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NUMERICAL MODELLING OF HEAT TRANSFER IN SPHERICAL DOMAINS BY MEANS OF THE BEM USING DISCRETISATION IN TIME

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A combined variant of the BEM called in literature the BEM using discretization in time consists in an approximation of the time derivative appearing in Fourier's equation by an adequate differential quotient. The next steps of mathematical manipulations and also the numerical algorithm are similar to a typical boundary element approach. In the paper the method is applied to numerical computations concerning a non-steady heat diffusion in homogeneous and non-homogeneous spherical domains. In the final part of the paper the results of computations are presented.

Key words: heat transfer, boundary element method

1. Introduction

At first, the well known linear Fourier equation for 3D domain oriented in Cartesian co-ordinate system is considered

$$\boldsymbol{x} \in \Omega: \qquad \frac{\partial T(\boldsymbol{x},t)}{\partial t} = a\sum_{e=1}^{3}\frac{\partial^2 T(\boldsymbol{x},t)}{\partial x_e^2} = a\nabla^2 T(\boldsymbol{x},t) \tag{1.1}$$

where $\boldsymbol{x} = (x_1, x_2, x_3)$, $a = \lambda/c$ is the heat diffusion coefficient (λ is the thermal conductivity, while c is the specific heat per unit volume), T, t denote temperature and time, respectively. On the outer surface Γ of the system boundary conditions are given, the initial condition is also known.

In this place a time grid with a constant step Δt must be introduced

$$0 = t^0 < t^1 < t^2 < ... < t^{f-1} < t^f < ... < t^F < \infty \tag{1.2}$$

Considering the transition $t^{f-1} \rightarrow t^f$ one transforms equation (1.1) to the form (Curan et al., 1980; Sichert, 1989)

$$\frac{T(\boldsymbol{x}, t^f) - T(\boldsymbol{x}, t^{f-1})}{\varDelta t} = a\nabla^2 T(\boldsymbol{x}, t^f) \tag{1.3}$$

or

$$\nabla^2 T(\boldsymbol{x}, t^f) - \frac{1}{a\varDelta t}T(\boldsymbol{x}, t^f) + \frac{1}{a\varDelta t}T(\boldsymbol{x}, t^{f-1}) = 0 \tag{1.4}$$

Using the weighted residual criterion one obtains (Brebbia et al., 1984; Majchrzak and Mochnacki, 1995)

$$\int\limits_{\varOmega} \Big[\nabla^2 T(\boldsymbol{x}, t^f) - \frac{1}{a\varDelta t}T(\boldsymbol{x}, t^f) + \frac{1}{a\varDelta t}T(\boldsymbol{x}, t^{f-1})\Big] U^*(d)\; d\varOmega = 0 \tag{1.5}$$

where U^* is a fundamental solution, in particular to the considered task it is the following function (Brebbia et al., 1984)

$$U^*(d) = \frac{1}{4\pi d}\exp\Big(-\frac{d}{\sqrt{a\varDelta t}}\Big) \qquad\qquad d = \sqrt{\sum_{e=1}^{3}(x_e - \xi_e)^2} \tag{1.6}$$

where $\boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3)$ is a point at which the concentrated heat source is applied (Brebbia et al., 1984).

The function U^* fulfills the equation

$$\nabla^2 U^*(d) - \frac{1}{a\varDelta t}U^*(d) = -\delta(\boldsymbol{\xi}, \boldsymbol{x}) \tag{1.7}$$

where $\delta(\boldsymbol{\xi}, \boldsymbol{x})$ is the Dirac function.

Using the 2nd Green formula and property (1.7) one obtains the WRM criterion in the form

$$\begin{aligned} T(\boldsymbol{\xi}, t^f) - \frac{1}{\lambda}\int\limits_{\varGamma} U^*(d) q(\boldsymbol{x}, t^f)\; d\varGamma &= \\ = \frac{1}{\lambda}\int\limits_{\varGamma} Q^*(d) T(\boldsymbol{x}, t^f)\; d\varGamma &+ \frac{1}{a\varDelta t}\int\limits_{\varOmega} U^*(d) T(\boldsymbol{x}, t^{f-1})\; d\varOmega \end{aligned} \tag{1.8}$$

where $q = -\lambda \partial T/\partial n$, $Q^* = -\lambda \partial U^*/\partial n$.

2. Homogeneous spherical domain

According to the idea presented by Brebbia et al. (1984) (the basic variant of the BEM and cylindrical object has been considered), equation (1.8) must be integrated assuming that Ω corresponds to the interior of the cylinder, while Γ to its surface. The similar approach has been proposed by Bokota (1989) in relation to the spherical domain.

In the paper this concept is applied in order to obtain the boundary equation in the case of BEM using discretization in time.

The spherical co-ordinate system should be introduced

$$x_1 = r\cos\varphi\sin\theta \qquad x_2 = r\sin\varphi\sin\theta \qquad x_3 = r\cos\theta \tag{2.1}$$

Additionally, we assume the position of Cartesian system for which $\boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3) = (0, 0, \xi)$ and then that the distance between this point and the point considered $\boldsymbol{x}$ (see Fig. 1) is equal to

$$d = \sqrt{r^2 + \xi^2 - 2r\xi\cos\theta} \tag{2.2}$$

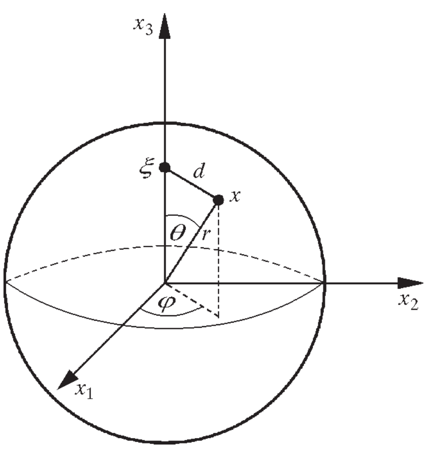

Fig. 1. Spherical co-ordinate system

The surface and volume elements of the sphere can be expressed as follows

$$d\Gamma = R^2\sin\theta\; d\theta d\varphi \qquad d\Omega = r^2\sin\theta\; d\theta d\varphi dr \tag{2.3}$$

where R is the radius of the sphere.

Thus, equation (1.8) takes the form

$$\begin{aligned}
&T(\xi,t^f)+\frac{R^2}{\lambda}q(R,t^f)\int\limits_0^{2\pi}\int\limits_0^{\pi}U^*(d)\sin\theta\;d\theta d\varphi= \\
&=\frac{R^2}{\lambda}T(R,t^f)\int\limits_0^{2\pi}\int\limits_0^{\pi}Q^*(d)\sin\theta\;d\theta d\varphi+ \\
&+\frac{1}{a\Delta t}\int\limits_0^{R}r^2\int\limits_0^{2\pi}\int\limits_0^{\pi}U^*(d)\sin\theta\;d\theta d\varphi T(r,t^{f-1})\;dr
\end{aligned}\tag{2.4}$$

or

$$\begin{aligned}
&T(\xi,t^f)+\frac{R^2}{\lambda}T^*(\xi,R)q(R,t^f)= \\
&=\frac{R^2}{\lambda}q^*(\xi,R)T(R,t^f)+\frac{1}{a\Delta t}\int\limits_0^{R}r^2T^*(\xi,r)T(r,t^{f-1}\;dr
\end{aligned}\tag{2.5}$$

where

$$\begin{aligned}
T^*(\xi,r)&=\int\limits_0^{2\pi}\int\limits_0^{\pi}U^*(d)\sin\theta\;d\theta d\varphi= \\
&=\frac{1}{4\pi}\int\limits_0^{2\pi}\int\limits_0^{\pi}\frac{1}{\sqrt{r^2+\xi^2-2r\xi}}\exp\Big(-\sqrt{\frac{r^2+\xi^2-2r\xi}{a\Delta t}}\Big)\sin\theta\;d\theta d\varphi \\
q^*(\xi,r)&=-\lambda\frac{\partial T^*(\xi,r)}{\partial r}
\end{aligned}\tag{2.6}$$

After the integration one obtains

$$\begin{aligned}
T^*(\xi,r)&=\frac{\sqrt{a\Delta t}}{2r\xi}\Big[\exp\Big(-\frac{|r-\xi|}{a\Delta t}\Big)-\exp\Big(-\frac{|r+\xi|}{a\Delta t}\Big)\Big] \\
q^*(\xi,r)&=\frac{\lambda}{r}T^*(\xi,r)+\frac{\lambda}{2\xi r}\Big[\operatorname{sgn}(r-\xi)\exp\Big(-\frac{|r-\xi|}{a\Delta t}\Big)- \\
&-\operatorname{sgn}(r+\xi)\exp\Big(-\frac{|r+\xi|}{a\Delta t}\Big)\Big]
\end{aligned}\tag{2.7}$$

The boundary equation (for $\xi \to R$) is of the form (cf. equation (2.5))

$$
\begin{aligned}
&T(R,t^f) + \frac{R^2}{\lambda} T^*(R,R) q(R,t^f) = \\
&= \frac{R^2}{\lambda} q^*(R,R) T(R,t^f) + \frac{1}{a\Delta t} \int\limits_0^R r^2 T^*(R,r) T(r,t^{f-1})\, dr
\end{aligned}
\tag{2.8}
$$

The first stage of numerical computations consists in determination of the boundary heat flux (if the boundary temperature is given) or boundary temperature (if the boundary heat flux is given) – equation (2.9). In the second stage the temperatures at the set of internal points $\xi \in (0,R)$ for time the t^f can be found on the basis of equation

$$
\begin{aligned}
&T(\xi,t^f) = \frac{R^2}{\lambda} q^*(\xi,R) T(R,t^f) - \\
&- \frac{R^2}{\lambda} T^*(\xi,R) q(R,t^f) + \frac{1}{a\Delta t} \int\limits_0^R r^2 T^*(\xi,r) T(r,t^{f-1})\, dr
\end{aligned}
\tag{2.9}
$$

The solution obtained constitutes a pseudo-initial condition for the next loop of computations. It should be pointed out that the integral appearing in equations (2.8) and (2.9) can be found using numerical methods e.g. Gaussian quadratures. The integral over the first internal cell $(r \in [0, \Delta r])$ is a singular one, but in a numerical realisation it does not cause essential difficulties.

3. Non-homogeneous spherical domain

The non-steady temperature field in the domain considered is described by a system of equations

$$
R_{m-1} < r < R_m : \qquad \frac{\partial T_m(r,t)}{\partial t} = \frac{a_m}{r^2} \frac{\partial}{\partial r} \Big[r^2 \frac{\partial T_m(r,t)}{\partial r} \Big]
\tag{3.1}
$$

where $m = 1, 2, ..., M$.

For $r = R_m$, $m = 1, 2, ..., M-1$ the continuity conditions in the form

$$
r = R_m : \qquad \begin{cases} -\lambda_m \dfrac{\partial T_m(r,t)}{\partial r} = -\lambda_{m+1} \dfrac{\partial T_{m+1}(r,t)}{\partial r} \\ T_m(r,t) = T_{m+1}(r,t) \end{cases}
\tag{3.2}
$$

are given. For $r = R_0$ and $r = R_M$ the boundary temperatures or boundary heat fluxes are known. For the time $t = 0$ the initial temperatures are also given.

Equation (2.5) for the spherical shell $r \in (R_{m-1}, R_m)$ takes the form

$$\begin{aligned} & T_m(\xi, t^f) + \Big[\frac{r^2}{\lambda_m} T_m^*(\xi, R) q_m(r, t^f)\Big]_{R_{m-1}}^{R_m} = \\ & = \Big[\frac{r^2}{\lambda_m} q_m^*(\xi, r) T_m(r, t^f)\Big]_{R_{m-1}}^{R_m} + \frac{1}{a_m \Delta t} \int\limits_{R_{m-1}}^{R_m} r^2 T_m^*(\xi, r) T_m(r, t^{f-1})\, dr \end{aligned} \tag{3.3}$$

Equation (3.3) can be written as follows

$$\begin{aligned} & T_m(\xi, t^f) + q_m(\xi, R_m) q_m(R_m, t^f) - q_m(\xi, R_{m-1}) q_m(R_{m-1}, t^f) = \\ & = h_m(\xi, R_m) T_m(R_m, t^f) - h_m(\xi, R_{m-1}) T_m(R_{m-1}, t^f) = p_m(\xi) \end{aligned} \tag{3.4}$$

where

$$q_m(\xi, r) = \frac{r^2}{\lambda_m} T_m^*(\xi, r) \qquad\qquad h_m(\xi, r) = \frac{r^2}{\lambda_m} q_m^*(\xi, r) \tag{3.5}$$

while

$$p_m(\xi) = \frac{1}{a_m \Delta t} \int\limits_{R_{m-1}}^{R_m} r^2 T_m^*(\xi, r) T_m(r, t^{f-1})\, dr \tag{3.6}$$

For $\xi \to R_{m-1}^+$ and $\xi \to R_{m-1}^-$ one obtains a system of equations

$$\begin{aligned} & \begin{bmatrix} g_m(R_{m-1}^+, R_{m-1}) & g_m(R_{m-1}^+, R_m) \\ g_m(R_m^-, R_{m-1}) & g_m(R_m^-, R_m) \end{bmatrix} \begin{bmatrix} q_m(r_{m-1}, t^f) \\ q_m(R_m, t^f) \end{bmatrix} = \\ & = \begin{bmatrix} h_m(R_{m-1}^+, R_{m-1}) - 1 & h_m(R_{m-1}^+, R_m) \\ h_m(R_m^-, R_{m-1}) & h_m(R_m^-, R_m) - 1 \end{bmatrix} \begin{bmatrix} T_m(R_{m-1}, t^f) \\ T_m(R_m, t^f) \end{bmatrix} + \\ & + \begin{bmatrix} p_m(R_{m-1}) \\ p_m(R_m) \end{bmatrix} \end{aligned} \tag{3.7}$$

or

$$\begin{bmatrix} g_{11}^m & g_{12}^m \\ g_{21}^m & g_{22}^m \end{bmatrix} \begin{bmatrix} q_m(R_{m-1}, t^f) \\ q_m(R_m, t^f) \end{bmatrix} = \begin{bmatrix} h_{11}^m & h_{12}^m \\ h_{21}^m & h_{22}^m \end{bmatrix} \begin{bmatrix} T_m(R_{m-1}, t^f) \\ T_m(R_m, t^f) \end{bmatrix} + \begin{bmatrix} p_1^m \\ p_2^m \end{bmatrix} \tag{3.8}$$

The final system for multi-layers domain results from the coupling of equations (3.8) by the continuity conditions given for $r = R_m$, $m = 1, ..., M-1$.

For example, we consider the two-layer system $(M = 2)$, Figure 2, and we assume the boundary conditions for $r = R_0$ and $r = R_2$ in the form

$$\begin{aligned} r = R_0: &\quad q_1(r, t^f) = q_b \\ r = R_2: &\quad q_2(r, t^f) = \alpha[T_2(r, t^f) - T^\infty] \end{aligned} \tag{3.9}$$

where α is the heat transfer coefficient and T^∞ is the ambient temperature. The continuity condition for $r = R_1$ can be written in the form (cf. equation (3.2))

$$r = R_1: \qquad \begin{cases} q_1(r, t^f) = q_2(r, t^f) = q(R_1, t^f) \\ T_1(r, t^f) = T_2(r, t^f) = T(R_1, t^f) \end{cases} \tag{3.10}$$

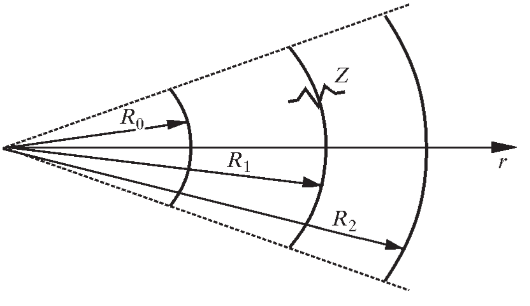

Fig. 2. Two-layer spherical domain

We put equations (3.10) to (3.8) for $m = 1, 2$. Additionally, taking into account the boundary conditions for $r = R_0$ and $r = R_2$ we have

$$\begin{bmatrix} g_{11}^1 & g_{12}^1 \\ g_{21}^1 & g_{22}^1 \end{bmatrix} \begin{bmatrix} q_b \\ q(R_1, t^f) \end{bmatrix} = \begin{bmatrix} h_{11}^1 & h_{12}^1 \\ h_{21}^1 & h_{22}^1 \end{bmatrix} \begin{bmatrix} T_1(R_0, t^f) \\ T(R_1, t^f) \end{bmatrix} + \begin{bmatrix} p_1^1 \\ p_2^1 \end{bmatrix}$$

$$\begin{bmatrix} g_{11}^2 & g_{12}^2 \\ g_{21}^2 & g_{22}^2 \end{bmatrix} \begin{bmatrix} q(R_1, t^f) \\ \alpha[T_2(R_2, t^f) - T^\infty] \end{bmatrix} = \begin{bmatrix} h_{11}^2 & h_{12}^2 \\ h_{21}^2 & h_{22}^2 \end{bmatrix} \begin{bmatrix} T(R_1, t^f) \\ T_2(R_2, t^f) \end{bmatrix} + \tag{3.11}$$

$$+ \begin{bmatrix} p_1^2 \\ p_2^2 \end{bmatrix}$$

Well-ordered systems (3.11) can be written in the form

$$
\begin{gathered}
\begin{bmatrix} -h_{11}^1 & -h_{12}^1 & g_{12}^1 \\ -h_{21}^1 & -h_{22}^1 & g_{22}^1 \end{bmatrix} \begin{bmatrix} T_1(R_0,t^f) \\ T(R_1,t^f) \\ q(R_1,t^f) \end{bmatrix} = \begin{bmatrix} -g_{11}^1 q_b + p_1^1 \\ -g_{21}^1 q_b + p_2^1 \end{bmatrix} \\
\begin{bmatrix} -h_{11}^2 & g_{11}^2 & \alpha g_{12}^2 - h_{12}^2 \\ -h_{21}^2 & g_{21}^2 & \alpha g_{22}^2 - h_{22}^2 \end{bmatrix} \begin{bmatrix} T(R_1,t^f) \\ q(R_1,t^f) \\ T_2(R_2,t^f) \end{bmatrix} = \begin{bmatrix} \alpha g_{12}^2 T^\infty + p_1^2 \\ \alpha g_{22}^2 T^\infty + p_2^2 \end{bmatrix}
\end{gathered}
\tag{3.12}
$$

Finally, one obtains the following system of equations

$$
\begin{bmatrix} -h_{11}^1 & -h_{12}^1 & g_{12}^1 & 0 \\ -h_{21}^1 & -h_{22}^1 & g_{22}^1 & 0 \\ 0 & -h_{11}^2 & g_{11}^2 & \alpha g_{12}^2 - h_{12}^2 \\ 0 & -h_{21}^2 & g_{21}^2 & \alpha g_{22}^2 - h_{22}^2 \end{bmatrix} \begin{bmatrix} T_1(R_0,t^f) \\ T(R_1,t^f) \\ q(R_1,t^f) \\ T_2(R_2,t^f) \end{bmatrix} = \begin{bmatrix} -g_{11}^1 q_b + p_1^1 \\ -g_{21}^1 q_b + p_2^1 \\ \alpha g_{12}^2 T^\infty + p_1^2 \\ \alpha g_{22}^2 T^\infty + p_2^2 \end{bmatrix}
$$

at the same time

$$
q_2(R_2,t^f) = \alpha[T_2(R_2,t^f) - T^\infty] \tag{3.13}
$$

The knowledge of boundary values for $r = R_0$, $r = R_1$ and $r = R_2$ allows one to find the internal temperatures at the time t^f using the equation (cf. formula (3.4))

$$
\begin{aligned}
T_m(\xi,t^f) &= g_m(\xi,R_{m-1})q_m(R_{m-1},t^f) - g_m(\xi,R_m)q_m(R_m,t^f) + \\
&+ h_m(\xi,R_m)T_m(R_m,t^f) - h_m(\xi,R_{m-1})T_m(R_{m-1},t^f) + p_m(\xi)
\end{aligned}
$$

The similar algorithm can be used in the case of non-zero thermal resistance Z between sub-domains considered. Then the continuity condition can be written in the form

$$
r = R_1: \qquad -\lambda_1 \frac{\partial T_1(r,t)}{\partial r} = \frac{T_1(r,t) - T_2(r,t)}{Z} = -\lambda_2 \frac{\partial T_2(r,t)}{\partial r} \tag{3.14}
$$

or

$$
r = R_1: \qquad \begin{cases} q_1(r,t^f) = q_2(r,t^f) = q(R_1,t^f) \\ T_2(r,t^f) = T_1(r,t^f) - Zq(R_1,t^f) \end{cases} \tag{3.15}
$$

It should be pointed out that for $Z = 0$ the last condition takes form (3.10).

The resolving system for the problem discussed can be written as follows

$$\begin{bmatrix} -h^1_{11} & -h^1_{12} & g^1_{12} & 0 \\ -h^1_{21} & -h^1_{22} & g^1_{22} & 0 \\ 0 & -h^2_{11} & g^2_{11}+Zh^2_{11} & \alpha g^2_{12}-h^2_{12} \\ 0 & -h^2_{21} & g^2_{21}+Zh^2_{21} & \alpha g^2_{22}-h^2_{22} \end{bmatrix} \begin{bmatrix} T_1(R_0,t^f) \\ T(R_1,t^f) \\ q(R_1,t^f) \\ T_2(R_2,t^f) \end{bmatrix} = \begin{bmatrix} -g^1_{11}q_b+p^1_1 \\ -g^1_{21}q_b+p^1_2 \\ \alpha g^2_{12}T^\infty+p^2_1 \\ \alpha g^2_{22}T^\infty+p^2_2 \end{bmatrix} \tag{3.16}$$

at the same time

$$T_2(R_1,t^f)=T_1(R_1,t^f)-Zq(R_1,t^f) \tag{3.17}$$

As previously, the internal temperatures in successive layers can be found using equation (3.15).

4. Examplary computations

The first example (Szopa, 1999) concerns a boundary initial problem for which a constant boundary temperature is known $T(R,t)=T_b$, while for $t=0: T(r,0)=0$. The problem can be solved in the exact way (Kącki, 1992). Test computations show, that a very good accuracy of numerical solutions can be obtained in a wide range of time steps, and they are practically the same as the exact result for the dimensionless time interval $\varDelta F_0 \in [0.001, 0.009]$.

In Figures 3 and 4 the heating curves at selected points ($r=0.05R$, $0.5R$, $0.75R$, $0.95R$) are shown. The first solution (Fig. 3) was obtained for $R=0.1\,\mathrm{m}$, $\lambda=35\,\mathrm{W/(mK)}$, $c=4.875\cdot 10^6\ \mathrm{J/(m^3K)}$, $T_b=100^\circ\,\mathrm{C}$, $\varDelta t=4.17\,\mathrm{s}$ ($\varDelta F_0=0.003$) and $\varDelta t=8.35\,\mathrm{s}$ ($\varDelta F_0=0.006$), while the second solution (Fig. 4) was obtained for $R=0.2\,\mathrm{m}$, $\lambda=1\,\mathrm{W/(mK)}$, $c=1.75\cdot 10^6\ \mathrm{J/(m^3K)}$, $T_b=100^\circ\,\mathrm{C}$, $\varDelta t=210\,\mathrm{s}$ ($\varDelta F_0=0.003$) and $\varDelta t=420\,\mathrm{s}$ ($\varDelta F_0=0.006$). In Figures 3 and 4 the exact solution is also marked.

The very good accuracy of numerical solution was also obtained in the case of the Robin boundary condition: $q(R,t)=\alpha[T(R,t)-T^\infty]$, where α is the heat transfer coefficient and T^∞ is the ambient temperature.

In Figure 5 the numerical and exact solution (symbols) for $R=0.2$, $\lambda=35\,\mathrm{W/(mK)}$, $c=4.875\cdot 10^6\ \mathrm{J/(m^3K)}$, $\alpha=350\,\mathrm{W/(m^2K)}$ (Biot's number Bi=1), $T^\infty=0$, initial temperature $T(r,0)=100^\circ\,\mathrm{C}$, $\varDelta F_0=0.002$, 0.003, 0.006, 0.01. The cooling curves correspond to points $r=0.05R$, $0.5R$, $0.75R$, $0.95R$.

The next examples concern non-homogeneous domains. We consider a sphere made from cast iron ($R_1=0.05\,\mathrm{m}$) which is spread within a steel

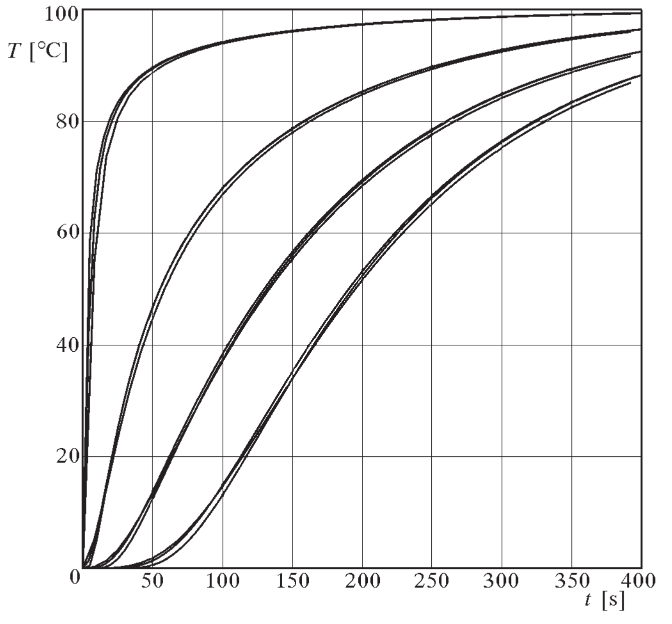

Fig. 3. Heating curves (example 1)

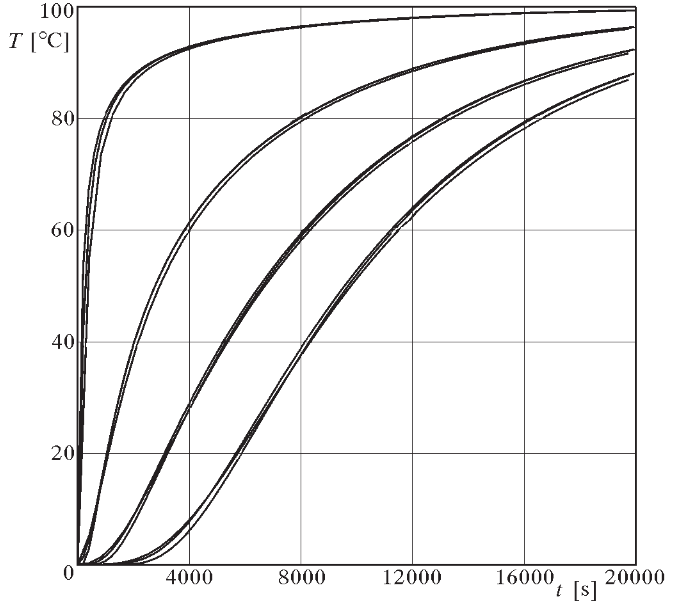

Fig. 4. Heating curves (example 2)

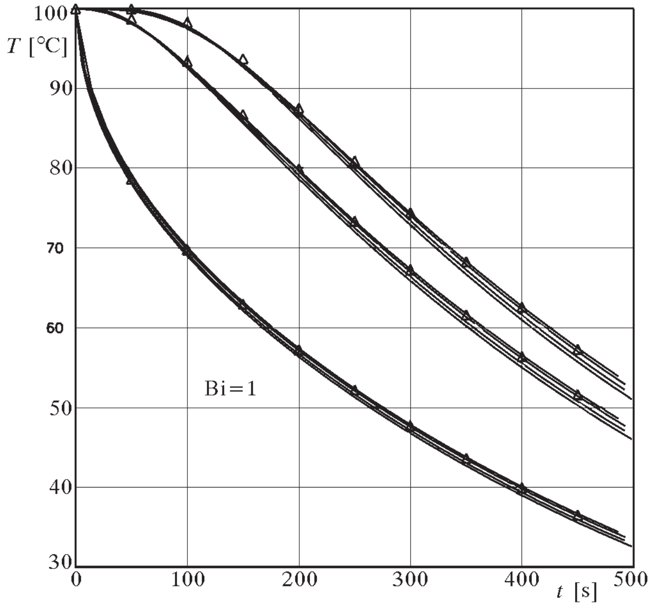

Fig. 5. Cooling curves

spherical shell of thickness 0.01 m. The initial temperature of the sphere (the internal radius $R_0 = 10^{-5}$ m) is equal $T_{10} = 20^\circ$ C, while the initial temperature of the shell $T_{20} = 300^\circ$ C. The differentiation of initial temperatures results from the need of assurring a good contact between the layers at the ambient temperature. The following thermophysical parameters of sub-domains are assumed: $\lambda_1 = 53\,\mathrm{W/(mK)}$, $c_1 = 3.917 \cdot 10^6\,\mathrm{J/(m^3K)}$, $\lambda_2 = 30\,\mathrm{W/(mK)}$, $c_2 = 4.875 \cdot 10^6\,\mathrm{J/(m^3K)}$. The heat transfer coefficient on the outer surface $r = R_2 : \alpha = 30\,\mathrm{W/(m^2K)}$, ambient temperature $T^\infty = 20^\circ$ C. For $r = R_0$ the adiabatic condition is assumed. The interior is divided into 25 linear elements, time step $\Delta t = 2.5$ s (Fig. 6) and $\Delta t = 1.25$ s (Fig. 7).

The results are compared with the numerical solution obtained using a repeatedly verified FDM program (symbols in Figures 6 and 7). It should be pointed out that the solutions are similar – a somewhat better agreement one obtains for the time step $\Delta t = 1.25$ s.

The last example concerns the problem of heat conduction in the domain considered for the case of non-zero thermal resistance between the sphere and shell. It is assumed that $Z = \mathrm{const} = 0.001\,\mathrm{m^2K/W}$. The geometry of the domain and the values of thermophysical parameters are the same as previously. For $r = R_0 : q(r,t) = 0$, for $r = R_2 : \alpha = 30\,\mathrm{W/(m^2K)}$, $T^\infty = 20^\circ$ C, the

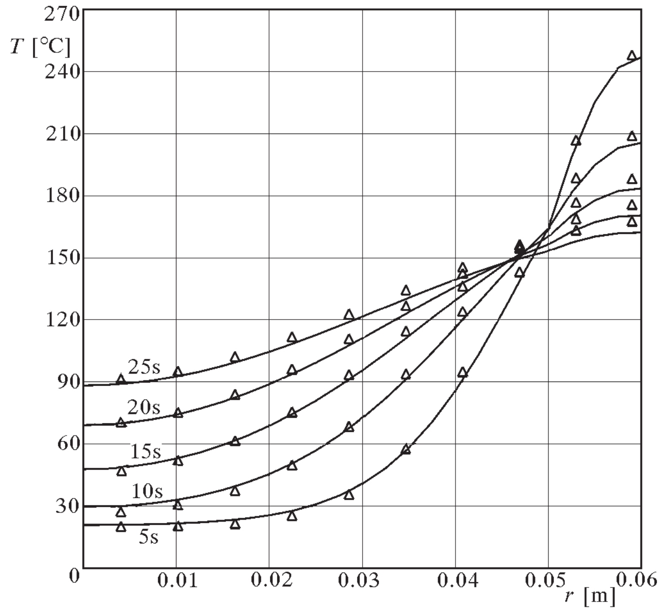

Fig. 6. Temperature field in domain considered ($\Delta t = 2.5\,\mathrm{s}$)

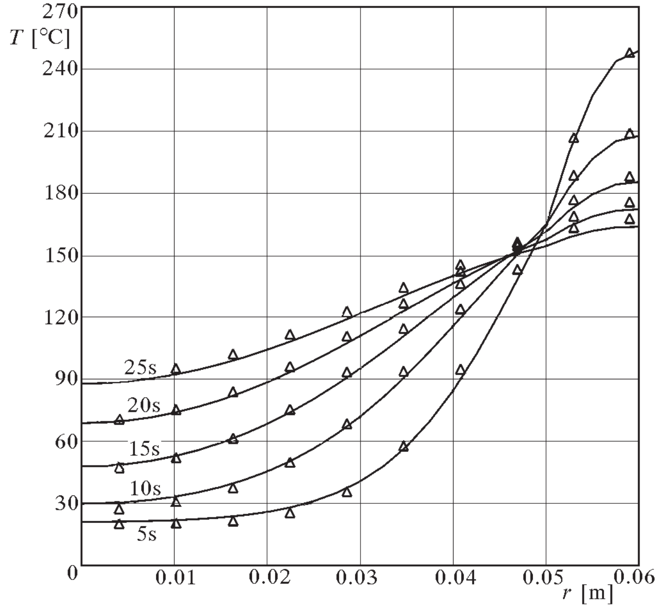

Fig. 7. Temperature field in domain considered ($\Delta t = 1.25\,\mathrm{s}$)

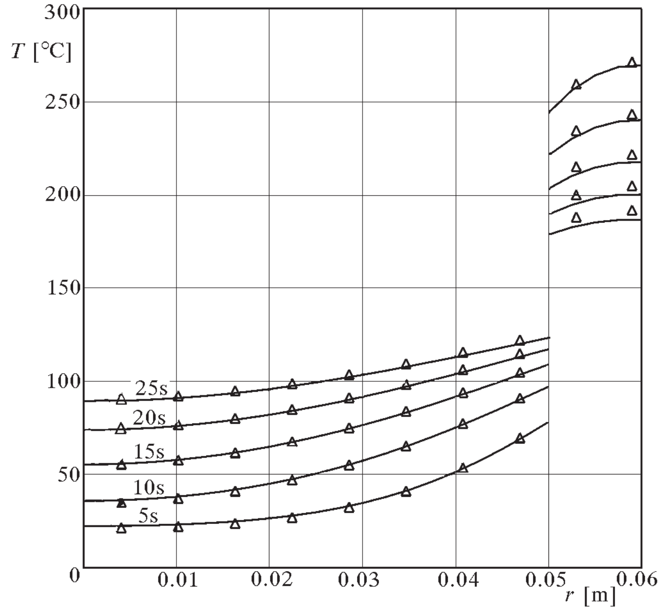

Fig. 8. The solution with thermal resistance ($\Delta t = 1.25\,\mathrm{s}$)

initial temperatures $T_{10} = 20°$ C, $T_{20} = 300°$ C. The results of computations are shown in Figure 8. The continuous lines illustrate the solution obtained, while the symbols the FDM solution. The agreement of these results is quite satisfactory.

Summing up, the algorithms presented in this paper can be used in the numerical modelling of heat conduction proceeding in spherical domains.

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Model numeryczny przepływu ciepła w obszarach sferycznych z wykorzystaniem kombinowanej metody elementów brzegowych

Streszczenie

Kombinowany wariant metody elementów skończonych, nazywany w literaturze MEB, z dyskretyzacją czasu polega na zastąpieniu wystepującej w równaniu Fouriera pochodnej temperatury po czasie odpowiednim ilorazem różnicowym. Dalsze etapy przekształceń matematycznych i konstrukcji algorytmu numerycznego nie odbiegają od typowego podejścia charakteryzującego klasyczną metodę elementów brzegowych. W pracy metodę kombinowaną wykorzystano do modelowania nieustalonej dyfuzji ciepła w obszarach sferycznych jednorodnych i niejednorodnych. W końcowej części przedstawiono przykłady obliczeń numerycznych.

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