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FATIGUE CRACK GROWTH PECULIARITIES AND MODIFICATIONS OF THE WHEELER RETARDATION MODEL (PART 2)

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Modifications of the Wheeler retardation model have been presented in the paper. The modifications improve description of data resulting from investigations on fatigue crack growth phenomena, in particular, for higher values of overloads.

Key words: crack growth, overloads, retardation, Wheeler model

1. Introduction

Fatigue crack growth under a variable amplitude loading is usually accompanied by the load interaction phenomenon, due to which the crack growth rate in a given cycle can differ from the growth rate observed for the same cycle in constant amplitude tests. The nature of the fatigue crack growth for various materials or conditions of loading – especially amplitude and frequency of overload occurrences, or for randomly variable loads, is different. Crack growth prediction methods can be grouped by various criteria. An obvious requirement for prediction models is their capability to estimate variable amplitude fatigue test results with a sufficient accuracy. It is also most desirable that the effect of any change in the load, material or geometry parameters on fatigue crack growth behaviour should be quantitatively predicted. The classical Wheeler model has been modified in terms of extending the description of the fatigue crack propagation phenomena, among other things, those discussed in part 1 of the paper and by Kłysz (1998), where some peculiarities concerning the problem of the fatigue crack propagation have been presented. The Wheeler model (Wheeler, 1972; Fuchs and Stephens, 1980; Kocańda and Szala, 1985) defines the retardation coefficient C_p in the following way (Fig. 1a)

$$C_p = \Big(\frac{r_{p,i}}{a_{ov} + r_{p,ov} - a_i}\Big)^n \tag{1.1}$$

where

$r_{p,i}, r_{p,ov}$	–	radii of plastic zones of current and overload cycles, respectively
a_i, a_{ov}	–	crack lengths of the current and overload cycles, respectively
n	–	exponent in the Wheeler model.

the range of using it is determined with the following condition: $a_i + r_{p,i} \leqslant a_{ov} + r_{p,ov}$.

According to this model, the retardation phase exists until the plastic zone $r_{p,i}$ related with the propagating crack (i.e. in the current cycle of the load) is contained within the plastic zone $r_{p,ov}$ originated from the overload previous to the given cycle. With the constant-amplitude loading applied in cycles after the overloads had been imposed, the retardation coefficient C_p changes monotonically and increases up to unity at the moment of reaching the overload plastic zone by the current plastic zone of the crack.

2. Modification I

The first modification has been based on the assumption that the crack growth retardation due to an overload is present up to the moment when the crack tip (and not the plastic zone spreading in front of it) reaches the boundary of the plastic zone produced by this overload. Hence, the condition of using the retardation model takes the form $a_i \leqslant a_{ov} + r_{p,ov}$, whereas the equation that defines the retardation coefficient assumes the following form

$$C_p = \Big(\frac{r_{p,i}}{a_{ov} + r_{p,ov} + r_{p,i} - a_i}\Big)^n \tag{2.1}$$

Treating the boundary of the overload plastic zone as a physical barrier to be overcome by the front of the propagating crack seems to be more justified than the approach presented in the initial form of the model. Not earlier than after the crack itself (not the plastic zone of the crack) has overcome this plastic zone, the retardation coefficient reaches unity and the crack grows again at the rate as if no overload has occurred. This modification does not generate

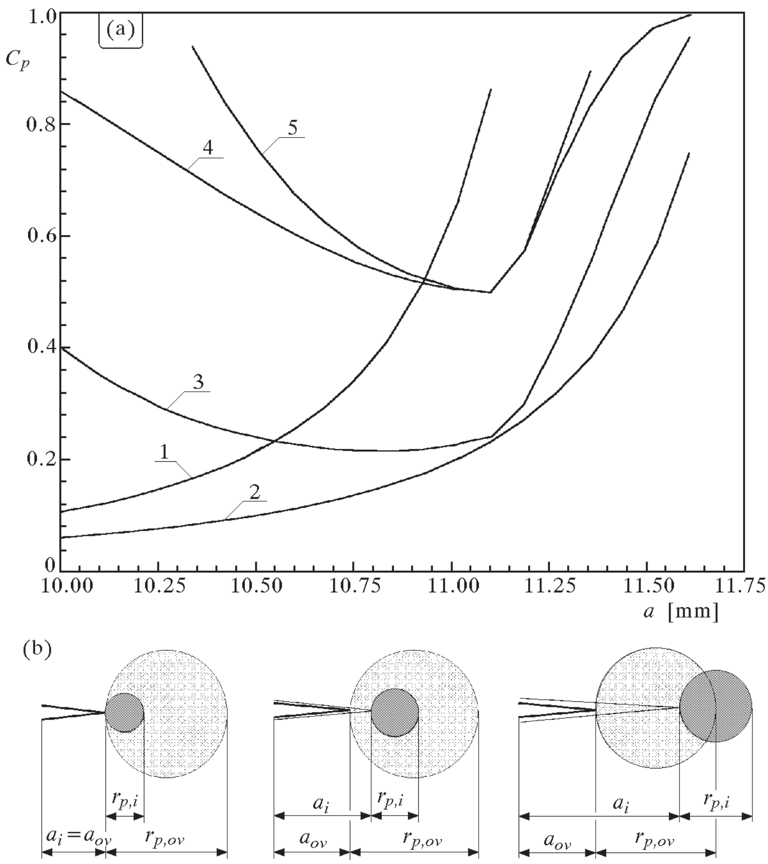

Fig. 1. Dependence of the retardation coefficient on the crack length (a) and plastic zones during overload crack propagation (b); curve No. 1 – initial Wheeler model, curve No. 2-No. 5 – Wheeler model after modification I-IV

any significant change in the nature of the dependence of the retardation coefficient upon the crack length. The only effect it produces is some delay (i.e. some elongation vs. crack length) of the range of retardation, i.e. up to reaching unity by the retardation coefficient. It is evident that in the case of the constant-amplitude loading after the overload has occurred it is the extension by a distance equal to the size of the current plastic zone. Fig. 1b shows some exemplary plots that illustrate these dependences for a selected computational case. Curve No. 1 corresponds to the initial form of the Wheeler model after modification. A significant difference between these two cases consists in the fact that the retardation model has not been based on the interdependence between the overload plastic zone and the current one only, but between the overload plastic zone and the front of the propagating crack. Thus, it allows some instances to be described, namely those when the crack growth retardation takes place just as after the first overload cycle in the course of executing a number of subsequent cycles of applying the overload (e.g. while replacing the load block of the L-H type with a block of a higher maximum load). In such situations, with the initial model incorparted, the size of the current plastic zone always exceeds the size of the plastic zone reached in the previous cycle. As the condition that implies validity of the retardation model had not been satisfied, no account was taken of the retardation phenomenon in computational analyses. In other words, the initial Wheeler model does not provide for any retardation in this case, because each subsequent overload cycle (i.e. a cycle from the block of a higher maximum load) generates a plastic zone beyond the zone originated in the course of the previously applied cycle (due to changes in both the crack lengths and the stress intensity factors in those cycles). The modified model relates in such situations the occurrence of the retardation phase to the sizes of crack increments in these overload cycles. If the crack increments are lower than the size of the plastic zone (e.g. at the initial stage of the crack growth), the retardation phase is to occur (before the crack front reaches this zone). On the other hand, if the increments are higher than the size of the plastic zone (like, for example, at the near-critical stage of the crack growth), the retardation phase will not occur at all. What can be referred to in this case, is the dependence of crack growth occurrence, or the crack growth not being retarded, on both: the size of the plastic zone in a given cycle – the retardation phase is to appear at large enough plastic zones (e.g. with plastic materials), and will not occur at small plastic zones (e.g. with brittle materials), and on purely fatigue properties of the material within which the crack propagates – the retardation phase is to occur with a slowly propagating crack, and will not occur with a quickly propagating one. In other words, the model comprises also the cases when – as the experimental knowledge shows – different materials respond in different ways to identical overloads applied (e.g. at the same time intervals and of the same size). The modified model conditions the occurrence and the duration of the retardation effect not only with differences in values of the yield points of a given material but additionally with taking account of the crack-propagation properties of the materials (by dint of including the current crack length, i.e. indirectly, the C and m coefficients of the equation of propagation, and the form of the stress intensity factor). The very problem of the material response to overload sequences (decreasing/increasing blocks) and their modes (number of cycles, succession of applied overload blocks, mean values, stress ratios, etc.) is also of great significance to evaluation of the fatigue life – it will not be, however, analysed any more in this paper.

Let us then return to the problem under discussion, i.e. a single overload. It is evident that this modification does not ensure ability to describe the crack growth, like it did in figures included in part 1 of the paper and by Kłysz (1998). There is still the monotone dependence of the delay coefficient upon the crack length within the whole range of the retardation occurrence. Therefore, the model describes only the cases featured with the regular increase in the crack growth rate.

3. Modification II

The second modification introduces the variable exponent n' of the Wheeler model, one that also depends (as the very retardation coefficient of the initial model) on the crack position and the current plastic zone previous to that originated in the overload cycle. Within the range, where the plastic zone of the propagating crack is totally included into the plastic zone originated in the overload cycle $(a_i+r_{p,i} \leqslant a_{ov}+r_{p,ov})$, assuming that the exponent n takes the form of n' described with one of the following relationships is suggested

$$n' = \Big(\frac{a_i + r_{p,i} - a_{ov}}{r_{p,ov}}\Big)n \tag{3.1}$$

or

$$n' = \Big(\frac{a_i + r_{p,i} - a_{ov}}{r_{p,ov}}\Big)^n \tag{3.2}$$

Within the range, where the plastic zone in front of the propagating crack crosses the plastic zone originated in the overload cycle $(a_i+r_{p,i} > a_{ov}+r_{p,ov})$, replacing the exponent n with the n' one described with one of the following relationships is recommended

$$n' = \Big(\frac{a_{ov} + r_{p,ov} - a_i}{r_{p,i,\max}}\Big)n \tag{3.3}$$

or

$$n' = \Big(\frac{a_{ov} + r_{p,ov} - a_i}{r_{p,i,\max}}\Big)^n \tag{3.4}$$

where $r_{p,i,\max}$ stands for the maximum current plastic zone generated in the course of subsequent cycles after the overload has been applied, i.e. the zone that has overcome the overload zone during the retardation effected with this overload.

As in the initial Wheeler model, the term in brackets in equations (3.1)-(3.6) illustrates just a fraction of the path that a crack has to cover to generate the plastic zone in front of it reaching the overload plastic zone (stage I) or escaping the retarded growth (stage II), and its value be contained in the (0,1) interval. The changes in variability of the retardation coefficient C_p against the crack length a_i (together with changes of curves for the initial (No. 1) and modified (No. 2) Wheeler models) have been shown in Fig. 1b. For the sake of simplicity in illustrating the problem, some exemplary values of the individual parameters ($r_{p,i}$, a_{ov}, $r_{p,ov}$, and $r_{p,i,\max}$) have been assumed, as well as the constant-amplitude loads to be applied in subsequent cycles to eliminate the shown changes (examplary plots) in the retardation coefficient due to load variability. Curve No. 3 corresponds with equations (3.1) and (3.3) (stage I and II) with the coefficient n in the form of a multiplier, whereas curve No. 4 corresponds with equations (3.2) and (3.4) with the coefficient n in the form of an exponent. In both the cases the retardation coefficient takes some value from the interval (0,1) and reaches the minimum at the instance of crossing the overload plastic zone by the plastic zone in front of the propagating crack. It means that the modified retardation model of fatigue crack growth permits the cases to be analysed when the crack growth does not show the uniformly monotonic nature after the overload has occurred (as in the initial Wheeler model). It provides for a period of gradual retardation of the crack after applying the overload (up to the moment of intersection of both plastic zones, i.e. the current and the overload ones), followed by an with the accelerated (but still slower than the crack propagation with no overload applied) escape from the phase of the growth retardation. This has found practical confirmation during testing the specimens.

4. Modification III

On the account of the mentioned in part 1 of this paper and in the work by Kłysz (1998) differences in crack length increments of 1-2 orders of magnitude experimentally observed in overload cycles and between the overloads application, it seems that the retardation model predicts these rapid crack length increments as well. In the considered model it is possible, if the retardation coefficient C_p could accept values within the range of 1-2 orders of magnitude.

The range of variability of C_p like of curves No. 3 and No. 4 in Fig. 1b does not render such a description possible.

As equations (3.1) and (3.3) in shortly expanded forms can be written down as

$$n' = \Big(1 - \frac{a_{ov} + r_{p,ov} - a_i - r_{p,i}}{r_{p,ov}}\Big)n \tag{4.1}$$

and

$$n' = \Big(1 - \frac{a_i + r_{p,i} - a_{ov} - r_{p,ov}}{r_{p,i,\max}}\Big)n \tag{4.2}$$

it is suggested using these equations in the following forms:

— at stage I

$$n' = 1 - \Big(\frac{a_{ov} + r_{p,ov} - a_i - r_{p,i}}{r_{p,ov}}\Big)n \tag{4.3}$$

— at stage II

$$n' = 1 - \Big(\frac{a_i + r_{p,i} - a_{ov} - r_{p,ov}}{r_{p,i,\max}}\Big)n \tag{4.4}$$

This makes the exponent n' capable of accepting positive and negative values (for specific positive values of n), and thus, the coefficient C_p capable of accepting the values higher than unity. Extending the range of possible applications of the retardation model is of crucial importance.

Curve No. 5 illustrates variability of the C_p coefficient in these cases. It is evident that immediately after applying an overload (i.e. for crack lengths close to a_{ov} at stage I) as well as before abandoning the retardation phase (i.e. for crack lengths close to $a_{ov} + r_{p,ov}$ at stage II) the retardation coefficient is greater than one. The case that takes place at stage II is of no interest to the model-based description (or at least difficult to interpret as far as the mechanism of the crack growth at this stage is concerned), therefore, it will not be taken into account any more. On the other hand, the changes in the retarded crack growth throughout stage I meet the requirement assumed at the beginning – they ensure variability of the C_p coefficient within the range of 1-2 orders of magnitude and exactly the same differences in the increments of the crack length. Hence the suggestion: let the following relationships be accepted as the retardation coefficient:

— in stage I (the left portion of curve No. 5)

$$C_p = \Big(\frac{r_{p,i}}{a_{ov} + r_{p,ov} - a_i}\Big)^{n'} = \Big(\frac{r_{p,i}}{a_{ov} + r_{p,ov} - a_i}\Big)^{1-\Big(\frac{a_{ov}+r_{p,ov}-a_i-r_{p,i}}{r_{p,ov}}\Big)n} \tag{4.5}$$

— in stage II (the right portion of curve No. 4)

$$C_p = \Big(\frac{r_{p,i}}{a_{ov} + r_{p,ov} - a_i}\Big)^{n'} = \Big(\frac{r_{p,i}}{a_{ov} + r_{p,ov} - a_i}\Big)^{\Big(\frac{a_{ov}+r_{p,ov}-a_i}{r_{p,i,\max}}\Big)^n} \tag{4.6}$$

The exponent of retardation n' can be then written down with only one equation of the following form

$$\begin{aligned} n' &= \operatorname{sgn}\left(a_i \leqslant a_{ov} + r_{p,ov} - r_{p,i}\right)\Big[1 - \Big(\frac{a_{ov} + r_{p,ov} - a_i - r_{p,i}}{r_{p,ov}}\Big)n\Big] + \\ &+ \operatorname{sgn}\left(a_i > a_{ov} + r_{p,ov} - r_{p,i}\right)\Big(\frac{a_{ov} + r_{p,ov} - a_i}{r_{p,i,\max}}\Big)^n \end{aligned} \tag{4.7}$$

The range of the retardation effect and changes in variability of the C_p coefficient in the cases under discussion depend, of course, on specific values of the individual parameters: $r_{p,i}$, a_{ov}, $r_{p,ov}$, and $r_{p,i,\max}$, i.e. on the crack length at the instance of the overload occurrence, on the overload size k_{ov}, and plasticity of the material.

The model still retains its essential, already known – owing to experimental work – property: when the overload increases, the crack length corresponding to stage I increases as well, and the crack over some considerable length escapes the retardation phase. Moreover, all equations for $n = 1$ are equivalents, which seems to confirm the theoretical roots common for all of them. For values of the exponent n from within the (0,1) interval, the variability of the retardation coefficient C_p increases monotonically – the model enables obtaining the properties like in the initial Wheeler model over the whole range of its applications; in other words, the initial Wheeler model is a special case of the modified model.

Fig. 2 shows a more complete display of the changes in the retardation coefficient C_p against the crack length for the suggested forms of the relationship defining this coefficient (equations (4.5)-(4.7)), for a wider interval of values of the coefficient n, i.e. from 0 up to 6. The conditions of the simulation were:

– application of constant-amplitude loads in subsequent cycles – to eliminate changes in the retardation coefficient due to variability of the load level, hence, $r_{p,i} = r_{p,i,\max} =$ const (although in practice the coefficient changes as the crack length increases – a constant value has been assumed for the sake of clarity of the figure)
– the same length of the crack at the moment of the overload a_{ov} occurrence
– values of $k_{ov} = 1.75$, 1.4, and 1.2
– values of $n = 0.5$, 2, 4.

So the extensive range of variability of the properties of the modified model ensures high flexibility in describing the phenomenon of fatigue crack growth retardation. Fig. 3 shows exemplary relationships of $a = f(N)$ determined

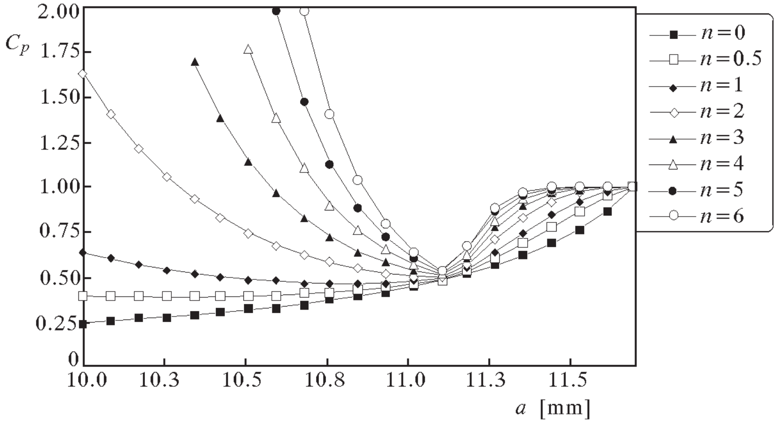

Fig. 2. Variations of the retardation coefficient as a result of modifications I-III (described in detail in the text); $a_{ov} = 10\,\text{mm}$, $r_{ov} = 1.695\,\text{mm}$, $r_i = 0.551\,\text{mm}$

according to the model described by Kłysz (1991), and the corresponding retardation coefficient C_p against the number of cycles, for simulation-based analyses of the crack propagation in a standard flat specimen with a single-edge notch (SEN). The calculations have been carried out for the following conditions:

– constant-amplitude load of the specimen, with overloads $k_{ov} = 1.75$ applied every 10000 cycles
– pre-determined values of the coefficients of the Paris equation: $m = 2$, $C = 1 \cdot 10^{-10}$
– materials with various yield points, i.e. 360 MPa
– variability of the coefficient n within the range (0,9).

It has become evident that the modified model enables description of a very wide spectrum of material behaviour under the above-mentioned conditions of loads application. For plastic materials (large plastic zones) no escape from the retardation phase occurred in a major part of the specimen life. The crack length increments between the overloads did not surmount the large overload plastic zone up to the moment just before the specimen failure, when the rate of propagation was the highest. The crack did not escape the retarded phase of growth. In the case of a material with a high yield point the situation is different. While between the overloads, the retardation manifested itself in its full range – the retardation coefficient was changing while passing through the

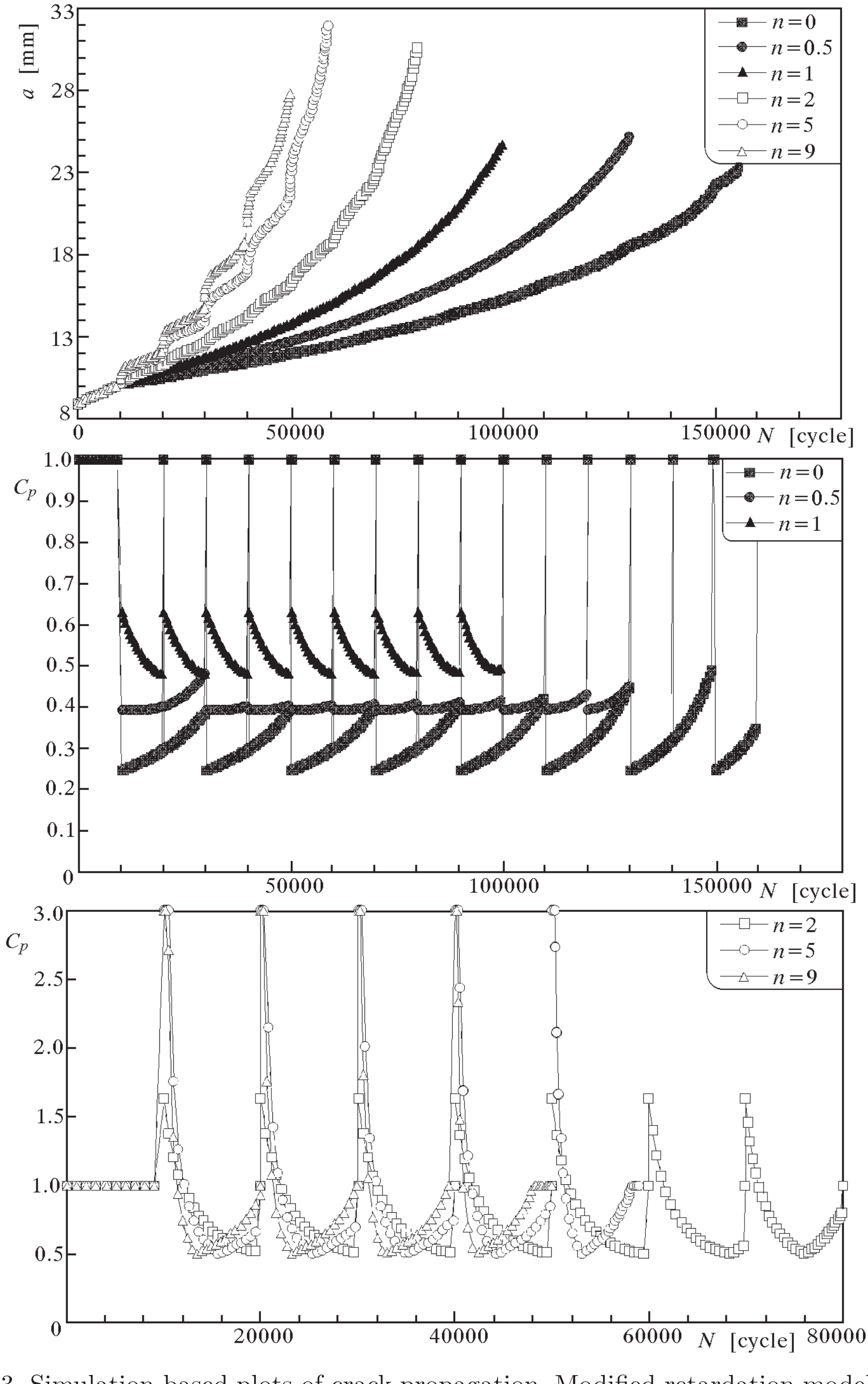

Fig. 3. Simulation-based plots of crack propagation. Modified retardation model, the yield point 360 MPa

minimum to reach unity before the subsequent overload occurred. An intermediate situation takes place in materials with mid-range values of the yield point. Therefore, the points of inflexion (or lack of them) and the directions of convexities on the $a = f(N)$ curves and, for the cases with high values of n, also considerable crack length increments in the overload cycles, are characteristic for those variants. All these features are easily observed in the course of fatigue tests. To make the comparisons easier, the $C_p = f(N)$ curves have been plotted up to the same scale; unfortunately, to the effect that the values of the C_p coefficient exceeding 2 (for $n = 5$ and $n = 6$) have been cut off. The occurrence of something like "double retardation phase" at $n = 0$ (up to 150000 cycles), and $n = 0.5$ (up to 30000 cycles), for a material with a low yield point, is an interesting detail of this simulation. The retardation coefficient C_p was not subjected to cyclic changes every pre-determined number of the cycles consistent with the frequency of overload occurrences, but every twice as large number of cycles. The crack length increment between the overload applications was small enough (as referred to the size of the overload plastic zone) not to escape the retarded phase of growth (the crack tip did not reach this zone) even at the moment of applying the subsequent overload. The result was that the previous retardation phase seemed to be continued. The retardation effect was repeated in cycles, every 20000 cycles instead of every 10000 cycles. It could be equivalent to a situation met in research practice, when the response of a given material to some overloads is evidently different from that to other ones (e.g. different nature of the relationships $a = f(N)$ or $COD = f(F)$, the crack opening displacement against the loading force, i.e. the specimen flexibility, or the relationship $da/dN = f(\Delta K)$, the crack propagation rate vs. range of stress intensity factor). It takes place particularly in the cases of high overloads, when the classical response to an overload does not occur after every overload application, but every second, third, or further one, and the material behaves between the overloads either in a different manner or "as if nothing had been happening". According to the terminology assumed in the paper, it may correspond to the above-mentioned: as if "double", "triple", etc. retardation phase.

Fig. 4 shows, according to a slightly different approach, exactly the same results for a pre-determined value of the coefficient n. The mid-range value has been selected from among the previously assumed values, i.e. $n = 2$. Thus, one of the essential factors that shape the retardation model (the coefficient n) has been eliminated for the sake of this analysis. From the above-shown plots it follows that the mentioned factor has originated a 300 per cent change in the specimen life and is responsible for physically different interpretation of

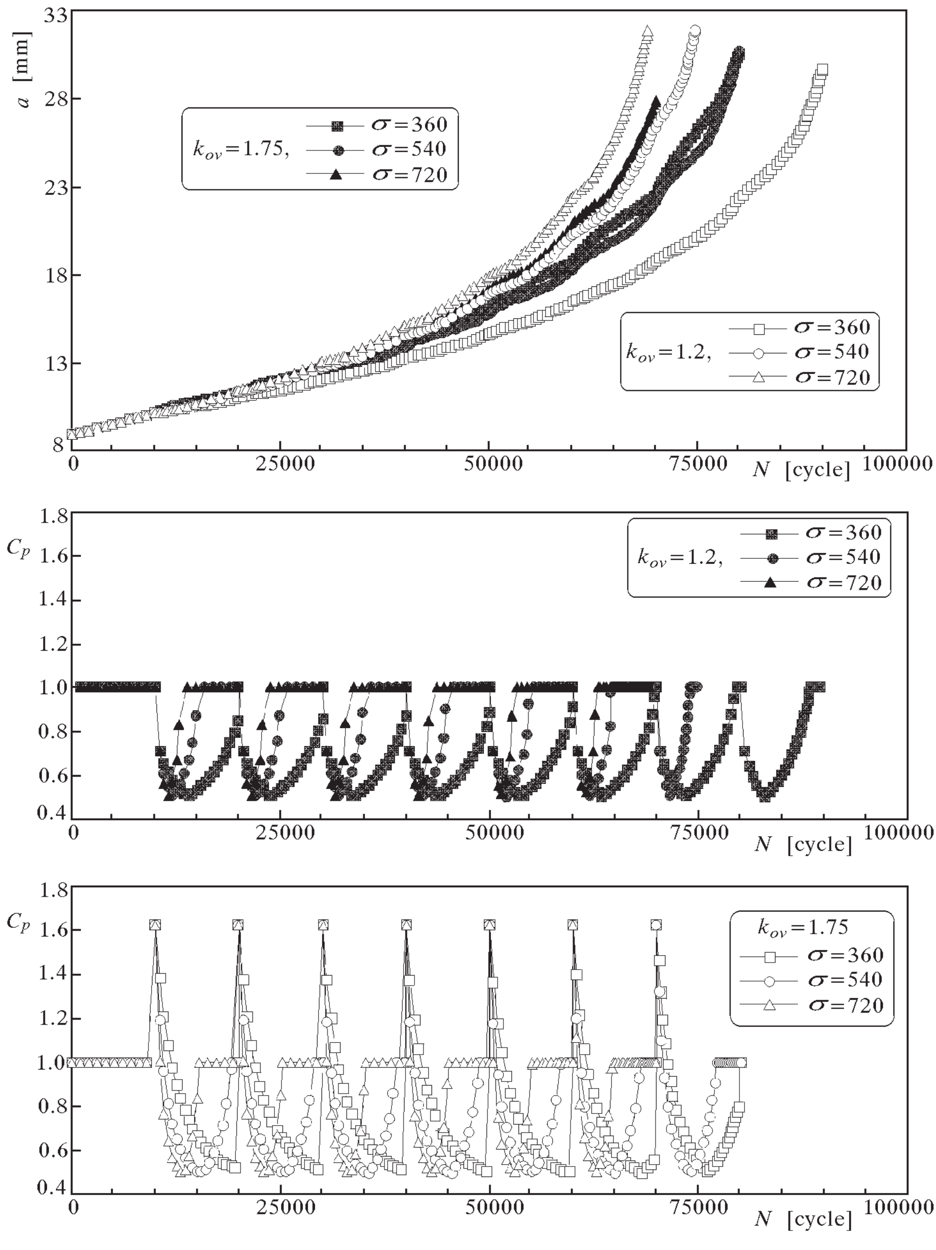

Fig. 4. Simulation-based plots of crack propagation. Modified retardation model, the coefficient $n = 2$

the changes in the retardation process of the individual curves $a = f(N)$. Therefore, the influence of the two other factors (i.e. overload quantity, and yield point) on description of the changes in the retardation process with the model under discussion has been presented. The plots confirm the above discussed relationships between the changes of the retardation coefficient in individual computational modes. It is evident that even for such a limited range of the analysed mode $(n = 2)$ the model is highly sensitive to the magnitude of the applied overload and the sort of material used. It proves suitability of the modified model to the analysis of fatigue crack growth under random loadings.

5. Modification IV

In all cases the minima of the curves $C_p = f(a)$ (Fig. 2) occurring for the crack length $a_i = a_{ov} + r_{p,ov} - r_{p,i}$ are either 0.5 (for curves No. 4 and No. 5) or $(1/2)^n$ (for curve No. 3) – the exponent n' takes – according to Eqs (4.5)-(4.7) – values 1 or n. For the ascending overload coefficients k_{ov} the model responses with a prolongation of stage I, like the initial Wheeler model does. The initial model reduces, however, the value of the retardation coefficient C_p – down to the minimum immediately after the overload cycle.

In improving the description of the experimental data it seems reasonable to make the minimum value of the C_p coefficient of the modified model change within a wider range, e.g. to make it decrease down to zero, or at least down to the range (0.5-0). It is suggested that the formula for C_p should take then the following form

$$C_p = \Big(\frac{1}{\alpha k_{ov}} \frac{r_{p,i}}{a_{ov} + r_{p,ov} - a_i}\Big)^{n'} \tag{5.1}$$

where α is to be found experimentally.

Fig. 5 shows some exemplary dependences of the retardation coefficient upon the crack lengths (based on Eqs (4.7) and (5.1) for some selected values of α and k_{ov}). Each curve models different "dynamics" of the changes in the retardation coefficient. Such behaviour can happen in reality for various materials (brittle, plastic, cyclic-hardening and cyclic-softening ones, at various stages of treatment), or different loading conditions (different values of DN, k_{ov}, R, but also randomness, multiaxiality, changes in temperature or frequency), which proves the universal nature of the suggested model. Same example of good fitting the above described theoretical model to experimental

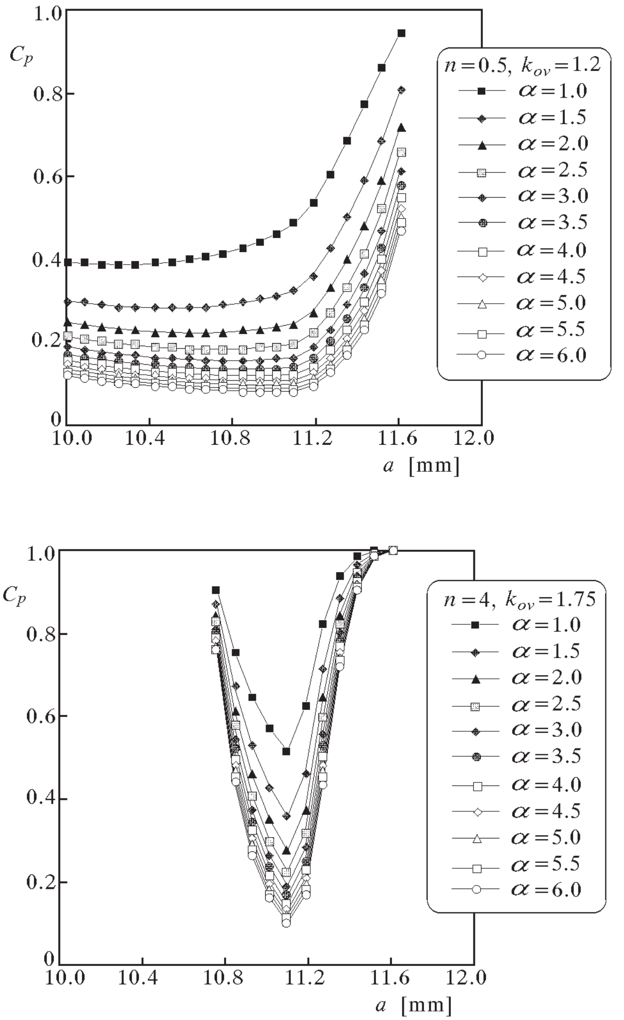

Fig. 5. Variability of the retardation coefficient after modification IV; $a_{ov} = 10$ mm, $r_{p,ov} = 1.695$ mm, $r_{p,i} = 0.551$ mm

data shows Fig. 6. It cannot be accomplished using the basic Wheeler retardation model.

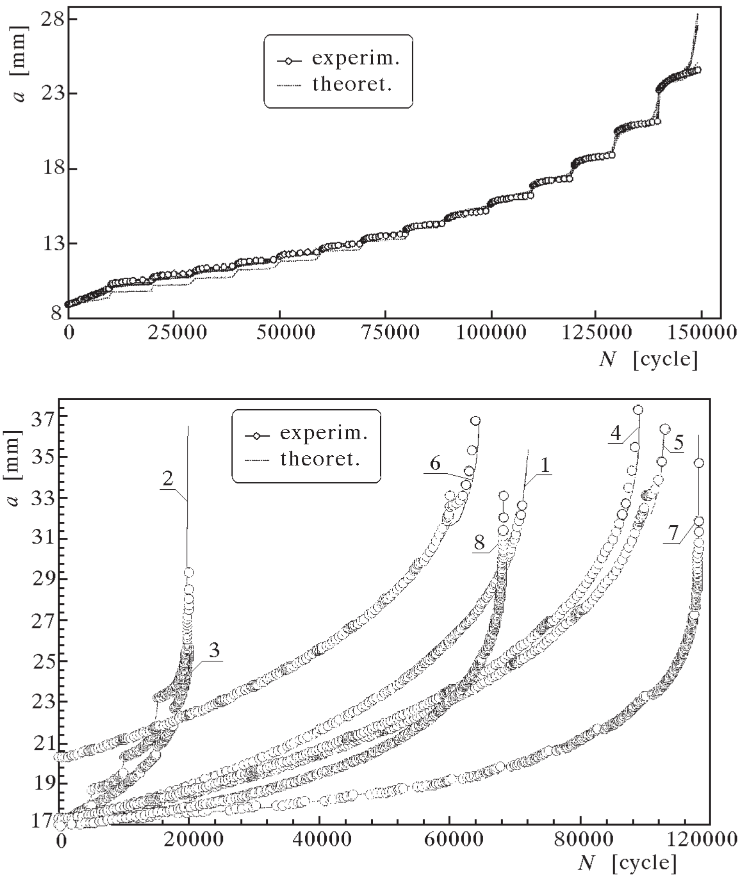

Fig. 6. Description of experimental data with the modified model

6. Conclusion

Crack growth prediction in structured elements in service can be based upon various techniques with different degrees of sophistication and varying life prediction accuracy. In rising order of complexity these are: rough estimates, analytical methods combined with the FEM, sophisticated crack modelling in 3D using automatic mesh-genetarion software and fracture mechanics analyses. The fracture behaviour of a given structure or material will depend on the stress level, material properties and the mechanisms by which the fracture progresses. The most successful approach in prediction and prevention of the fracture is modelling the crack growth rate, especially when under a service loading. The different features of $a = f(N)$ and $da/dN = f(\Delta K)$ relationships (presented in part 1) make a uniform theoretical description rather difficult, and sometimes preclude simple mathematical models from being applied.

A number of models have been developed to describe the observed variability in the crack growth. The modifications of the Wheeler retardation model have been proposed in the paper. They widen the range of possibilities of describing the experimental data using this model. They also enable application of it to any of the dependences $a = f(N)$ or $da/dN = f(\Delta K)$ found in the research practice. This model has shown the ability to characterise both constant and variable amplitude fatigue crack growth.

The above-introduced model guarantees good representations of the complex, experimentally determined relationships $a = f(N)$ and, which is probably of fundamental importance, very precise estimations of the coefficients of the equations of fatigue crack growth rate propagation, as well as the resultant fatigue life. The possibility of matching the theoretical description with experimental data is well confirmed. Hence, with such precise representations of the experimental data the model makes the estimations of dispersion of these parameters possible (e.g. while analysing some specific kinds of tests with a suitable number of the specimens). All mentioned features are very useful in the two main approaches to the design of critical structural elements with respect to fatigue safe-life design and damage-tolerance design. The first approach aims at withdrawal of the elements from serwice before the crack is detectable. The second approach tolerates a growing crack and aims at removal of the part when the crack is detectable or has the critical size.

All the problems discussed above give grounds for correct analysis of fatigue life of specimens and structural elements. It is a very important problem especially for all kinds of devices responsible for peoples' life. It is hard to find tasks in engineering practice being more critical than the prediction of service lives of critical structural elements and prediction of their fatigue strength.

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Osobliwości rozwoju pęknięć zmęczeniowych i modyfikacja modelu opóźnień Wheelera – część 2

Streszczenie

Przedstawiono modyfikacje modelu opóźnień Wheelera rozwoju pęknięć zmęczeniowych. Modyfikacje te poprawiają możliwości opisu danych doświadczalnych z badań propagacji pęknięć zmęczeniowych, w szczególności dla dużych przeciążeń. Przedstawiono przykłady opisu danych doświadczalnych przy użyciu zmodyfikowanego modelu opóźnień Wheelera.

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