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ANALYTICAL SOLUTION FOR MAGNETOHYDRODYNAMIC STAGNATION POINT FLOW AND HEAT TRANSFER OVER A PERMEABLE STRETCHING SHEET WITH CHEMICAL REACTION

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This work deals with the problem of steady two-dimensional magnetohydrodynamic (MHD) stagnation point flow towards a permeable stretching sheet with chemical reaction. The fundamental equations of the boundary layer are transformed into ordinary differential equations, which are then solved analytically using the Optimal Homotopy Asymptotic Method (OHAM). Comparisons are made between the results of the proposed method and the numerical method (fourth-order Runge-Kutta) in solving this problem, and excellent agreement has been observed. Subsequently, effects of different involved parameters on the temperature profiles, concentration profiles, local Nusselt number, local Sherwood number and skin-friction coefficient are presented and discussed in detail.

Key words: MHD stagnation point flow, permeable stretching sheet, heat and mass transfer, Optimal Homotopy Asymptotic Method (OHAM)

1. Introduction

The study of boundary layer flow over a stretching sheet has long been studied, as it is a representative model problem for numerous engineering applications such as in the polymer processing of a chemical engineering plant, metallurgy for the metal processing of glass fiber, paper production, liquid films in the condensation process and filament extrusion from a dye. Due to high applicability of this problem in such industrial phenomena, it has attracted attention of many researchers. Perhaps, Sakiadis (1971) were pioneers in formulating this problem to study a steady two-dimensional boundary layer flow due to a stretching sheet. Many investigators have extended the work of Sakiadis (1971) in order to study heat and mass transfer under various physical situations (e.g. Gupta and Gupta, 1997; Datta, *et al.*, 1985; McLeod and Rajagopal, 1987; Chiam, 1996).

Nevertheless, all the above-mentioned researches do not consider situations, where hydro--magnetic effects arise. The study of hydrodynamic flow and heat transfer over a stretching sheet may find its application to sheet extrusion in order to make flat plastic sheets. In doing so, it is important to investigate cooling and heat transfer for the improvement of the final products. The conventional fluids such as water and air are amongst the most widely used fluids as the cooling medium. However, the rate of heat exchange achievable by the above fluids is realized to be unsuitable for certain sheet materials. Thus, in recent years, it has been proposed to alter flow kinematics that it leads to a slower rate of solidification, as compared with water. Among the techniques to control flow kinematics, the idea of using magnetic fields appears to be the most attractive one both because of its ease of implementation and also because of its non-intrusive nature. In view of this, the study of MHD flow of Newtonian/non-Newtonian flow over a stretching sheet was carried out by many researchers (Yazdi *et al.*, 2011; Hayat and Sajid, 2007;

Takhar *et al.*, 2000; Afify, 2004; Babaelahi *et al.*, 2010; Alizadeh-Pahlavan, *et al.*, 2009; Cortell, 2005; Nandeppanavar *et al.*, 2011; Robert *et al.*, 2011; Farzaneh-Gord *et al.*, 2010; Mamun *et al.*, 2008).

Analytical solutions have been done for boundary layer flow over a stretching sheet. Yazdi *et al.* (2008) investigated friction and heat transfer in the slip flow boundary layer at constant heat flux boundary conditions. It has been found that suction makes a significant effect on the velocity adjacent to the wall in the presence of slip. Rana and Bhargava (2012) studied the problem of steady laminar boundary fluid flow, which results from non-linear stretching of a flat surface in a nanofluid. Javed *et al.* (2010) studied heat transfer of a viscous fluid over a non-linear shrinking sheet in the presence of a magnetic field, where they have obtained dual solutions for the exact and numerical solutions in the shrinking sheet problem. Hydrodynamic nano-boundary layer flow over a permeable stretching surface by employing Homotopy Analysis Method (HAM) and Boundary Value Problem solver (BVP) was studied by Van Gorder *et al.* (2010). Kechil and Hashim (2008) studied the boundary-layer equation of flow over a nonlinearly stretching sheet in a magnetic field with chemical reaction. Fang *et al.* (2009) investigated analytically the hydrodynamic boundary layer of slip MHD viscous flow over a stretching sheet. They have concluded that the wall drag force increases with the magnetic parameter. Recently, Ishak *et al.* (2009) studied numerically steady two-dimensional MHD stagnation point flow towards a stretching sheet with variable surface temperature. They found that the heat transfer rate at the surface increases with the magnetic parameter, when the free stream velocity exceeds the stretching velocity. More recently, Nadeem *et al.* (2010) presented HAM solution for unsteady boundary layer flow in the region of the stagnation point towards a stretching sheet.

There have been many theoretical models developed to describe MHD flow towards a stretching sheet. However, to the best of our knowledge, no investigation has been made yet to analyze the MHD stagnation point flow over a permeable stretching surface in the presence of chemical reaction. In industrial processes involving heat and mass transfer over a stretching surface, the diffusing species can be generated or absorbed due to some kind of chemical reaction with the ambient fluid. This generation or absorption of species can affect the flow and, accordingly, the properties and quality of the final product. This fact motivates the present study to provide an investigation taking into consideration the diffusion equation with a chemical reaction source or a sink term. The flow is not caused only by the stretching sheet, as previously considered by many authors, but also due to the external stream. The highly non-linear momentum equation, to gather with the heat and mass transfer equations, are solved analytically using the Optimal Homotopy Asymptotic Method (OHAM) proposed by Marinca and Herisanu (2008) and successfully examined by some authors (Joneidi *et al.*, 2009; Dinarvand and Hosseini, 2011; Marinca *et al.*, 2009).

2. Governing equations

Let us consider the steady, two-dimensional flow of an incompressible electrically conducting fluid near the stagnation point on a permeable stretching sheet as shown in Fig. 1. The stretching velocity and the free stream velocity are assumed to vary proportional to the distance x from the stagnation point, i.e. $U_w(x) = ax$ and $U_\infty(x) = bx$. It is also assumed that a uniform magnetic field of strength B_0 is applied in the positive y-direction normal to the sheet. The induced magnetic field due to motion of the electrically conducting fluid and the pressure gradient are neglected. The temperature and the species concentration are maintained at the prescribed constant values T_w, C_w, respectively at the sheet. This model can be utilized for simulating spray drying of milk, fluidized bed catalysis and many more industrial and engineering applications of transport processes over a stretching sheet to study the mass and momentum transfer

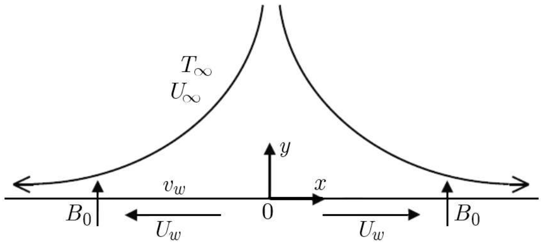

Fig. 1. Geometry of problem under investigation

of chemically reactive species. The two-dimensional equations, governing the flow of a steady, laminar and incompressible viscous fluid are

$$\begin{array}{ll} \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0 & u\dfrac{\partial u}{\partial x} + v\dfrac{\partial v}{\partial y} = U_\infty \dfrac{dU_\infty}{dx} + \nu \dfrac{\partial^2 v}{\partial y^2} + \dfrac{\sigma B_0^2}{\rho}(U_\infty - u) \\ u\dfrac{\partial T}{\partial x} + v\dfrac{\partial T}{\partial y} = \alpha \dfrac{\partial^2 T}{\partial y^2} & u\dfrac{\partial C}{\partial x} + v\dfrac{\partial C}{\partial y} = D\dfrac{\partial^2 C}{\partial y^2} - \kappa_0 C^m \end{array} \tag{2.1}$$

where κ_0 is the reaction rate constant, u, v – velocity components in the x and y direction, respectively and m is the order of chemical reaction. The boundary conditions for this problem are

$$\begin{array}{lllll} y = 0 & T = T_w & u = U_w(x) & v = v_w & C = C_w \\ y \to \infty & T = T_\infty & u = U_\infty(x) & C = C_\infty & \end{array} \tag{2.2}$$

Equation of continuity $(2.1)_1$ is satisfied by introducing a stream function ψ such that

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x}$$

Introducing the similarity transformation

$$\eta = \sqrt{\frac{a}{\nu}}\, y \qquad f(\eta) = \frac{\psi}{\sqrt{\alpha\nu}\, x} \qquad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \qquad \varphi = \frac{C - C_\infty}{C_w - C_\infty} \tag{2.3}$$

where α denotes thermal diffusivity, ν – kinematic viscosity and η similarity variable. The subscripts w and ∞ mean: at the wall and in infinity, respectively.

After transformation, we have

$$\begin{array}{ll} f''' + ff'' - f'^2 + \varepsilon^2 + M(\varepsilon - f') = 0 & \dfrac{1}{\mathrm{Pr}}\theta'' + f\theta' - f'\theta = 0 \\ \varphi'' + \mathrm{Sc} f\varphi' - \mathrm{Sc}\gamma\varphi^m = 0 & \end{array} \tag{2.4}$$

where f is the similarity function, ε – velocity ratio, ϕ – dimensionless concentration, θ – dimensionless temperature.

The associated boundary conditions are,

$$\begin{array}{llll} f(0) = f_w & f'(0) = 1 & \theta(0) = 1 & \varphi(0) = 1 \\ f'(\infty) = \varepsilon & \theta(\infty) = 0 & \varphi(\infty) = 0 & \end{array} \tag{2.5}$$

where $\varepsilon = b/a$ is the velocity ratio parameter, $M = \sigma B_0^2/(\rho a)$is the magnetic parameter, in which denotes conductivity and ρ is density, $\mathrm{Pr} = \nu/\alpha$ is the Prandtl number, $\mathrm{Sc} = \nu/\alpha$ is the Schmidt number, $f_w = -v_w/\sqrt{U_w \nu_\infty / x}$ is the suction/injection parameter, $\gamma = (\kappa_0/a)C_w^{m-1}$ is the non-dimensional chemical reaction parameter. The skin friction coefficient C_f, the local Nusselt number Nu_x and the local Sherwood number Sh_x can be writen as

$$C_f = \frac{\mu\Big(\frac{\partial u}{\partial y}\Big)_{y=0}}{\rho U_w^2/2} \qquad \mathrm{Nu}_x = -\frac{x\Big(\frac{\partial T}{\partial y}\Big)_{y=0}}{T_w - T_\infty} \qquad \mathrm{Sh}_x = -\frac{x\Big(\frac{\partial C}{\partial y}\Big)_{y=0}}{C_w - C_\infty} \tag{2.6}$$

Using non-dimensional variables (2.3), we obtain

$$\frac{1}{2}C_f\sqrt{\mathrm{Re}_x} = f''(0) \qquad \frac{\mathrm{Nu}_x}{\sqrt{\mathrm{Re}_x}} = -\theta'(0) \qquad \frac{\mathrm{Sh}_x}{\sqrt{\mathrm{Re}_x}} = -\varphi'(0) \tag{2.7}$$

3. Solution using Optimal Homotopy Asymptotic Method

In this section, we will apply OHAM to solve Eqs. (2.4), subjected to boundary conditions Eqs. (2.5). According to OHAM, we can construct a Homotopy of Eqs. (2.4) as follows

$$\begin{aligned}
&(1-p)(f''+f') - H_1(p)[f''' + ff'' - f'^2 + \varepsilon^2 + M(\varepsilon - f')] = 0\\
&(1-p)(\theta'+\theta) - H_2(p)\Big(\frac{1}{\mathrm{Pr}}\theta'' + f\theta' - f'\theta\Big) = 0\\
&(1-p)(\varphi'+\varphi) - H_3(p)(\varphi'' + \mathrm{Sc}f\varphi' - \mathrm{Sc}\gamma\varphi^m) = 0
\end{aligned} \tag{3.1}$$

where primes denote differentiation with respect to η. We consider f, θ, φ, $H_1(p)$, $H_2(p)$ and $H_3(p)$ as follows

$$\begin{aligned}
&f = f_0 + pf_1 + p^2 f_2 \qquad &&\theta = \theta_0 + p\theta_1 + p^2\theta_2 \qquad &&\varphi = \varphi_0 + p\varphi_1 + p^2\varphi_2\\
&H_i(p) = pc_{i1} + p^2 c_{i2} \qquad &&i = 1,2,3
\end{aligned} \tag{3.2}$$

Substituting Eqs. (3.2) into Eqs. (3.1) and making some simplification and rearrangement based on the powers of p-terms, we have

$$\begin{aligned}
&p^0:\ f_0' + f_0'' = 0 \qquad &&f_0(0) = f_w \qquad f_0'(0) = 1\\
&\theta_0' + \theta_0 = 0 &&\theta_0(0) = 1\\
&\varphi_0' + \varphi_0 = 0 &&\varphi_0(0) = 1
\end{aligned} \tag{3.3}$$

and

$$\begin{aligned}
&p^1:\ -f_0' - f_0'' - c_{11}\varepsilon^2 + f_1' + f_1'' - c_{11}\varepsilon M + c_{11}Mf_0' - c_{11}f_0 - c_{11}f_0f_0''\\
&\qquad + c_{11}(f_0'')^2 - c_{11}f_0''' = 0\\
&f_1(0) = 0 \qquad f_1'(0) = 0\\
&-\theta_0' + \theta_1 + c_{21}\,\mathrm{Pr}\,f_0'\theta_0 + \theta_0' - c_{21}\theta_0'' - c_{21}\,\mathrm{Pr}\,f_0\theta_0' - \theta_0 = 0\\
&\theta_1(0) = 0\\
&-c_{31}\varphi_0'' - \varphi_0 + \varphi_1 + \varphi_1' - \varphi_0' - c_{31}\mathrm{Sc}\gamma\varphi_0 - c_{31}\mathrm{Sc}f_0\varphi_0' = 0\\
&\varphi_1(0) = 0\\
&\vdots
\end{aligned} \tag{3.4}$$

Solving Eqs. (3.3) and (3.4) with the boundary conditions, we obtain

$$f_0(\eta) = f_w + 1 - \mathrm{e}^{-\eta} \qquad \theta_0(\eta) = \mathrm{e}^{-\eta} \qquad \phi_0(\eta) = \mathrm{e}^{-\eta} \tag{3.5}$$

and

$$\begin{aligned}f_1(\eta) &= c_{11}\varepsilon^2\eta + c_{11}M\varepsilon\eta + c_{11}(M+f_w)(\eta\mathrm{e}^{-\eta}+\mathrm{e}^{-\eta}) + (c_{11}\varepsilon^2 + c_{11}M\varepsilon)\mathrm{e}^{-\eta}\\ &\quad - c_{11}(M+f_w+M\varepsilon+\varepsilon^2)\\ \theta_1(\eta) &= -c_{21}(-\eta + \mathrm{Pr}\, f_w\eta + \mathrm{Pr}\,\eta)\mathrm{e}^{-\eta}\\ \varphi_1(\eta) &= c_{31}(\eta - \mathrm{Sc}\gamma\eta - \mathrm{Sc}f_w\eta - \mathrm{Sc}\eta - \mathrm{Sc}\mathrm{e}^{-\eta} + \mathrm{Sc})\mathrm{e}^{-\eta}\\ &\vdots\end{aligned} \tag{3.6}$$

The other terms are too large to be mentioned. The final expression for $f(\eta)$, $\theta(\eta)$ and $\varphi(\eta)$ would be

$$\begin{aligned}f(\eta) &= f_0(\eta) + f_1(\eta) + f_2(\eta) + \ldots \qquad \theta(\eta) = \theta_0(\eta) + \theta_1(\eta) + \theta_2(\eta) + \ldots\\ \varphi(\eta) &= \varphi_0(\eta) + \varphi_1(\eta) + \varphi_2(\eta) + \ldots\end{aligned} \tag{3.7}$$

Now, we should construct the following expressions. Note that R_i, $i = 1,2,3$ are the residual of Eqs. (2.4)

$$J_i(c_{i1}, c_{i2}) = \int_0^\infty R_i^2(\eta, c_{i1}, c_{i2})\, d\eta \qquad i = 1,2,3 \tag{3.8}$$

We can find the constants c_{11}, c_{12}, c_{21}, c_{22}, c_{31}, c_{32} by solving the following equations simultaneously

$$\frac{\partial J_i(c_{i1}, c_{i2})}{\partial c_{i1}} = \frac{\partial J_i(c_{i1}, c_{i2})}{\partial c_{i2}} = 0 \qquad i = 1,2,3 \tag{3.9}$$

In the specific case

$$\begin{aligned}M &= \varepsilon = f_w = \gamma = 0.1 & \mathrm{Pr} &= \mathrm{Sc} = 1\\ c_{11} &= -0.7305331575 & c_{12} &= 2.949971023\\ c_{21} &= -1.412920644 & c_{22} &= 7.105993798\\ c_{31} &= 0.2434693959 & c_{31} &= 0.04693256846\end{aligned} \tag{3.10}$$

4. Results and discussion

The system of Eqs. (2.4), along with boundary conditions of Eqs. (2.5), has been solved by using the OHAM. To check the validity of our results, we have made a comparison between the present analytical solution and a numerical solution through the fourth-order Runge-Kutta method that is depicted in Table 1. It can be realized that our analytical approximations are in good agreement with the numerical results.

The non-dimensional temperature distribution $\theta(\eta)$ for various parameters including the velocity ratio parameter ε, the magnetic parameter M and the wall suction/injection parameter f_w are plotted in Figs. 2a-2c. As shown in Fig. 2a, an increase in the velocity ratio parameter escalates temperature. In addition, the boundary layer thickness tends to increase when the velocity parameter increases. The effect of the magnetic parameter M on the temperature profile $\theta(\eta)$ is illustrated in Fig. 2b. It is clear that an increase in the magnetic parameter tends to increase temperature. On the other hand, the thermal boundary layer thickens by increasing M. Figure 2c shows the variation of temperature profiles $\theta(\eta)$ across the boundary layers for different values of the suction/injection parameter f_w. It is worth mentioning that suction corresponds to $f_w > 0$, injection to $f_w < 0$, and $f_w = 0$ to impermeable stretching. As seen, suction reduces

Table 1. Comparison between the present analytical solution and numerical solution for $\theta(\eta)$ when $\varepsilon = 0.1$, $\mathrm{Pr} = \mathrm{Sc} = 1$ and $\gamma = 0.1$

	$\theta(\eta)$					
	$M = 0.4$ and $f_w = 0.2$			$M = 4$ and $f_w = -0.2$		
	OHAM solution	Numerical solution	Relative error	OHAM solution	Numerical solution	Relative error
0.00	1.000000	1.000000	0.000000	1.00000	1.00000	0.00000
0.50	0.584463	0.583593	0.000870	0.68391	0.68344	0.00047
1.00	0.343715	0.342955	0.000760	0.47312	0.47276	0.00036
1.50	0.202815	0.202195	0.000620	0.32521	0.32493	0.00028
2.00	0.119858	0.119338	0.000520	0.22072	0.22046	0.00026
2.50	0.070856	0.070396	0.000460	0.14773	0.14748	0.00025
3.00	0.041869	0.041539	0.000330	0.09758	0.09732	0.00026
3.50	0.024716	0.024456	0.000260	0.06369	0.06348	0.00021
4.00	0.014570	0.014300	0.000270	0.04113	0.04098	0.00015
4.50	0.008575	0.008335	0.000240	0.02631	0.02617	0.00014
5.00	0.005037	0.004857	0.000180	0.01670	0.01659	0.00011
5.50	0.002952	0.002812	0.000140	0.01051	0.01041	0.00010
6.00	0.001726	0.001616	0.000110	0.00658	0.00651	0.00007
6.50	0.001006	0.000936	0.000070	0.00409	0.00403	0.00006
7.00	0.000585	0.000555	0.000030	0.00253	0.00249	0.00004

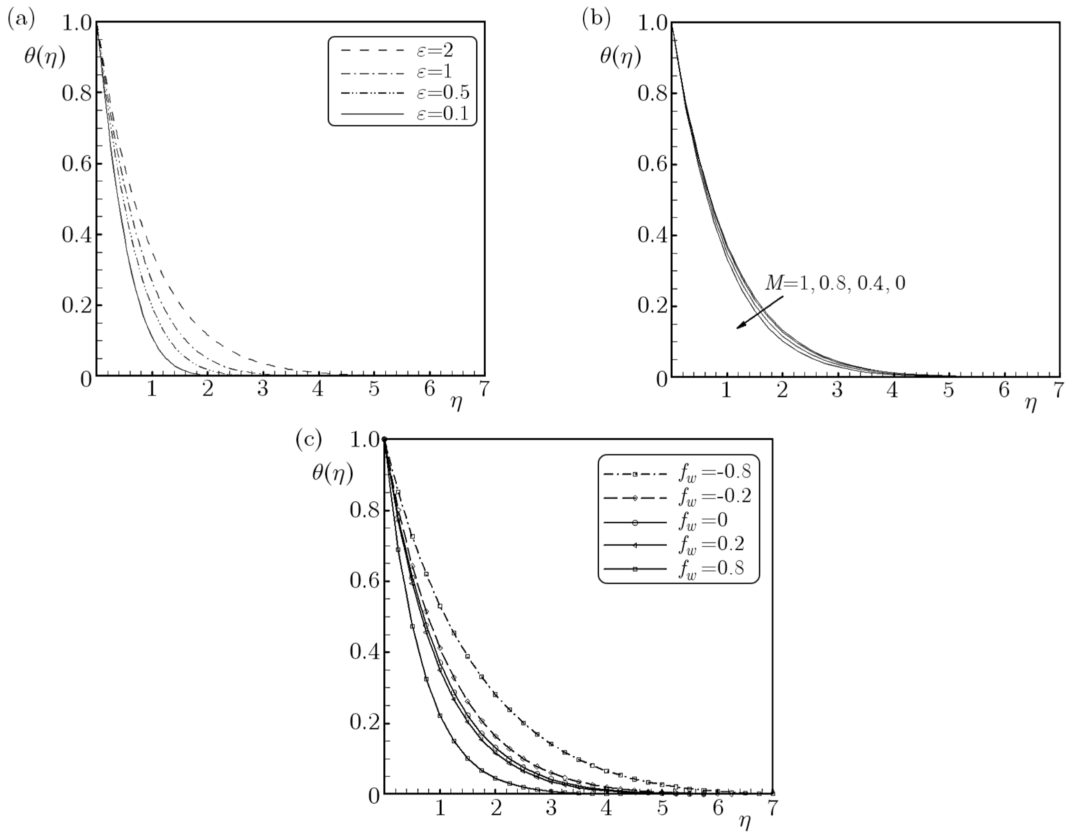

Fig. 2. Temperature profiles as function of η for various values of (a) velocity ratio parameter ε, (b) magnetic parameter M, (c) suction/injection parameter f_w at $M = 0.4$, $f_w = 0.2$, $\mathrm{Pr} = \mathrm{Sc} = 1$ and $\gamma = 0.1$

the boundary layer thickness sharply. It is also found that increasing the value of f_w results in a decrease in temperature.

Figures 3a-3d illustrate the effects of important parameters on the concentration profiles across the boundary layers. The influence of the reaction rate parameter on the concentration is shown in Fig. 3a when $M = \varepsilon = 0.1$, $f_w = 0.2$ and $\mathrm{Pr} = \mathrm{Sc} = 1$. Note that the reaction rate parameter is positive for destructive $(\gamma > 0)$ and negative due to generative $(\gamma < 0)$ chemical reactions. It is seen that increasing the reaction rate parameter tends to decrease the concentration in the case of destructive chemical reaction especially for a higher value of γ. Figure 3b indicates the variation of the velocity ratio parameter ε with the concentration profile. It is realized that the concentration increases as the velocity ratio parameter goes up. The effect of the magnetic parameter M on the concentration profiles is also demonstrated in Fig. 3c. By increasing the magnetic parameters, the concentration intensifies. It is also interesting to know the variation the suction/injection parameter f_w with the concentration profile, so it has been shown in Fig. 3d. It can be seen that the concentration is lower in the case of suction, in particular for a higher absolute value of f_w.

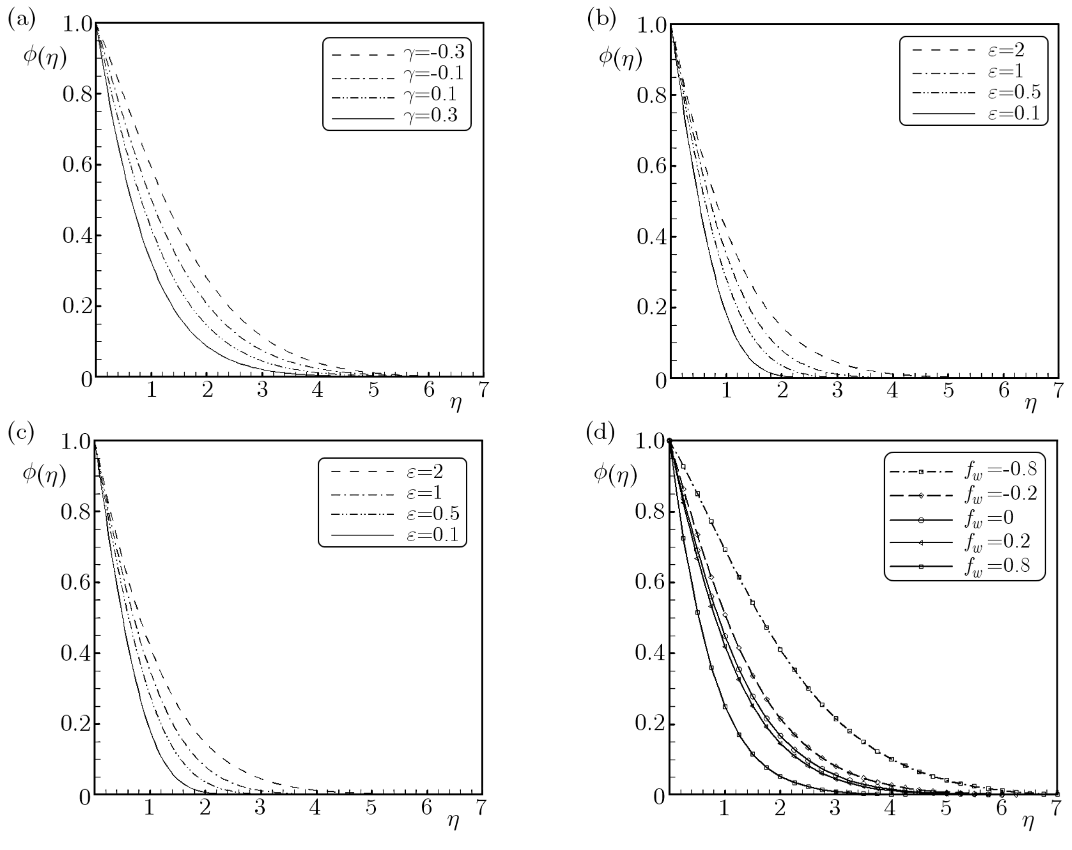

Fig. 3. Concentration profiles as function of η for various values of (a) chemical reaction parameter γ, (b) velocity ratio parameter ε, (c) magnetic parameter M, (d) suction/injection parameter f_w at $M = 0.4$, $f_w = 0.2$, $\mathrm{Pr} = \mathrm{Sc} = 1$ and $\varepsilon = 0.1$

Figures 4a-4c show the variation of the heat transfer rate at the wall versus the magnetic parameter for various values of ε, f_w and Pr. As shown in Fig. 4a, it is concluded that the rate of heat flux increases with the velocity ratio parameter. Moreover, a deterioration of heat transfer can be seen with an increase in the magnetic parameter for a lower value of the velocity ratio parameter. By contrast, the effect of the magnetic parameter on the heat transfer rate, for higher values of ε, is almost infinitesimal. The effect of the suction/injection parameter on the heat transfer rate has been shown in Fig. 4b. It is indicated that increasing suction $(f_w > 0)$

at the wall results in a higher heat transfer rate, whereas injection $(f_w < 0)$ decreases the heat transfer rate for all values of the magnetic parameter. Figure 4c represents the variation of the heat transfer rate with the magnetic parameter for different values of the Prandtl number when $M = \gamma = \varepsilon = 0.1$, $f_w = 0.2$ and $\mathrm{Sc} = 1$. The rate of heat transfer is predicted to experience an increase with the Prandtl number. More importantly, the magnetic parameter due to the applied Lorenz force on the fluid field causes the wall temperature gradient to decrease. As a result, the Nusselt number decreases with the magnetic parameter. Generally, to reach a higher heat transfer rate, a working fluid with higher Prandtl number, along with lower magnetic parameter, is needed.

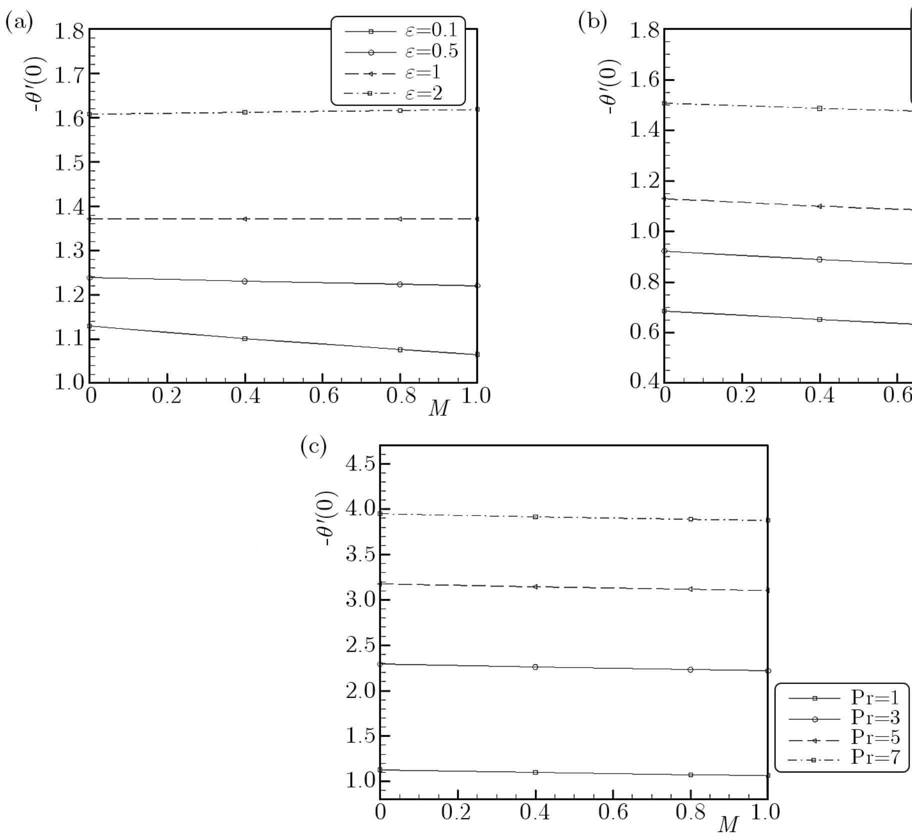

Fig. 4. Temperature gradient at the wall as function of M for various values of (a) velocity ratio parameter ε, (b) suction/injection parameter f_w, (c) Prandtl number Pr at $f_w = 0.2$, $\mathrm{Pr} = \mathrm{Sc} = 1$ and $\gamma = 0.1$

Variations of the concentration gradient magnitude at the wall versus the magnetic parameter for other parameters, namely γ, ε, Sc and f_w, have been depicted in Figs. 5a-5d. As shown in Fig. 5a, the magnetic parameter tends to decrease the mass transfer slightly for both destructive chemical reactions and generative chemical reactions when $\varepsilon = 0.1$, $f_w = 0.2$ and $\mathrm{Pr} = \mathrm{Sc} = 1$. Furthermore, increasing the reaction rate parameter in absolute sense results in a decrease in the concentration gradient at the wall for generative chemical reaction. However, the opposite effect is observed in the case of destructive chemical reaction. Figure 5b shows different trends, where $-\varphi'(0)$ increases as M increases for $\varepsilon > 1$, whereas it decreases with M for $\varepsilon < 1$. It is also revealed that an increase in the velocity ratio parameter results in an increase in the concentration gradient at the surface. Similarly, the mass transfer at the wall has considerably increased with the Schmidt number (see Fig. 5c). Regarding the effect of the suction/injection parameter on the concentration gradient at surface, it is realized that the mass transfer at the wall is intensified in the case of suction, which has been shown in Fig. 5d. It also can be concluded that higher values of f_w lead to an increase in the mass transfer at the wall.

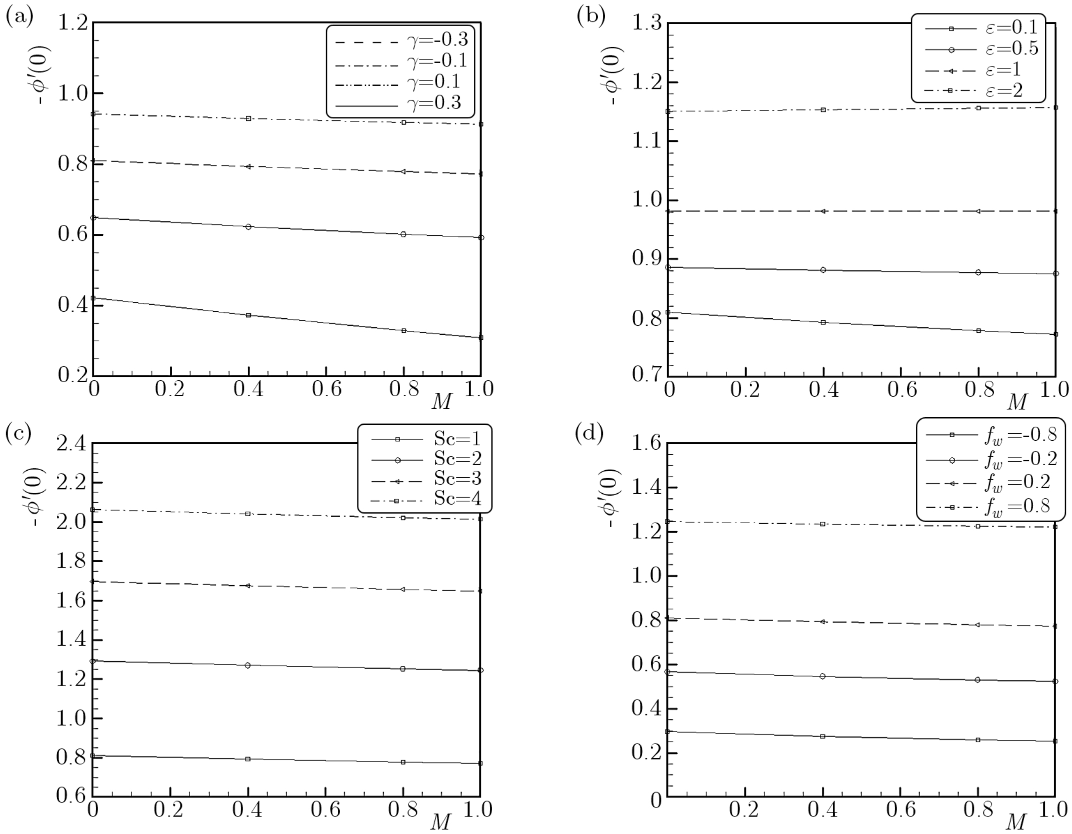

Fig. 5. Concentration gradient at the wall as function of M for various values of (a) chemical reaction parameter γ, (b) velocity ratio parameter ε, (c) Schmidt number Sc, (d) suction/injection parameter f_w at $f_w = 0.2$, Pr = Sc = 1 and $\gamma = 0.1$

Figure 6 illustrates the variation of the wall shear stress versus the magnetic parameter for various values of the suction/injection parameter when $\gamma = \varepsilon = 0.1$ and Pr = Sc = 1. It is found that the wall shear stress increases with an increase in suction, whereas in absolute sense the injection tends to decrease the wall shear stress. Additionally, for both cases of suction and injection, it is realized that a rise in the magnetic parameter increases the skin friction coefficient.

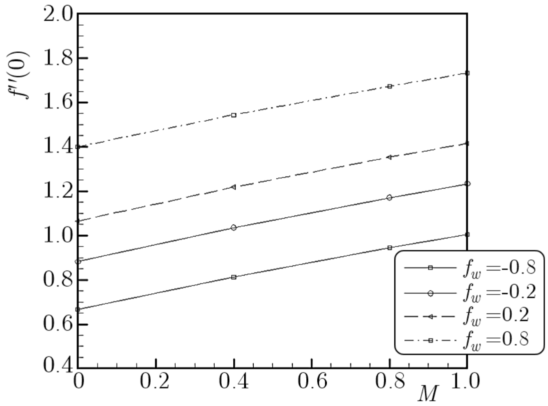

Fig. 6. The effects of the magnetic parameter on $f''(0)$ for different values of the suction/injection parameter f_w, when $\varepsilon = 0.1$, Pr = Sc = 1 and $\gamma = 0.1$

5. Conclusions

The flow field and heat transfer as well as mass transfer due to first order chemical reaction have been investigated for an MHD stagnation point flow towards a permeable stretching sheet immersed in a viscous fluid at the CST boundary condition. The governing partial differential equations were converted into ordinary differential equations by using a suitable similarity transformation, which were analytically solved by employing the OHAM technique. Comparisons with the numerical solution were performed and the results were found to be in good agreement. The results suggest that:

- The rate of heat transfer at the sheet surface increases due to an increase in the velocity ratio parameter, suction parameter and Prandtl number, while the local Nusselt number decreases with the injection parameter and the magnetic parameter. Moreover, the effect of the magnetic parameter on the heat transfer rate, for higher values of ε, is nearly negligible.
- High concentration gradients are possible at high reaction rate parameters for destructive chemical reaction, high Schmidt number, high suction/injection parameter in the case of suction, low magnetic parameter, when $\varepsilon > 1$ and a high magnetic parameter when $\varepsilon < 1$.
- The skin friction coefficient increases with an increase in the magnetic parameter and f_w in the case of suction, whereas opposite trend has been observed in the case of injection.

The analytical solution presented here is potentially influential in controlling the wall shear stress as well as the local Nusselt and Sherwood numbers. It is hoped that this work will serve as a vehicle for understanding of more complex problems involving various physical effects investigated in the present paper.

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