

COMPARISON OF NUMERICAL TESTING METHODS IN TERMS OF IMPULSE LOADING APPLIED TO STRUCTURAL ELEMENTS

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This paper presents comparison of numerical testing methods of an impulse loading which comes from a detonation process, i.e. blast wave propagation in a gas medium. Investigations were carried out using an analytical and numerical model based on the Finite Element Method. In order to reduce computational time, the substitute analytical model with one degree of freedom was implemented, which replaced a chosen actual system (I-section steel column). For structure modelling, the constitutive model was used, which included the strain rate effect. From the performed analyses, an acceptable similarity was noticed, although the discrete model due to greater forces gave inflated results. Nevertheless, it should be pointed out that simplified methods do not take any wave and flow around effects into consideration, which have an influence on the dynamical response of the structure and are possible to implement in the gas medium coupling.

Key words: blast wave, FEM, Eulerian mesh

1. Introduction

Within mechanics, some analyses are carried out which enclose the effects of pressure wave propagation and their impact on encountered structures. Hence, two main groups of methods for numerical implementation of pressure wave propagation can be distinguished, i.e. methods based on analytical models and methods which use discrete models. The first ones are methods which substitute the problem into a system with one degree of freedom (SDOF) or multiple degrees of freedom (MDOF) (US Army Corps of Engineers, 2008). The others are based on the Finite Element Method (FEM), which use several algorithms for determination of dynamical loading of the structure, e.g. ConWep blast wave function, Smoothed Particle Hydrodynamics (SPH) method and Arbitrary Lagrangian-Eulerian formulation (ALE). A detonation process and pressure wave propagation and its interaction with structures were thoroughly investigated by other authors (Lu *et al.*, 2003; Małachowski, 2010a,b; Mazurkiewicz and Małachowski, 2011).

The presented paper encloses comparison analysis of numerical methods based on discrete models, where the detonation process was simulated using ALE formulation, ConWep function and incorporate analytical model with one degree of freedom (SDOF). The first two methods are available in LS-Dyna code, whereas the system with SDOF is implemented in “Single degree of freedom Blast Effects Design Spreadsheet” (SBEDS) software developed by US Army Corps of Engineers (US Army Corps of Engineers, 2008). The main objective of the presented paper is to assess effectiveness and reliability of the mentioned methods on the example of a structural element subjected to a dynamic impulse.

2. Theoretical basis of implemented methods

As stated before, there are two methods of analysis of a structure subjected to an impulse loading: methods based on analytical models (beam elements) and methods which use discrete models reflecting full three-dimensional actual geometry of the structure.

2.1. Analytical models

The simplest models for dynamical responses of structural elements are models which use systems with one degree of freedom which replaces the actual structural element with lumped stiffness, inertia and damping (Fig. 1).

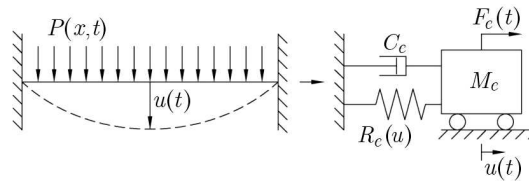


Fig. 1. Loaded beam model replaced by a system with SDOF (US Army Corps of Engineers, 2008)

Consequently, the following dynamic equation of dynamic equilibrium is formulated (US Army Corps of Engineers, 2008)

$$M_e \ddot{u}(t) + C_e \dot{u}(t) + R_e(u)u(t) = F_e(t) \quad (2.1)$$

where M_e is the effective mass of the equivalent SDOF system, C_e – effective viscous damping coefficient (constant), $R_e(u)$ – effective stiffness, $\ddot{u}(t)$ – acceleration, $\dot{u}(t)$ – velocity, $u(t)$ – displacement of the mass, $F_e(t)$ – effective load acting on the equivalent SDOF system, t – time.

The equivalent SDOF system is defined as a system that has the same energy in terms of work energy, strain energy and kinetic energy as the real beam element subjected to the pressure wave from a detonation. This is achieved through relating the applicable values of the blast loaded component and the corresponding effective parameters of the equivalent SDOF system with the relevant transformation coefficients, as shown in (2.2) (US Army Corps of Engineers, 2008)

$$K_{Lm} M_c \ddot{u}(t) + C_c \dot{u}(t) + R_c(u)u(t) = F_c(t) \quad (2.2)$$

where M_c is the mass of the blast-loaded component, C_c – viscous damping constant, $R_c(u)$ – stiffness resistance, $F_c(t)$ – history of load acting on the blast-loaded component, K_{Lm} – load-mass factor equal to K_m/K_L , K_m – mass transformation factor, K_L – load transformation factor.

The mass transformation factor K_m is calculated by comparing the work of external forces for the component and SDOF system. Consequently, for the component, this work is obtained by (US Army Corps of Engineers, 2008)

$$WE(t)_{element} = \int_0^L P(x,t)U(x,t) dx \quad (2.3)$$

where L is the length of the blast-loaded component, $U(x,t)$ – its displacement, $P(x,t)$ – dynamic load acting on the blast-loaded component.

Assuming that $U(x, t) = U_{max}(t)\phi(x)$ and $P(x, t) = P_{max}(t)p(x)$, which gives: $\phi(x) = U(x, t)/U_{max}(t)$ and $p(x) = P(x, t)/P_{max}(t)$, the following equation is obtained

$$WE(t)_{element} = P_{max}(t)U_{max}(t) \int_0^L p(x)\phi(x) dx \quad (2.4)$$

Then, for the SDOF system, the work of external forces can be described by

$$WE(t)_{SDOF} = F_e(t)u(t) = K_L F_c(t)u(t) \quad (2.5)$$

Assuming that $u(t) = U_{max}(t)$ and $F_c(t) = P_{max}(t) \int_0^L p(x) dx$ it gives

$$WE(t)_{SDOF} = K_L P_{max}(t)U_{max}(t) \int_0^L p(x) dx \quad (2.6)$$

By comparing equations $WE(t)_{element} = WE(t)_{SDOF}$, after some transformations, K_L coefficient is obtained

$$K_L = \frac{\int_0^L p(x)\phi(x) dx}{\int_0^L p(x) dx} \quad (2.7)$$

Analogously, by comparing kinetic energy for the component and SDOF, the system K_m coefficient is obtained

$$K_m = \frac{\int_0^L m(x)\phi^2(x) dx}{\int_0^L m(x) dx} \quad (2.8)$$

The formulated equivalent system is then loaded with a pressure impulse.

2.2. FE analyses

Numerical simulations were carried out using an LS-Dyna explicit code dedicated for dynamic simulations using the Finite Element Method (FEM) central difference scheme with modified time integration of the equation of motion. The basis of this method was frequently described in the literature (Belytschko *et al.*, 2000; Hallquist, 2003). The main assumption of it is to replace the continuous component by a discrete one which consists of finite elements connected by nodes. For the defined model, a well-known matrix equation of motion is defined by (Hallquist, 2003)

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}(u)\mathbf{u}(t) = \mathbf{F}(t) \quad (2.9)$$

where \mathbf{M} is the mass matrix, \mathbf{C} – damping matrix, $\mathbf{K}(u)$ – stiffness matrix, $\ddot{\mathbf{u}}(t)$ – acceleration vector, $\dot{\mathbf{u}}(t)$ – velocity vector, $\mathbf{u}(t)$ – displacement vector, $\mathbf{F}(t)$ – vector of external forces, t – time.

3. Blast wave loading modelling

3.1. SBEDS pressure function

Both in the analytical and discrete model, the dynamic pressure loading is described by a mathematical function and an idealized shape of the pressure pulse at a point caused by the shock wave from a high explosive detonation, see Fig. 2.

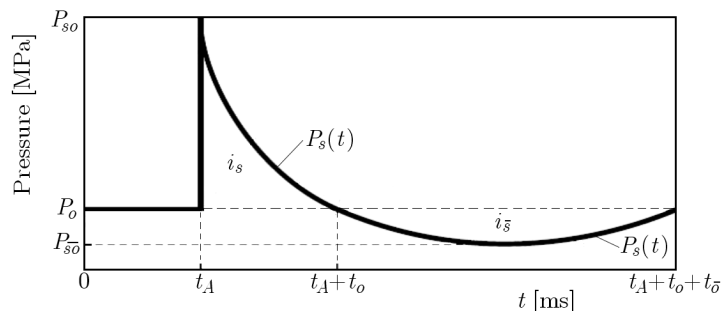


Fig. 2. Typical pressure-time history of the pressure wave from an explosion (US Army Corps of Engineers, 2008); i_s $i_{\bar{s}}$ – positive and negative impulse, respectively, t_o $t_{\bar{o}}$ – positive and negative phase duration, respectively

The shape of the positive phase blast load from an open air explosion in Fig. 2 can be represented mathematically with Eq. (3.1) (US Army Corps of Engineers, 2008)

$$P_s(t) = P_{so} \left[1 - \left(\frac{t - t_A}{t_o} \right) \right] \exp \left(- \frac{t - t_A}{\theta} \right) \quad t_A \leq t \leq t_A + t_o \quad (3.1)$$

where $P_s(t)$ is the shock overpressure as a function of time [MPa], P_{so} – peak side-on overpressure [MPa], t – detonation time [ms], t_A – arrival time of the initial shock front [ms], t_o – positive phase duration [ms], θ – shape constant of the pressure waveform.

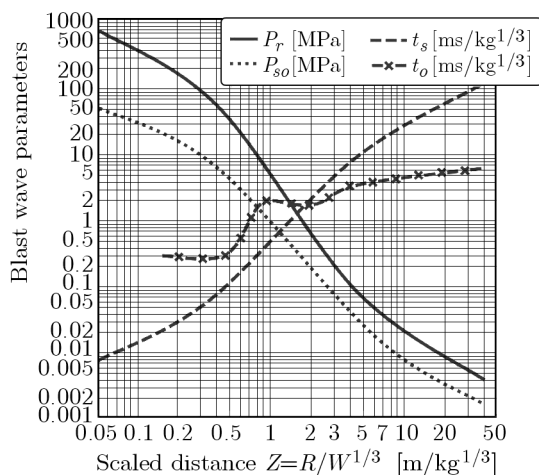


Fig. 3. Positive phase parameters of the blast wave for a spherical TNT explosion in air at the sea level (US Army Corps of Engineers, 2008)

In order to obtain peak pressure values, the characteristics obtained from empirical tests are used with the reduced distance of the charge (Z parameter) which is described by Eq. (3.2) (US Army Corps of Engineers, 2008). Characteristics of exemplary explosion parameters with the side-on (subscript so) and reflected (subscript r) pressure wave are presented in Fig. 3

$$\frac{R_1}{R_2} = \sqrt[3]{\frac{W_1}{W_2}} \quad \rightarrow \quad Z = \frac{R}{\sqrt[3]{W}} \quad (3.2)$$

where W_i is mass of the explosive [kg] for the case i , R_i – standoff distance from the detonation point of W_i , Z_i – scaled standoff for the case i .

3.2. ConWep function pressure impulse loading

This method is similar to the one mentioned before and used in this paper, i.e. the SBEDS system. It was developed by Randers-Person and Bannister in 1997. In this case, the load blast is described by the ConWep function which substitutes the wave propagation with a pressure function. In order to simulate the blast loading, it is necessary to determine the equivalent of TNT mass, the type of explosion (on the surface or in the air), charge localization and surfaces segments interacting with the blast wave. The main difference between those two methods is that the ConWep function applies the loading separately on every finite element of a model. Thus the effect of pressure wave propagation within the loaded structure can be observed from elements which are closest to the explosive. ConWep approach allows generation of the equivalent pressure value (Hyde, 1988; Hallquist, 2003)

$$P(t) = P_{ref} \cos^2 \theta + P_{in}(1 + \cos^2 \theta - 2 \cos \theta) \quad (3.3)$$

where θ is the incidence angle of the wave, P_{ref} – overpressure value of the reflected pressure wave, P_{in} – overpressure value of the incident pressure wave.

3.3. Arbitrary Lagrangian-Eulerian formulation (ALE) with ConWep implementation

Another way to represent the blast wave interaction with a structure is the Arbitrary Lagrangian-Eulerian formulation, where it is necessary to define an Eulerian air domain, in which the explosive pressure wave propagates. In the presented case, the pressure impulse was applied using the ConWep model, Eq. (3.3), to the Eulerian air domain surface. The air is considered as a simple ideal gas with a linear polynomial equation of state (Hallquist, 2003)

$$p = (C_4 + C_5 \mu) E \quad (3.4)$$

where $\mu = \rho/\rho_0$ is density, ρ_0 is the initial density, C_4 and C_5 are polynomial equation coefficients, E is the internal energy.

The ALE procedure consists of two major steps: the classical Lagrangian step and the advection Eulerian step. The advection step is carried out with the assumption that the displacements of nodes are very small in comparison to characteristics of the elements surrounding these nodes, e.g. dimensions. Moreover, in this procedure, a constant topology of the mesh is provided.

The governing equations for the fluid domain (Euler domain) describe the conservation of mass, momentum and energy (Hallquist, 2003)

$$\begin{aligned} \frac{dM}{dt} &= \frac{d}{dt} \int_{V(t)} \rho \mathbf{v} dV = \oint_{S(t)} \rho (\mathbf{w} - \mathbf{v}) \cdot \mathbf{n} dS \\ \frac{d\mathbf{Q}}{dt} &= \frac{d}{dt} \int_{V(t)} \rho \mathbf{v} dV = \oint_{S(t)} \rho \mathbf{v} [(\mathbf{w} - \mathbf{v}) \cdot \mathbf{n}] dS - \int_{V(t)} \nabla p dV + \int_{V(t)} \nu \mathbf{g} dV \\ \frac{dE}{dt} &= \frac{d}{dt} \int_{V(t)} \rho e dV = \oint_{S(t)} \rho e (\mathbf{w} - \mathbf{v}) \cdot \mathbf{n} dS - \int_{S(t)} p \mathbf{v} \cdot \mathbf{n} dS + \int_{V(t)} \rho \mathbf{g} \cdot \mathbf{v} dV \end{aligned} \quad (3.5)$$

where ρ is fluid mass density, p – pressure, \mathbf{g} – acceleration of gravity, e – total specific energy. The quantities M , \mathbf{Q} and E are the total mass, total momentum and total energy, respectively, of the control volume $V(t)$ bounded by a surface S which moves in the fluid (gas-air) with an arbitrary velocity \mathbf{w} which may be zero in Eulerian coordinates or \mathbf{v} in Lagrangian coordinates. The vector \mathbf{n} is the outwards normal to the surface S .

4. Object of investigations

The research object in the present paper is an I-section steel column $203 \times 203 \times 86$ (British standards) made of A992 steel subjected to a blast load generated by 250 kg of TNT at 10 m distance from the component. Also, static axial loading 300 000 N is applied and fixed-simple boundary conditions are assumed (Fig. 4)

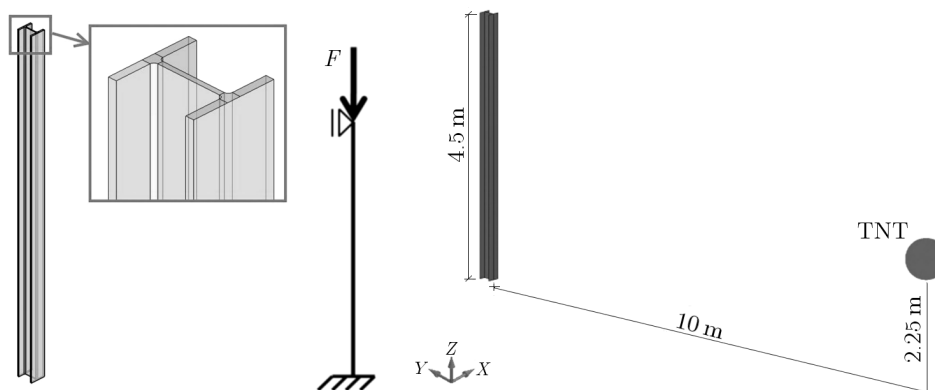


Fig. 4. I-section steel column used in the analysis

4.1. FE model

A discrete model of the mentioned column (Fig. 5) has been developed using Belytschko-Lin-Tsay shell elements (Belytschko *et al.*, 2000). These elements are effective in simulating several numerical problems due to a reduced number of important mathematical operations that have to be performed (Hallquist, 2003)

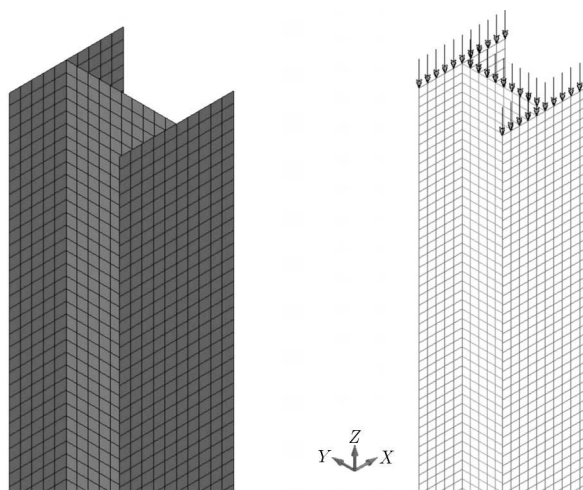


Fig. 5. Numerical model of the column with the applied axial loading

4.2. Material model

The load due to outbreak causes very rapid changes of the strain field in the structure. Therefore, an elastic-plastic material model with isotropic hardening and strain rate effects has been applied in order to describe properties of the beam material. Thus, simplified Johnson-Cook (J-C) model was implemented, which provides a prediction of the flow stress σ_{flow} for large strains and high strain rates, when its dependence on the strain rate is linear in a semi-logarithmic scale. Material parameters presented in Table 1 for the simplified J-C material were

taken from the literature (Małachowski, 2010a,b). The mathematical formula which describes this model is as follows (Hallquist, 2003)

$$\sigma_{flow} = [A + B(\varepsilon^p)^n](1 + C \ln \dot{\varepsilon}_*^p) \quad (4.1)$$

where A, B, C, n are material constants, ε^p – effective plastic strain, $\dot{\varepsilon}_*^p$ – effective plastic strain rate.

Table 1. Steel parameters for the simplified Johnson-Cook model used in the analysis

ρ [kg/m ³]	A [MPa]	B [MPa]	C	n
7850	365	510	0.0936	0.9

5. Numerical analyses

5.1. Pre-stress (static) simulation

Simulation of two numerical models (ConWep and by ALE + ConWep cases) was preceded by the so called pre-stress stage. In this stage, the I-beam structure was subjected to nominal load P_n , which was equal to the load carried by the pillar during its normal service. Incremental static analysis was performed using the full Newton-Rapshon algorithm. The equation solved at this stage has the following form (Hallquist, 2003)

$$\mathbf{K}_i \Delta x_{i-1} = \Delta Q_i \quad (5.1)$$

where \mathbf{K}_i is the stiffness matrix, Δx_{i-1} – displacement vector, ΔQ_i – external force vector.

Convergence of the solution was controlled by two criteria: displacement and energy relative convergence tolerance.

In the next stages, the results from static analysis were taken into account as the pre-stress field present in the column. It was obtained using a dynamic relaxation procedure. Dynamic relaxation allows one to quickly reach the preloaded state by the linear ramping nodal displacement field to prescribed values over 100 time steps. It should be noted that the comparison of the stress field taken from the static analysis and the stress field generated by the dynamic relaxation procedure showed small differences introduced by the latter. On the other hand, the procedure allows for application of the predefined stress field on the selected part of the FE model in a very effective manner.

5.2. ConWep function loading – conditions of analysis

As mentioned before, the ConWep function simulates the pressure wave loading from an explosive by generating an applicable pressure value $P(X, t)$, $X = X(x, y, z)$. Consequently, the pressure is then distributed on chosen surface segments with time and element localization taken into consideration (Fig. 6).

5.3. Lagrangian-Eulerian with ConWep – conditions of analysis

The most advanced case of the proposed analysis is Arbitrary-Lagrangian-Eulerian formulation where Euler elements define surrounding air. The interaction between the Lagrangian structure, which in this case is the I-section column, and air is obtained through numerical coupling of these two media (Małachowski, 2010a,b; Mazurkiewicz, 2011).

The pressure wave was applied analogously to the previous case with the ConWep function, but with one major difference: it was applied to the surrounding air surface, not directly to the column surface. Thus, the pressure $P(X, t)$ was a particular boundary condition on one of

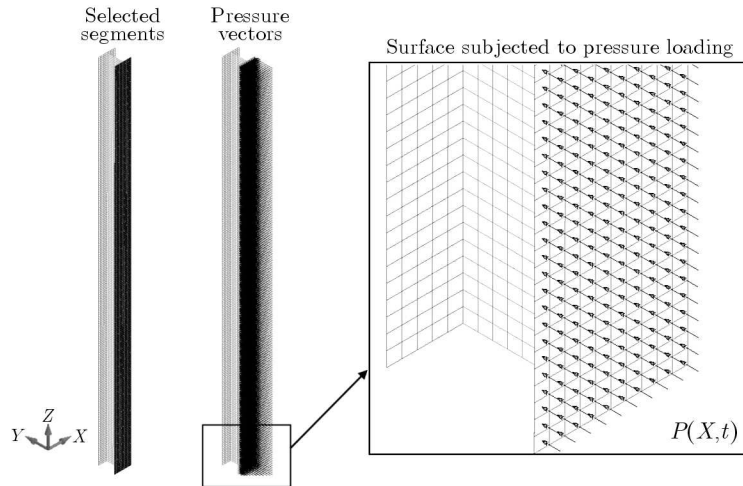


Fig. 6. Pressure impulse loading – schematic presentation

the outer surfaces of the gas medium described by Eulerian coordinates (Fig. 7). Additionally, on the outer walls of it, a non-reflecting option was applied, which considers the flow of the pressure outside the domain. The blast wave propagated in the air domain and interacted with the structure, which can be noticed in Fig. 8.

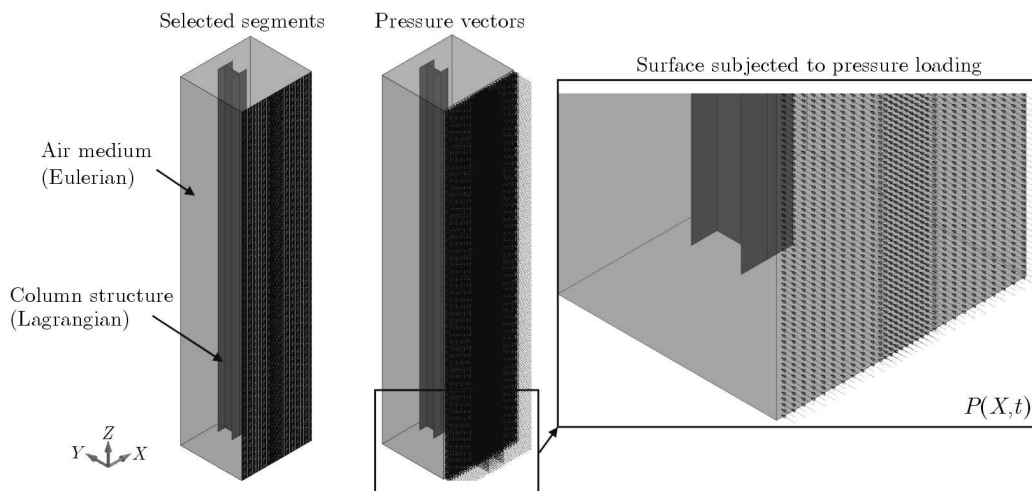


Fig. 7. Numerical model of the column inside the Euler air medium

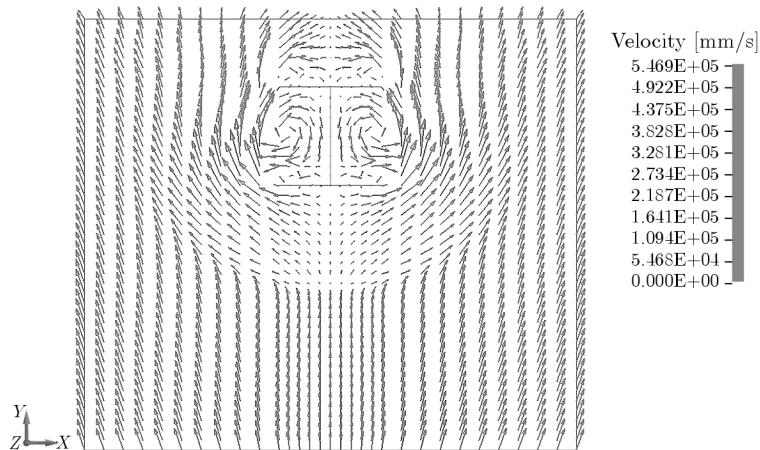


Fig. 8. Pressure wave flow effect in the air (Eulerian) medium

6. Results and conclusions

From the performed analysis, dynamic responses and deformations of the structural element were obtained. The column structure deflection versus time characteristics for all three cases are presented in Fig. 9, which also shows the dynamic response of these components. It should be pointed out that during the analysis, the column structure did not exceed the yield stress. From the carried out numerical simulations, it can be concluded that both mixed ALE-ConWep and ConWep models give similar results.

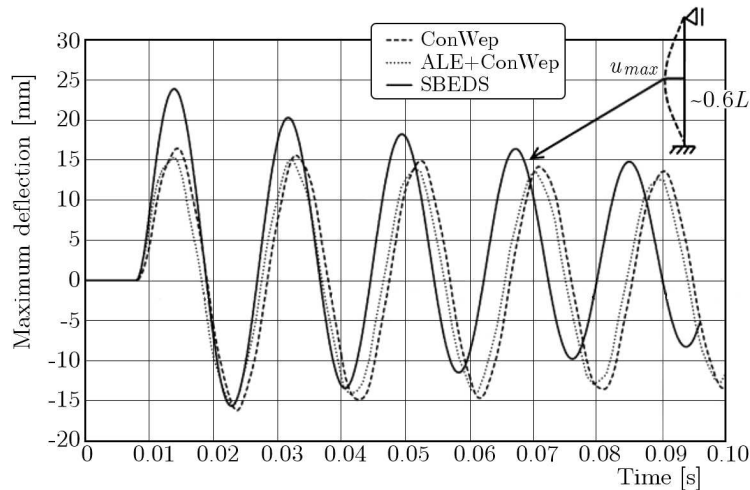


Fig. 9. Column deflection versus time characteristics for all studied cases

For the SBEDS system, the obtained dynamic response is higher due to a greater pressure impulse energy (longer duration time) generated by the algorithm. It was confirmed through further analyses, where the pressure impulse was measured in all methods (Fig. 10). Due to the wave effects implemented in ALE + ConWep case, the incident and reflected pressure can be noticed, and also the pressure wave characteristic is not ideal as in the other methods. The initial value of pressure was caused by the atmospheric pressure, which was defined by the polynomial EOS for the air Euler medium. Taking a closer look at Fig. 10, it can be concluded that both mixed ALE-ConWep and SBEDS models give similar results ($p_{max} \approx 2.2$ MPa), whereas in the ConWep model $p_{max} \approx 1.3$ MPa.

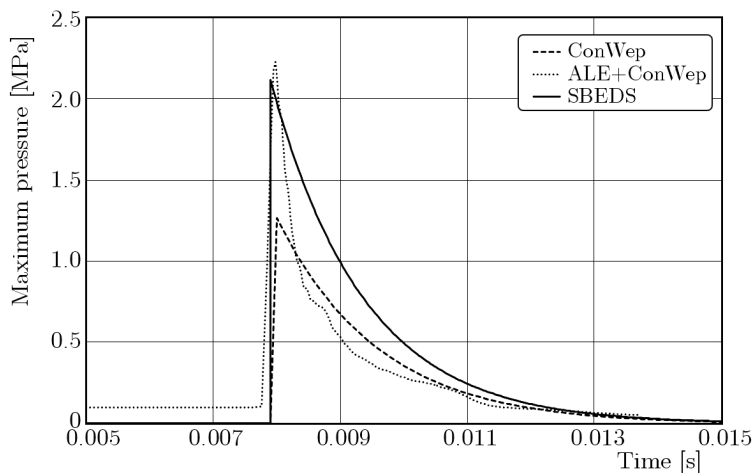


Fig. 10. Pressure impulse versus time – comparison of the methods

It is worth to mention the effectiveness of implemented methods. The ALE formulation, due to the greater number of elements modeling the air medium and its complex mathematical background, is much more computationally expensive – the analyses were carried out for ~ 33 min. For the ConWep model, the simulation ended after 10 min, whereas for the SBEDS system, it was approximately 1 min (Fig. 11). It is also noteworthy that in order to perform simulations using discrete models, there is a need to develop geometry of the structure and, consequently, a discrete model with finite elements, apply boundary initial conditions and implement a constitutive material model suitable for a specific type of the simulated problem. In contrast with the SBEDS system, the overall model is chosen from the available software database.

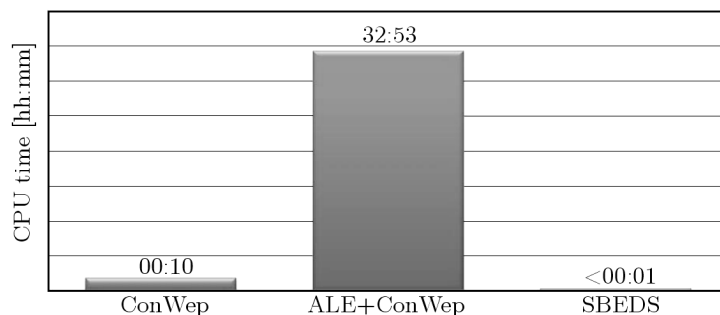


Fig. 11. Comparison of CPU time for all cases

The presented above comparison results show great efficiency of the analytical models. Nevertheless, the scope of application of this method is very limited and only allows for analysing basic structural elements. For the preliminary analyses of the response of components subjected to the pressure impulse from an explosion, the ConWep function is adequate enough. It gives much more information about the structure deformation (local deformation of the element), allows for more complex material models (with strain rate taking into account) and does not have so many limitations of its usage. The most advanced Arbitrary-Lagrangian-Eulerian formulation, where Euler elements define the surrounding air, is much more computationally expensive than the other two. Although, with wave and flow around effects taken into consideration, it has an influence on analysis of more complex structures, which was confirmed by many authors (Włodarczyk, 1995; Lu *et al.*, 2003; Małachowski, 2010a,b; Mazurkiewicz and Małachowski, 2011).

The carried out studies allow for choosing the most effective and reliable method for finding the response of a structure subjected to dynamic loading. This will give the possibility for further simulations at the developing stage of safety structures.

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