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DECOHESIVE CARRYING CAPACITY OF CIRCULAR SANDWICH PLATE

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The problem of critical values of external loadings for a circular plate subject to bending and radial tension is discussed. The deformation process of a perfectly elastic-plastic sandwich plate clamped on a rigid shaft terminates, when radial strain in one of the layers tends to infinity. Then the plate reaches its decohesive carrying capacity, because further increase of loadings leads to inadmissible discontinuity of the radial displacement. For the plate subject to tension and bending (uniformly distributed moment over the outer edge), corresponding curves of elastic and decohesive carrying capacities are found.

Key words: plate, perfect plasticity, decohesive carrying capacity

1. Introduction

For some perfectly elastic-plastic structures, attempts at finding the limit carrying capacity, connected with certain mechanism of plastic collapse, fail. The limit carrying capacity cannot be reached, because earlier some inadmissible discontinuities of displacements occur. The corresponding external loadings were called by Szuwalski and Życzkowski (1973) decohesive carrying capacity.

This effect is observed due to an infinite increase in one of the strains – derivative of displacement. The process cannot be continued, as the increase of external loadings would result in a displacement jump; i.e., division of the structure into two parts.

The problem was investigated for bar systems, and disks by Szuwalski (1980), (1986). For beams the decohesive carrying capacity is connected with formation of the first plastic hinge (full plastification of the first cross-section).

For statically determined beams it coincides with the limit carrying capacity, while for statically indetermined beams it is smaller than limit carrying capacity, calculated with the help of plastic hinge concept. Tran-Le and Życzkowski (1976) using the concept of decohesive carrying capacity, clarified the well known Stussi-Kollbrunner paradox. Other examples may be found in the survey by Szuwalski (1990).

In the present paper, for the first time, the possibility of the decohesive carrying capacity occurance in the case of two-dimensional bending is investigated. The problem of disk with a circular rigid inclusion, subject to in-plane tension, discussed earlier by Szuwalski (1979), is generalized by adding out-of-plane bending, as well. As the integration over the thickness of the plate would involve to significant complications, the sandwich structure is assumed.

2. Elastic carrying capacity

The circular plate of perfectly elastic-plastic material, clamped at the inner perimeter is investigated. The plate is subject to uniform tension at the outer radius p, and bending with the moment m uniformly distributed over the outer radius b. It has two thin load carrying layers, located at a constans distance H (Fig.1). In both layers of the thickness h the plane stress is assumed.

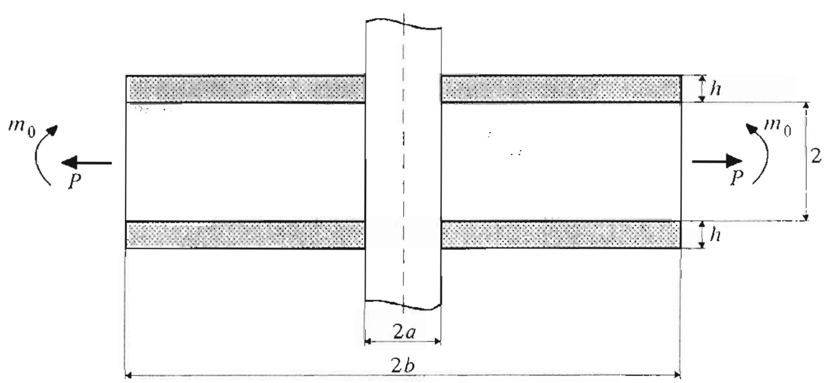

Fig. 1. Circular sandwich plate under combined loadings

The internal equilibrium conditions in polar coordinates, for the discussed plate have the form

$$n_r' + \frac{1}{r}(n_r - n_\theta) = 0 \qquad\qquad m_r' + \frac{1}{r}(m_r - m_\theta) = t_r \tag{2.1}$$

where

n_r, n_θ	–	intensities of radial and circumferential normal forces
m_r, m_θ	–	intensities of bending moments
t_r	–	shear force intensity (in the plate under consideration equal to zero)
$(\cdot)'$	–	derivatives with respect to the radius.

For the sandwich plate the relations between generalized internal forces, and stresses have the form

$$\begin{aligned} m_r &= (\sigma_r^- - \sigma_r^+)Hh \qquad & m_\theta &= (\sigma_\theta^- - \sigma_\theta^+)Hh \\ n_r &= (\sigma_r^- + \sigma_r^+)h \qquad & n_\theta &= (\sigma_\theta^- + \sigma_\theta^+)h \end{aligned} \tag{2.2}$$

the subscript "+" corresponds to the upper load carrying layer, while "-" to the lower one.

Substituting Eq (2.2) into Eq (2.1) leads to the internal equilibrium equations, expressed in stresses

$$\begin{aligned} \sigma_r^{-\prime} + \frac{1}{r}(\sigma_r^- - \sigma_\theta^-) &= \frac{t_r}{2Hh} \\ \sigma_r^{+\prime} + \frac{1}{r}(\sigma_r^+ - \sigma_\theta^+) &= -\frac{t_r}{2Hh} \end{aligned} \tag{2.3}$$

Taking advantage of the Love-Kirchhoff hypothesis, the strains in both layers may be expressed in terms of the radial displacement u and the vertical one (deflection) w

$$\begin{aligned} \varepsilon_r^+ &= \lambda_r - \kappa_r H = u' + Hw'' \\ \varepsilon_\theta^+ &= \lambda_\theta - \kappa_\theta H = \frac{u}{r} + H\frac{w'}{r} \\ \varepsilon_r^- &= \lambda_r + \kappa_r H = u' - Hw'' \\ \varepsilon_\theta^- &= \lambda_\theta + \kappa_\theta H = \frac{u}{r} - H\frac{w'}{r} \end{aligned} \tag{2.4}$$

The first one of the internal equilibrium conditions (2.3), after making use of Hooke's law, can be rewritten as

$$u'' + \frac{1}{r}u' - \frac{1}{r^2}u = 0 \tag{2.5}$$

defining the radial diplacement

$$u(r) = Ar + \frac{B}{r} \tag{2.6}$$

The second one of Eqs (2.3) helps to find the deflection w

$$w^{IV} + \frac{2}{r}w''' - \frac{1}{r^2}w'' + \frac{1}{r^3}w' = \frac{q(r)}{D} \tag{2.7}$$

where $q(r)$ stands for the surface loading intensity, in our case equal to zero, and

$$D = \frac{2EhH^2}{1-\nu^2} \tag{2.8}$$

is the plate stiffness.

The general solution of Eq (2.7) may be written as follows

$$w = F_1 + F_2 r^2 + F_3 r^2 \ln r + F_4 \ln r \tag{2.9}$$

The integration constants can be found from the boundary conditions

$$\begin{array}{llll} \text{for } r = a & w = 0 & w' = 0 \\ \text{for } r = b & m_r = -m & t_r = 0 \end{array} \tag{2.10}$$

The stress intensity according to the Huber-Mises-Hencky hypothesis, for the loadings shown in Fig.1, is maximal at the inner radius a in the upper layer. When this maximum takes the value of yield stress σ_o

$$\sigma_r^2 + \sigma_\theta^2 - \sigma_r\sigma_\theta = \sigma_o^2 \tag{2.11}$$

the first plastic deformations occur and the elastic carrying capacity is exhausted. This condition may be rewritten in terms of the internal forces intensities

$$n_r(a) \mp \frac{m_r(a)}{H} = \pm\frac{2h\sigma_o}{\sqrt{1-\nu+\nu^2}} \tag{2.12}$$

where the upper signs apply to the upper layer, while the lower ones to the other; ν stands for the Poisson ratio.

Expressing the internal forces acting at the radius a, by the external loadings, we come to the equation of elastic carrying capacity in the loading plane

$$\overline{\psi} \pm \overline{\mu} = \pm\frac{\beta^2(1+\nu) + (1-\nu)}{2\beta^2\sqrt{1-\nu+\nu^2}} \tag{2.13}$$

where the dimensionless external loadings were introduced

$$\psi = \frac{p}{2h\sigma_o} \qquad \mu = \frac{m}{2Hh\sigma_o} \tag{2.14}$$

and β stands for the ratio between radii $\beta = b/a$. In the ψ, μ coordinates Eq (2.13) describes four straight lines, forming a square (Fig.2). The diagonal of it Δ_e has a length equal to the doubled value of the right hand side of Eq (2.13).

3. Elastic-plastic range

For positive values of loadings (Fig.1), first plastic deformations occur in the upper layer in the vicinity of inner radius a. In the plastic zone, for $a \leqslant r \leqslant r_1$, the stresses must satisfy the Huber-Mises-Hencky yield condition. This will be ensured, by application of the Nadai-Sokolovsky parametrization

$$\sigma_r^+ = \frac{2}{\sqrt{3}}\sigma_o \sin\zeta \qquad \sigma_\theta^+ = \frac{2}{\sqrt{3}}\sigma_o \sin\Big(\zeta + \frac{\pi}{3}\Big) \tag{3.1}$$

Distribution of the parameter ζ may be determined from the second equation of (2.1) in the form of reversed function

$$r = C_1 \frac{\exp\Big(\frac{\sqrt{3}}{2}\zeta\Big)}{\sqrt{\sin\Big(\zeta - \frac{\pi}{3}\Big)}} \tag{3.2}$$

This parameter may take values $\pi/2 \leqslant \zeta \leqslant 2\pi/3$.

To establish the integration constant C_1, the boundary condition

$$\varepsilon_\theta(a) = 0 \tag{3.3}$$

formulated for strain, not stress, must be used. Therefore, the strains in the plastic zone must be found. To this end, the deformational theory assuming proportionality of stress and strain deviators, combined with the law of elastic volume change is used. They finally lead to the equation

$$\frac{d\varepsilon_\theta}{d\zeta} + \sqrt{3}\varepsilon_\theta = \frac{2\sigma_o}{3\sqrt{3}K}\sin\Big(\zeta + \frac{\pi}{6}\Big) \tag{3.4}$$

solution of which has the form

$$\varepsilon_\theta^+ = \frac{\sigma_o}{3\sqrt{3}K}\sin\zeta + C_2\exp(-\sqrt{3}\zeta) \tag{3.5}$$

where $K = E/[3(1-2\nu)]$ is the bulk modulus.

Taking advantage of the compatibility condition

$$\varepsilon_r = \varepsilon_\theta + r\varepsilon_\theta' \tag{3.6}$$

and Eq (3.5) the radial strain may be found

$$\varepsilon_r^+ = \frac{\sigma_o}{3\sqrt{3}K}\sin\Big(\zeta+\frac{\pi}{3}\Big) + \frac{C_2}{\cos\zeta}\sin\Big(\zeta-\frac{\pi}{6}\Big)\exp(-\sqrt{3}\zeta) \tag{3.7}$$

It is worth noticing, that for $\zeta \to \pi/2$ this strain will tend to infinity.

To determine displacements of the plate, the lower layer (which remains elastic) must be taken into account. Expressing strains in this layer by the displacements (Eq (2.4)), and taking advantage of Hooke's law, from the equation of internal equilibrium (2.3), we come to

$$rw''' + w'' - \frac{1}{r}w' = \frac{1}{2H}[(r\varepsilon_r^+)' - \varepsilon_\theta^+] \tag{3.8}$$

This equation may be treated, as the second order equation with respect to the deflection angle $\alpha = w'$. After integration we arrive at the solution

$$\alpha = Sr + \frac{T}{r} + \frac{r}{2H}\varepsilon_\theta^+ \tag{3.9}$$

with two integration constants: T and S. From the second one of Eqs (2.3), the radial displacement may be found

$$u = -H\Big(Sr + \frac{T}{r}\Big) + \frac{r}{2}\varepsilon_\theta^+ \tag{3.10}$$

The described above one-side plastified zone $a \leqslant r \leqslant r_1$ is surrounded by the totally elastic one $r_1 \leqslant r \leqslant b$, in which the elastic solutions (2.6) and (2.9) found earlier may be applied.

The complete elastic-plastic solution may be found with the help of twelve boundary conditions

$$\begin{array}{llll}
\text{for } r = a & u^{(p)} = 0 & w'^{(p)} = 0 & \zeta = \zeta_a \\
\text{for } r = r_1 & u^{(p)} = u^{(e)} & w'^{(p)} = w'^{(e)} & \zeta = \zeta_1 \\
 & n_r^{(p)} = n_r^{(e)} & m_r^{(p)} = m_r^{(e)} & \sigma_i^{+(e)} = \sigma_o \\
\text{for } r = b & m_r^{(e)} = -m & n_r^{(e)} = p & t_r^{(e)} = 0
\end{array} \tag{3.11}$$

They enable one to determine the following twelve unknowns: integration constants in elastic zone A, B, F_2, F_3, F_4 and in the plastic zone C_1, C_2, S, T; values of parameters ζ_1 and ζ_a, and radius dividing both zones r_1.

When the magnitude of one of external loadings is much bigger than the other, it is possible, that plastic deformations may occur also in the lower layer of the plate. Then three zones in the plate should be distinguished: both-side plastification $a \leqslant r \leqslant r_I$, one-side plastification $r_I \leqslant r \leqslant r_{II}$, and elastic zone for $r_{II} \leqslant r \leqslant b$. It turns out, that in the layer plastified up to the radius r_{II}, from the continuity coditions on the radius r_I it results, that the constants C_1 in Eq (3.2) and C_2 in Eq (3.6) are the same on the both sides of r_I. Consequently, in this layer only one plastic zone for $a \leqslant r \leqslant r_{II}$ may be discussed, no matter if the other layer is also plastified, or not.

4. Decohesive carrying capacity

The first plastification always takes place at the clamped edge, for $r = a$. Then the parameter ζ is

$$\tan \overline{\zeta}_a = \frac{\sqrt{3}}{2\nu - 1} \tag{4.1}$$

and may take values from $\pi/2$ for $\nu = 0.5$, to $2\pi/3$ for $\nu = 0$.

Further increase in external loadings is associated with the occurance of plastic zone in the vicinity of radius a, where the parameter ζ is always smaller than $\overline{\zeta_a}$. The parameter in this zone is a monotonic increasing function of the radius, so it reaches its minimal value at the inner radius, but it never can be smaller than $\pi/2$.

The denominator of the formula for radial strain ε_r^+ (3.7) contains $\cos\zeta$, while the constant C_2 in the numerator is always nonzero. As a result, when the parameter at the inner radius ζ_a reaches the value $\pi/2$, the radial strain tends to infinity. According to the geometrical relation

$$\varepsilon_r = \frac{du}{dr} \tag{4.2}$$

it means infinitely large derivative of radial displacement u. Continuation of the process (increase in external loadings) is impossible, because it would result in a jump of radial displacement at the radius a, inadmissible from the viewpoint of continuous medium. Consequently it leads to separation of the upper layer from the rigid shaft. The external loadings for which the continuous solution ceases to exist, represent the decohesive carrying capacity, the set of maximal admissible loadings.

To establish the decohesive carrying capacity of the plate, the complete elastic-plastic problem, with the help of twelve boundary and continuity conditions (3.10), must be solved. One of the unknowns will be already given, because the parameter ζ_a may be assumed equal to $\pi/2$, as it is in the moment of decohesion. Instead of it, the relationship between the external loadings at the moment of exhaustion of decohesive carrying capacity may be found.

After some transformations, this relation may be rewritten as

$$\widehat{\psi} + \widehat{\mu} = \frac{3\beta^2 + \rho_1^2}{2\sqrt{3}\beta^2} \sin\zeta_1 - (1 - 2\nu)\frac{\beta^2 - \rho_1^2}{2\sqrt{3}\beta^2} \exp\Big[\sqrt{3}\Big(\frac{\pi}{2} - \zeta_1\Big)\Big] \tag{4.3}$$

where $\beta = b/a$ represents the width of the plate, while ρ_1 stands for the dimensionless radius separating the elastic and plastic zones in the upper layer of the plate

$$\rho_1 = \frac{r_1}{a} = \frac{\exp\Big[\frac{\sqrt{3}}{2}\Big(\zeta_1 - \frac{\pi}{2}\Big)\Big]}{\sqrt{2\sin\Big(\zeta_1 - \frac{\pi}{3}\Big)}} \tag{4.4}$$

The value of the parameter at this radius ζ_1 is established by the transcendental equation

$$\nu = \frac{1}{2} + \frac{\sqrt{3}}{2} \cos\zeta_1 \exp\Big[\sqrt{3}\Big(\zeta_1 - \frac{\pi}{2}\Big)\Big] \tag{4.5}$$

From the above equations it can be seen that, the width of the plastic zone, at the moment of decohesion, depends only on elastic constant – Poisson ratio ν, and is independent of plate dimensions. The solution of Eq (4.5) for $\nu = 0.5$ is $\zeta_1 = \pi/2$. Consequently $\rho_1 = 1$, i.e. the plastic zone cannot spread out and the decohesive carrying capacity coincides with the elastic carrying capacity – instant decohesion. The plastic zone is widest for $\nu = 0$, when $\rho_1 = 1.084$.

Taking into account all possible combinations of the external loadings senses, from Eq (4.3) the equations of four straight lines may be derived. These lines create a square in $\psi\mu$ plane, completing the decohesive carrying capacity curve. Both the elastic and decohesive carrying capacity curves are presented in Fig.2.

Let us notice that such a formulation of the problem admits the possibility of termination for the plastic deformation process, also due to compression, as $\varepsilon_r \to -\infty$. Such a criterion is a purely mathematical one, however, within the framework of the small strain theory, it establishes termination of the continuous elastic-plastic solution.

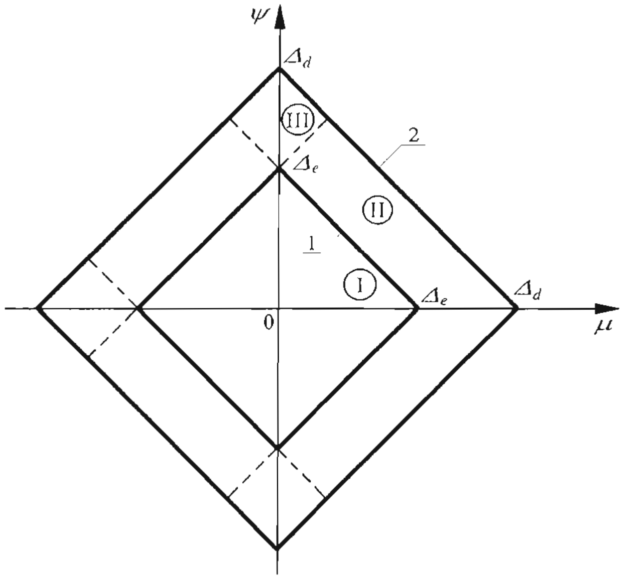

Fig. 2. Elastic and decohesive carrying capacity for the plate subject to tension ψ, and uniformly distributed moment μ; I – elastic range, II – elastic-plastic range (one layer plastified), III – elastic-plastic range (two layers plastified), 1 – elastic carruing capacity, 2 – decohesive carrying capacity

Parameter ζ in the plastic zone on the compressive side takes values from $3\pi/2$ to $5\pi/3$, and at the moment of decohesion is equal to $3\pi/2$. Therefore, Eq (4.4) must be slightly changed

$$\rho_1 = \frac{\exp\Big[\frac{\sqrt{3}}{2}\Big(\zeta_1 - \frac{3}{2}\pi\Big)\Big]}{\sqrt{2\sin\Big(\zeta_1 + \frac{2}{3}\pi\Big)}} \tag{4.6}$$

and Eq (4.5) should be replaced by

$$\nu = \frac{1}{2} - \frac{\sqrt{3}}{2}\cos\zeta_1 \exp\Big[\sqrt{3}\Big(\zeta_1 - \frac{3}{2}\pi\Big)\Big] \tag{4.7}$$

In the first and third quadrants in Fig.2 the limiting states are reached in the lower layer, while in the second and the fourth – in the upper layer. In the elastic-plastic range regions of one-side and both-sides plastification are distinguished. In the vicinity of axis ψ stresses (and strains) in both layers have the same sign, while in the vicinity of axis μ – different signs.

The dimensions of diagonals of squares in Fig.2 depend on material of plate (Poisson's ratio ν) and its geometry (width β). The courses: of Δ_e for elastic carrying capacity, and Δ_d for decohesive carrying capacity are presented in Fig.3 in the form of spatial diagram.

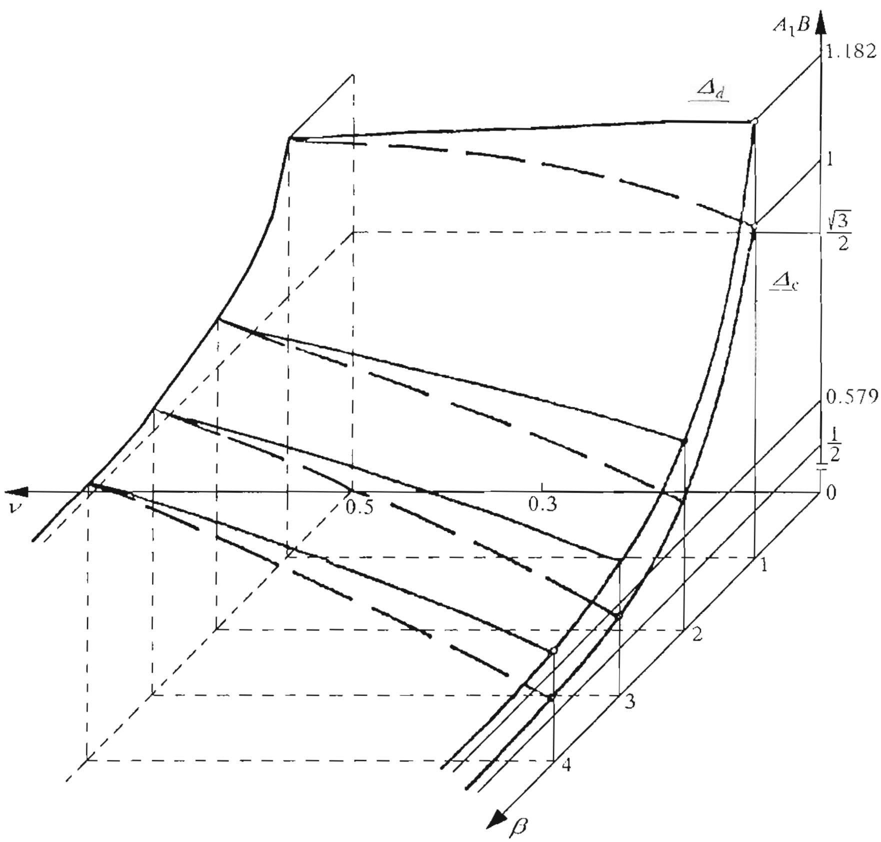

Fig. 3. Dimensions of the limiting curves for various materials (ν) and disks (β)

With an increase in the plate width β, both capacities decrease, and for an infinite plate they asymptotically tend to the minimal value, depending on ν. The curve for $\nu = 0.5$ is common for both surfaces – elastic and decohesive carrying capacities coincide. The differences between decohesive, and elastic carrying capacities decrease, with an increase of Poisson's ratio ν (up to zero for incompressible material $\nu = 0.5$). The values of both capacities simultaneously increase.

The case of very narrow plate should be discussed separately. When the ratio between radii β is smaller than the dimensionless radius separating the elastic and plastic zones, at the moment of decohesion ρ_1 (4.4), the whole plate (at least one of the layers) is fully plastified before decohesion. In spite of this, further work of plate is possible. It can carry external loadings, even of larger magnitudes than at the moment of full plastification. The loading increase results in redistribution of stresses, but the displacements may be precisely determined – the mechanism of plastic collapse does not exist.

The process may be continued up to the moment, when the parameter ζ_a at the clamped edge reaches the value $\pi/2$. So it is terminated by the infinite increase of the radial strain (exhaustion of decohesive carrying capacity), described by the equation

$$\widehat{\psi} + \widehat{\mu} = \frac{2}{\sqrt{3}} \sin \widehat{\zeta}_b \tag{4.8}$$

The value of parameter $\widehat{\zeta}_b$ at the outer radius b, may be found at this moment from the relationship

$$\exp\Big[\frac{\sqrt{3}}{2}\Big(\widehat{\zeta}_b - \frac{\pi}{2}\Big)\Big] = \beta\sqrt{\sin\Big(\widehat{\zeta}_b - \frac{\pi}{3}\Big)} \tag{4.9}$$

In this case the decohesive carrying capacity does not depend on the elastic material constants (Poisson's ratio ν), as it was observed earlier Eq (4.3). It results from the fact, that elastic zone in the layer, in which the parameter reaches the value of $\pi/2$ (or $3\pi/2i$), vanishes.

In the present paper the case of rather simple external loadings (Fig.1) is investigated, because only then the shear force is equal zero, what makes it possible to find the analytic solution. Such a solution was necessary for examination whether the continuous solution, may be prolonged up to the limit carrying capacity. It turned out, that earlier inadmissible discontinuity of the displacement occured and solution vanishes.

Determination of the curves of decohesive carrying capacity for plates with non-zero shear force t_r (2.1) is much more difficult. It needs a numerical approach and will be presented separately. The most important is the qualitative conclusion from here presented analysis, that continuous elastic-plastic solution will be terminated then, again, when parameter at radius a reaches the critical value $\pi/2$ or $3\pi/2$. The range of admissible loadings will be bounded by four segments of curves (not straight lines). However, it may be proved, that in the limiting case of incompressibility, these curves become straight and coincide with the elastic carrying capacity. The plastic zone cannot spread out then, and instant decohesion is obtained.

5. Concluding remarks

The problem of decohesive carrying capacity, associated with termination of the elastic-plastic deformations process, was earlier discussed for disks subject to tension in its plane. In the present paper, for the first time, the loadings perpendicular to the plane of disk, causing bending, were taken into account. For simplicity, the sandwich structure was discussed. It turned out, that the process terminated when, in one of the layers, radial strain tended to infinity. It means that the derivative of the radial strain becomes infinitely large, so continuation of the process must result in jump of radial displacement and separation of this layer from the rigid shaft.

The decohesive carrying capacity, in contrast with the limit carrying capacity, depends on elastic material constants. In the case of incompressible material (Poisson's ratio $\nu = 0.5$), it coincides with the elastic carrying capacity. The plastic zone cannot spread out then, and $\varepsilon_r \to \infty$ at the moment of first plastification.

The next step in investigations, should be connected with the replacing the sandwich structure by another one. It would cause significant complications, as a simple addition should be then replaced by integration over the plate thickness. One may anticipate, that the process will be limited, as it was observed in beams. Application of the finite strain theory would not introduce major qualitative changes. As it was proved by Życzkowski and Szuwalski (1982), the decohesive carrying capacity again will occur, and will be even slightly smaller. This time it will be caused by inadmissible discontinuity of the stress field.

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Nośność rozdzielcza kołowej płyty sandwiczowej

Streszczenie

W pracy zajęto się problemem wyznaczania krytycznych wartości obciążeń zewnętrznych dla kołowej płyty poddanej rozciąganiu i zginaniu. Proces odkształceń dla idealnie sprężysto-plastycznej sandwiczowej płyty osadzonej na sztywnym wale kończy się, gdy odkształcenie promieniowe w jednej z warstw zmierza do nieskończoności. Układ osiąga wówczas swoją nośność rozdzielczą, ponieważ dalszy wzrost obciążeń prowadziłby do niedopuszczalnych nieciągłości przemieszczenia promieniowego. Dla płyty poddanej równomiernemu rozciąganiu w kierunku promieniowym i zginaniu równomiernie rozłożonym na obwodzie zewnętrznym momentem wyznaczono odpowiednie krzywe nośności sprężystej i nośności rozdzielczej.

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