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SOME METHODS FOR MULTICRITERIA DESIGN OPTIMIZATION USING EVOLUTIONARY ALGORITHMS

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In this paper new multicriteria design optimization methods are discussed. These methods are evolutionary algorithm based methods, and their aim is to make the process of generating the Pareto front very effective. Firstly, the multistage evolutionary algorithm method is presented. In this method, in each stage only a bicriterion optimization problem is solved and then an objective function is transformed to the constrain function. The process is repeated till all the objective functions are considered. Secondly, the preference vector method is presented. In this method, an evolutionary algorithm finds the ideal vector. This vector provides the decision maker with the information about possible ranges of the objective functions. On the basis of this information the decision maker can establish the preference vector within which he expects to find a preferred solution. For this vector, a set of Pareto solutions is generated using an evolutionary algorithm based method. Finally, the method for selecting a representative subset of Pareto solutions is discussed. The idea of this method consists in reducing the set of Pareto optimal solutions using the indiscernibility interval method after running a certain number of generations. To show how the methods discussed work each of them in turn is applied to solve a design optimization problem. These examples show clearly that using the proposed methods the computation time can be reduced significantly and that the generated solutions are still on the Pareto front.

Key words: multicriteria design optimization, evolutionary algorithms, Pareto front

1. Introduction

While running evolutionary algorithms for multicriteria optimization a set of Pareto optimal solutions is generated (see Deb, 2001; Coello *et al.*, 2002). For some problems the final result of running a computer program is the Pareto set which contains hundreds or even thousands of solutions. For the decision maker it is difficult and tiresome to analyze all these solutions. In addition, the computing time rapidly grows while dealing with this type of problems. To overcome these difficulties, three methods are proposed in this paper. These methods can be very useful considering both the computing time and the decision-making problem while solving different design optimization problems.

The paper deals with design optimization problems which can be modelled by means of nonlinear programming, which is formulated as follows:
find $\boldsymbol{x}^* = [x_1^*, x_2^*, ..., x_I^*]$ which will satisfy the K inequality constraints

$$g_k(\boldsymbol{x}) \geqslant 0 \qquad \text{for} \quad k = 1, 2, \dots, K \tag{1.1}$$

and optimize the vector function

$$f(x^*) = [f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), ..., f_N(\boldsymbol{x})] \tag{1.2}$$

where $\boldsymbol{x} = [x_1, x_2, ..., x_I]$ is the vector of decision variables.

2. Multistage evolutionary algorithm based method

2.1. Description of the method

The decision-making problem is fairly easy when two criteria are considered. This process becomes more difficult when more than two criteria should be considered and when the set of Pareto optimal solutions is large. Making up a decision on the basis of this set is a fairly difficult task. Thus, in this Section a new multistage optimization process is proposed. The outline of this process is as follows:

Step 1. Order the objective functions according to their significance for the design process.

Step 2. Set $n = 1$, where n is the considered stage of the optimization process.

Step 3. Find the Pareto set for the objective functions $f_n(\boldsymbol{x})$ and $f_{n+1}(\boldsymbol{x})$. Illustrate graphically this set in the space of objectives.

Step 4. Consider the function $f_n(\boldsymbol{x})$ as an additional constraint of the form $f_n(\boldsymbol{x}) < F_{nu}$ for minimized functions or $f_n(\boldsymbol{x}) > F_{nl}$ for maximized functions, where F_{nu} and F_{nl} are the upper and lower restrictions on the nth objective function given by the designer.

Step 5. Set $n = n + 1$, if $n < N - 1$, go to Step 3, otherwise go to Step 6.

Step 6. Check the obtained results, and if they are satisfied terminate the calculations, otherwise make a new order of objective functions and repeat the procedure from Step 2.

Verbally, this method can be described as follows. At all stages bicriteria optimization models are solved giving in each case Pareto optimal solutions, which can be graphically illustrated in the space of objectives. At each stage from the obtained set of the Pareto optimal solutions, the designer decides how to change one of the two objective functions into a constraint and which new criterion can be considered in the next stage. In particular, all decisions of the designer consist in choosing the most preferable ranges of objectives. Note that the results of the optimization process depend on the ordering of the objective functions, i.e., which one is considered as the first objective function, the second and so on. To solve bicriterion optimization models in each step, an evolutionary algorithm based method called the constraint tournament selection method (Osyczka and Krenich, 2000) is applied.

2.2. An example of multistage method for the robot gripper mechanism

Let us consider an example of a mechanism of the commercial robot gripper. For this gripper, the kinematical scheme is presented in Fig. 1.
The outline of the model can be described as follows:

- Vector of decision variables $\boldsymbol{x} = [a, b, c, e, f, l, \delta]^\top$, where a, b, c, e, f, l are linear dimensions of the gripper and δ is the angle between element b and c.
- Objective functions

$f_1(\boldsymbol{x})$	–	difference between maximum and minimum griping forces
$f_2(\boldsymbol{x})$	–	force transmission ratio between the gripper actuator and the gripper ends

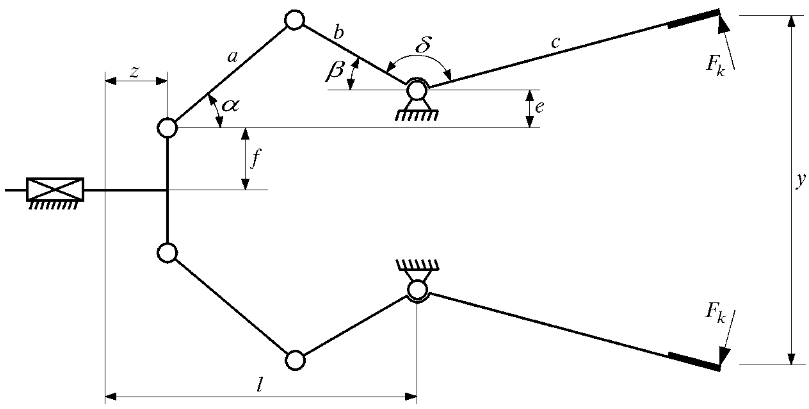

Fig. 1. Scheme of a robot gripper mechanism

$f_3(\boldsymbol{x})$	–	shift transmission ratio between the gripper actuator and the gripper ends
$f_4(\boldsymbol{x})$	–	length of all the elements of the gripper
$f_5(\boldsymbol{x})$	–	maximal force in the joints
$f_6(\boldsymbol{x})$	–	mechanical losses in the gripper mechanism

Note that the functions $f_1(\boldsymbol{x})$, $f_4(\boldsymbol{x})$, $f_5(\boldsymbol{x})$, $f_6(\boldsymbol{x})$ are to be minimized, whereas the function $f_2(\boldsymbol{x})$ and $f_3(\boldsymbol{x})$ are to be maximized.

- Constraints

  There are 11 constraints which refer to geometrical constraints, shear stress constraints, and a minimum gripping force constraint. The full description of the optimization model is given in Krenich (2002).

Results of the optimization process

For the model given above the optimization process was considered as a continuous programming problem. The data for the optimization process were as follows:

— Data for the evolutionary algorithm

population size = 400 number of generations = 400

crossover rate = 0.6 mutation rate = 0.08.

— Lower and upper bounds for the decision variables

$$10 \leqslant a \leqslant 250 \qquad 10 \leqslant b \leqslant 250$$

$$10 \leqslant c \leqslant 300 \qquad 0 \leqslant e \leqslant 250$$

$$0 \leqslant f \leqslant 250 \qquad 0 \leqslant \delta \leqslant \pi$$

Optimization procedure

Consider the problem of optimum design of the robot gripper presented in Fig. 1. Assuming that the objective functions are ordered as it is given in the optimization model, the stages of the optimization process are as follows:

Stage 1

In this stage, the following two criteria are considered:

- $f_1(\boldsymbol{x})$ – function which describes the difference between the maximum and minimum griping forces for the assumed range of displacement of the gripper ends

$$f_1(\boldsymbol{x}) = \max_z F_k(\boldsymbol{x}, z) - \min_z F_k(\boldsymbol{x}, z) \tag{2.1}$$

- $f_2(\boldsymbol{x})$ – function which describes the force transmission ratio between the gripper actuator and the gripper ends

$$f_2(\boldsymbol{x}) = \frac{\min\limits_z F_k(\boldsymbol{x}, z)}{P} \tag{2.2}$$

The constraints are given by the equations from the basic model. After solving the above bicriterion optimization problem, the generated by an evolutionary algorithm set of Pareto optimal solutions is as presented in Fig. 2.

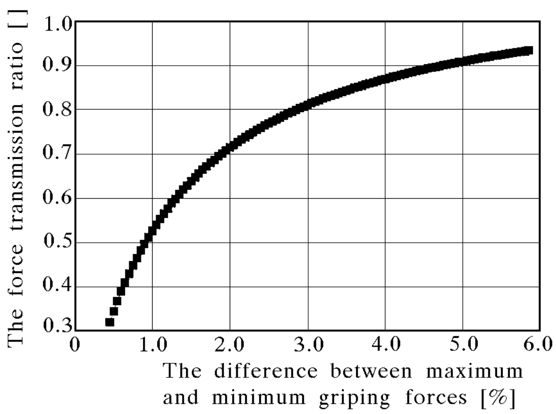

Fig. 2. Set of Pareto optimal solutions from stage 1

Stage 2

At this stage, the designer decides on which level the first objective function will be treated as the constraint. Assuming that the difference between the maximum and minimum griping forces should not be less than $F_{1g} = 4$, the following constraint is added to the existing set

$$g_{12}(\boldsymbol{x}) = F_{1g} - \Big[\max_z F_k(\boldsymbol{x}, z) - \min_z F_k(\boldsymbol{x}, z)\Big] \geqslant 0 \qquad (2.3)$$

where F_{1g} is the assuming upper limit on the first objective function.

The remaining constraints are as considered at Stage 1. A new objective function is introduced into the optimization model, and now the bicriterion optimization problem is as follows:

- $f_1(\boldsymbol{x})$ – function which describes the shift transmission ratio between the gripper actuator and the gripper ends

$$f_1(\boldsymbol{x}) = \left| \frac{y(\boldsymbol{x}, Z_{\max}) - y(\boldsymbol{x}, Z_{\min})}{Z_{\max} - Z_{\min}} \right| \qquad (2.4)$$

- $f_2(\boldsymbol{x})$ – function which describes the force transmission ratio between the gripper actuator and the gripper ends

$$f_2(\boldsymbol{x}) = \frac{\min_z F_k(\boldsymbol{x}, z)}{P} \qquad (2.5)$$

The results of optimization process for the above model are presented in Fig. 3.

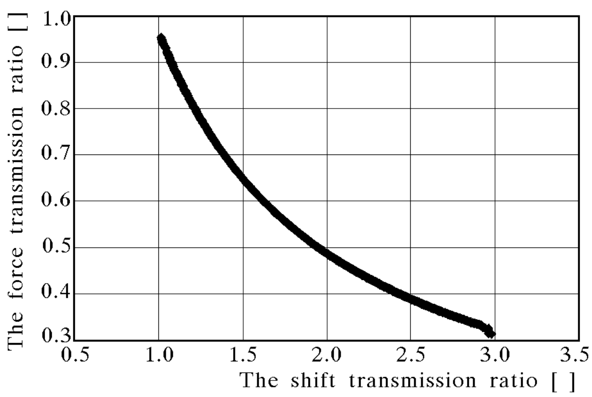

Fig. 3. Set of Pareto optimal solutions from Stage 2

Stage 3

At this stage of the optimization process it is assumed that the force transmission ratio will be considered as a constraint with the assumed lower bound $F_{2d} = 0.5$. Thus, an additional constraint is added to the model. This constraint has the form

$$g_{13}(\boldsymbol{x}) = \frac{\min\limits_{z} F_k(\boldsymbol{x}, z)}{P} - F_{2d} \geqslant 0 \tag{2.6}$$

The remaining constraints are as considered at Stage 2.

The objective functions at this stage are:

- $f_1(\boldsymbol{x})$ – function which describes the shift transmission ratio

$$f_1(\boldsymbol{x}) = \left| \frac{y(\boldsymbol{x}, Z_{\max}) - y(\boldsymbol{x}, Z_{\min})}{Z_{\max} - Z_{\min}} \right| \tag{2.7}$$

- $f_2(\boldsymbol{x})$ – function which describes the length of all elements of the gripper

$$f_2(\boldsymbol{x}) = a + b + c + e + f + l \tag{2.8}$$

The results of the optimization process for this stage of calculations are presented in Fig. 4.

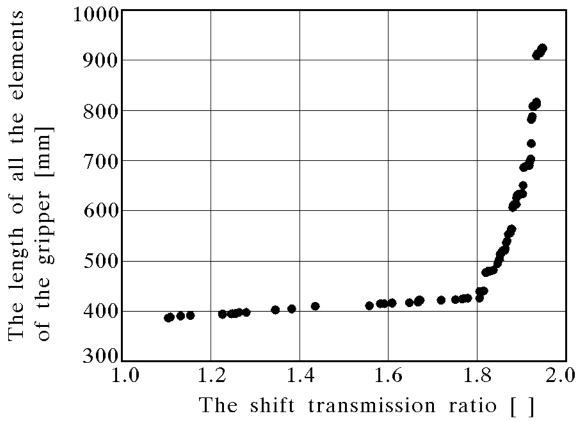

Fig. 4. Set of Pareto optimal solutions from Stage 3

Stage 4

At this stage of the optimization process it is assumed that the shift transmission ratio will be considered as a constraint with the assumed lower bound $F_{3d} = 1.5$. Thus, an additional constraint is added to the model. This constraint has the form

$$g_{14}(\boldsymbol{x}) = \left|\frac{y(\boldsymbol{x}, Z_{\max}) - y(\boldsymbol{x}, Z_{\min})}{Z_{\max} - Z_{\min}}\right| - F_{3d} \geqslant 0 \tag{2.9}$$

The remaining constraints are as considered at Stage 3.

The objective functions at this stage are:

- $f_1(\boldsymbol{x})$ – function which describes the length of all elements of the gripper
$$f_1(\boldsymbol{x}) = a + b + c + e + f + l \tag{2.10}$$
- $f_2(\boldsymbol{x})$ – function which describes the maximum force in the joints
$$f_2(\boldsymbol{x}) = \max_j \{R_j\} \tag{2.11}$$

The results of the optimization process for this stage of calculations are presented in Fig. 5.

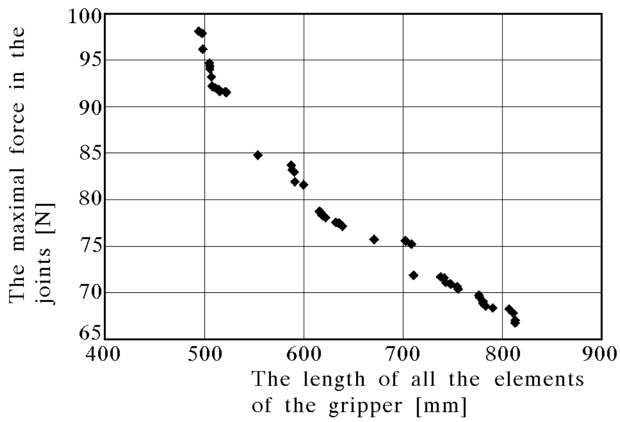

Fig. 5. Set of Pareto optimal solutions from Stage 4

Stage 5

At this stage of the optimization process it is assumed that the length of all elements of the gripper will be considered as a constraint with the assumed upper bound $F_{4d} = 600$. Thus, an additional constraint is added to the model. This constraint has the form

$$g_{15}(\boldsymbol{x}) = F_{4g} - (a + b + c + e + f + l) \geqslant 0 \tag{2.12}$$

The remaining constraints are as considered at Stage 4.

The objective functions at this stage are:

- $f_1(\boldsymbol{x})$ – function which describes the maximum force in the joints

$$f_1(\boldsymbol{x}) = f_5(\boldsymbol{x}) = \max_j \{R_j\} \tag{2.13}$$

- $f_2(\boldsymbol{x})$ – function which describes the efficiency of the gripper mechanism

$$f_2(\boldsymbol{x}) = \sum_{z=0}^{Z_{\max}} [F_k^{BT}(\boldsymbol{x}, z) - F_k^T(\boldsymbol{x}, z)] \tag{2.14}$$

The results of the optimization process, i.e. the set of Pareto optimal solutions, obtained at this stage of calculations are presented in Fig. 6.

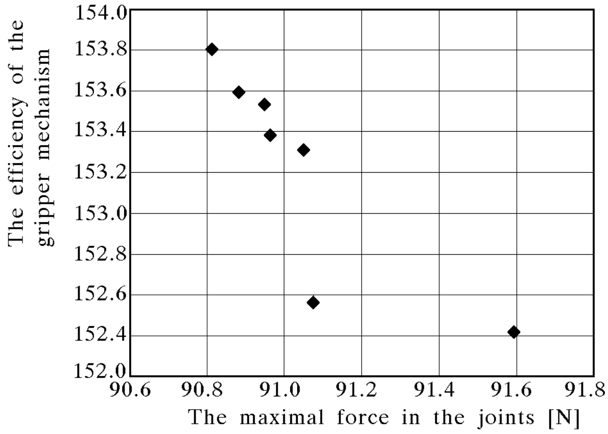

Fig. 6. Set of Pareto optimal solutions from Stage 5

Two solutions from this set are presented in Table 1. As it is for most multicriteria optimization problems, the final decision regarding the choice of the solution belongs to the designer. If none of the solutions from the last stage of calculations satisfies the designer, he may repeat calculation from any stage assuming another values of the limits for the objective function.

Table 1. Two solutions from the Pareto set obtained at 5th stage of the optimization process

It.	Objective functions $f(\boldsymbol{x})$	Decision variables						
		a	b	c	e	f	l	δ
1	[3.01,0.50,1.67,495.61,90.83,153.85]	134.5	89.12	100.1	0.05	1.04	170.80	1.57
2	[3.05,0.50,1.67,499.81,91.59,152.40]	135.0	90.53	102.2	0.00	1.28	170.57	1.80

where $f(\boldsymbol{x}) = [f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), f3(\boldsymbol{x}), f4(\boldsymbol{x}), f5(\boldsymbol{x}), f6(\boldsymbol{x})]$

3. The preference vector method

3.1. Description of the method

The method consists in reducing the set of Pareto solutions to only those which are in the space restricted by the preference vector given by the decision maker. The main steps of the method are as follows:

Step 1. Find the ideal vector for the given multicriteria optimization problem, in other words find the vector $\boldsymbol{f}^0(\boldsymbol{x}) = [f_1^0(\boldsymbol{x}), f_2^0(\boldsymbol{x}), ..., f_N^0(\boldsymbol{x})]^\top$ for which the nth element of this vector defines the separately attainable minimum of the ith objective function which can be evaluated as follows

$$f_n^0(\boldsymbol{x}) = \min_{\boldsymbol{x} \in X} f_n(\boldsymbol{x}) \tag{3.1}$$

This vector is found after running N times an evolutionary single-criterion optimization method.

Step 2. Set $p = 1$, where p is the number of iteration.

Step 3. Give a preference vector $\boldsymbol{f}^p(\boldsymbol{x}) = [f_1^p(\boldsymbol{x}), f_2^p(\boldsymbol{x}), ..., f_N^p(\boldsymbol{x})]^\top$ according to the designer's preferences.

Step 4. Introduce new constraints to the optimization model

– for minimized functions

$$g_{K+n}(\boldsymbol{x}) = f_n^p(\boldsymbol{x}) - f_n(\boldsymbol{x}) \geqslant 0 \tag{3.2}$$

or

– for maximized functions

$$g_{K+n}(\boldsymbol{x}) = f_i(\boldsymbol{x}) - f_n^p(\boldsymbol{x}) \geqslant 0 \tag{3.3}$$

where $n = 1, 2, ..., N$.

These constraints limit the space of search to the solutions which are within the space defined by the assumed preference vector.

Step 5. For a new model generate a set of Pareto optimal solutions using an evolutionary multicriteria optimization method.

Step 6. If it is possible to make the final decision regarding the choice of the preferred solution on the basis of the obtained set of Pareto optimal solutions, stop the calculations. Otherwise set $p = p+1$ and go to Step 4.

Each new preference vector should limit the set of Pareto solutions and move it closer to the user's preference solution. Note that the search area is limited by the ideal vector and by the front of Pareto optimal solutions. The preference vector reduces the search area and, at the same time, reduces the computing time. A more restricted area of search leads to shorter computing time for generating the set of Pareto solutions. The area of search restricted too much may lead to a model in which there is no feasible domain.

3.2. An example of multiple clutch break design

Let us consider an example of a multiple clutch brake, the configuration of which is shown in Fig. 7. The optimization model is as follows:

- Decision variables

  The vector of the decision variables is $\boldsymbol{x} = [R_i, R_o, A, F, Z]^\top$, where

  R_i – inner radius [mm]
  R_o – outer radius [mm]
  A – thickness of the discs [mm]
  F – actuating force [N]
  Z – number of friction surfaces.

- Objective functions

  The vector of objective functions is $f(\boldsymbol{x}) = [f_1(\boldsymbol{x}), f_2(\boldsymbol{x})]^\top$, where

  $f_1(\boldsymbol{x})$ – mass of the brake [kg]
  $f_2(\boldsymbol{x})$ – stopping time [s].

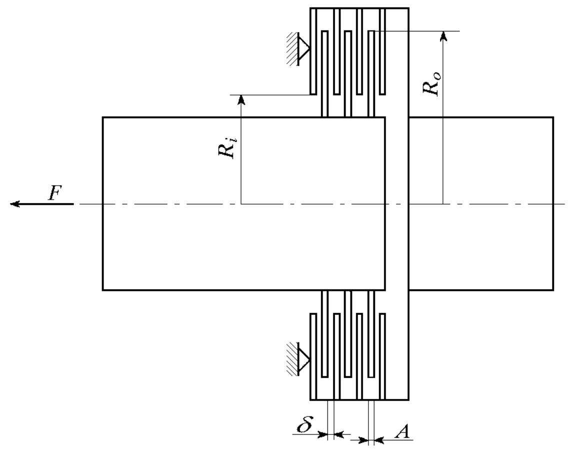

Fig. 7. Scheme of the multiple clutch brake

Both objective functions are to be minimized. The decision variables are under 16 constraints which are:

a) shear stress constraint
b) temperature constraint
c) relative speed of the slip-stick constraint
d) geometrical constraints.

The full description of the optimization model is given in Osyczka, 2002.

Results of the optimization process

The given optimization problem is considered as a mixed continuous and integer programming problem. The data for the optimization process were as follows:

— Data for the evolutionary algorithm for single and multicriteria problems
population size = 400 number of generations = 400
crossover rate = 0.6 mutation rate = 0.08.

— Lower and upper bounds on the first four decision variables

$$35.0 \leqslant x_1 \leqslant 80.0 \qquad 60.0 \leqslant x_2 \leqslant 110.0$$

$$1.5 \leqslant x_3 \leqslant 10.0 \qquad 600.0 \leqslant x_4 \leqslant 1000$$

The set of integer values for the fifth decision variable

$$X_5 = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

- Optimization procedure

  To solve both single and multicriteria optimization problems, the constraint tournament selection method is used (Osyczka and Krenich, 2000). Firstly, the separately attainable minima of the objective functions are found. These minima are calculated using the single criterion optimization method, and they define the ideal vector which is as follows $\boldsymbol{f}^0(\boldsymbol{x}) = [0.37, 3.36]$. Two case studies for two different preference vectors are considered:

  - The first preference vector $\boldsymbol{f}^{p^I}(\boldsymbol{x}) = [0.6, 9.0]^\top$
  - The second preference vector $\boldsymbol{f}^{p^{II}}(\boldsymbol{x}) = [1.25, 5.0]^\top$

For the first case, the following constraints are introduced to the model

$$g_{17}(\boldsymbol{x}) = 0.6 - f_1(\boldsymbol{x}) \geqslant 0$$
$$g_{18}(\boldsymbol{x}) = 9.0 - f_2(\boldsymbol{x}) \geqslant 0 \tag{3.4}$$

For the second case, the constraints are as follows

$$g_{17}(\boldsymbol{x}) = 1.25 - f_1(\boldsymbol{x}) \geqslant 0$$
$$g_{18}(\boldsymbol{x}) = 5.0 - f_2(\boldsymbol{x}) \geqslant 0 \tag{3.5}$$

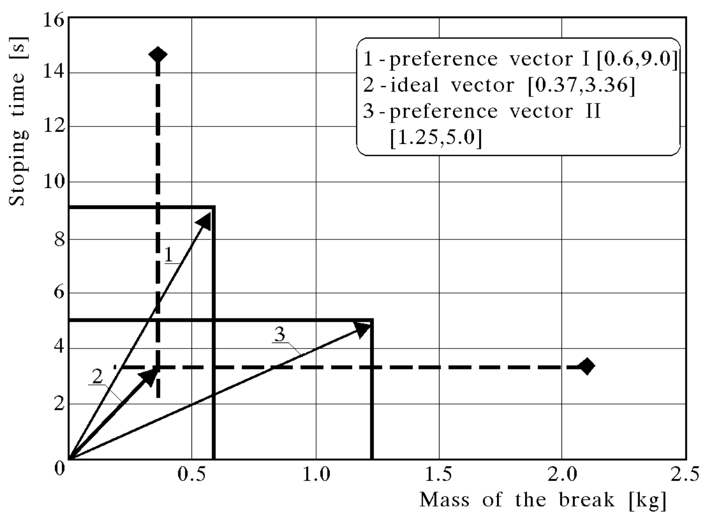

Fig. 8. The ideal and preference vectors in the objective space

The ideal vector and both preference vectors are illustrated graphically in Fig. 8. Finally, comparison between the traditional optimization method and the proposed method is made. For the traditional optimization method, the full set of Pareto solutions is generated. For the proposed method, the set of Pareto optimal solutions lying only within the assumed ideal vector is generated. The results of calculations are shown in Table 2 and illustrated graphically in Fig. 9. It is clearly seen in Fig. 9 that using the proposed method the obtained sets of Pareto solutions lie on the Pareto front. In both case studies, the calculation time needed for obtaining these sets is significantly lower.

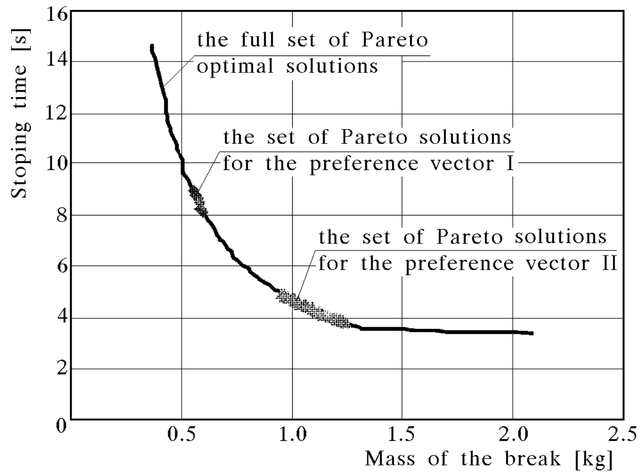

Fig. 9. The sets of Pareto optimal solutions for the multiple clutch brake

Table 2. Results of the optimization process of the multiple clutch brake

Approach	Results	Computing time [s]
Common procedure of generating the full set of Pareto solutions	No. of solutions 1678	342
Separately attainable minima of the functions $f_1(\boldsymbol{x})$ and $f_2(\boldsymbol{x})$	Ideal vector [0.37,3.36]	14+15=29
The set of Pareto solutions for the preference vector $\boldsymbol{f}^{p^I}(\boldsymbol{x})$	No. of solutions 16	39
The set of Pareto solutions for the preference vector $\boldsymbol{f}^{p^{II}}(\boldsymbol{x})$	No. of solutions 60	126

4. Evolutionary algorithm method with selecting the representative subset of Pareto solutions

4.1. Description of the method

The idea of the method consists in reducing the set of Pareto optimal solutions using the indiscernibility interval method after running a certain number of generations. This process can be called a filtration process in which less important Pareto optimal solutions are removed from the existing set.

The steps of the method are as follows:

Step 1. Set $t = 1$, where t is the number of the currently run generation.

Step 2. Generate the set of Pareto optimal solutions using any evolutionary algorithm method.

Step 3. Is the criterion for filtration the set of Pareto solutions satisfied? If yes, select a representative subset of Pareto solutions using the indiscernibility interval method and go to Step 4. Otherwise, go straight to Step 4.

Step 4. Set $t = t + 1$ and if $t \leqslant T$, where T is the assumed number of generations, go to Step 2. Otherwise, terminate the calculations.

Note that if in Step 3 the answer is *yes*, we start the process, which can be called the filtrating process since we filtrate and retain in the Pareto set only these solutions which are not close to each other in the space of objectives. Note also that in Step 3 the term *criterion for filtration* is introduced. Three types of criteria can be used here:

Type 1. The number of solutions in the Pareto set exceeds the assumed number P, for example 100.

Type 2. The number of solutions in the Pareto set is assumed as P. The first filtration is made if the number of solutions in the Pareto set exceeds this number. The following filtration is made when P new Pareto optimal solutions are added to the set.

Type 3. The filtration is made after running the assumed number of generations P, and in this case, the number of solutions in the Pareto set is not controlled.

These three types of criteria may produce slightly different results, but generally all of them reduce the computation time significantly. The choice of the criterion depends on the problem to be solved. Using these three criteria, the choice of P should be made with great care. If P is too small, the number of Pareto solutions might not be representative for the problem and the evolutionary algorithm may not reach the real Pareto frontier. If P is too large, we lose the effect of reducing the calculation time. Also the choice of the indiscernibility interval u_i is very important. If u_i is too small, the number of rejected solutions is also too small and there is no effect in reducing the set of Pareto solutions, whereas too big value of u_i may make the subset of the obtained solutions too small to be representative.

4.2. An example of shaft design

Let us consider the problem of optimum design of a shaft, the scheme of which is presented in Fig. 10. The shaft is under loading as presented in Fig. 11.

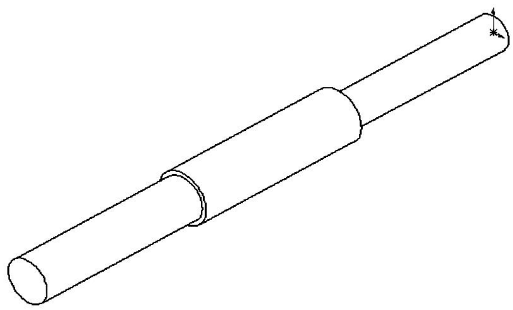

Fig. 10. Scheme of the shaft

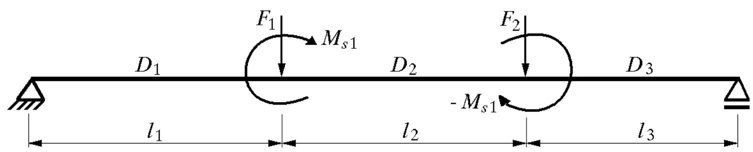

Fig. 11. Scheme of bending and torsion load of the shaft

The multicriteria optimization problem is formulated as follows:

- The vector of decision variables is

$$\boldsymbol{x} = [l_1, l_2, l_3, D_1, D_2, D_3]^\top \tag{4.1}$$

- The objective functions are
  - volume of the shaft

$$f_1(\boldsymbol{x}) = l_1\pi\Big(\frac{D_1}{2}\Big)^2 + l_2\pi\Big(\frac{D_2}{2}\Big)^2 + l_3\pi\Big(\frac{D_3}{2}\Big)^2 \tag{4.2}$$

– the first form of eigenfrequency along the direction of the bending load, generated by Finite Element Method (FEM), system ANSYS, $f_2(\boldsymbol{x})$.

The function $f_1(\boldsymbol{x})$ is to be minimized and the second function $f_2(\boldsymbol{x})$ is to be maximized.

There are three stress constraints built on the basis of average stress values according to Huber-Mises-Hencky's hypothesis given by formulas (4.3) and generated by FEM system ANSYS. For three different cross-sections, the relevant equations are as follows

$$\delta_{red} = \sqrt{\frac{1}{2}\Big[(\delta_x - \delta_y)^2 + (\delta_y - \delta_z)^2 + (\delta_z - \delta_x)^2 + 3(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\Big]}$$
$$\delta = \frac{M_b R}{J} \qquad \tau = \frac{M_s R}{J_o} \tag{4.3}$$

where: δ_{red} – reduced stress, δ – bending stress, τ – torsional stress, M_b – bending moment, M_s – torque, J – moment of interia, Jo – polar moment of interia, R – radius of cross-section.

Results of the optimization process

For the model given above, the optimization process was considered as a continuous programming problem. The data for the optimization process were as follows:

— Lower and upper bounds for the decision variables

$$200 \leqslant l_1 \leqslant 260 \qquad 200 \leqslant l_2 \leqslant 260$$
$$200 \leqslant l_3 \leqslant 260 \qquad 18 \leqslant D_1 \leqslant 30$$
$$20 \leqslant D_2 \leqslant 32 \qquad 18 \leqslant D_3 \leqslant 30$$

— Loading values

$$F_1 = 1300 \text{ N} \qquad F_2 = 2500 \text{ N}$$
$$M_{s1} = 50 \text{ Nm} \qquad Re = 300 \text{ MPa}$$

yield point of material St 30

— Data for the evolutionary algorithm

population size = 400 number of generations = 400
crossover rate = 0.6 mutation rate = 0.08.
penalty rate $r = 10^5$

The experiments were carried out using the indiscernibility interval $u_i = 5\%$ for $i = 1, 2$ and for $P = 100$. The results of experiments are shown in Fig. 12, in which Pareto frontiers for solutions without filtration and with filtration are compared. The solutions depicted by black points are obtained while running the evolutionary algorithm without the filtration process, whereas those depicted by almost white points are obtained while running the evolutionary algorithm with the assumed type of filtration.

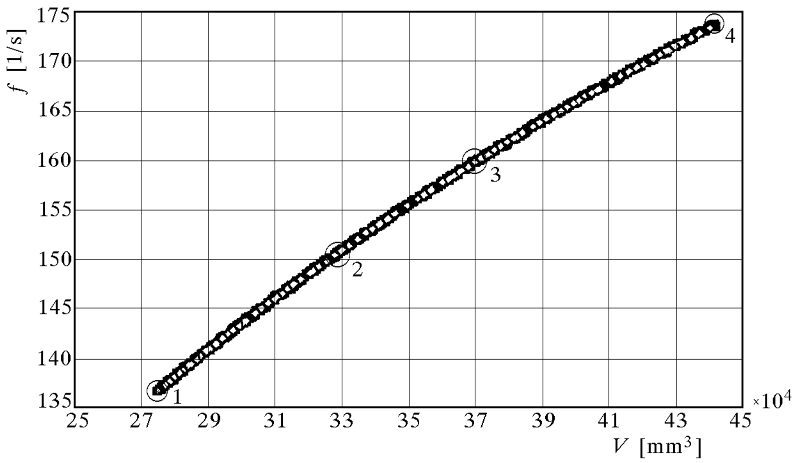

Fig. 12. Sets of Pareto optimal solutions for the shaft design problem

In Table 3, the number of generated Pareto optimal solutions obtained during each experiment and the computing time for each experiment are presented. From these experiments, it is clear that using the indiscernibility interval method with the evolutionary algorithm, the computation time may be reduced several times and still almost the same Pareto frontier is obtained, which in some parts is even better than the one obtained by the ordinary evolutionary algorithm.

Table 3. Comparison of the results for the shaft design problem

Method	Number of Pareto solution	Time [h]
Without filtration	1013	~ 42
Filtration Type 1	166	~ 6

In Table 4, the dimensions of the shaft for four Pareto optimal solutions from the generated set are presented. The numbers of solutions are as denoted in Fig. 12.

Table 4. Four Pareto optimal solutions from the generated set

No.	$f_1(\boldsymbol{x})$ [mm^3]	$f_2(\boldsymbol{x})$ [1/s]	l_1 [mm]	l_2 [mm]	l_3 [mm]	D_1 [mm]	D_2 [mm]	D_3 [mm]
1	275052.5	136.7796	200.0	200.5	200.3	22.68	25.30	24.37
2	325083.3	149.7082	200.0	200.1	200.0	25.52	27.95	25.23
3	375254.6	160.6884	200.0	200.0	200.1	27.11	29.97	27.48
4	441487.5	173.6689	200.0	200.1	200.1	29.97	31.80	30.00

5. Conclusions

In the paper, new multicriteria design optimization methods based on evolutionary algorithms are presented. The main aims of these methods is to reduce the computing time while running an evolutionary algorithm program and to facilitate the decision making process. This means that the methods make the process of seeking the preferred solution more effective with respect to both the computation time and the decision-making problem. The methods can be very useful for design optimization problems with computationally expensive functions. Examples presented in this paper show that the methods can be used to solve different design optimization problems.

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Pewne metody optymalizacji wielokryterialnej w projektowaniu technicznym przy wykorzystaniu algorytmów ewolucyjnych

Streszczenie

W artrykule przedstawiono nowe metody optymalizacji wielokryterialnej w projektowaniu technicznym. Metody te oparte są na algorytmach ewolucyjnych, a ich celem jest znaczne zwiększenie efektywności procesu generowania rozwiązań Pareto optymalnych. Najpierw zaprezentowano metodę wieloetapowego algorytmu ewolucyjnego. W metodzie tej na każdym etapie realizowany jest jedynie problem optymalizacji dwukryterialnej, po rozwiązaniu którego jedna z funkcji celu jest przekształcana do postaci ograniczenia. Proces ten jest powtarzany aż do momentu rozpatrzenia wszystkich funkcji celu. Następnie omówiono metodę wektora preferencji. W metodzie tej w pierwszym etapie algorytm ewolucyjny znajduje wektor idealny. Wektor ten dostarcza decydentowi informacji o możliwym zasięgu wszystkich funkcji celu. Na podstawie tej informacji decydent może oszacować wektor preferencji, wewnątrz którego spodziewa się znaleźć preferowane rozwiązanie. Dla tego wektora preferencji generowany jest za pomocą algorytmu ewolucyjnego zbiór rozwiązań Pareto optymalnych. Ostatnią z omawainych metod jest metoda redukcji zbioru rozwiązań Pareto optymalnych po przebiegu założonej liczby generacji realizowanych przez algorytm ewolucyjny. W celu pokazania sposobu działania omawianych metod, każda z nich została zilustrowana innym przykładem zadania optymalnego projektowania. Przykłady te wskazują, że zaproponowane metody mogą znacząco zredukować czas obliczeń komputerowych nie pogarszając wyników.

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