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ANALYSIS OF THE INFLUENCE OF ELASTICITY CONSTANTS AND MATERIAL DENSITY ON THE BASE FREQUENCY OF AXI-SYMMETRICAL VIBRATIONS WITH VARIABLE THICKNESS PLATES

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In the paper, the influence of Young's modulus and Poisson ratio as well as mass density of a material on the base frequency of circular plates of the diaphragm type with variable thickness is discussed. In order to solve the boundary-value problem, the Cauchy function method and double Bernstein-Kieropian estimators were applied. An analytical form of Cauchy's influence function was found and used to construct the characteristic equation in the form of a power series with respect to a frequency parameter. Application of this method allowed a functional dependency of the base frequency on material constants of the plates to be established. The results of calculations for plates made of duralumin and tin were mentioned as examples. Comparison of the obtained results with those found in scientific literature indicated high accuracy of the method applied therein.

Key words: circular plates, variable thickness, boundary-value problem, Cauchy function method

1. Introduction

Boundary-value problems of axi-symmetrical vibrations of circular plates fixed around their circumference can be solved exactly for some values of the variable thickness coefficient m and Poissons ratio when a solution to equation (2.2) can be presented with the help of Bessel's function (cf. Kantham, 1958; Hondkiewič, 1964; Leissa, 1969; Conway, 1958). This problem can also be solved by means of characteristic series and partial discretisation utilizing double sided estimators as well as Bernsteina-Kieropiana's tables (Bernštein and Kieropian, 1960). It was exhibited in studies concerning long cylinders and beams (Zoryj and Jaroszewicz, 2000a; Jaroszewicz and Zoryj, 2000) as well as discs (Zoryj and Jaroszewicz, 2000b, 2002; Jaroszewicz, 2000). The theorem of vibrations of continuous-discrete, linearly elastic systems (based on Cauchy's influence functions and the characteristic series method), which has been developed in this work, is useful for constructing and studying universal frequency equations (Zoryj, 1982). In order to study such systems, it is necessary to solve boundary problems, as a result of which, we obtain appropriate characteristic determinants of multi-parametric problems. By equating a determinant to zero, it is possible to calculate its roots by numerical or analytical means through the use of double sided estimators. For constructing characteristic determinants, a form of a general solution of the entry-level differential equation is required, which is rarely known, by means of special functions, like for instance, the Bessel functions. That is why approximate methods are most commonly used to determine the basic frequencies. Among them one mentiones: Bubnov-Galerkin's, Rayleygh-Ritz's, consecutive approximations and differences or finite element method (Vibracii v tiehnike, 1978). The usage of approximate methods does not, however, address the question of accuracy of solutions. That problem may be resolved either having the exact solution, which is possible only in few particular cases (by means of Euler's equation for example) or by supplying double sided estimators, whose difference in value depends on the number of used terms of the characteristic series.

The greatest advantage of the method developed in this work is the general form of the power series of the characteristic equation, which gives functional dependency of proper frequencies on materials constants of considered plates, where Berstein's double sided estimators can be easily applied to.

2. Problem formulation

We consider a clamped circular plate, with the radius R flexural rigidity D and thickness described by power functions of the radial coordinate r

$$
\begin{aligned}
&D = D_0\Big(\frac{r}{R}\Big)^m \qquad\qquad h = h_0\Big(\frac{r}{R}\Big)^{\frac{m}{3}} \qquad\qquad 0 < r \leqslant R \\
&D_0 = \frac{Eh_0^3}{12(1-\nu^2)}
\end{aligned}
\tag{2.1}
$$

where D_0, h_0, m – constants.

Investigation of free, axi-symmetrical vibrations of such a plate consists of the boundary problem (cf. Conway, 1958; Hondkiewič, 1964; Zoryj and Jaroszewicz, 2002)

$$L_0[u] - pr^{-\frac{2}{3}m}u = 0 \qquad p = \frac{\rho h_0}{D_0}R^{\frac{2}{3}m}\omega^2 \tag{2.2}$$

$$u(R) = 0 \qquad u'(R) = 0 \tag{2.3}$$

$$\begin{aligned} L_0[u] \equiv u^{IV} + \frac{2}{r}(m+1)u^{III} + \frac{1}{r^2}(m^2+m+\nu m-1)u^{II} + \\ + \frac{1}{r^3}(m-1)(\nu m-1)u^I \end{aligned} \tag{2.4}$$

where

u	–	flexural amplitude, $u = u(r)$
ρ	–	density
ω	–	frequency parameter
ν	–	Poisson's ratio
E	–	Young's flexural modulus.

The value $m = 0$ refers to a plate with constant thickness; $m > 0$ - to plates of the diaphragm type; $m < 0$ – disc type plates (Hondkiewič, 1964).

We shall analyze the dependence of eigen frequencies on the constants ρ, E and ν. In problem (2.1)-(2.2), a limitation of solutions and their first derivatives with respect to the independent variable r is required (Conway, 1958).

3. Constant thickness plate ($m = 0$)

The basic frequency of such a plate can be calculated on the basis of a well known formula. For example, from an equation found in the work by Vasylenko (1992)

$$\omega_1 = \gamma\frac{h_0}{R^2}\sqrt{\frac{E}{12\rho(1-\nu^2)}} \tag{3.1}$$

where $\gamma = 10.214$. Now, we will compare frequencies of two plates of the same thickness h_0 and radius R but made of different materials

$$\frac{(\omega_1)_{\rm I}}{(\omega_1)_{\rm II}} = \sqrt{\frac{E_{\rm I}}{E_{\rm II}}\frac{\rho_{\rm II}}{\rho_{\rm I}}\frac{(1-\nu_{\rm II}^2)}{(1-\nu_{\rm I})^2}} \tag{3.2}$$

where I and II correspond to the first and second material. If, for sake of the study, we assume the first one (I) to be duralumin and the other one (II) to be tin, we will obtain

$$\Big(\frac{E}{\rho}\Big)_{\mathrm{I}} = 2.65 \qquad \Big(\frac{E}{\rho}\Big)_{\mathrm{II}}^{-1} = 7.09 \qquad \frac{1-\nu_{\mathrm{II}}^2}{1-\nu_{\mathrm{I}}^2} \approx \frac{0.81}{0.88} \approx 0.92$$

and the corresponding ratio of frequencies (3.2)

$$\begin{aligned} &\frac{(\omega_1)_{\mathrm{I}}}{(\omega_1)_{\mathrm{II}}} \approx \sqrt{2.65 \cdot 7.09 \cdot 0.92} = 4.16 \\ &\nu_{\mathrm{I}} \approx 0.34 \qquad \nu_{\mathrm{II}} \approx 0.44 \end{aligned} \tag{3.3}$$

Remarks

- Let the frequency equation be presented in the following form (as a characteristic series) (Zoryj, 2000a)

$$a_0 - a_1 pR^2 + a_2(pR^2)^2 - \ldots = 0 \qquad p = \rho\frac{h_0}{D_0}\omega^2$$

In that case, double sided Bernstein-Keropian's estimators for the coefficient γ in formula (3.1) should by calculated as follows

$$\sqrt{\frac{a_0}{\sqrt{a_1^2 - 2a_0a_2}}} < \gamma < \sqrt{\frac{2a_0}{a_1 + \sqrt{a_1^2 - 4a_0a_2}}}$$

where $a_0 = 1$, $a_1 = 1/96$, $a_2 = 1/122880$ (cf. Jaroszewicz, 2004), hence

$$10.204 < \gamma < 10.224$$

- The simplest lower estimator (γ_-) has been calculated from the equation $a_0 - a_1pR^2 = 0$ (cf. Jaroszewicz, 1997), therefore $\gamma_- = \sqrt{96}$, which is lower than the exact value $(\gamma = 10.214)$ by 4%.

As we can see, the properties of material (ρ, E, ν) significantly influence the basic frequency of the constant thickness plate.

4. Plate with parabolically variable rigidity

Let $m = 2$ (Hondkiewič, 1964; Conway, 1958). In this case, on the basis of (2.4), we obtain

$$L_0[u] \equiv u^{IV} + \frac{6}{r}u^{III} + \frac{5+2\nu}{r^2}u^{II} + \frac{2\nu-1}{r^3}u^{I} \tag{4.1}$$

Substituting $u = r^\mu$ in (4.1), we pass to the corresponding algebraic equation

$$\mu^4 + (-2+2\nu)\mu^2 = 0 \tag{4.2}$$

with the following roots

$$\mu_1 = \sqrt{2-2\nu} \qquad \mu_2 = -\mu_1 \qquad \mu_3 = \mu_4 = 0 \tag{4.3}$$

Thus we obtain linearly-independent solutions to equation (4.1) for arbitrary $\nu[\nu(00.5)]$, which are functions (see example) described by Zoryj and Jaroszewicz (2002)

$$u_1 = r^{\mu_1} \qquad u_2 = r^{-\mu_1} \qquad u_3 = 1 \qquad u_4 = \ln r \tag{4.4}$$

The Cauchy function corresponding to this example will be determined by the formula

$$K_0(r,\alpha) = -\frac{\mu_1}{8(1-\nu^2)}(r^{-\mu}\alpha^{3+\mu} - r^\mu\alpha^{3-\mu}) - \frac{\alpha^3}{2(1-\nu)}\ln\frac{r}{\alpha} \tag{4.5}$$

In the formula, the subscript "1" at can be omitted, and that is why it will not be used afterwords.

In the above case, equation (2.2) will take the following form

$$L_0[u] - pr^{-\frac{4}{3}}u = 0 \tag{4.6}$$

That is why the limited solution to equation (4.6) with $m = 2$ can be determined by the S_1 and S_2 series, which are constructed on the basis of formulas

$$S_j = S_{j0} + pS_{j1} + p^2S_{j2} + \dots \qquad p = \frac{\rho h_0}{D_0}R^{\frac{4}{3}}\omega^2 \tag{4.7}$$

and

$$\begin{aligned} S_{jK} &= \int\limits_0^r K_0(r,\tau)\tau^{-\frac{4}{3}}S_{j,K-1}(\tau)\,d\tau \qquad K = 1,2,\dots \qquad j = 1,2 \\ S_{10} &= 1 \qquad S_{20} = \tau^{\mu_1} \end{aligned} \tag{4.8}$$

where the function $K_0(r,\alpha)$ is described by formula (4.5).

Having determined the first two integrals (4.8) we obtain

$$S_{11}(r) = ar^{\frac{8}{3}} \qquad a = \frac{3^4}{2^7(23+9\nu)} \tag{4.9}$$

and

$$S_{21}(r) = br^{\mu+\frac{8}{3}} \qquad b = \frac{1}{4(1-\nu)}\Big(\frac{3}{4\Big(2\mu+\frac{8}{3}\Big)} - \frac{2}{\Big(\mu+\frac{8}{3}\Big)^2}\Big)$$
$$\mu = \sqrt{2-2\nu} \tag{4.10}$$

So, we can see

$$S_1(r) = 1 + pS_{11}(r) + \dots \qquad S_2(r) = r^{\mu} + pS_{21}(r) + \dots \tag{4.11}$$

Having found the dependencies in forms (4.9)-(4.11), one can determine the simplest lower estimator for the basic frequency of vibrations. In order to accomplish that, we need to determine

$$S_1' = \frac{8}{3}par^{\frac{5}{3}} \qquad S_2' = \mu r^{\mu-1} pb\Big(\mu+\frac{8}{3}\Big)r^{\mu+\frac{5}{3}}$$
$$\begin{vmatrix} S_1 & S_2 \\ S_1' & S_2' \end{vmatrix} = \mu r^{\mu-1}\Big[1 + pb\Big(\mu+\frac{8}{3}\Big)\frac{1}{\mu}r^{\frac{8}{3}} + par^{\frac{8}{3}} - \frac{8}{3}\frac{a}{\mu}r^{\frac{8}{3}} + \dots\Big] = 0$$

Hence, considering that $r = R$, we obtain the first two elements of the characteristic series (the frequency equation) of the problem defined by expressions (2.2) and (2.3)

$$1 + pR^{\frac{8}{3}}\Big(b\frac{\mu+\frac{8}{3}}{\mu} + a - \frac{8}{3}\frac{a}{\mu}\Big) + \dots = 0 \tag{4.12}$$

From (4.12), for $\nu = 1/9$, $= 4/3$, we obtain

$$1 + pR^{\frac{8}{3}}(3b - a) + \dots = 0 \tag{4.13}$$

Considering (4.6) as well as the fact that

$$3b - a = \frac{3^3}{2^{11}} - \frac{3^3}{2^{10}} = -\frac{3^3}{2^{11}}$$

we under estimate the basic frequency for this case

$$\omega_- = \gamma_-(\nu)\Big|_{\nu=\frac{1}{9}}\frac{h_0}{R^2}\sqrt{\frac{E}{12\rho(1-\nu^2)}} \tag{4.14}$$

where $\gamma_-(1/9) = 8.71$ is about 8% smaller than the corresponding exact value $\gamma(1/9) = 9.46$ (cf. Conway, 1958).

For $\nu = 0$ $(\mu = \sqrt{2})$, on the basis of (4.9)-(4.12), we calculate

$$1 - \frac{8}{3\sqrt{2}} = -0.8856181 \qquad a\Big(1 - \frac{8}{3\sqrt{2}}\Big) = -0.0233513$$

$$\Big(1 - \frac{8}{3\sqrt{2}}\Big)b = 0.0118247 \qquad \gamma_-(0) = 9.3$$

In the same fashion, for $\nu = 1/2$ $(\mu = 1)$ we obtain $\gamma_-(0.5) = 7.8$, which amounts to about 84% of the value of 9.3.

As we can see, for plates characterized by the parabolic variable rigidity of $m = 2$, formula (4.14) differs from (3.1) not only by its multiplier $\gamma_-(\nu)$

$$7.8 < \gamma_-(\nu) < 9.3 \tag{4.15}$$

Considering that $\nu_{\rm I} = 0.34$, $\nu_{\rm II} = 0.44$ we calculate $(\gamma_{\rm I})_- \approx 9$, $(\gamma_{\rm II})_- \approx 8$ which agrees with (4.15). Hence, the corresponding ratios

$$\frac{\gamma_-(\nu_{\rm I})}{\gamma_-(\nu_{\rm II})} = \frac{9}{8} \approx 1.125 \qquad \frac{\omega_-(\nu_{\rm I})}{\omega_-(\nu_{\rm II})} = 1.125 \cdot 4.16 = 4.66 \tag{4.16}$$

Summing it all up, in the case of $m = 2$ for the above mentioned materials (I-duralumin, II-tin) the ratio of basic frequencies $\omega_{\rm I}/\omega_{\rm II}$ increased by 12% as compared to (3.3) for $m = 0$, where the multiplier γ did not depend on ν.

Let us notice that formula (4.10) can also be written down as follows

$$S_{21}(r) = \frac{1}{4(1-\nu)} \frac{81x(x+1)^2}{4(14x+6)(11x+3)^2} \tag{4.17}$$

where $x \in (1 + \sqrt{2}, \infty)$.

In that case

$$\mu = \frac{x+1}{x} \qquad \nu = \frac{1}{2x^2}(x^2 - 2x - 1) \tag{4.18}$$

for $\nu \in [0, 1/2]$, $\mu \in [\sqrt{2}, 1]$.

Set forth dependencies (4.17) are extremely useful in calculations. In particular, it is easy to see that in inequalities (4.15), $\gamma_-(\nu)$ changes in a monotone fashion with the Poisson ratio ν.

Remarks. The coefficients of the corresponding characteristic series (see remarks from Section 3 for case $m = 0$) are as follows (Jaroszewicz, 2004) $m = 2$, $\nu = 1/9$

$$a_0 = 1 \qquad a_1 = 3^3 \cdot 2^{-11} \qquad a_2 = 3^5 \cdot 5^{-1} \cdot 2^{-21}$$

Hence, the double sided Bernstein-Keropian estimators give

$$9.412 < \gamma\Big(\frac{1}{9}\Big) < 9.495$$

which is in agreement with the simplest lower estimator, see (4.14), (4.15).

5. Disc type plate

Let $m = -1$ (Zoryj and Jaroszewicz, 2002). On the basis of (2.4), we obtain

$$L_0[u] \equiv u^{IV} - \frac{1+\nu}{r^2}u^{II} + \frac{2(1+\nu)}{r}u^I \tag{5.1}$$

and equation (2.2) takes the following form

$$L_0[u] - pr^{-\frac{2}{3}}u = 0 \qquad\qquad p = \frac{\rho h_0}{D_0}R^{-\frac{2}{3}}\omega^2 \tag{5.2}$$

The linearly-independent solutions to equation $L_0[u] = 0$ are

$$1,\ r^3,\ r^{\mu_1},\ r^{\mu_2} \tag{5.3}$$

where

$$\mu_1 = \frac{1}{2}(3 + \sqrt{5+4\nu}) \qquad\qquad \mu_2 = \frac{1}{2}(3 - \sqrt{5+4\nu}) \tag{5.4}$$

Let us notice that all solutions (5.3) are limited for $r = 0$. That is why it is enough to consider the first two solutions together with their derivatives. The necessary solutions, which correspond to them, are constructed with the aid of formulas (4.7), (4.8), in which

$$K_0(r,\alpha) = \frac{1}{2\sqrt{5+4\nu}}(r^{\mu_2}\alpha^{\mu_1} - r^{\mu_1}\alpha^{\mu_2}) + \frac{1}{6}(r^3 - \alpha^3) \tag{5.5}$$

and

$$\begin{aligned} &S_{jK} = \int\limits_0^r K_0(r,\tau)\tau^{\frac{2}{3}}S_{j,K-1}(\tau)\;d\tau \qquad\qquad K = 1,2,\dots \qquad\qquad j = 1,2 \\ &S_{10} = 1 \qquad\qquad S_{20} = r^3 \end{aligned} \tag{5.6}$$

Having determined the first two integrals (5.6), we obtain

$$S_1(r) = 1 + pS_{11}(r) + \dots \qquad\qquad S_2(r) = r^3 + pS_{21}(r) + \dots \tag{5.7}$$

where

$$
\begin{aligned}
S_{11}(r) &= a(\nu)r^{\frac{14}{3}} \qquad & a(\nu) &= \frac{81(1-\nu)}{140(79-9\nu)} \\
S_{21}(r) &= c(\nu)r^{\frac{23}{3}} \qquad & c(\nu) &= \frac{81(1-\nu)}{23\cdot 28(331-9\nu)}
\end{aligned}
\tag{5.8}
$$

Continuing in the same manner as in Section 3 (constant thickness plate ($m = 0$)), we come to the first two elements of characteristic series (the frequency equation) of the problem given by expressions (2.2) and (2.3) for $m = -1$

$$
3R^2 + \Big(\frac{5}{3}a(\nu) - \frac{23}{3}c(\nu)\Big)pR^{\frac{20}{3}} + \ldots = 0
$$

After simplification by means of R^2 reduction, and having considered formula (5.2, we obtain

$$
3 + \frac{1}{3}[5a(\nu) - 23c(\nu)]\frac{\rho h_0}{D_0}R^4\omega^2 = 0 \tag{5.9}
$$

hence, making use of formulas (5.8), we calculate the coefficient

$$
-a_1(\nu) = \frac{1}{3}[5a(\nu) - 23c(\nu)] \tag{5.10}
$$

for values $\nu = 0$, $\nu_{\rm I} = 0.34 \approx 3/9$, $\nu_{\rm II} = 0.44 \approx 4/9$ and $\nu = 0.5$ we have respectively

$$
0.0092928,\ 0.0064991,\ 0.0055045,\ 0.0049951
$$

From (5.9), we come to formulas (4.12) and (4.13), then to estimators

$$
17.97 < \gamma_-(\nu) < 24.5 \tag{5.11}
$$

and then to relations

$$
\frac{\gamma_{\rm I}}{\gamma_{\rm II}} = \frac{21.48}{23.35} = 0.92 \tag{5.12}
$$

So, in the case of $m = -1$ instead of (4.16), we get

$$
\frac{\omega_-(\nu_{\rm I})}{\omega_-(\nu_{\rm II})} = 0.92 \cdot 4.16 = 3.83 \tag{5.13}
$$

From (5.13), it follows that the ratio between basic frequencies decreased by 8% as compared to the result obtained from (3.3) for fixed thickness ($m = 0$). It is worth noticing that for $m = -1$ the exact values of basic frequencies are known (Hondkiewič, 1964). Having them applied, instead of (5.12), we obtain

$$a_2\Big|_{\nu=\frac{3}{9}} = 0.0000009 \qquad a_2\Big|_{\nu=\frac{4}{9}} = 0.0000007$$

$$a_1\Big|_{\nu=\frac{3}{9}} = 0.0064991 \qquad a_1\Big|_{\nu=\frac{4}{9}} = 0.0055045$$

($a_1(\nu)$ is determined by formulas (5.8), (5.10)).

Therefore

$$\frac{\gamma_{\rm I}}{\gamma_{\rm II}} = \frac{22.25}{24.25} = 0.92 \tag{5.14}$$

6. Conclusions

- As far as diaphragm type plates $(m > 0)$ are considered, materials with large values of ν exhibit lower basic frequencies as compared to fixed thickness plates. Likewise, disc type plates $(m < 0)$ have higher basic frequencies. The decrease in the diaphragm type plate of $m = 2$ with respect to the fixed thickness plate $(m = 0)$, is from 9% for $\nu = 0$ up to 24% for $\nu = 0.5$. For disc plates of $m = -1$ we observe an increase in the frequency from 83% for $\nu = 0$ and 247% for $\nu = 0.5$.

- The simplest lower estimators calculated with the first two elements of the series taken into account, allow us to credibly observe the effect of E, ν and ρ on the frequencies of axi-symmetrical vibrations of circular plates, whose thickness or rigidity change along the radius according to ae power function.

- In the case of fixed thickness plates $(m = 0)$, the coefficient of basic frequencies γ does not depend on Poisson's ratio ν. Thus $\gamma_{\rm I}/\gamma_{\rm II} = 1$ for materials having large values of $\nu \sim 0.5$, as because of the conditions $(E/\rho)_{\rm I} > 1$, $(\rho/E)_{\rm II} < 1$ they correspond to smaller frequencies.

Selected calculation results for two different materials $\nu_{\rm I} \approx 0.34$, $\nu_{\rm II} \approx 0.44$, $(E/\rho)_{\rm I} = 2.65$, $(\rho/E)_{\rm II} = 7.09$ and three types of plates are presented as follows:

— Bernstein-Keropian double sided estimators

$$
\begin{array}{ll}
9.412 < \gamma(1/9) < 9.495 & \text{for} \quad m = 2 \\
10.204 < \gamma < 10.224 & \text{for} \quad m = 0 \\
22.232 < \gamma(4/9) < 22.262 & \text{for} \quad m = -1 \\
24.233 < \gamma(1/3) < 24.272 & \text{for} \quad m = -1
\end{array}
$$

— basic frequency parameter ratio

$$
\frac{\gamma_-(\nu_{\rm I})}{\gamma_-(\nu_{\rm II})} = \begin{cases} 1.125 & \text{for} \quad m = 2 \\ 1 & \text{for} \quad m = 0 \\ 0.92 & \text{for} \quad m = -1 \end{cases}
$$

— basic frequency ratio

$$
\frac{(\omega_{\rm I})_{\rm I}}{(\omega_{\rm I})_{\rm II}} = \begin{cases} 4.66 & \text{for} \quad m = 2 \\ 4.16 & \text{for} \quad m = 0 \\ 3.83 & \text{for} \quad m = -1 \end{cases}
$$

— simple under estimate estimators of the basic frequency parameter for Poisson's ratio $0 \leqslant \nu \leqslant 0.5$

$$
\begin{array}{ll}
7.8 < \gamma_-(\nu) < 9.3 & \text{for} \quad m = 2 \\
\gamma_-(\nu) = \sqrt{96} & \text{for} \quad m = 0 \\
17.97 < \gamma_-(\nu) < 24.5 & \text{for} \quad m = -1
\end{array}
$$

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Analiza wpływu stałych sprężystych i gęstości materiału na częstość podstawową drgań osiowosymetrycznych płyt kołowych o zmiennej grubości

Streszczenie

W pracy zbadano wpływ modułu sprężystości Younga i liczby Poissona, a także gęstości materiału na częstość podstawową płyt o zmiennej grubości typu diafragmy i dysku. Do rozwiązania zagadnienia brzegowego zastosowano metodę funkcji wpływu Cauchy, najprostszy estimator z niedostatkiem i dwustronne estymatory Bernstejna-Kieropiana. Znaleziono analityczną postać funkcji wpływu Cauchy, z pomocą której zbudowano równanie charakterystyczne w postaci szeregu potęgowego względem parametru częstotliwości. Zastosowanie metody pozwoliło wyprowadzić funkcjonalną zależność częstości podstawowej od stałych materiałowych wymienionych płyt. W charakterze przykładu przytoczono wyniki obliczeń dla płyt wykonanych z duraluminium i z cyny. Porównanie wyników obliczeń ze znanymi z literatury potwierdziły wysoką dokładność metody.

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