIAL IMPLEMENTATION OF MULTIAXIAL HIGH-CYCLE FATIGUE CRITERION TO STRUCTURAL OPTIMIZATION

Mirosław Mrzygłód

Institute of Rail Vehicles, Cracow University of Technology
e-mail: mrzyglod@mech.pk.edu.pl

Andrzej P. Zieliński

Institute of Machine Design, Cracow University of Technology
e-mail: apz@mech.pk.edu.pl

Modern multiaxial high-cycle fatigue criteria were investigated with respect to their application in structural optimization procedures coupled with finite element codes. As a result of tests carried out for several fatigue criteria, the Dang Van hypothesis was used for the detailed numerical study. A way of respective adapting the high-cycle load history was also suggested. The complete algorithm of the fatigue optimization was illustrated by applying the proposed procedures to vehicle parts which are subject to high-cycle loadings. The finite element code ANSYS® was used in the structural modeling.

Key words: high-cycle fatigue, multi-axial fatigue criterion, structural optimization, finite element method

1. Introduction

The optimization of structures subjected to Multiaxial High-Cycle Fatigue (MHCF) is a very important problem for both aircraft and ground vehicle design. Among problems, which impede development of this area of study, is the lack of a unique hypothesis for the MHCF phenomena. Equally important seems to be the problem of suitable adaptation of an experimental high-cycle load history. It is also necessary to note that the optimization of this type of problems enounters a barrier of long computational times. However, this last obstacle seems to be less important, considering the rapid development of computing power.

Before starting the investigations, certain necessary assumptions were made. They can be divided into two groups, the first of which concerns the range of the analyzed phenomena:

- only high-cycle fatigue and infinite fatigue life are taken into consideration,
- examined hypotheses concern multi-axial loading conditions.

The second group takes into account a method of modelling and a level of simplification:

- state of stresses and deformations in structures is modelled using the finite element method,
- particular components of loads have in-phase or reverse character,
- frequency of loads is considerably lower than the first eigen-frequency of studied structures,
- inertial effects are not taken into account,
- real time load history is transformed into a form admissible for the analysis and optimization,
- possible short term violations of the established load regime are accounted for by a suitable safety factor.

The above mentioned simplifications are not very restrictive and can sometimes be easily weakened, for example, by adding inertial forces to the load history. However, taking into account less important influence of the simplified phenomena on the behavior of a large family of the considered structures, they were not taken into account in the present analysis.

2. Multiaxial high-cycle fatigue criteria

Experimental investigations in recent decades resulted in numerous hypotheses of the Multiaxial High-Cycle Fatigue (MHCF). In literature, many proposals of such criteria can be found (Garud, 1981; Ballard *et al.*, 1995; Papadopoulos *et al.*, 1997; Dang Van and Papadopoulos, 1999). From the point of view of applications to numerical optimizations, the most convenient are two groups: criteria based on stress state invariants and criteria using average stresses or deformations in an elementary volume. For the final selection of a suitable fatigue hypothesis, the following three formulae were taken into consideration: Sines (1959), Crossland (1956) and Dang Van *et al.* (1989). The first two belong to the invariant group and the third one is based on the average value concept. The components of these criteria are easy to obtain from FEA software. The invariant formulae usually consist of quantities related to hydrostatic and octahedral stresses. The use of these hypotheses allows one to determine the initiation point of fatigue cracks. However, the orientation of potential cracks with these criteria cannot be defined. Sines (1959) and Papadopoulos *et al.* (1997) analyzed the influence of different combinations of variable bending and torsion stresses on the fatigue life of a structure. On this basis, the formulated a criterion including the amplitude of octahedral shear stresses and an average value of normal hydrostatic stresses. The Sines hypothesis shows satisfactory correlations with experimental investigations. It has the following form

$$\sqrt{J_{2,a}} + \kappa\sigma_{H,m} \leqslant \lambda \tag{2.1}$$

where κ and λ are material parameters, $\mathbf{J}_2$ is the second invariant of stress deviator tensor

$$J_{2,a} = \frac{1}{6}[(\sigma_{1a} - \sigma_{2a})^2 + (\sigma_{2a} - \sigma_{3a})^2 + (\sigma_{3a} - \sigma_{1a})^2]$$

and

$$\sigma_{H,m} = \frac{1}{3}[\sigma_{1m} + \sigma_{2m} + \sigma_{3m}]$$

σ_{ia}, $i = 1, 2, 3$, are amplitude-type principal stresses; symbol a means corresponding amplitudes of loads, σ_{im}, $i = 1, 2, 3$ are mean-type principal stresses; symbol m means the average (mean) value of loads.

A criterion formulated by Crossland (1956) and Ballard *et al.* (1995) is very close to the Sines formula. The difference in the approach of both researchers concerns the hydrostatic stresses σ_H which, according to Crossland, should be represented by their maximum value

$$\sqrt{J_{2,a}} + \kappa\sigma_{H,max} \leqslant \lambda \tag{2.2}$$

where

$$\sigma_{H,max} = \sigma_{H,a} + \sigma_{H,m}$$

Criteria based on average stresses in an elementary volume V take into consideration the average value of shear and normal stresses in this volume. Dang Van (1989) and Dang Van and Papadopoulos (1999) formulated his hypothesis by observing local plastic deformations on a microscopic scale, on the level of crystallites. They can initiate micro-cracks even then, when a studied structure remains in the macroscopic scale in a range of elastic strains. According to Dang Van, the fatigue damage appears in a definite time, when the combination of local shear stresses $\tau(t)$ and the hydrostatic stress $\sigma_H(t)$ cuts the borders of an admissible fatigue area. Initially complicated, the calculation method of shear stress amplitude became simplified in a new form of the Dang Van criterion (Ballard *et al.*, 1995)

$$\max_A[\tau(t) + \kappa\sigma_H(t)] \leqslant \lambda \tag{2.3}$$

where A is the area of the studied object

$$\tau(t) = \frac{\sigma_1(t) - \sigma_3(t)}{2} \qquad\qquad \sigma_H(t) = \frac{1}{3}[\sigma_1(t) + \sigma_2(t) + \sigma_3(t)]$$

The material parameters can, in principle, be expressed by data from two high-cycle fatigue tests: reversed bending (fatigue limit f_{-1}) and reversed torsion (fatigue limit t_{-1}). For all the criteria $\lambda = t_{-1}$. However, κ is different for the particular hypothesis:

— Sines

$$\kappa = \frac{3t_{-1}(\sigma_f + f_{-1})}{f_{-1}\sigma_f} - \sqrt{6} \tag{2.4}$$

where σ_f is the ultimate tensile strength of a material,

— Crossland

$$\kappa = \frac{3t_{-1}}{f_{-1}} - \sqrt{3} \tag{2.5}$$

— Dang Van

$$\kappa = \frac{3t_{-1}}{f_{-1}} - \frac{3}{2} \tag{2.6}$$

3. Load time history transformation

The Sines and Crossland criteria require calculations of appropriate stress invariants in the whole structure for equivalent average and amplitude loads. The equivalent loads could be determined in different ways. In our case, we decided to base the computations on extreme values of these loads from the available time load history of investigated objects. This would increase the safety factor of the new, improved structures.

Dang Van criterion (2.3) proposes the analysis with respect to variable time. For optimization the following pattern of reduction of the load time history was proposed:

- determination of the equivalent amplitude and mean load values in the way described above,
- calculation of five load levels, which would be checked in every single fatigue analysis (see Fig. 1).

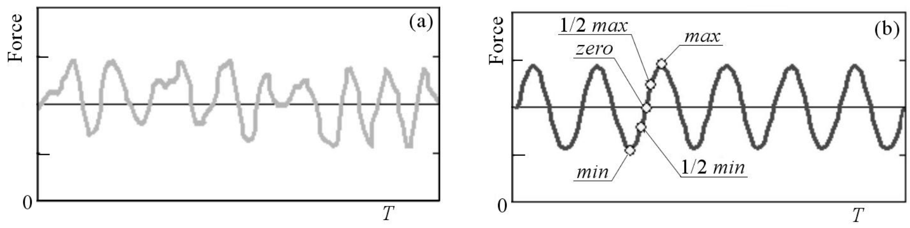

Fig. 1. An example of the load time history $q(t)$ (a) transformed to a convenient form for the fatigue Dang Van criterion (b)

The whole fatigue analysis of five load cases according to the proposed rules was carried out in an automatic way.

4. Numerical implementation of fatigue criteria

For computer calculations, a convenient notion of the von Mises equivalent stress can be introduced to Sines criterion (2.1)

$$\frac{1}{\sqrt{3}}\sigma_{eq,a(von\ Mises)} + \kappa\sigma_{H,m} \leqslant \lambda \tag{4.1}$$

Regarding (2.4), the Sines equation has now the following form

$$\frac{1}{\sqrt{3}}\sigma_{eq,a(von\ Mises)} + \Big(\frac{3t_{-1}}{f_{-1}} + \frac{3t_{-1}}{\sigma_f} - \sqrt{6}\Big)\sigma_{H,m} \leqslant t_{-1} \tag{4.2}$$

Analogically, the Crossland criterion with the von Mises equivalent stress can be written as follows

$$\frac{1}{\sqrt{3}}\sigma_{eq,a(von\ Mises)} + \kappa\sigma_{H,max} \leqslant \lambda \tag{4.3}$$

and, taking into account material parameters (2.5), it has the final form

$$\frac{1}{\sqrt{3}}\sigma_{eq,a(von\ Mises)} + \Big(\frac{3t_{-1}}{f_{-1}} - \sqrt{3}\Big)\sigma_{H,max} \leqslant t_{-1} \tag{4.4}$$

The numerically convenient form of the Dang Van equation uses the Tresca equivalent stress

$$\max_{A}\Big[\frac{1}{2}\sigma_{eq(Tresca)}(t)+\kappa\sigma_H(t)\Big]\leqslant\lambda \tag{4.5}$$

After introduction the material parameters (2.6)

$$\max_{A}\Big[\frac{1}{2}\sigma_{eq(Tresca)}(t)+\Big(\frac{3t_{-1}}{f_{-1}}-\frac{3}{2}\Big)\sigma_H(t)\Big]\leqslant t_{-1} \tag{4.6}$$

For the numerical implementation of the criteria, a scripting language APDL of the program ANSYS® (Ansys Inc. 2004) was used.

APDL makes it possible to build macros with the following functions:

- to record current values of stress tensor components and a certain predefined quantity (e.g. an equivalent stress),
- to calculate the value of the hydrostatical stress σ_H,
- to calculate the material parameters κ and λ,
- to calculate the value of the equivalent stresses according to Sines (4.2), Crossland (4.4) and Dang Van (4.6) criteria,
- to automatically record the results of analysis in the form of contour maps.

Two versions of the fatigue analysis programs were prepared. The first version, for the Sines and Crossland criteria, contained two series of static calculations: for the equivalent average and amplitude loads. From the FEM analysis, the invariants (i.e. the criteria components) were obtained in the whole structure. The second version of the program, for the Dang Van criterion, calculated five load cases, according to the accepted assumptions (Fig. 1). The whole fatigue program was executed automatically, including generation of the graphical map for each criterion. Full listings of fatigue APDL macros of the Sines, Crossland and Dang Van criteria are enclosed in Appendices A and B.

5. Test of fatigue criteria

To compare the fatigue criteria discussed above, certain FEM models were proposed and suitable analysis programs were prepared. The programs were introduced into the FE code ANSYS v. 8.1. A car front suspension arm (Fig. 2a) was chosen as a test example. The Multi-Body Simulation (MBS) using the program CarDyn 1.0 was conducted for determination of the load history (Walczak, 2003). From these results, the equivalent loads for fatigue analysis were prepared according to the rules assumed in the previous section.

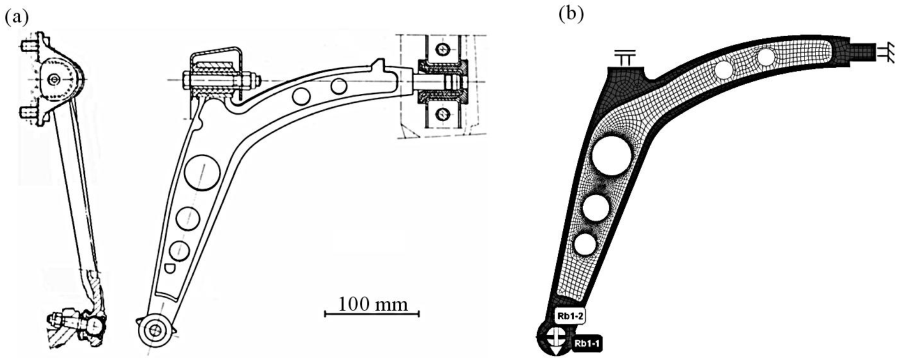

Fig. 2. The front suspension arm of a vehicle (a) and its FE model (b)

A three-dimensional shape of the structure was simplified to a 2D model with varying thickness. The analysis was carried out with the help of the ANSYS SHELL63 element (see the model in Fig. 2b).

For the assumed material model ($f_{-1} = 260\,\mathrm{MPa}$, $t_{-1} = 160\,\mathrm{MPa}$, $\sigma_f = 580\,\mathrm{MPa}$), the analyzed fatigue criteria take the following form (compare Eqs (2.4)-(2.6)):
— Sines

$$\sigma_{eqv,S} = 0.58\sigma_{eq,a(von\ Mises)} + 0.224\sigma_{H,m} \leqslant 160\,\mathrm{MPa} \tag{5.1}$$

— Crossland

$$\sigma_{eqv,C} = 0.58\sigma_{eq,a(von\ Mises)} + 0.114\sigma_{H,\max} \leqslant 160\,\mathrm{MPa} \tag{5.2}$$

— Dang Van

$$\sigma_{eqv,DV} = 0.5\sigma_{eq(Tresca)}(t) + 0.346\sigma_{H}(t) \leqslant 160\,\mathrm{MPa} \tag{5.3}$$

The results of conducted fatigue analysis of the studied structure are presented in the form of contour-maps of equivalent stresses accordingly to the investigated criteria (Fig. 3). The contour-map of the Dang Van criterion (Fig. 3c) represents the maximum value of the equivalent stress obtained in the five-step load history.

As it resulted from the numerical investigations, the Dang Van hypothesis turned out to be the most rigorous formulation. It leads to the most demanding requirements ensuring, however, the highest safety level. The Dang Van

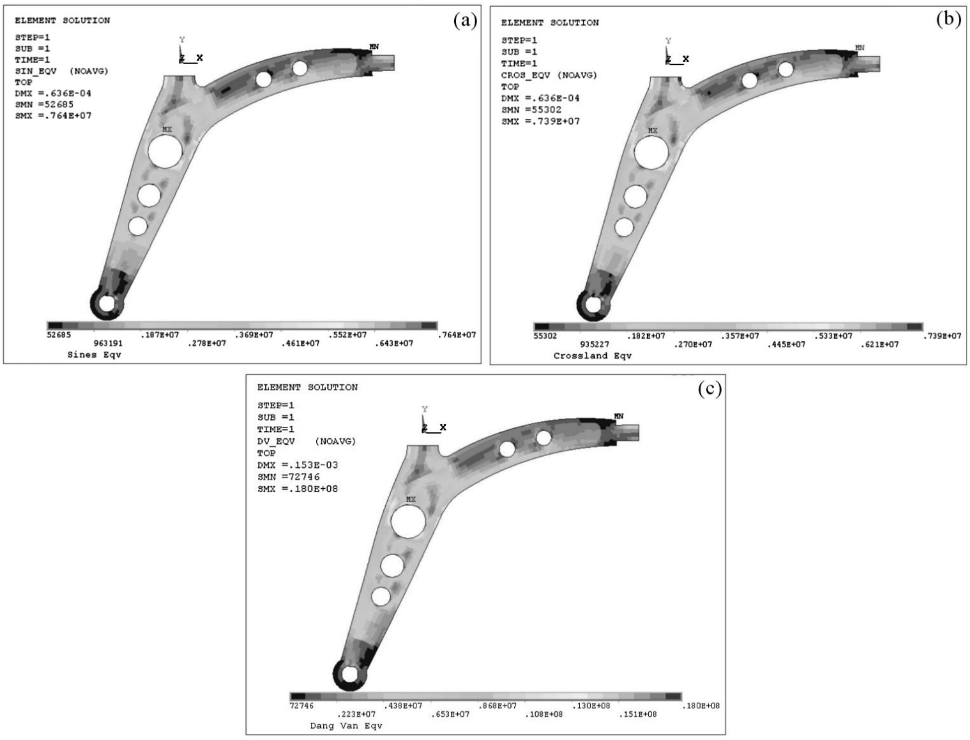

Fig. 3. Distribution of equivalent stresses [Pa] according to the Sines, Crossland and Dang Van criteria; SMX – maximal equivalent stress [Pa]

hypothesis is used in the next stage of the investigations concerning the optimization of a structure working in high-cycle load conditions.

For the process of optimization, the method of probabilistic search based on evolutionary algorithms was chosen and the software package Evolutionary Optimization System (EOS) (Osyczka, 2000) was applied. It should be also noted that the parallel computing used by the authors to accelerate calculations presented in this paper (Mrzygłód, 2005) was an effective and very convenient tool in the optimization algorithms.

6. Examples of structural optimization

6.1. Example I

A front suspension arm of a car was chosen as the first example of fatigue optimization. The regime of work as well as geometry of this part was discussed in the previous chapter. The structure presented in Fig. 2a was subjected to variable, high-cycle load conditions. The objective of the optimization was to decrease the mass of the suspension arm by parametric adaptation of the geometry of the object with maintaining its fatigue endurance on the same level. The safety factor for the optimized part was accepted by assuming, as the admissible limit, the extreme equivalent stress obtained in the fatigue test of the initial form of the structure (Fig. 3c).

Considering the geometrical form of the part as well as the way of its loading, it was possible to simplify the examined problem to two dimensions. This permitted a considerable acceleration of the optimization process with a minor decrease in the quality of the results.

As a FEM model of the examined component, the earlier prepared model of the fatigue criteria test was adapted. To the non-parametric fatigue analysis of the suspension arm an option of a parametric change of the geometry was introduced. Moreover, functions of automatic choosing the maximum value of the state parameter were added. This means investigation of five load cases and calculation of five values of the equivalent stresses according to Dang Van equation (4.5). The accepted state parameter bound 18 Mpa was taken from the preliminary analysis of the original form of the examined part (see Fig. 3c).

The optimization problem was formulated as follows:

- Eight decision variables were considered (four radii of holes in the left side of the suspension arm, position of the first hole on a secant line and, respectively, three distances between edges of the remaining holes, see Fig. 4)

$$\boldsymbol{x} = [x_1, x_2, \dots, x_n] \qquad n = 8 \tag{6.1}$$

  The decision variables were limited by their upper and lower bounds

$$\underline{x}_i \leqslant x_i \leqslant \overline{x}_i \qquad i = 1, 2, \dots, 8 \tag{6.2}$$

- One state variable (the maximum equivalent stress according to the Dang Van criterion) was considered, with the upper limit

$$\max_A g_i(\boldsymbol{x}) \leqslant \overline{g}_i \qquad i = 1 \tag{6.3}$$

- The mass of the arm was chosen as the main objective function $W(x)$.

In this example of optimization, a decrease of about 4.11% mass of the examined structure was obtained (in reference to the original FEM model).

For a comparative optimization procedure of the considered structure, the von Mises criterion was applied. In this case, a mass reduction of 6.14% was

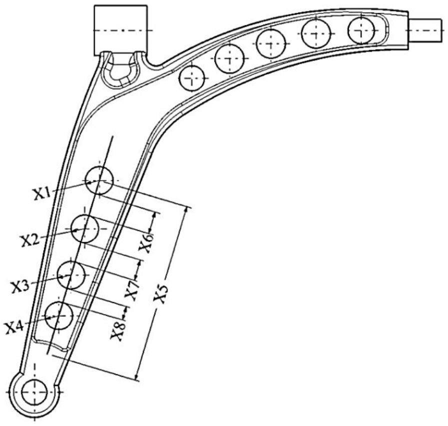

Fig. 4. Description of design variables in the first optimization example

obtained. The results of the optimization processes for the Dang Van and von Mises criteria are compared in Fig. 5. A considerably bigger value of the von Mises stress (Fig. 5a) in relation to the Dang Van equivalent stress (Fig. 5b) can be observed. However, it should be noted that the von Mises stress is usually compared to the yield stress limit σ_y, whereas the fatigue equivalent stress – to the reversed torsion fatigue limit t_{-1}. In Fig. 5, a low level of admissible stresses can be noticed. This is due to the large safety factor, which was taken from the analysis of the original suspension arm.

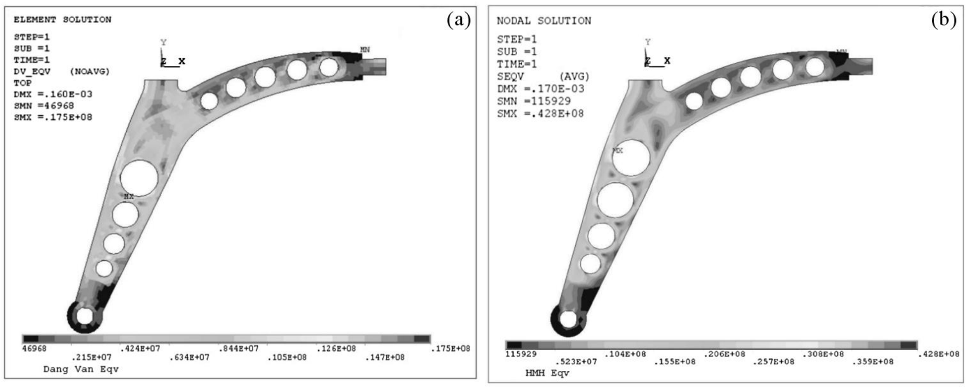

Fig. 5. Comparison of results of the first optimization example between the Dang Van (a) and von Mises (b) criteria; maps of equivalent stresses [Pa]; SMX – maximal equivalent stress [Pa]

6.2. Example II

A more complex object – a rear suspension arm of a car (Fig. 6a and Fig. 7a) was chosen as a second numerical example. In a vehicle, this element is also subjected to high-cycle load conditions (the load scheme is presented in Fig. 4b, see also Walczak (2003)). In the example, the mass of the structure was optimized with the Dang Van fatigue equivalent stress as a state parameter. For the examined element, MBS simulations were also carried out to obtain the load history of close to standard work conditions (Walczak, 2003). Based on this simulation, the history of 12 components of loads was determined.

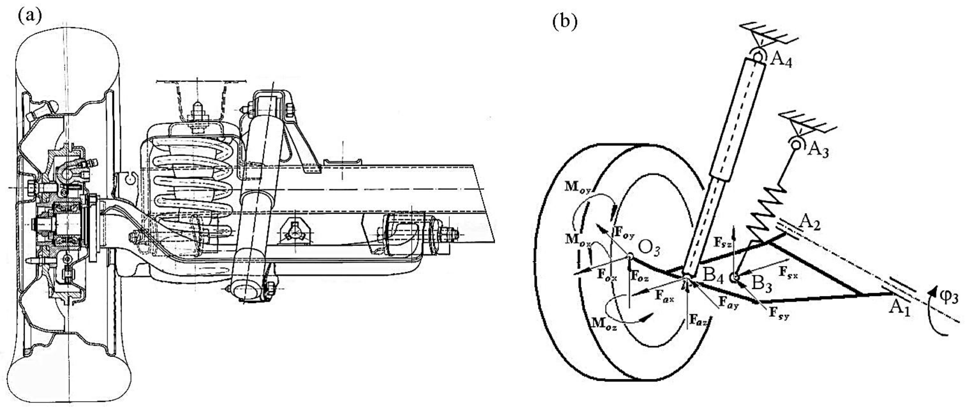

Fig. 6. A car suspension system with the optimized part (a) and a scheme of loads acting on the optimized suspension arm (b) (Walczak, 2003)

The optimized suspension arm is built from two thin sheet draw pieces assembled by welding. Hence, the shell-type finite elements (SHELL63) were used here in the FEM modeling. This allowed us to obtain acceptable results in a relatively small computational time. Using the APDL scripting language, the parametric FE model of the arm was coded. Because of the limited modeling capabilities of the ANSYS® program, the original form of the suspension arm had to undergo certain simplifications (Fig. 7b). The process of simplification was carried out to obtain results on the safe side for the studied object.

The Dang Van fatigue subroutine was added to the FEA program. Also the automatic recording sequences of the state parameter and the objective function were included into the text file. The load time history was transformed according to the assumptions of the Dang Van criterion. The five cases from the equivalent time history (see Fig. 1b) were calculated for all twelve load components. Moreover, to find the limit value of the state parameter (the

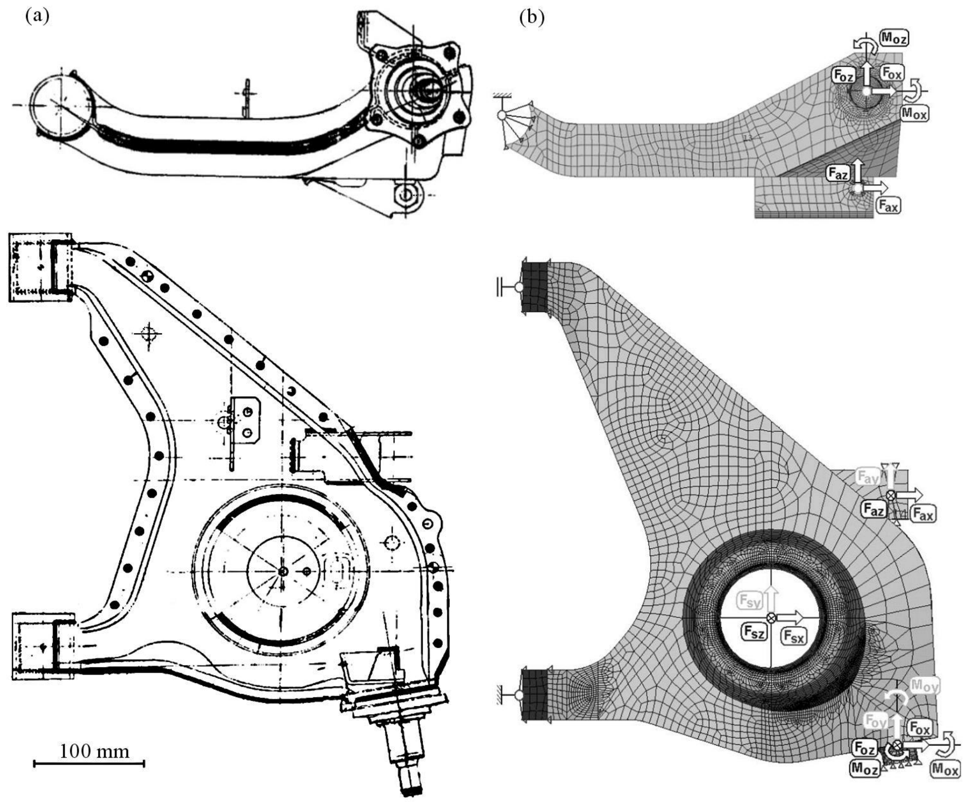

Fig. 7. The rear suspension arm (a) and its FE model with loads and boundary conditions (b)

Dang Van equivalent stress), a preliminary fatigue analysis for the original shape of the suspension arm was done.

The optimization task was formulated in the following form:

- Five decision variables were selected

$$\boldsymbol{x} = [x_1, x_2, \dots, x_n] \qquad n = 5 \tag{6.4}$$

  Remark: Initially selected twelve (see Fig. 8) decision variables were reduced to five during the process of sensitivity analysis (Mrzygłód and Zieliński, 2005).

- The decision variables were limited by their upper and lower bounds

$$\underline{x}_i \leqslant x_i \leqslant \overline{x}_i \qquad i = 1, 2, \dots, 5 \tag{6.5}$$

- One state variable (the maximum equivalent stress according to the Dang Van criterion) was considered, with the upper limit

$$\max_{A} g_i(\boldsymbol{x}) \leqslant \overline{g}_i \qquad i = 1 \tag{6.6}$$

- The mass of the arm was chosen as the main objective function $W(x)$.

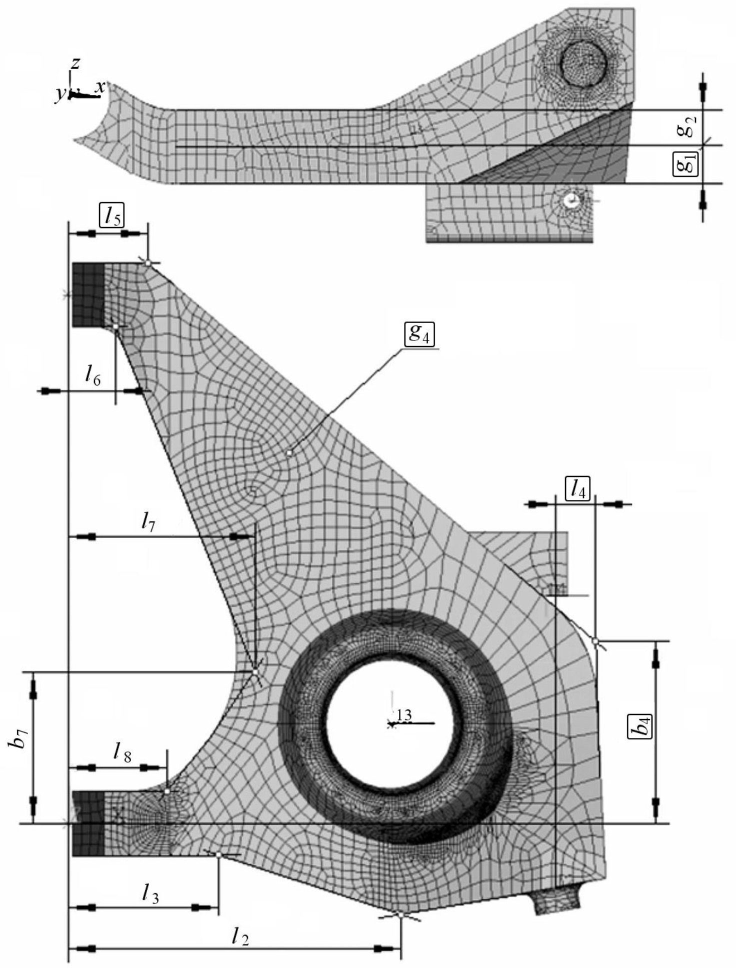

Fig. 8. Decision variables initially accepted for the optimization process; variables selected by sensitivity analysis marked with the boxes

As a result of the optimization, a decrease of ca 11.3% mass of the studied structure was obtained. Figure 9 presents the initial and optimal forms of the investigated structure.

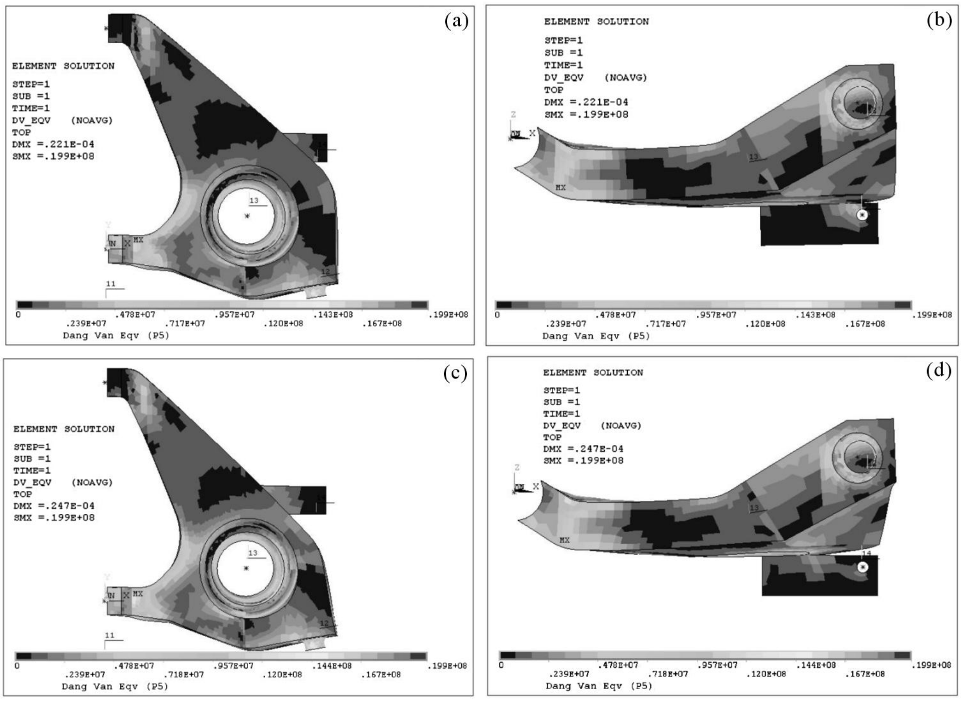

Fig. 9. Comparison of the initial (upper) and optimal (bottom) geometry of the structure; maps of equivalent stresses [Pa]; SMX – maximal equivalent stress [Pa]

7. Conclusions

In this paper, proposals of modelling of multiaxial high-cycle fatigue phenomena with the use of the finite element method are presented. As a result of the discussed tests, the Dang Van fatigue criterion was chosen and implemented to investigate the structural optimization process. The proposed strategy of adaptation of the time load history considerably accelerated the whole numerical cycle of the fatigue analysis and optimization.

The examples of structural optimization described in the work refer to the use of the fatigue criterion as a constraint (state parameter). However, this is only one of the possible applications. The examples of numerical implementations of MHCF criteria in the ANSYS® APDL code (enclosed) show the possibility of their easy application to any fatigue numerical analysis. Hence, the proposed methodology of fatigue analysis and optimization can be used in a wide spectrum of high-cycle fatigue problems.

Appendix A. The ANSYS APDL macro of fatigue analysis according to the Sines and Crossland MHCF criteria

```
!==============================
! ANSYS APDL macro of fatigue analysis according
! Sines & Crossland MHCF criteria
! Cracow University of Technology
! Miroslaw Mrzyglod 2005 (C)
! mrzyglod@mech.pk.edu.pl
!==============================
/prep7 ! preprocessing
/SHOW,WIN32C
! definition of material model
! e.g. :
Rm=580e6
zgo=260e6
zso=160e6
! definition of geometry, FE mesh and boundary conditions
finish
!==============================
!---
! 1
!---
/solution
! definition of loads: mean values
! e.g.
Fk,43,Fx,(forcex_m/2)
Fk,40,Fx,(forcex_m/2)
Fk,41,Fy,(forcey_m/2)
Fk,42,Fy,(forcey_m/2)
SOLVE
finish
!==============================
/post1 ! postprocessing
/CONT,1,25,
! Hydrostatic mean stress
etable,m_s_1,s,1 ! S 1
etable,m_s_2,s,2 ! S 2
etable,m_s_3,s,3 ! S 3
```

```
SADD,m_s12,m_s_1,m_s_2,,, ! S 1 + S 2
SADD,m_s123,m_s12,m_s_3,,,! S 1 + S 2 + S 3
! Sigma H,m = 1/3 * (S 1 + S 2 + S 3) (hydrostatic mean
! stress)
sadd,m_s_h,m_s123,,(1/3),
! print of contour map of stress Sigma H,m to screen and ! ! to file
/title, Sigma H,m
PLetable,m_s_h,
/ui,copy,save,tiff,graph,color,reverse,portrain,yes,,
finish
!=============================
!—
! 2
!—
/solution
FDELE,ALL,ALL
! definition of loads: amplitude
! e.g.
Fk,43,Fx,(forcex_a/2)
Fk,40,Fx,(forcex_a/2)
Fk,41,Fy,(forcey_a/2)
Fk,42,Fy,(forcey_a/2)
SOLVE
finish
!=============================
/post1
/CONT,1,25,
! Amplitude of von Mises eqv. stress
etable,a_s_hmh,s,eqv
/title, Sigma EQV,a (von Mises)
PLetable,a_s_hmh,
/ui,copy,save,tiff,graph,color,reverse,portrain,yes,,
! Amplitude of hydrostatic stress
etable,m_s_1a,s,1 ! S 1a
etable,m_s_2a,s,2 ! S 2a
etable,m_s_3a,s,3 ! S 3a
SADD,m_s12a,m_s_1a,m_s_2a,,, !S 1a + S 2a
SADD,m_s123a,m_s12a,m_s_3a,,,! S 1a + S 2a + S 3a
! Sigma H,a = 1/3 * (S 1a + S 2a + S 3a)
```

```
sadd,m_s_ha,m_s123a,,(1/3),
! Maximal hydrostatic stress
! Sigma H,max = Sigma H,a + Sigma H,m
SADD,max_s_h,m_s_ha,m_s_h,,,
/title, Sigma H,max
PLetable,max_s_h,
/ui,copy,save,tiff,graph,color,reverse,portrain,yes,,
!————————————————————
! Sines criterion
!————————————————————
sines_kappa=(((3*zso)/zgo)+((3*zso)/Rm)-(sqrt(6)))
sadd,sin_eqv,a_s_hmh,m_s_h,(1/(sqrt(3))),sines_kappa,
/title, Sines Eqv
PLETAB,sin_eqv,
/ui,copy,save,tiff,graph,color,reverse,portrain,yes,,
!————————————————————
! Crossland criterion
!————————————————————
crossland_kappa=(((3*zso)/zgo)-(sqrt(3)))
sadd,cros_eqv,a_s_hmh,max_s_h,0.577350269,crossland_kappa,
/title, Crossland Eqv
PLETAB,cros_eqv,
/ui,copy,save,tiff,graph,color,reverse,portrain,yes,,
!————————————————————
finish
```

Appendix B. The ANSYS APDL macro of fatigue analysis according to the Dang Van MHCF criterion

```
!=============================
! ANSYS APDL macro of fatigue analysis according
! Dang Van MHCF criterion
! Cracow University of Technology
! Miroslaw Mrzyglod 2005 (C)
! mrzyglod@mech.pk.edu.pl
!=============================
/prep7 ! preprocessing
```

```
/SHOW,WIN32C
! definition of material model
! e.g. :
zgo=260e6
zso=160e6
! definition of geometry, FE mesh and boundary
conditions finish
!—————-1—————
/solution
! definition of loads: level 1
!e.g.:
F,29003,FX,(Fox_ level1) ! etc.
SOLVE
/post1
etable,max_s_1,s,1 ! sigma 1
etable,max_s_2,s,2 ! sigma 2
etable,max_s_3,s,3 ! sigma 3
etable,max_s_TG,s,int ! Tresca stress intensity
SADD,max_s12,max_s_1,max_s_2,,,
SADD,max_s123,max_s12,max_s_3,,,
sadd,max_s_h,max_s123,,(1/3), ! Sigma H,max
! Dang Van criterion
dang_van_kappa=(((3*zso)/zgo)-(3/2))
sadd,dv_eqv,max_s_tg,max_s_h,0.5,dang_van_kappa,
ESORT,ETAB,dv_eqv,,1
*GET,SMAX_1,SORT,,MAX
!—————-2—————
/solution
FDELE,ALL,ALL
! definition of loads: level 2
!e.g.:
F,29003,FX,(Fox_ level2) ! etc.
SOLVE
/post1
etable,max_s_1,s,1 ! sigma 1
etable,max_s_2,s,2 ! sigma 2
etable,max_s_3,s,3 ! sigma 3
etable,max_s_TG,s,int ! Tresca stress intensity
SADD,max_s12,max_s_1,max_s_2,,,
```

```
SADD,max_s123,max_s12,max_s_3,,,
sadd,max_s_h,max_s123,,(1/3), ! Sigma H,max
! Dang Van criterion
dang_van_kappa=(((3*zso)/zgo)-(3/2))
sadd,dv_eqv,max_s_tg,max_s_h,0.5,dang_van_kappa,
ESORT,ETAB,dv_eqv,,1
*GET,SMAX_2,SORT,,MAX
!——————-3——————
/solution
FDELE,ALL,ALL
! definition of loads: level 3
!e.g.:
F,29003,FX,(Fox_ level3) ! etc.
SOLVE
/post1
etable,max_s_1,s,1 ! sigma 1
etable,max_s_2,s,2 ! sigma 2
etable,max_s_3,s,3 ! sigma 3
etable,max_s_TG,s,int ! Tresca stress intensity
SADD,max_s12,max_s_1,max_s_2,,,
SADD,max_s123,max_s12,max_s_3,,,
sadd,max_s_h,max_s123,,(1/3), ! Sigma H,max
! Dang Van criterion
dang_van_kappa=(((3*zso)/zgo)-(3/2))
sadd,dv_eqv,max_s_tg,max_s_h,0.5,dang_van_kappa,
ESORT,ETAB,dv_eqv,,1
*GET,SMAX_3,SORT,,MAX
!——————-4——————
/solution
FDELE,ALL,ALL
! definition of loads: level 4
!e.g.:
F,29003,FX,(Fox_ level4) ! etc.
SOLVE
/post1
etable,max_s_1,s,1 ! sigma 1
etable,max_s_2,s,2 ! sigma 2
etable,max_s_3,s,3 ! sigma 3
etable,max_s_TG,s,int ! Tresca stress intensity
```

```
SADD,max_s12,max_s_1,max_s_2,,,
SADD,max_s123,max_s12,max_s_3,,,
sadd,max_s_h,max_s123,,(1/3), ! Sigma H,max
! Dang Van criterion
dang_van_kappa=(((3*zso)/zgo)-(3/2))
sadd,dv_eqv,max_s_tg,max_s_h,0.5,dang_van_kappa,
ESORT,ETAB,dv_eqv,,1
*GET,SMAX_4,SORT,,MAX
!————————-5————————
/solution
FDELE,ALL,ALL
! definition of loads: level 5
!e.g.:
F,29003,FX,(Fox_ level5) ! etc.
SOLVE
/post1
etable,max_s_1,s,1 ! sigma 1
etable,max_s_2,s,2 ! sigma 2
etable,max_s_3,s,3 ! sigma 3
etable,max_s_TG,s,int ! Tresca stress intensity
SADD,max_s12,max_s_1,max_s_2,,,
SADD,max_s123,max_s12,max_s_3,,,
sadd,max_s_h,max_s123,,(1/3), ! Sigma H,max
! Dang Van criterion
dang_van_kappa=(((3*zso)/zgo)-(3/2))
sadd,dv_eqv,max_s_tg,max_s_h,0.5,dang_van_kappa,
ESORT,ETAB,dv_eqv,,1
*GET,SMAX_5,SORT,,MAX
!—————————————————
smax=smax_1
*IF,smax,LT,smax_2,THEN smax=smax_2
*ENDIF
*IF,smax,LT,smax_3,THEN smax=smax_3
*ENDIF
*IF,smax,LT,smax_4,THEN smax=smax_4
*ENDIF
*IF,smax,LT,smax_5,THEN smax=smax_5
*ENDIF
! print of maximum value of Dang Van equivalent stress in 5
```

```
! steps fatigue analysis
/OUTPUT,par_state_var,txt,,
max_eq_stress=smax
/OUTPUT
```

References

1. ANSYS Inc., 2004, Release 8.1 Documentation
2. BALLARD P., DANG VAN K., DEPERROIS A., PAPADOPOULOS Y. V., 1995, High cycle fatigue and a finite element analysis, *Fatigue and Fracture of Engineering Materials and Structures*, **18**, 397-411
3. CROSSLAND B., 1956, Effect of large hydrostatic pressures on the torsional fatigue strength of an alloy steel, *Proceedings of the International Conference on Fatigue of Metals*, Institution of Mechanical Engineers, 138-149, London
4. DANG VAN K., GRIVEAU B., MESSAGE O., 1989, On a new multiaxial fatigue limit criterion: theory and application, *Biaxial and Multiaxial Fatigue*, EGF 3. Mechanical Engineering Publications, London, 479-496
5. DANG VAN K., PAPADOPOULOS I.V. (EDIT.), 1999, *High Cycle Metal Fatigue, From Theory to Applications*, C.I.S.M. Courses and Lectures No. 392, Springer-Verlag Company
6. GARUD Y.S., 1981, Multiaxial fatigue: a survey of the state of the art, *Journal of Testing and Evaluation*, **9**, 165-178
7. MRZYGŁÓD M., 2005, *Parametrical Optimization of Structures Working in High-Cycle Load Conditions*, PhD Thesis, Cracow University of Technology, Cracow
8. MRZYGŁÓD M., ZIELIŃSKI A.P., 2005, Parametric structural optimization with multiaxial high-cycle fatigue criterion, *6th World Congress on Structural and Multidisciplinary Optimization*, Rio de Janeiro
9. OSYCZKA A., 2001, *Evolutionary Algorithms for Single and Multicriteria Design Optimization*, Springer-Verlag Company
10. PAPADOPOULOS I.V., DAVOLI P., GORLA C., FILIPPINI M., BERNASCONI A., 1997, A comparative study of multiaxial high-cycle fatigue criteria for metals, *International Journal of Fatigue*, **19**, 219-235
11. SINES G., 1959, Behaviour of metals under complex static and alternating stresses, In: *Metal Fatigue*, McGraw Hill, New York, 145-169

12. Walczak S., 2003, *Analysis of Dynamic Loads Occurring in Different Types of Independent Car Suspension Systems*, PhD Thesis, Cracow University of Technology, Cracow

Numeryczne badania kryterium wieloosiowego wysoko-cyklowego zmęczenia pod kątem zastosowania do optymalizacji konstrukcji

Streszczenie

W artykule przedstawiono badania nad zastosowaniem nowoczesnych kryteriów wieloosiowego, wysoko-cyklowego zmęczenia materiału do procedur optymalizacji konstrukcji analizowanych metodą elementów skończonych. Po przeprowadzeniu testów kilku kryteriów zmęczeniowych do szczegółowych badań numerycznych wybrana została hipoteza Dang Vana. W pracy zaproponowano także metodę odpowiedniej adaptacji wysoko-cyklowych przebiegów obciążeń. Całkowity algorytm optymalizacji zmęczenia został zilustrowany przez zastosowanie proponowanych procedur do optymalizacji części pojazdu, które są szczególnie narażone na obciążenia wysoko-cyklowe. Do modelowania konstrukcji został użyty program metody elementów skończonych ANSYS®.

Manuscript received December 21; accepted for print March 15, 2006